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STABILIZATION OF A FURUTA PENDULUM BY USING A VARIANT DISTURBANCE REJECTION CONTROL APPROACH

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Abstract. *This work addresses the stabilization problem of a rotary inverted pendulum mechanism by using a robust control strategy, which is robust to unmodeled dynamics, parametric uncertainties and external disturbances. As the rotating pendulum is a nonminimum phase system, it is proposed to use an extension of the MP-ADRC controller recently proposed in literature. Through a cascade control structure, in which the dynamics of the inverted pendulum system is represented as two second-order systems in cascade, two MP-ADRC controllers are designed, one for solving a tracking problem and the other one for the stabilization problem of a nonminimum phase system. The stability analysis of the resulting closed-loop configuration is discussed. Simulation tests are performed with the control strategy proposed and the obtained results are discussed.*

Keywords: *Nonminimum phase, rotary pendulum, robust control, cascade structure.*

1. INTRODUCTION

The control of uncertain dynamical systems has received significant attention of the control community motivated by the difficulty of obtaining the exact model for some systems, particularly involving situations in which their parameters are not perfectly known. The problem of dealing with parametric uncertainties is of great interest in the control systems community and represents a challenging benchmark for industrial and academic applications. Among several control strategies proposed in the literature, many contributions have been reported concerning at linear and nonlinear plants, such as the recent ones involving Backstepping (Chen *et al.*, 2015), Model Reference Adaptive Control (MRAC) (Selfridge and Tao, 2016) and the references therein. In this context, the Active Disturbance Rejection Control (ADRC) method has been a subject of continuous research in the control systems community, since the pioneering work of Han (Jingqing, 1998), as can be seen in (Gao *et al.*, 2001; Meng and Moore, 2016; Meng *et al.*, 2019; Wei *et al.*, 2019; Patelski and Dutkiewicz, 2020; Han *et al.*, 2021). The ADRC strategy uses an Extended State Observer (ESO), which consists of considering all external disturbances and unmodeled dynamics gathered together in one state variable that is to be estimated by the ESO and then used in a stabilizing control law. The relevant characteristics of ADRC strategy are, mainly, its robustness to unmodeled dynamics, external disturbance, and the relatively simple implementation when compared to other adaptive robust control strategies.

However, the vast majority of works on ADRC strategy only deals with systems with stable zero dynamics, which are also known as minimum phase systems. Moreover, many practical dynamical systems such as general aircrafts (Wang *et al.*, 2019; Sir Elkhatem *et al.*, 2021), vessels (Sun *et al.*, 2021), drones (Li *et al.*, 2020), all have their equations of motion with nonminimum phase characteristics. It is well-known in control system's literature that the stabilization of nonminimum phase systems is challenging since the zeros of the system cannot be allocated or canceled by classical feedback control. The limitations introduced by unstable zero dynamics are structural and, this way, they cannot be avoided without changing the system structure or re-formulating the tracking problem (Aguiar *et al.*, 2005). This challenge is even greater if there are uncertainties in the system's parameters, non-modeled dynamics, and external disturbances. In the case of ADRC strategy, some works approach the nonminimum phase systems, but the zeros were considered known in most of them (see Xue *et al.* (2016) and references therein).

Another important issue to be highlighted is the poor robustness of most proposed ADRC strategies with respect to large level uncertainties in the control coefficient (or control gain) of the plant. In general, the inverse value of this parameter is required to compute the control law. When it is not exactly known, some approximate value is adopted in the control design. In this context, Zachi *et al.* (2019) has proposed the MP-ADRC, an extension of the ADRC, in which a structural change is introduced in the plant, so that the ESO no longer depends on the plant control gain, making the MP-ADRC more robust to uncertainties in this parameter. Although there is an advantage over the traditional ADRC method, the application of MP-ADRC is still restricted to minimum phase systems, until now.

In this work, it is proposed an extension of MP-ADRC to nonminimum phase systems. The idea is to implement a partial feedback so that it is possible to represent the nonminimum phase system as a minimum phase one with a parallel integrator. Thus, the MP-ADRC can be applied, maintaining its good robustness to external disturbances and unmodeled dynamics. The proposed strategy is applied in the stabilization problem of a rotary inverted pendulum, which possesses nonminimum phase characterístics. In short, the current paper's contributions are: (1) introduction to a rigorous discussion on the extension of the MP-ADRC strategy applied to nonminimum phase system; (2) comprehensive discussion about the numerical simulation results obtained with the application of the proposed strategy in the rotary inverted pendulum.

The paper is organized as follows: In Section 2, the Linear ADRC controller (LADRC) is discussed, where some basic concepts are explained. The MP-ADRC controller is presented in Section 3and its extension to systems with finite zeros is discussed in Section 4. In Section 5, the MP-ADRC is extended to nonminimum phase systems. In Section 6, the simulation results are presented, where the cascade structure is used to apply the MP-ADRC in the control of the inverted rotating pendulum.

2. BACKGROUND AND RELATED WORKS

Consider the class of second-order nonlinear systems that can be described by

$$\ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) + d(t) = bu(t)\,, \tag{1}$$

in which a_1 and a_0 are real uncertain constant parameters, $d(t)$ is a function that gathers the system's nonlinearities and unknown external disturbances, and b is a real uncertain constant that represents the plant control coefficient or control gain. The objective is to define an adequate control law for $u(t) \equiv u \in \mathbb{R}$ in Eq. (1) such that the plant output $y(t) \equiv y \in \mathbb{R}$ can track the desired reference trajectory $y_r(t) \equiv y_r \in \mathbb{R}$. In addition, we consider that the only available data from the system are the input and output variables u and y, respectively.

By adopting the ADRC formalism (Jingqing, 1998; Gao *et al.*, 2001; Madoński *et al.*, 2015; Wu *et al.*, 2021), the plant in Eq. (1) can also be described by

$$\ddot{y} = bu \overbrace{-a_1\dot{y} - a_0 y - d(t)}^{f}\,, \tag{2}$$

An ideal control law u^* that could solve the tracking control problem for Eq. (2) is given by

$$u \;=\; u^* \;=\; -\frac{1}{b}\left[f + \lambda_1(\dot{y} - \dot{y}_r) + \lambda_0(y - y_r) + \ddot{y}_r\right]\,, \tag{3}$$

where λ_1 and λ_0 are design constants chosen so that the closed loop system is stable. However, as the generalized disturbance term f is unknown, then Eq. (3) cannot be implemented directly. In order to overcome this difficulty, the authors in Gao et al. (Gao *et al.*, 2001) proposed a linear version of the ADRC method, called LADRC (Linear Active Disturbance Rejection Control). The main idea is to incorporate f as an additional plant state variable and use an Extended State Observer (ESO) to estimate it online.

2.1 The LADRC basic method

By defining $X = [y, \dot{y}, f]^T$ as the extended state vector, and also considering the parameter b known, the representation of Eq. (2), in the controllable canonical form, assumes the following format

$$\dot{X} = \overbrace{\begin{bmatrix} 0 & 1 & 0 \\ -\lambda_0 & -\lambda_1 & 1 \\ 0 & 0 & 0 \end{bmatrix}}^{A} X + \overbrace{\begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}}^{B} u + \overbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}^{\Lambda} \dot{f}\,. \tag{4}$$

Then, applying the well-known Luenberg's Observer (Ogata, 2000) for Eq. (4), we will have the ESO described by (Gao *et al.*, 2001)

$$\dot{\hat{X}} = (A - LC)\hat{X} + Bu + Ly\,, \tag{5}$$

in which $L \in \mathbb{R}^3$ is the observer's gain vector, $C = [1, 0, 0]$ and $\hat{X} = [\hat{y}, \dot{\hat{y}}, \hat{g}]^T$ is the observer's estimated state vector. It is worth remembering that L is calculated so that the matrix $A - LC$ has eigenvalues in the left half-plane (LHP). If we define the estimation error vector by $\tilde{X} = X - \hat{X}$, then its dynamical equation will be given by

$$\dot{\tilde{X}} = (A - LC)\tilde{X} + \Lambda \dot{f} \,. \tag{6}$$

Note that if the derivative $\dot{f}$ of the generalized disturbance f is bounded, then so are the state estimation errors. Judging by the asymptotic convergence properties of the matrix $(A - LC)$, we can conclude that $\tilde{X} \to 0$ if $\dot{f} \to 0$. Then, Gao *et al.* (2001) proposed the control law described by

$$u = -\frac{1}{b}\left[\hat{f} + \lambda_1(\dot{\hat{y}} - \dot{y}_r) + \lambda_0(\hat{y} - y_r) + \ddot{y}_r\right] \,. \tag{7}$$

The stability analysis and robustness properties applied to the closed-loop control system composed by Eqs. (7) and (2) have already been studied and stated in several works over the years (Jingqing, 1998; Gao *et al.*, 2001; Zheng *et al.*, 2009b,a; Zachi *et al.*, 2019; Madoński *et al.*, 2015; Teixeira *et al.*, 2021; Zhou *et al.*, 2021).

Although the LADRC method just discussed in this section presents good robustness to the parametric uncertainties, unmodeled dynamics, and external disturbances, it has little robustness to large parametric uncertainty in the plant control coefficient b. The uncertainties that may occur in b are an issue of great concern in several practical control applications. Particularly, in those plants in which the value of b depends on physical and geometrical constants, such as mass, inertia moment, angles, and lengths, which possibly can vary during the system operation. This problem is discussed in Zachi *et al.* (2019), in which a new ADRC structure is proposed, denoted by *Modified Plant ADRC* (MP-ADRC). The MP-ADRC will be presented below.

3. THE MODIFIED-PLANT ADRC

The main idea proposed by Zachi *et al.* (2019) is to perform a structural transformation on the original plant for obtaining a resulting dynamical system with an advantageous format. The strategy is to introduce an adjustable constant gain β in series with the plant output error $e(t) = y(t) - y_r(t)$, and a linear stable filter Q_0 in parallel with them, as illustrated in Figure 1. The output gain β is defined as:

$$\beta = \bar{K}_0 \, sign(b) \,, \tag{8}$$

in which $\bar{K}_0 > 0 \in \mathbb{R}$ is arbitrary design constant. The positive design constant $\gamma \in \mathbb{R}$ of the filter is chosen such that $(s + \gamma)^2 = s^2 + \alpha_1 s + \alpha_0$. Based on the configuration of Figure 1, the new output error $z(t)$ can be written as:

$$z(t) = \beta e(t) + u_f(t) \,, \tag{9}$$

$$\ddot{u}_f = -\alpha_1 \dot{u}_f - \alpha_0 u_f + \dot{u}(t) \,. \tag{10}$$

So, by differentiating (9) two times, the dynamics of the new output error variable $z(t)$, now with $b_p = \beta b$, will be given by:

$$\ddot{z} = \beta[-a_1\dot{y} - a_0 y - d + bu(t)] - \alpha_1 \dot{u}_f - \alpha_0 u_f + \dot{u} \,. \tag{11}$$

As, from Eq. (9),

$$u_f^{(i)} = z^{(i)}(t) - \beta e^{(i)}(t) \,, \quad (i = 0, 1) \,, \tag{12}$$

then, after a simple algebraic manipulation, we have

$$\ddot{z} + \alpha_1 \dot{z} + \alpha_0 z = \overbrace{\beta[(\alpha_1 - a_1)\dot{y} + (\alpha_0 - a_0)y - \ddot{y}_r - \alpha_1 \dot{y}_r - \alpha_0 y_r - d + bu(t)]}^{\Omega(t)} + \dot{u} \,. \tag{13}$$

Since the homogeneous part of the ODE in the left-hand side of Eq.(13) inherits the coefficients of filter $Q_0(s)$ in Figure 1, it becomes exponentially stable by construction. Then, a viable stabilizing control law can be given by:

$$\dot{u}(t) = -\Omega(t) \,. \tag{14}$$

Remark 1 *In the modified system of Eq. (13), the new control input $\dot{u}$ has a unitary coefficient. This last fact means that the control law in Eq. (14) is not dependent on the original plant parameter b in Eq. (2), which contrasts with the LADRC control law in Eq. (7). At first glance, this may consist of a significant improvement in the method's robustness property.*

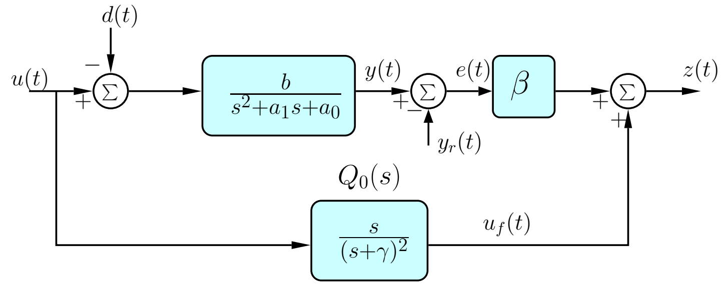

Figure 1. Modified-plant block diagram of the ADRC scheme (Zachi *et al.*, 2019).

3.1 Extended State Observer (ESO) design

As $\Omega(t)$ is not known, the control law (14) cannot be directly implemented. Thus, similar to LADRC from Section 2, an ESO is implemented to estimate $\Omega(t)$. Then, the following extended state vector is defined:

$$\zeta(t) := [\zeta_1, \zeta_2, \zeta_3]^T = [z(t), \dot{z}(t), \Omega(t)]^T \ . \tag{15}$$

By assuming that $\Omega(t)$ is differentiable, the state-space representation of (13), in controllable canonical form, is defined as:

$$\dot{\zeta} = \overbrace{\begin{bmatrix} 0 & 1 & 0 \\ -\alpha_0 & -\alpha_1 & 1 \\ 0 & 0 & 0 \end{bmatrix}}^{A_m} \zeta + \overbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}^{B_\zeta} \dot{u}(t) + \overbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}^{\Gamma} \dot{\Omega}(t) \,, \quad z(t) = C\zeta \,. \tag{16}$$

Based on Eq (16), the full-order ESO assumes the following implementation format:

$$\begin{cases} \dot{\hat{\zeta}} = A_m\hat{\zeta} + B_\zeta \, \dot{u}(t) + Le_z \,, \\ \hat{z} = C\hat{\zeta} \,, \end{cases} \tag{17}$$

with $e_z := z(t) - \hat{z}(t)$, $L = [L_1, \ L_2, \ L_3]^T$, and for which

$$\alpha_e(s) = \det[sI - (A_m - LC)] \tag{18}$$

is a stable characteristic polynomial. By defining the ESO state error as

$$e_\zeta = \zeta - \hat{\zeta} \,, \tag{19}$$

then the ESO closed-loop dynamics can be computed from (16) and (17), resulting in:

$$\begin{cases} \dot{e}_\zeta = \underbrace{(A_m - LC)}_{\bar{A}_m} e_\zeta + \Gamma \, \dot{\Omega}(t) \,, \\ e_z = Ce_\zeta \,. \end{cases} \tag{20}$$

Note that $\hat{\zeta}_3 = \hat{\Omega}(t)$, the estimated generalized disturbance. Thus, since the ESO performance can be conveniently adjusted by the choice of the design constant vector L, the following control law expression is proposed based on Eq. (14) (Zachi *et al.*, 2019):

$$\dot{u}(t) = - \, \hat{\zeta}_3 \quad \text{or} \quad u(t) = - \int_0^t \hat{\zeta}_3(\tau) \, d\tau \,. \tag{21}$$

Remark 2 *Note that:*

- *if* $\hat{\zeta}_3 \to \hat{\Omega}$*, then* $z(t) \to 0$*;*
- *if* $\dot{u} \to 0$ *and* $z(t) \to 0$*, then, from Eq. (10), so does* u_f *and* $e(t)$*.*

In (Zachi et al.*, 2019) it is shown that if* y_r *and* $d(t)$ *are constants, then the above conditions are satisfied and* $e(t) \to 0$*. Furthermore, it is also shown that if the highest frequency* ω_0 *of the ESO poles is chosen much higher than the highest frequency of* $d(t)$*, then* $e(t)$ *tends to a residual set around zero that can be reduced as much as you want by increasing* ω_0*. Due to a lack of space, this demonstration will not be included here.*

The MP-ADRC proposed above is only applicable to systems without finite zeros, which is not the case for many practical systems. In this context, the next section discusses the extension of the MP-ADRC to a class of Minimum Phase Systems (MPS) with finite zeros, which have all their zeros in the Left Half of the Complex Plane (LHP).

4. MP-ADRC APPLIED TO MINIMUM PHASE SYSTEMS WITH FINITE ZEROS

Consider the system described by

$$\ddot{y} + a_1\dot{y} + a_0 y = \bar{b}(\dot{u} + b_0 u) \tag{22}$$

where $b_0 > 0$ and $\bar{b}$ are real constants. Since the system of Eq. (22) has finite zeros, it is proposed a change in the structure of the modified plant represented in Figure 1, as is shown in Figure 2, where is inserted an integrator in series with the plant. Implementing a similar mathematical development to Subsection 3. it can be conclude that

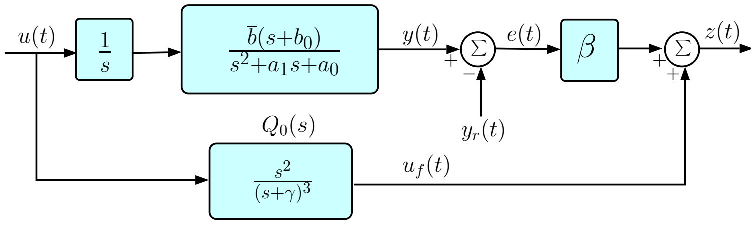

Figure 2. Modified-plant block diagram of the MP-ADRC scheme with finite zeros (Zachi *et al.*, 2019).

$$\dddot{z} = \bar{f} + \ddot{u}\,, \tag{23}$$

where $\bar{f}$ is the new generalized disturbance described by

$$\bar{f} = -\alpha_2\ddot{z} - \alpha_1\dot{z} - \alpha_0 z + \beta\left[(\alpha_2 - a_1)\ddot{y} + (\alpha_1 - a_0)\dot{y} + \alpha_0 y - \dddot{y}_r - \alpha_2\ddot{y}_r - \alpha_1\dot{y}_r - \alpha_0 y_r + \bar{b}\dot{u} + \bar{b}bu\right] \tag{24}$$

in which α_0, α_1 and α_2 are calculated using

$$(s+\gamma)^3 = s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0\,. \tag{25}$$

Then, similar to (Filho *et al.*, 2017), integrating twice the Eq. (23), we have

$$\dot{z} = \overbrace{\iint \bar{f}(t)dt}^{\bar{\Omega}(t)} + u\,. \tag{26}$$

Note that $\bar{\Omega}(t)$ depends on $u(t)$. Consequently, $\bar{\Omega}(t)$ can grow exponentially, resulting in an unbounded $\dot{\bar{\Omega}}(t)$. In order to mitigate this problem, it is proposed to increase the order of the extended state observer (ESO), estimating not only $\bar{\Omega}(t)$, but also its derivatives. For this, the extended system states are defined as $\zeta_1 = z$, $\zeta_2 = \bar{\Omega}$, $\zeta_3 = \dot{\bar{\Omega}}$ and $\zeta_4 = \ddot{\bar{\Omega}}$, resulting in the following state space representation:

$$\dot{\zeta} = \overbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}^{A_m}\zeta + \overbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}}^{B_\zeta} u(t) + \overbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}^{\Gamma} \dot{\bar{f}}(t)\,, \tag{27}$$

$$z(t) = C\zeta\,.$$

Thus, the ESO described by Eq. (17) and characteritic polynomial of Eq. (18) can be used again, being that now $L = [L_1,\ L_2,\ L_3,\ L_4]^T$. Note that $\dot{\bar{f}}$ is a function of the derivatives of $u(t)$, which solves the problem of integrating $u(t)$ into $\bar{\Omega}$. Finally, from Eq. (26), the following stabilizing control law can be proposed for the modified plant of Figure 2:

$$u(t) = -\hat{\zeta}_2 - k_1 z\,, \tag{28}$$

where $\hat{\zeta}$ is the estimate of $\bar{\Omega}$ and k_1 is a design parameter that should be chosen so that

$$\dot{z} + k_1 z = 0 \tag{29}$$

is stable. There are systems that, in addition of having finite zeros, are nonminimum phase systems (zeros in the Right Half of the Complex Plane (RHP)). In the next section, an extension of the MP-ADRC for a class of nonminimum phase systems is proposed. The idea is to use preliminary feedback so that the closed-loop system can be represented as a minimum-phase system in parallel with an integrator.

5. THE EXTENSION OF MP-ADRC APPLIED TO NONMINIMUM PHASE SYSTEMS

Consider the system described by

$$\ddot{y} + a_1\dot{y} + a_0 y = \bar{b}(\dot{u} - bu) \tag{30}$$

where $b > 0$ and $\bar{b}$ are real constants. Then, representing the system in the frequency domain, we have

$$H(s) = \frac{\bar{b}(s-b)}{s^2 + a_1 s + a_0}\,. \tag{31}$$

Note that, as $b > 0$, we have a nonminimum phase system, since its zero belongs to the RHP.

Controlling nonminimum phase systems is challenging since system zeros cannot be allocated or canceled by feedback control. This challenge is even greater if there are uncertainties and/or unmodeled dynamics parameters and external disturbances. To overcome this challenge, it is proposed to represent Eq. (31) as

$$H(s) = \frac{\bar{b}}{s} - \overbrace{\frac{\bar{b}\,[(b+a_1)s + a_0]}{s^3 + a_1 s^2 + a_0 s}}^{\bar{H}(s)}\,. \tag{32}$$

From Eq. (32), it can be seen that $\bar{H}(s)$ will be a minimum phase system if one of the conditions

$$(b + a_1) > 0 \quad \text{and} \quad a_0 > 0\,, \tag{33}$$

$$(b + a_1) < 0 \quad \text{and} \quad a_0 < 0 \tag{34}$$

is satisfied. However, many systems do not satisfy the above conditions. Then, in order to obtain a more general proposal, it is used the preliminary feedback

$$u(t) = K_1\dot{y}(t) + K_0 y(t) + r(t) \tag{35}$$

where K_1 and K_0 are design constants. Therefore, replacing Eq. (35) in Eq. (30), we obtain, after an algebraic manipulation:

$$\ddot{y} + \overbrace{\frac{(a_1 - \bar{b}K_0 + \bar{b}bK_1)}{1 - \bar{b}K_1}}^{\bar{a}_1}\dot{y} + \overbrace{\frac{(a_0 + \bar{b}bK_0)}{1 - \bar{b}K_1}}^{\bar{a}_0} y = \overbrace{\frac{\bar{b}}{1 - \bar{b}K_1}}^{\bar{b}_1}(\dot{r} - br)\,, \tag{36}$$

which, in the frequency domain, can be described by

$$Y(s) = \overbrace{\frac{\bar{b}_1(s-b)}{s^2 + \bar{a}_1 s + \bar{a}_0}}^{G(s)} R(s)\,, \tag{37}$$

where $R(s)$ is the Laplace's transform of $r(t)$. Thus, similar to Eq. (32), $G(s)$ can be rewritten as

$$G(s) = \frac{\bar{b}_1}{s} - \overbrace{\frac{\bar{b}_1\,[(b+\bar{a}_1)s + \bar{a}_0]}{s^3 + \bar{a}_1 s^2 + \bar{a}_0 s}}^{\bar{G}(s)}\,. \tag{38}$$

Consequently, the conditions of Eqs. (33) and (34) can now be described in terms of $\bar{a}_0$ and $\bar{a}_1$:

$$(b + \bar{a}_1) > 0 \quad \text{and} \quad \bar{a}_0 > 0\,, \tag{39}$$

$$(b + \bar{a}_1) < 0 \quad \text{and} \quad \bar{a}_0 < 0\,. \tag{40}$$

Therefore, from Eq. (36), it can be seen that K_0 and K_1 can always be chosen such that $\bar{G}(s)$ is a minimum phase system.

Defining

$$\bar{Y}(s) = \bar{G}(s)R(s)\,, \tag{41}$$

it can be concluded that

$$\bar{Y}(s) = \frac{\bar{b}_1}{s}R(s) - Y(s) \tag{42}$$

in which $Y(s)$ is the Laplace's transform of $y(t)$. Note that $\bar{y}$ is known and can be calculated using

$$\bar{y}(t) = \bar{b} \int r(t)dt - y(t)\,. \tag{43}$$

Therefore, since the conditions of Eqs. (39) or (40) are always satisfied by an apropriated chosen of K_0 and K_1, it is possible to use the ADRC strategy(Section 4.) for Eq. (41). Note, from Eq. (35), which $\dot{y}$ is used in preliminary feedback, which can be a problem as $\dot{y}$ cannot always be measured. However, this can be overcome by noting, when applying Eq. (9) to the minimum phase system of Eq. (41), that

$$z = \beta\bar{y} + r_f\,, \tag{44}$$

where the reference input $\bar{y}_r = 0$ for simplicity and $R_f(s) = Q_0(s)R(s)$, being $R_f(s)$ and $R(s)$ the Laplace's transform of r_f and r respectively. Then, using Eq. (29) in the derivative of (44) with respect to time, we have

$$\dot{\bar{y}} = -\frac{1}{\beta}(k_1 z + \dot{r}_f)\,. \tag{45}$$

Note, from (42), that

$$\dot{y} = \bar{b}_1 r - \dot{\bar{y}}\,. \tag{46}$$

Then, substituting Eqs. (44) and (45) into Eq. (46), we have

$$\dot{y} = \bar{b}_1 r + k_1\bar{y} + \frac{1}{\beta}\left(k_1 r_f + \dot{r}_f\right)\,. \tag{47}$$

Therefore, Eq. (47) can be used in the preliminary feedback of Eq. (35) if $\dot{y}$ is not measurable. In this case, only the output y must be measured, since r_f, $\dot{r}_f$, and r are calculated by the control strategy. The final control structure is shown in Figure 3.

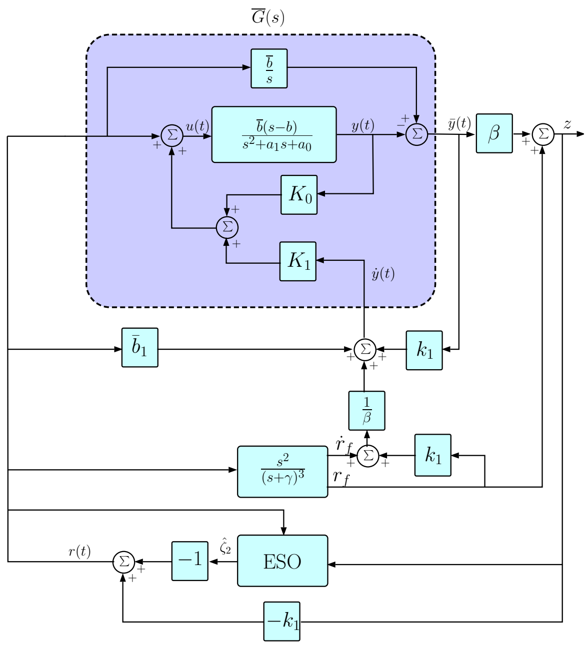

Figure 3. MP-ADRC strategy for nonminimum phase system without measuring $\dot{y}$.

6. SIMULATION RESULTS - MP-ADRC APPLICATION ON A ROTARY INVERTED PENDULUM

The inverted pendulum control has been widely researched because of its nonlinear dynamic, being an important system to validate control strategies. Besides that, there are important practical systems that can be represented as an inverted pendulum, such as the control of rockets and Segway-type vehicles. In order to implement the controller, the dynamics of the pendulum were represented as two second-order systems connected in series configuration. A state change of variable is used so that a cascaded control strategy can be implemented without restrictions. Thus, an MP-ADRC is designed for each second-order system in the cascade structure, emphasizing that one of them is a non-minimum phase system, due to the rotary inverted pendulum dynamical characteristics.

The nonlinear dynamical model of the rotary inverted pendulum system shown in Figure 4 is described by (Quanser, 2023):

$$\left(m_pL_r^2 + \frac{1}{4}m_pL_p^2 - \frac{1}{4}m_pL_p^2\cos(\alpha)^2 + J_r\right)\ddot{\theta} - \left(\frac{1}{2}m_pL_pL_r\cos(\alpha)\right)\ddot{\alpha} + \left(\frac{1}{2}m_pL_p^2\sin(\alpha)\cos(\alpha)\right)\dot{\theta}\dot{\alpha} + \left(\frac{1}{2}m_pL_pL_r\sin(\alpha)\right)\dot{\alpha}^2 + B_r\dot{\theta} = \tau\,, \quad (48)$$

$$-\frac{1}{2}m_pL_pL_r\cos(\alpha)\ddot{\theta} + \left(J_p + \frac{1}{4}m_pL_p^2\right)\ddot{\alpha} - \frac{1}{4}m_pL_p^2\cos(\alpha)\sin(\alpha)\dot{\theta}^2 - \frac{1}{2}m_pL_pg\sin(\alpha) + B_p\dot{\alpha} \;= 0\,, \quad (49)$$

in which τ is the torque applied at the base of the rotary arm, α the pendulum angle, θ the arm angle, $m_p = 0.127kg$ the pendulum mass, $L_p = 0.337m$ the pendulum total length, $L_r = 0.216$ the rotary arm length from pivot to tip, $J_r = 0.002kg.m^2$ the inertia rotary arm moment about its mass center and $B_p = 0.0024N.m.s/rad$ the pendulum viscous damping coefficient. Since the τ is generated by a DC motor, it is used

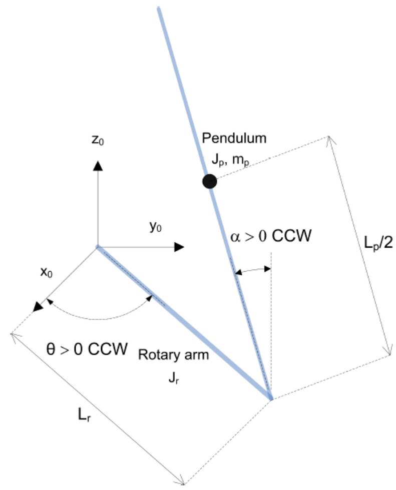

Figure 4. Rotary inverted pendulum (Quanser, 2023).

$$\tau = \frac{\eta_g K_g \eta_m k_t (V_m - K_g k_m \dot{\theta})}{R_m} \quad (50)$$

in order to consider the motor armature voltage V_m as input, where $R_m = 2.6$, $k_t = 0.0077$, $k_m = 0.0077$, $K_g = 70$, $\eta_g = 0.9$ and $\eta_m = 0.69$. Following the Quanser® specification, it was considered $|V_m|$ bounded in $5V$ ($|V_m| \leq 5V$). Using Taylor's series approximation to Linearize the dynamic model of Eqs (48) and (49) around the unstable equilibrium point $(\theta_0,\, \alpha_0,\, \dot{\theta}_0,\, \dot{\alpha}_0) = (0,\, 0,\, 0,\, 0)$, we have

$$\ddot{\theta} \;=\; -0.97\,\dot{\theta} - 0.93\,\dot{\alpha} + 81.4\,\alpha + 403.5\,V_m\,, \qquad \ddot{\alpha} \;= 0.93\,\dot{\theta} - 1.4\,\dot{\alpha} + 122.1\,\alpha + 388.3\,V_m\,. \quad (51)$$

Since the system dynamics are described by two second-order differential equations, a natural approach is to use a cascade strategy for representing the plant as two single systems connected in series. However, it is not possible do make it directly from the expressions in Eq. (51), because the control input V_m acts on both. This difficulty can be circumvented by proposing the following change of variable:

$$\upsilon = \theta - 1.04\,\alpha\,. \quad (52)$$

Then, after some algebraic manipulations, we have that

$$\ddot{\theta} \;=\; -1.86\,\dot{\theta} + 78.3\,\theta + +0.89\,\dot{\upsilon} - 78.3\,\upsilon + 403.5\,V_m\,, \quad (53)$$

$$\ddot{\upsilon} \;=\; 43.72\,\upsilon - 0.5\,\dot{\upsilon} - 43.72\,\theta + 0.5\,\dot{\theta}\,. \quad (54)$$

Thus, by representing the Eqs (53) and (54) in the frequency domain, the linearized dynamical equation can be described by two second-order systems connected in series, as shown in Figure 5. Note that $H_2(s)$ is a nonminimum phase system.

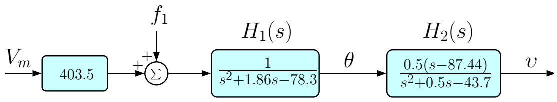

Figure 5. Cascade representation of the rotary inverted pendulum dynamics.

Therefore, the use of the MP-ADRC strategy for nonminimum phase systems, proposed in Section 5. is justified. Besides that, the cascade control strategy can now be applied. The idea of this strategy is to design a tracking control for $H_1(s)$ such that $\theta(t) \to \theta_r(t)$, where $\theta_r(t)$ is a reference signal which is a stabilizing control law for $H_2(s)$. Therefore, when $\theta(t) = \theta_r$, the closed loop system behaves as if θ_r acted on $H_2(s)$ directly. Figure 6 shows the implemented cascade structure. The used control parameters tuning values are shown in Table 1 and the considered initial conditionals are $[\theta(0),\ \alpha(0),\ \dot{\theta}(0),\ \dot{\alpha}(0)] = [0,\ 0,\ 0,\ 0]$. It was considered the sinusoidal voltage signal $d(t) = 0.2\cos(5t)V$ as input disturbance.

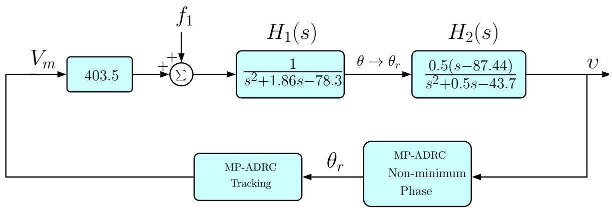

Figure 6. Cascade Control Strategy.

Table 1. Control parameters tuning values.

MP-ADRC								
	K_0	K_1	k_1	L_1	L_2	L_3	L_4	β
Tracking - $H_1(s)$	–	–	–	1760	1009200	2.16×10^8	–	2
nonminimum Phase - $H_2(s)$	1.14	0	0.1	16	78	112	49	0.15

Figure 7 shows the evolution of the pendulum angle α and the arm angle θ over time. Note that both angles converge to zero, which confirms the stabilization of the system and the robustness to input disturbance. The control signal V_m is shown in Figure 8.

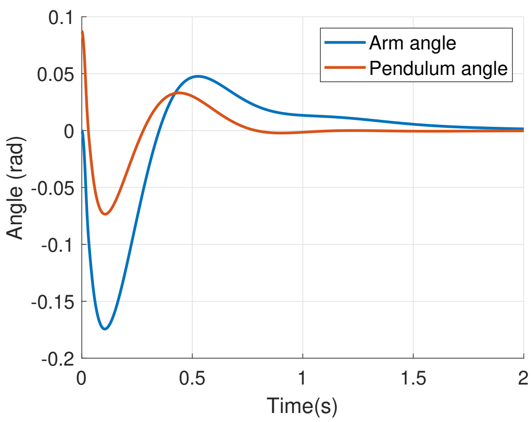

Figure 7. Arm and Pendulum angles.

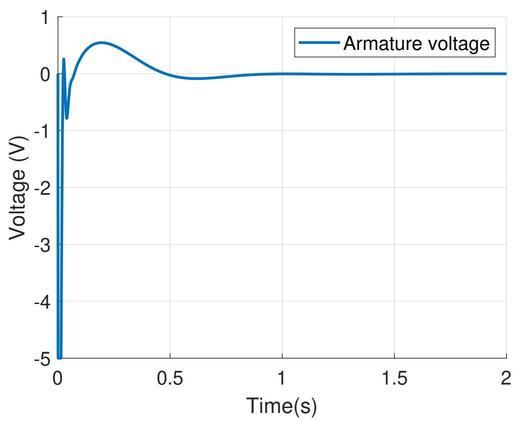

Figure 8. Motor armature voltage.

7. CONCLUDING REMARKS

This work addresses the control problem of a rotating inverted pendulum by using a robust ADRC control scheme. The strategy is based on a variant ADRC method called Modified-Plant ADRC (MP-ADRC) in which the plant appears modified in the view of the extended state observer (ESO). It is shown that by adopting such a modification, the controller synthesis becomes independent of the plant's control coefficient while giving the closed-loop system greater robustness to parametric uncertainties that may occur in such a constant. The presented strategy is applied in the inverted pendulum mechanism whose dynamics present some challenges since it is a nonlinear system, under-actuated, and whose linear part presents nonminimum phase characteristics. The pendulum's fourth-order system is represented as a cascade of two second-order systems in which two ADRC controllers are applied, one to solve the tracking problem, and the other to solve the stabilization problem of a nonminimum phase system. By adopting such a scheme, a resulting stable closed-loop configuration is then obtained. Numerical simulations are presented and discussed to illustrate and confirm the efficiency of the proposed method.

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9. RESPONSIBILITY NOTICE

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