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SPECTRAL ENTROPY OF TURBULENCE IN A FLUID MODEL OF FUSION PLASMAS DURING A LOW-TO-HIGH CONFINEMENT TRANSITION

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Abstract. *Nuclear fusion is regarded as a clean and safe solution to the growing global energy demand. The most promising device for achieving fusion energy is the tokamak, which confines a plasma using magnetic fields. However, turbulence dominates the radial transport at the edge region of tokamak plasmas, reducing magnetic confinement in fusion experiments, and its control remains a challenge in physics and engineering. The spectral entropy is a useful tool from information theory that can characterize the degree of order/disorder of turbulent fluids and plasmas. We analyze numerical simulations of a fluid model of fusion plasmas in tokamaks. By varying a control parameter we construct a bifurcation diagram of a transition from a turbulent regime to a regime dominated by zonal flows, in which turbulence is suppressed. This transition is then characterized by computing the normalized spectral entropy of the turbulent patterns observed in the numerical simulations. Our results show that the turbulent regime displays a higher degree of entropy, the regime dominated by zonal flows is characterized by lower values of entropy, and the transition from the low-to-high confinement occurs abruptly.*

Keywords: *Fluid turbulence. Plasma. Information Theory. Bifurcation. Nonlinear dynamics.*

1. INTRODUCTION

Thermonuclear fusion refers to a reaction that commonly occurs in stars, such as the Sun, and is regarded as a clean and safe solution to the growing energy demand as the global population evolves and finite resources such as fossil fuels are depleted (Dewhurst, 2010). In this reaction, the nuclei fuse together, releasing energy. For this to occur an electrostatic barrier must be overcome, due to the integration of attractive nuclear forces with the positively charged nuclei, thus causing mutual electrostatic repulsion. The nuclear fuel is heated to high temperatures and completely ionized. The resulting plasma can be then confined by means of a magnetic field.

Maintaining the confinement of fusion plasmas is a major engineering challenge. A significant experimental breakthrough in the field of nuclear fusion research involved the detection of a low-to-high (L-H) transition of plasma confinement (Wagner *et al.*, 1982). This transition markedly diminishes particle and energy losses from the core of magnetically confined plasma, thereby enhancing the prospects for successful nuclear fusion. Subsequently, L-H transitions have become commonplace observations in numerous contemporary tokamaks and stellarators. The designs of advanced facilities such as ITER place significant importance on achieving H-mode operation, largely influenced by these findings (Pushkarev, 2013).

One of the explanations the L-H transition involves the suppression of turbulence through $\vec{E} \times \vec{B}$ flow shearing. This suppression effect can be attributed to mean $\vec{E} \times \vec{B}$ flows and/or to zonal flows (Burrell, 1997). The theoretical characterization of the L-H transition and non-linear turbulent states in fusion devices is a considerable challenge since this complexity arises from the multitude of crucial physical parameters, scales of motion and intricate magnetic field geometries (Krommes, 2012).

Aiming to improve plasma confinement, the understanding of the low-to-high (L-H) confinement transition is fundamental for controlling plasma in fusion experiments, since it enhances the confinement by suppressing anomalous or turbulent plasma particles as well as heat fluxes. In particular, emergent zonal flows are known to be of fundamental importance for the high-confinement regime, because they reduce anomalous transport by absorbing energy from drift waves and sorting out eddies that mediate turbulent transport (Diamond *et al.*, 2005).

Employing a simplified model of turbulence in tokamak plasmas, the L-H confinement transition can be derived from the interaction of three energy subsystems, namely, the kinetic energy of turbulence as well as shear flow, and the potential energy arising from density or pressure gradients (Numata *et al.*, 2007). Characterized by processes such as the generation of turbulence by drift waves, destabilization, and self-organization of zonal flows, the instabilities resulting from these changes are called a bifurcation of the equilibrium solution (Numata *et al.*, 2007). These instabilities can be classified into primary instabilities that originate from an accumulation of potential energy generating turbulence, secondary instabilities that feed the shear flows as well as the zonal flows using the kinetic energy of the turbulence, and finally, the tertiary instabilities that destabilize both flows (Numata *et al.*, 2007).

This article aims to characterize the degree of order/disorder of turbulent fusion plasmas during its transition from low-to-high confinement through numerical simulation of a simplified model. We apply the spectral entropy from information theory, to quantify the degree of disorder in the spatial patterns in the numerical simulations. This paper is organized as follows. The model is presented in section 2, followed by the numerical results obtained and explained in section 3, and section 4 presents the concluding remarks.

2. METHODOLOGY

2.1 Hasegawa-Wakatani equations

The Hasegawa-Wakatani equations describe a simplified model of the drift wave turbulence behavior in the edge region of tokamak, considering a plasma magnetically confined and a non-uniform backgroud density. Therefore, the equations can be obtained from the derivatives of the Braginskii fluid equations (Braginskii, 1965). Thus, the Hasegawa-Wakatani model is described by

$$\frac{\partial}{\partial t}\zeta + [\varphi, \zeta] \quad = \quad \alpha\left(\varphi - n\right) - D\nabla^4\zeta, \tag{1}$$

$$\frac{\partial}{\partial t}n + [\varphi, \zeta] \quad = \quad \alpha\left(\varphi - n\right) - \kappa\frac{\partial \varphi}{\partial y} - \nabla^4 n, \tag{2}$$

being ζ is the vorticity, α is the adiabacity operator described by Eq. (4), and D is the dissipation coefficient. The square brackets indicate Poisson bracket notation, and represent nonlinear terms. For two scalar fields A and B, this notation represents

$$[A, B] = \frac{\partial A}{\partial x}\frac{\partial B}{\partial y} - \frac{\partial A}{\partial y}\frac{\partial B}{\partial x}. \tag{3}$$

The parameters α and κ in Eqs. (1) and (2) are

$$\alpha = \frac{Tk^2}{n_0 e^2 \eta \omega_{ci}},\ \kappa = -\frac{\partial \ln n_0}{\partial x}, \tag{4}$$

where α is the adiabaticity operator associated with the resistive coupling force between density, n, and the electrostatic potential (φ) of the plasma. In Eq. 4, T represent the plasma temperature, k is the wave number in the parallel direction, n_0 is the equilibrium plasma density, e elementary charge, η is the resistivity, ω_{ci} is the cyclotron frequency, and the parameter κ corresponds to the background density gradient.

2.2 Modified Hasegawa-Wakatani

Restricted to two dimensions, the model proposed by Hasegawa and Wakatani (1983) does not take into account the presence of zonal flows. Numata *et al.* (2007) present a modified model that emphasizes the presence of these flows. Thus, the zonal and non-zonal components of any variable f are given by

$$zonal : \langle f \rangle \equiv \frac{1}{L_y}\int f dy, \tag{5}$$

$$non-zonal : \tilde{f} \equiv f - \langle f \rangle. \tag{6}$$

The square brackets $\langle ... \rangle$ denotes an average in the poloidal direction, and $(\ \tilde{}\ )$ represent the fluctuating quantity from which the averaged component has been removed. Therefore, the modified Hasegawa-Wakatani equations can be written as

$$\frac{\partial}{\partial t}\zeta + [\varphi, \zeta] \quad = \quad \alpha\left(\widetilde{\varphi} - \widetilde{n}\right) - D\nabla^4\zeta, \tag{7}$$

$$\frac{\partial}{\partial t}n + [\varphi, \zeta] \quad = \quad \alpha\left(\widetilde{\varphi} - \widetilde{n}\right) - \kappa\frac{\partial \varphi}{\partial y} - \nabla^4 n, \tag{8}$$

In this paper we set $D = 10^{-4}$ and $\kappa = 10^{-1}$, and choose α as a control parameter. Equations (7)-(8) are solved using the finite-differences method for the spatial derivatives, and the fourth-order Runge-kutta method for the time integration.

A spatial grid of 256x256 in 2D allows for accurately solving the equations while maintaining a reasonable computing time. The solver is implemented in Fortran language. The Poisson brackets are solved using the Arakawa method, which conserves energy and entropy with a 3rd order accuracy. The initial conditions are set as small amplitude random noise. Equations (7)-(8) are solved for 10^8 time steps, which is enough to reach a steady-state solution.

2.3 Analysis Tools

2.3.1 Fourier Transform

The Fourier transform is a fundamental component of signal processing, enabling the representation of periodic functions through Fourier series. According to Press (2007) a physical process can be described either in the spatial domain, using a quantity u as a function of x, $u(x)$, or in the wavenumber domain, where the process is characterized by a complex variable $\hat{U}$ as a function of wavenumber k, denoted as $\hat{U}(k)$. Therefore, $u(x)$ and $\hat{U}(k)$ are two distinct representations of the same function. It can be described as

$$\hat{U}(k) = \int_{-\infty}^{\infty} u(x)e^{ikx}\,dx \tag{9}$$

$$u(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{U}(k)e^{-ikx}\,dk. \tag{10}$$

2.3.2 Spectral Entropy

The Shannon entropy is defined in the context of information theory as the quantization of the information contained in a message, relating it to the occurrence of groups of transmitted symbols (Shannon, 1948). Applying to the present context, the spectral entropy employs the information of amplitudes of Fourier modes to measure the degree of spatial disorder (Rempel *et al.*, 2007). It can be written as

$$S(t) = -\sum_{k=1}^{N} p_{k,t}\ln p_{k,t} \tag{11}$$

where $p_{k,t}$ is the relative Fourier weight of mode k, which is represented by

$$p_{k,t} = \frac{|\hat{U}(k,t)|^2}{\sum_k |\hat{U}(k,t)|^2} \tag{12}$$

where $\hat{U}(k,t)$ is a real function obtained from the Fourier transforms (Eq. (10)). Normalizing Eq. (12) we infer that $p_{k,t} \in [0,1]$ and $\sum_{k=1}^{N} p_{k,t} = 1$. Thus, we can conclude that entropy will be maximum when $p(k,t) = 1/N$, that is, when the distribution is uniform.

For a better understanding of the concept, Figure 1 depicts a distribution of energy among five wavenumbers. The left-side panel of Figure1 shows that all the energy is contained in a single wavenumber, therefore, the entropy in this case is zero, because the resulting signal will be a sinusoidal function with k = 1, which is fully predictable. The right-side panel of Figure 1 displays a uniform distribution of energy among modes. In this case the entropy is maximum (the degree of disorder is maximum), because the resulting signal will have fluctuations in all wavenumbers, resulting in an unpredictable series.

Figure 1. Examples of energy distribution among five wavenumbers. The left-side distribution corresponds to zero entropy, whereas the right-side distribution represents maximum entropy.

3. RESULTS

The electrostatic potential φ is shown in figure 2 for two different values of α, namely $\alpha = 0.01018$ and $\alpha = 0.01104$. For $\alpha = 0.01018$ (left-side panel), φ displays turbulent patterns with the presence of vortices that can be recognized as regions of local minima and maxima of φ. For $\alpha = 0.01104$ (right-side panel), a large-scale pattern can be easily recognized, due to the presence of zonal flows. The zonal flows can suppress turbulence and radial mass flow towards the walls of the tokamak, enhancing plasma confinement. The first regime ($\alpha = 0.01018$) is called the turbulent regime, and the second regime ($\alpha = 0.01104$) is called the zonal-flow regime. In this model, the transition between these two regimes is regarded as a L-H confinement transition.

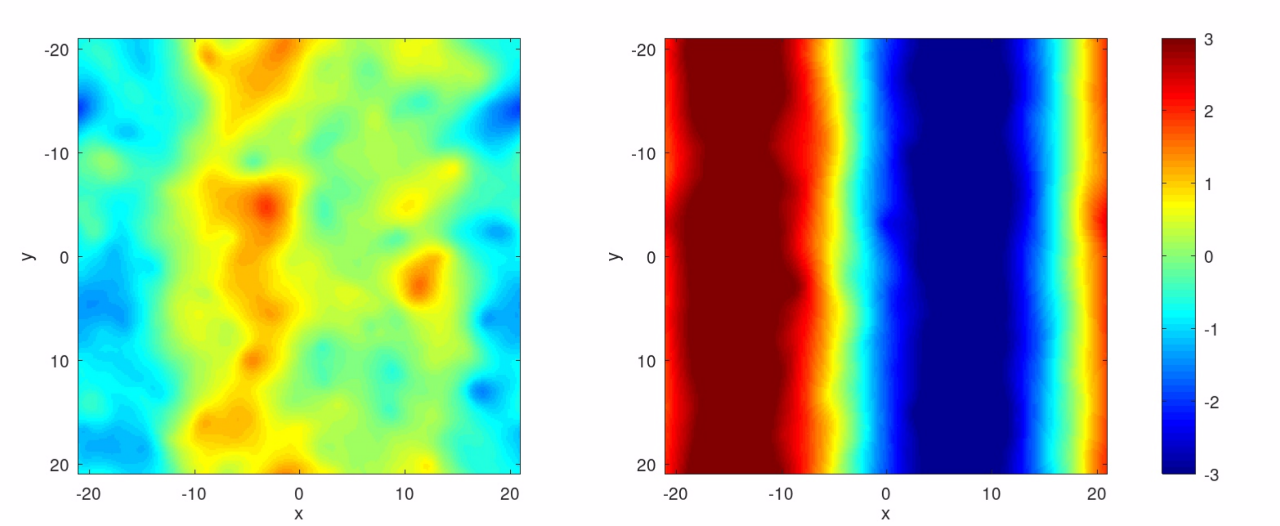

Figure 2. Plasma electrostatic potential obtained from the numerical simulations. On the left is the turbulent regime, on the right is the zonal flow.

We study the L-H confinement transition in detail by constructing a bifurcation diagram. This diagram is obtained by setting an initial value of α, solving Eqs. (7)-(8) from small amplitude random initial conditions, and then increasing α using small step values to solve (7)-(8) again. The transition from the turbulent regime to the zonal-flow regime can be clearly elucidated by computing the ratio between the kinetic energy of the zonal flow K_Z and the total kinetic energy K_T (Numata *et al.*, 2007). The energy of zonal flows can be obtained by

$$K_Z = \frac{1}{2}\int \left(\frac{\partial \langle\varphi\rangle_y}{\partial x}\right)^2 dx, \tag{13}$$

where $\langle\varphi\rangle_y$ represent the average of φ in the y direction. The total kinetic energy by

$$K_T = \frac{1}{2}\int\int |\nabla\varphi|\, dxdy. \tag{14}$$

The upper panel of Figure 3 shows the average of 500 values of K_Z/K_T, computed from each value of α after reaching a steady-state. From this panel it is clear that, in the turbulent regime ($\alpha < 0.0103$), K_Z/K_T displays relatively low values, whereas in the zonal-flow regime ($\alpha > 0.0103$), K_Z/K_T displays values close to unity, which confirms that this regime is dominated by the zonal flows. Therefore, the transition between the turbulent and the zonal-flow regimes occurs at approximately $\alpha \sim 0.0103$. The vertical lines represent the error bars, which are obtained by computing the standard deviation of the 500 values of K_Z/K_T. Note that the error bars in the turbulent regime are larger than those in the zonal-flow regime. This indicates that the variability of the kinetic energy is larger in the turbulent regime than in the zonal-flow regime.

The lower panel of Figure 3 shows the average of 500 values of the normalized spectral entropy applied to the electrostatic potential φ, for the same values of the parameter α shown in the upper panel of the same figure. The averaged spectral entropy has low values (less than 0.3) in the turbulent and the zonal flow regimes, which indicates that the electrostatic potential displays ordely patterns in both regimes. Since Eqs. (7)-(8) do not have stochastic terms (i.e., deterministic), the degree of disorder of the numerical solutions (e.g., the disorder of the spatial patterns of φ) is expected to be low. A comparison of the averaged spectral entropy before and after the L-H transition ($\alpha \sim 0.0103$) shows that the turbulent regime is characterized by values of the entropy higher than the values of the zonal-flow regime. This demonstrates that the emergence of zonal flows for $\alpha > 0.0103$ results in a decrease of the degree of disorder of the electrostatic potential. This is in agreement with the fact that zonal flows are responsible for the suppression of turbulence, acting as barriers against the flux of plasma particles in the horizontal direction, therefore enhancing plasma confinement.

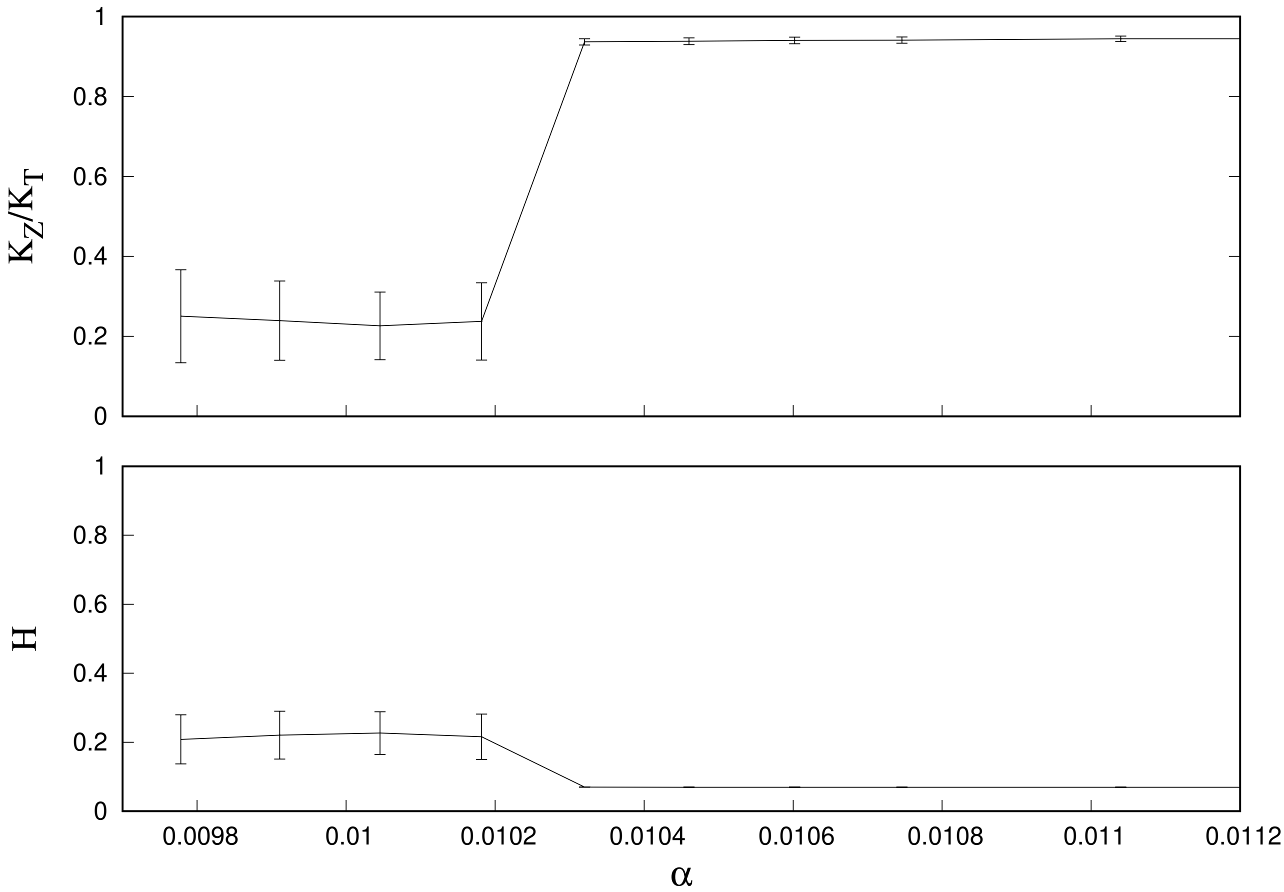

Figure 3. Bifurcation diagram of the ratio between the kinetic energy of zonal flows and the total kinetic energy, and the spectral entropy, as a function of the control parameter α.

4. CONCLUSION

In this paper we studied numerical solutions of the Hasegawa-Wakatani equations, which is a model of drift-wave turbulence in tokamak plasmas. By changing the value of a control parameter related to adiabaticity, a transition from a low-confinement regime to a high-confinement regime has been identified. The low-confinement regime displays turbulent patterns of the electrostatic potential, whereas the high-confinement regime is dominated by zonal flows that suppress plasma transport in the radial direction.

We have quantified the degree of spatial disorder of the electrostatic potential φ by applying the spectral entropy, which is the Shannon entropy applied to the amplitudes of Fourier modes. A bifurcation diagram of the transition was constructed by varying the value of the control parameter in small steps. The bifurcation diagram shows that the ratio between the kinetic energy contained in the zonal modes and the total kinetic energy increases abruptly during the transition between the turbulent regime and the zonal-flow regime. The turbulent regime displays higher values of entropy compared to the zonal flow regime. This is in agreement with the spatial patterns of the electrostatic potential φ, in which the turbulent regime displays disordered patches of φ, whereas the zonal-flow regime is characterized by a large-scale coherent structure that suppresses turbulence and acts as barriers of horizontal flux.

The dynamics of turbulent flows and the transition to turbulence in fluids and plasmas has important applications to mechanical engineering. Therefore, we believe that the tools described in this paper can contribute to the understanding of the turbulent patterns obtained from experiments and numerical simulations of fluids and plasmas.

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6. REFERENCES

Bittencourt, J.A., 2004. *Fundamentals of plasma physics*. Springer Science & Business Media.

Braginskii, S.I., 1965. "Transport Processes in a Plasma". *Reviews of Plasma Physics*, Vol. 1, p. 205. URL `https://ui.adsabs.harvard.edu/abs/1965RvPP....1..205B`. Provided by the SAO/NASA Astrophysics Data System.

Burrell, K., 1997. "Effects of e× b velocity shear and magnetic shear on turbulence and transport in magnetic confinement devices". *Physics of Plasmas*, Vol. 4, No. 5, pp. 1499–1518.

Dewhurst, J.M., 2010. *Statistical description and modelling of fusion plasma edge turbulence*. Ph.D. thesis, University of Warwick. URL `http://wrap.warwick.ac.uk/3903/`.

Diamond, P.H., Itoh, S.I., Itoh, K. and Hahm, T.S., 2005. "Zonal flows in plasma—a review". *Plasma Physics and Controlled Fusion*, Vol. 47, No. 5, pp. R35–R161. doi:10.1088/0741-3335/47/5/r01. URL `https://doi.org/10.1088/0741-3335/47/5/r01`.

Dolan, T.J., Moir, R.W., Manheimer, W., Cadwallader, L.C. and Neumann, M.J., 2013. *Magnetic fusion technology*. Springer.

Gallagher, S.J., 2013. *Zonal flow generation through four wave interaction in reduced models of fusion plasma turbulence*. Ph.D. thesis, University of Warwick. URL `http://wrap.warwick.ac.uk/59703/`.

Hasegawa, A. and Wakatani, M., 1983. "Plasma edge turbulence". *Physical Review Letters*, Vol. 50, pp. 682–686. ISSN 0031-9007. doi:10.1103/PhysRevLett.50.682. URL `https://link.aps.org/doi/10.1103/PhysRevLett.50.682`.

Krommes, J.A., 2012. "The Gyrokinetic Description of Microturbulence in Magnetized Plasmas". *Annual Review of Fluid Mechanics*, Vol. 44, No. 1, pp. 175–201. doi:10.1146/annurev-fluid-120710-101223.

Magossi, J.C., Abreu, P.H.C.d., Barros, A.C.d.C. and Paviotti, J.R., 2021. "A medida de informação de shannon: Entropia". *Revista Brasileira de História da Matemática*, Vol. 21, No. 41, pp. 45–72. doi: 10.47976/RBHM2021v20n4145-72. URL `https://www.rbhm.org.br/index.php/RBHM/article/view/337`.

Miranda, R.A., Rempel, E.L. and Chian, A.L., 2015. "On-off intermittency and amplitude-phase synchronization in keplerian shear flows". *Monthly Notices of the Royal Astronomical Society*, Vol. 448, No. 1, pp. 804–813.

Numata, R., Ball, R. and Dewar, R.L., 2007. "Bifurcation in electrostatic resistive drift wave turbulence". *Physics of Plasmas*, Vol. 14, No. 10, p. 102312.

Powell, G.E. and Percival, I.C., 1979. "A spectral entropy method for distinguishing regular and irregular motion of hamiltonian systems". *Journal of Physics A: Mathematical and General*, Vol. 12, No. 11, pp. 2053–2071. doi: 10.1088/0305-4470/12/11/017. URL `https://doi.org/10.1088/0305-4470/12/11/017`.

Press, W.H., 2007. *Numerical recipes 3rd edition: The art of scientific computing*. Cambridge university press.

Pushkarev, A., 2013. *Self-organization of isotopic and drift-ware turbulence*. Ph.D. thesis, Ecully, Ecole centrale de Lyon.

Rempel, E.L., Chian, A.C.L. and Miranda, R.A., 2007. "Chaotic saddles at the onset of intermittent spatiotemporal chaos". *Physical Review E*, Vol. 76, No. 5, p. 056217.

Rempel, E.L., Miranda, R.A. and Chian, A.C.L., 2009. "Spatiotemporal intermittency and chaotic saddles in the regularized long-wave equation". *Physics of Fluids*, Vol. 21, No. 7, p. 074105. doi:10.1063/1.3183590. URL `https://doi.org/10.1063/1.3183590`.

Shannon, C.E., 1948. "A mathematical theory of communication". *The Bell System Technical Journal*, Vol. 27, No. 3, pp. 379–423. doi:10.1002/j.1538-7305.1948.tb01338.x.

Wagner, F., Becker, G., Behringer, K., Campbell, D., Eberhagen, A., Engelhardt, W., Fussmann, G., Gehre, O., Gernhardt, J., Gierke, G.v., Haas, G., Huang, M., Karger, F., Keilhacker, M., Klüber, O., Kornherr, M., Lackner, K., Lisitano, G., Lister, G.G., Mayer, H.M., Meisel, D., Müller, E.R., Murmann, H., Niedermeyer, H., Poschenrieder, W., Rapp, H., Röhr, H., Schneider, F., Siller, G., Speth, E., Stäbler, A., Steuer, K.H., Venus, G., Vollmer, O. and Yü, Z., 1982. "Regime of improved confinement and high beta in neutral-beam-heated divertor discharges of the asdex tokamak". *Phys. Rev. Lett.*, Vol. 49, pp. 1408–1412. doi:10.1103/PhysRevLett.49.1408. URL `https://link.aps.org/doi/10.1103/PhysRevLett.49.1408`.

Xi, H. and Gunton, J.D., 1995. "Spatiotemporal chaos in a model of rayleigh-bénard convection". *Phys. Rev. E*, Vol. 52, pp. 4963–4975. doi:10.1103/PhysRevE.52.4963. URL `https://link.aps.org/doi/10.1103/PhysRevE.52.4963`.

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