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RELIABILITY COMPARISONS AND DIFFERENCES BETWEEN DARCY-WEISBACH, COLEBROOK-WHITE AND MOODY-ROUSE EQUATIONS

Rodrigo Andrade Ferreira
Dawson Tadeu Izola
Centro Universitário da Fundação Hermínio Ometto, Araras, SP, Brazil
rodrigoaer@alunos.fho.edu.br, dawson@fho.edu.br

Abstract. *Head losses in circular hydraulic tubes can be determined for different flow regimes. In turbulent regimes, there is more than one way to estimate such loss based on mathematical models. Energy losses are related to the shear stress between the fluid and the tube walls, depending on the roughness of the material and the viscosity of the fluid related to the speed and consequently the flow regime. The head loss in acrylic were measured and the result were compared with three different mathematical models. To determine the friction coefficient, the Darcy-Weisbach model, the Colebrook-White iterative model and the Moody-Rouse Diagram were used. The head loss was determined by the Darcy model considering the friction coefficient obtained by the three methods described above and the, the results, for each situation, were compared with the experimental solution. Errors of less than 20% were observed and the pressure loss values are very close in the three models..*

Keywords: *Fluid Mechanics, Head Loss, Colebrook-White, Moody-Rouse, Darcy-Weisbach*

1. INTRODUCTION

Bistafa (2010) says that for flows in the laminar regime the friction factor is independent of the relative roughness. However, in transitional and turbulent regimes, the friction factor can be determined through interactions using the Colebrook-White equation, by the Darcy method or by the Moody-Rouse graphical method.

Brunetti (2008) lists head loss in two classifications. The first is the distributed head loss, which occurs along long straight pipes due to the friction of fluid particles against each other. The second is localized head loss, which occurs in places where the fluid undergoes sudden disturbances, such as changes in direction, passing through valves, etc.

Çengel and Cimbala (2007) says that a quantity of interest in pipe flow analysis is the pressure drop since it is directly related to the pump power. The pressure drop occurs due to the head loss present in the piping and these are related to the viscosity and shear stress on the wall.

Izola (2023) suggests the uncertainty obtaining the friction factor from Darcy's equation or Moody's diagram is less than 15%.

2. METHODS

In this section, the analysis procedure used to carry out the distributed load loss experiment will be presented. To collect velocity information, a Pitot tube was used connected to an Omega HHC280 differential manometer, thus allowing the use of Bernoulli's equation to determine the flow velocity. The pressure loss in the section under analysis is found through the use of a Gulpress micromanometer – capable of measuring between (-1000 to +3000) Pa with a resolution of 1 Pa. The system used for analysis is a circuit with water recirculation , using PVC and acrylic tubes, with a 372.85 W pump. To determine the roughness of the tubes, a Mitutoyo roughness meter was used. The water temperature is checked using the Minipa MT-350 sensor with a range from -30 to 550°C. With each new test, the water temperature was measured and the corresponding density and viscosity were determined. On Tab. 1 you can see the data obtained for the flow in the acrylic tube. The complete circuit can be seen in the Fig. 1.

The determination of the flow regime classification was obtained through the Reynolds number by the Eq. (1).

$$Re = \frac{\rho.V.d}{\mu}, \tag{1}$$

Where ρ is the density of the fluid used, d is the pipe diameter, μ is dynamic viscosity and V is the fluid flow speed. For Reynolds numbers lower than 2000, the flow is classified as laminar, for $2000 < Re < 4000$, the flow is classified as transitional and for Reynolds numbers greater than 4000, the flow is classified as Turbulent. Within the parameters necessary to determine the Reynolds number, only the velocity (V) of the fluid will be a variable parameter, unlike the

Table 1. Acrylic's Data.

Variable	**Value**	**Unit**
Temperature (room)	20	°C
Absolute Roughness (ε)	0,17x10^{-4}	m
Diameter (d)	19,42x10^{-3}	m
Water Density(ρ)	998,21	kg/m^3
Lenght(L)	303,2x10^{-3}	m
Dynamic Viscosity(μ)	1,003x10^{-3}	Pa.s

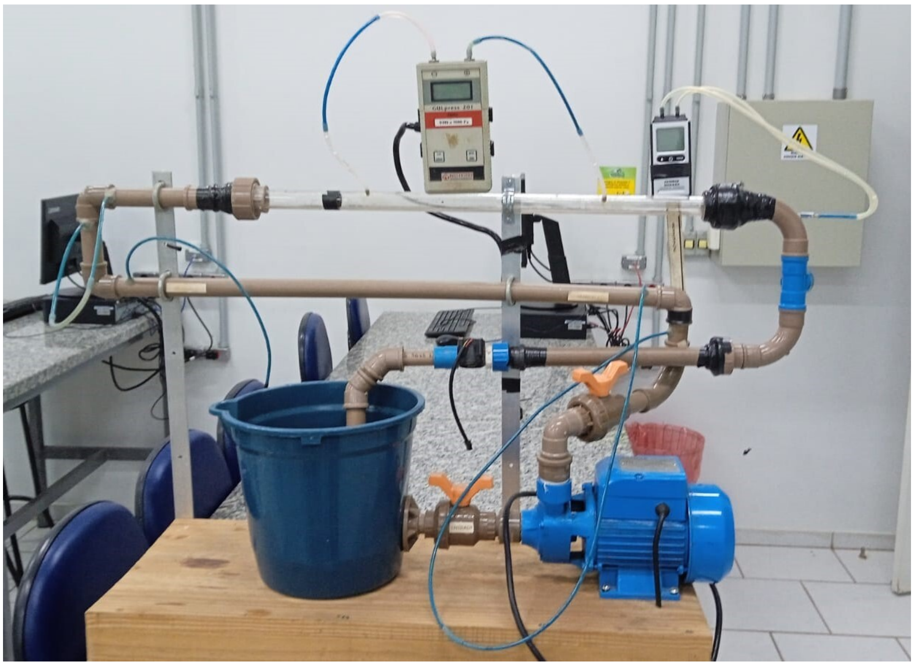

Figure 1. Circuit used for the experiment.

others that will be considered constant in this experiment. The fluid velocity can be obtained using the Eq. (2), derived from the Bernoulli equation, for Pitot tubes.

$$V = \sqrt{\frac{2.\Delta P}{\rho}}, \tag{2}$$

To obtain data from different Reynolds numbers, a ball valve was used coupled before the suction of the water pump, thus being able to decrease and increase the flow and consequently vary the flow speed. The data obtained with the valve in different positions can be seen in the Tab. 2, Tab. 3 and Tab. 4.

The first method adopted to determine the friction factor is the Moody-Diagram presented in Fig. 2. The relative roughness($\varepsilon_{relative}$) can be found by the Eq. (3), where ($\varepsilon_{absolute}$) is the absolute roughness of the acrylic and (d) is the diameter of the pipe. It is important to note that both must be used in the same unit of measurement.

$$\varepsilon_{relative} = \frac{\varepsilon_{absolute}}{d}, \tag{3}$$

The relative roughness will represent the different curves present on the right axis of the diagram, Fig. 2, and relating them to the flow regime it is possible to draw a straight line that represents the friction factor on the left axis of the diagram.

The most well-known method to calculated friction factor(f), the Darcy-Weisbach is presented Eq. (4), as described in (Çengel and Cimbala, 2007).

$$f = \frac{0,25}{\left[log\left(\frac{\varepsilon_{absolute}}{3,7.d} + \frac{5,74}{Re^{0,9}}\right)\right]^2}, \tag{4}$$

Table 2. Fully Open Valve Flow Data.

Measured Quantities	**Sensor 1 (Section Under Analysis) [Pa] Differential Pressure Between P1 and P2**	**Sensor 2 (Pitot) [Pa] Dynamic Pressure**
1	301	1129
2	320	1039
3	310	1030
4	351	1081
5	320	1052
6	302	1001
7	333	1091
8	310	1070
9	318	1025
10	311	1039

Table 3. Data obtained with Valve Half Open (1/2 Flow).

Measured Quantities	**Sensor 1 (Section Under Analysis) [Pa] Differential Pressure Between P1 and P2**	**Sensor 2 (Pitot) [Pa] Dynamic Pressure**
1	301	785
2	302	873
3	301	865
4	306	799
5	310	972
6	306	869
7	308	884
8	316	868
9	316	891
10	311	861

Table 4. Data obtained with Valve Open at Approximately (¼ of the Flow).

Measured Quantities	**Sensor 1 (Section Under Analysis) [Pa] Differential Pressure Between P1 and P2**	**Sensor 2 (Pitot) [Pa] Dynamic Pressure**
1	107	100
2	77	138
3	110	298
4	115	285
5	126	256
6	114	273
7	115	306
8	117	289
9	109	278
10	112	275

The 3° method used is the Colebrook-White Eq. (5), which requires a little more mathematical work, due to the fact that is a transcendental equation, requiring interactive calculations to obtain a result. However, through computational means, such as Excel, for example, it becomes much easier and faster.

$$\frac{1}{\sqrt{f}} = -2log\left(\frac{\varepsilon_{absolute}}{3,7.d} + \frac{2,51}{Re\sqrt{f}}\right), \tag{5}$$

The distributed head loss can be found using the equation below and is known as the Darcy Eq.(6).

$$J = f\frac{L}{d}\frac{V^2}{2g}, \tag{6}$$

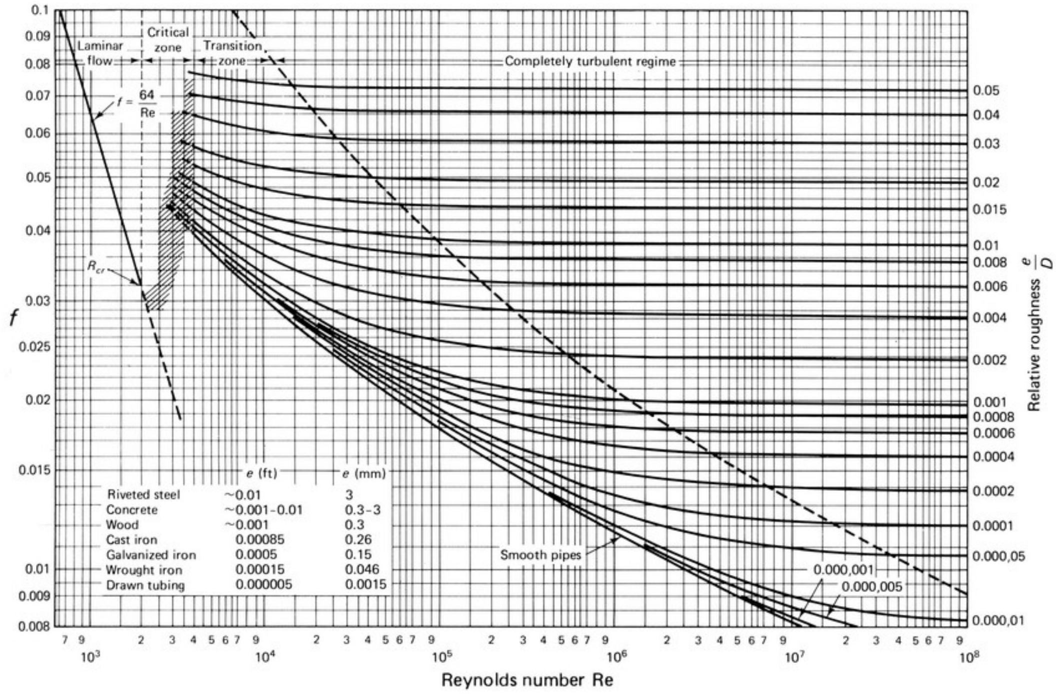

Figure 2. Moody's Diagram.

Where L is the length of the section under analysis and g is the gravitational constant adopted as 9,81m/s².

The error to be obtained between the average calculated pressure loss and the average pressure loss found with the pressure sensor, installed in the section under analysis, is described by the Eq.(7).

$$Error(\%) = \left| \frac{J_{Theoretical} - J_{Practical}}{J_{Theoretical}} \right| \times 100, \tag{7}$$

3. RESULTS AND DISCUSSION

This section presents the calculated pressure loss and the friction factors found for the conditions described previously. The friction factors found with fully open valve are represented in Tab. 5 and the head loss values obtained for the respective data collected through the sensor can be seen in the Fig. 3. The average of the pressure loss values obtained by the sensor results in $307,700Pa$. Comparing with the average of the results obtained analytically, it is possible to estimate the error between the theoretical and the real (measured) value, as demonstrated in the Tab. 6.

Table 5. Fully Open Valve - Acrylic.

Variable	Moody-Rouse	Darcy-Weisbach	Colebrook-White
f_1	0,025	0,023577009	0,023682389
f_2	0,025	0,023813	0,023915
f_3	0,025	0,023838	0,02394
f_4	0,025	0,02369992	0,023803625
f_5	0,025	0,0237	0,023804
f_6	0,025	0,02392	0,02402
f_7	0,025	0,023673788	0,023777854
f_8	0,025	0,023729	0,023832
f_9	0,025	0,023852	0,023953
f_{10}	0,025	0,024635774	0,024725061

The friction factors found with half open (1/2 Flow) valve are represented in Tab. 7 and the head loss values obtained for the respective data collected through the sensor can be seen in the Fig. 4. The average of the head loss values obtained by the sensor results in $307,700Pa$. Comparing with the average of the results obtained analytically, it is possible to estimate the error between the theoretical and the real (measured) value, as demonstrated in the Tab. 8.

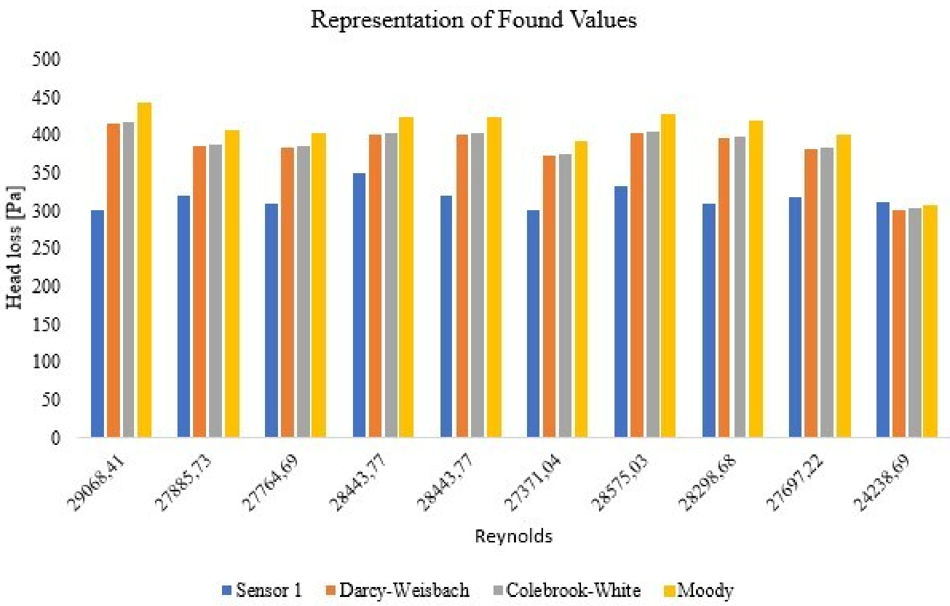

Figure 3. Values Found with the Valve Fully Open

Table 6. Average between the Values Obtained Analytically and the Error in Comparison with the Theoretical.

Variable	Moody-Rouse	Darcy-Weisbach	Colebrook-White
J [Pa]	403, 750	384,682	386,328
Error [%]	21,338	17,438	17,790

Table 7. Valve Half Open (1/2 Flow).

Variable	Moody-Rouse	Darcy-Weisbach	Colebrook-White
f_1	0,025	0,024635774	0,024725061
f_2	0,025	0,024319	0,024413
f_3	0,025	0,024346	0,02444
f_4	0,025	0,02458262	0,024672805
f_5	0,025	0,024005	0,024104
f_6	0,025	0,024332	0,024427
f_7	0,025	0,024377029	0,024377029
f_8	0,025	0,024336	0,02443
f_9	0,025	0,024259	0,024354
f_{10}	0,025	0,024356375	0,024450276

Table 8. Average between the Values Obtained Analytically and the Error in Comparison with the Theoretical.

Variable	Moody-Rouse	Darcy-Weisbach	Colebrook-White
J [Pa]	338, 725	329,731	331,007
Error [%]	9,159	6,682	7,041

The friction factors found with approximately (¼ of the Flow) are represented in Tab. 9 and the head loss values obtained for the respective data collected through the sensor can be seen in the Fig. 5. The average of the pressure loss values obtained by the sensor results in $110, 200Pa$. Comparing with the average of the results obtained analytically, it is possible to estimate the error between the theoretical and the real (measured) value, as demonstrated in the Tab. 10.

Through the errors obtained from the differences in results found by the three methods used, in a turbulent regime, the Moody-Rouse graphic method is the one that presented the greatest difference in values in relation to the others. The Colebrook-White and Darcy-Weisbach analytical methods are those that presented the closest values.

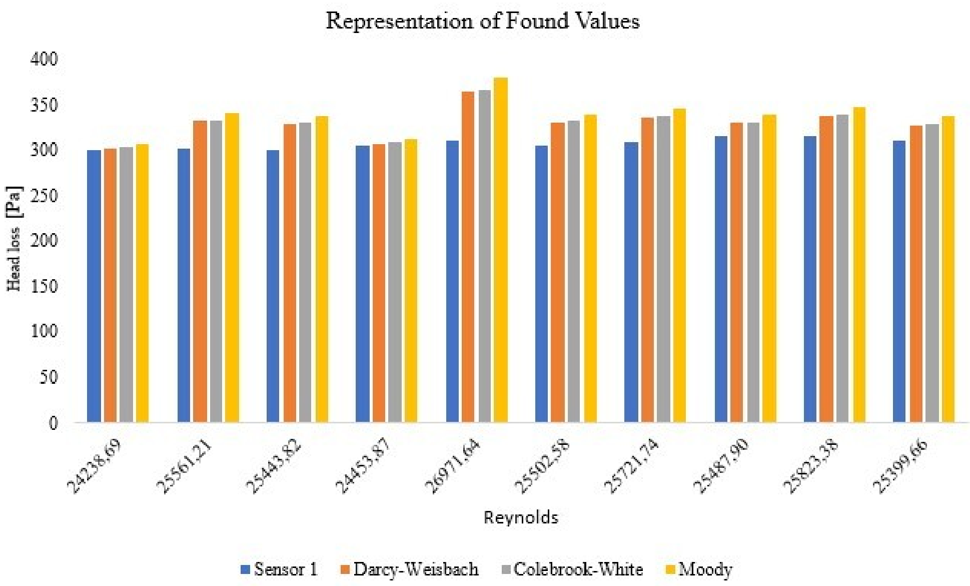

Figure 4. Valve Half Open (1/2 Flow)

Table 9. Valve Open at Approximately (¼ of the Flow).

Variable	Moody-Rouse	Darcy-Weisbach	Colebrook-White
f_1	0,029	0,029339489	0,032114826
f_2	0,030	0,030851	0,030772
f_3	0,029	0,027837	0,027855
f_4	0,029	0,027998826	0,028011542
f_5	0,029	0,028011542	0,028396
f_6	0,028	0,028157	0,028164
f_7	0,029	0,02774085	0,027762097
f_8	0,029	0,027948	0,027962
f_9	0,029	0,0280899	0,0280998
f_{10}	0,029	0,028129799	0,028138435

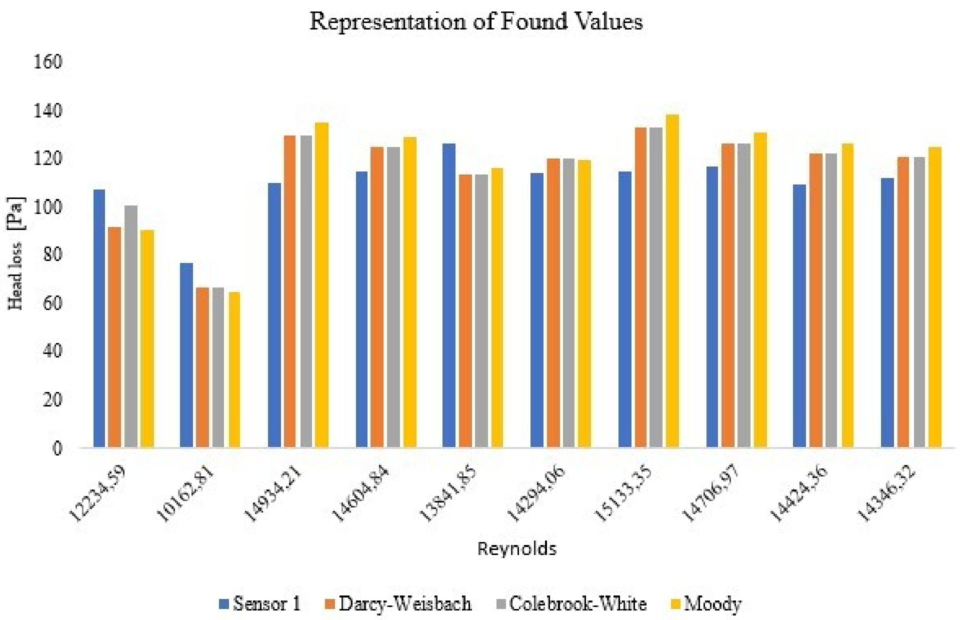

Figure 5. Valve Open at Approximately (¼ of the Flow)

Table 10. Average between the Values Obtained Analytically and the Error in Comparison with the Theoretical.

Variable	Moody-Rouse	Darcy-Weisbach	Colebrook-White
J [Pa]	117, 556	114,836	115,728
Error [%]	6,258	4,037	4,777

4. CONCLUSION

The error between models is relatively small. The values observed in this study, despite still being considered in a range where there is some suspicion regarding, the measurements (turbulent flow), the results observed are consistent with the literature and the uncertainty regarding these values is a percentage rate small as described in (Çengel and Cimbala, 2007) and in (Izola, 2023).

5. REFERENCES

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6. RESPONSIBILITY NOTICE