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HEAT FLUX ESTIMATION IN THE TURNING PROCESS USING THE SEQUENTIAL METHOD WITH NON-CONSTANT FUTURE TIMES

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Abstract. *In machining processes, the measurement of heat flux at the contact zone between the cutting tool and the work-piece represents a significant challenge in experimental analysis. Utilizing mathematical methods, it becomes feasible to estimate the heat flux within this region by leveraging temperature data obtained from thermocouples positioned adjacent to the contact zone. The objective of this study is to solve, by Finite Elements Method, Sequential Procedure and analytical equations, the heat flux estimation problem on machining. Temperature data, measured by a referenced author, was used as basis for the inverse technique to find the unknown heat flux parameter. With this objective, the boundary conditions of the referenced experimental study were considered and analyzed through the COMSOL software programming for the computational modeling of the system. Through the Sequential Procedure iterations, using the MATLAB software, it was possible to estimate the heat flux caused by the friction in the cutting process. The temperature distribution was calculated using the estimated heat flux on software COMSOL and was compared this temperature result to the referenced temperature. A modified version of the Sequential Procedure was devised, incorporating a dynamic parameter for future time steps. Various initial values for this parameter were tested to evaluate their impact on the procedure. It was reassured with this analysis the relevant precision of the numerical method to estimate the heat flux in this process using the inverse technique.*

Keywords: *Turning, Heat Flux, Sequential Function, Contact Area, Numerical Simulation, Inverse Problem.*

1. INTRODUCTION

Machining processes are a fundamental part of contemporary industrial production, making it of considerable relevance to study the operating parameters used to optimize performance based on them. It was emphasized, in Figueiredo *et al.* (2022) study, that it is possible to achieve a more precise production and machining tool useful life by controlling its temperature and operating conditions. From the analysis and experimentation of parameters and boundary conditions of these production methods, it is possible to obtain possibilities of operation with lower cost and greater efficiency (da Silva *et al.*, 2021).

The turning method is one of the most used machining processes due to its low cost for mass production and high precision. One of the main limitations of turning is the determination of the heat flux on the contact surface between the cutting tool and the machined part, owing to the limited access to the unexposed contact region of the workpiece and the machining tool at the time of cutting.

It is possible to solve the problem of accurately determining the heat flux in regions of the shear zone between the machined part and the cutting tool with the approximation of the results in a comparison between the experimental results of temperature measured in the cutting tool and the analytical and numerical results of temperature determination in the same point. The numerically calculated temperature comes from the estimation of the heat flux generated in the contact zone.

Experimental methods have a limitation in measuring the temperature in the contact zone between the cutting tool and the workpiece due to factors such as the difficulty of reaching the unexposed contact region of the workpiece and the tool (Oliveira *et al.*, 2021). These temperature measurement methods have been widely explored by the industry in recent decades, such as radiation measurement using infrared sensors, that, however, have limitations regarding measurement interference due to the emissivity of cutting fluids, high equipment cost, and difficulty of emissivity calibration (Figueiredo

et al., 2022). Another measurement method is through thermocouples installed at strategic points of the cutting tool to measure the temperature in this point and estimate the thermal field generated in the system of shear zones. However, this second presents a limitation in measuring the highest and lowest temperatures, only an approximate temperature gradient, as confirmed by the study just mentioned.

Numerical methods for determining the thermal field in the turning process are part of an emerging field of study for optimization of parameters and operating boundary conditions due to the low implementation cost and considerable accuracy (Custódio, 2016). These methods are of fundamental importance for the industry, as they make it possible to predict multiphysics events with relative accuracy (da Silva *et al.*, 2021). Thus, it enables experimentation of parameters and materials, process optimization, and more due to the low execution cost, fast test execution, and parameter adaptation (Hu *et al.*, 2014). Numerical methods can compare and evaluate error factors between temperature results of experimental and computational methods, adapting the heat flux parameter to decrease the difference in temperature results.

Considering these limitations was developed a model of the cutting tool-chip system through analytical expressions, modeling the boundary conditions that govern the problem. These formulations were in the implicit mathematics of COMSOL software, which uses numerical methods, such as the Finite Element Method, to estimate the heat flux in the system of the part/tool contact region. However, the model has limitations due to the indefinite variation, or complex modeling, of parameters and boundary conditions that influence the determination of heat flow in thermal transfer processes.

This work aims to adopt a convergence in the heat flux parameter of the system through the analytical method for solving the problem of estimating the heat generated on the chip-tool contact surface through an Inverse Method known as the Sequential Function Specification Method (SFSM) or Sequential Procedure Farzan *et al.* (2017). This method uses experimental temperature-measured values that were collected from Kaminise *et al.* (2012) study for comparison and adaptation of the heat flux. The mathematical techniques present a precise and low-cost solution to estimate the heat flux on the contact area between the cutting tool and the piece.

2. MATERIALS AND METHODS

Machining is one of the most conventional methods of manufacturing, currently. However, the metal cutting process is still not completely understood by the state of the art of deformation and temperature fields in certain situations inherent to this process, as summarized by Brito *et al.* (2015). He complements by pointing out that the study of the influence of temperature on the machining process began many years ago and is still the focus of current research. The complexity of this study is mainly due to the difficulty in accurately measuring the thermal field in the contact region of the chip with the tool in machining processes.

The aforementioned study uses as a solution for characterizing the thermal field in this contact region, two numerical methods, in principle: the Direct Method and the Inverse Method, considering that the second can be used in problems that cannot be solved by the first. For the modeling of the problem in a computational-thermal system, the analytical definition of the boundary conditions acting in the real problem is necessary.

2.1 Thermal Model

The geometry was modeled using a real turning tool as basis, using SolidWorks software for building. It was imported to the COMSOL software, as demonstrates Fig. 1, to be analyzed the temperature distribution on it. For the geometry physical and chemical parameters, it was configured with WC (Tungsten and Carbon) with transient properties in COMSOL. This material was chosen to simulate the hard metal properties, with an initial heat flux inputted, which was later adapted.

The software COMSOL calculates the temperature distribution over the tool. The temperature found on the coordinate equivalent to the thermocouple position is loaded in the MATLAB algorithm and compared to the temperature measured in Kaminise *et al.* (2012) in that same moment.

To characterize the boundary conditions in the turning machining problem, it is necessary to consider three shear zones: primary, secondary and tertiary, with their own conditions in each. The generation of heat and the thermal field in these regions depend on parameters such as cutting speed, depth of cut, tool geometry characteristics, material properties of the part and tool, coating properties (if used), among others. The study of Santos *et al.* (2014) analyzes and compares the effects of these cutting parameters on the temperature in the cutting region, using the Inverse Method. FERREIRA (2017) compares the thermal effects of using ceramic as a coating for turning tools, through numerical studies with the COMSOL software. As this work shows, the insulating coating tends to increase the temperature in the contact zone of the part with the tool, and it is necessary to pay attention to the consequences of this phenomenon.

Carvalho *et al.* (2006) develops the Direct Method through the three-dimensional equation of transient heat diffusion in Eq. (1), where k is the thermal conductivity, ρ is the density, c_p is the heat capacity and T the temperature:

$$\frac{\delta}{\delta x}(k_i \frac{\delta T}{\delta x}) + \frac{\delta}{\delta y}(k_i \frac{\delta T}{\delta y}) + \frac{\delta}{\delta z}(k_i \frac{\delta T}{\delta z}) = (\rho c_p)_i \frac{\delta T}{\delta t} \qquad (1)$$
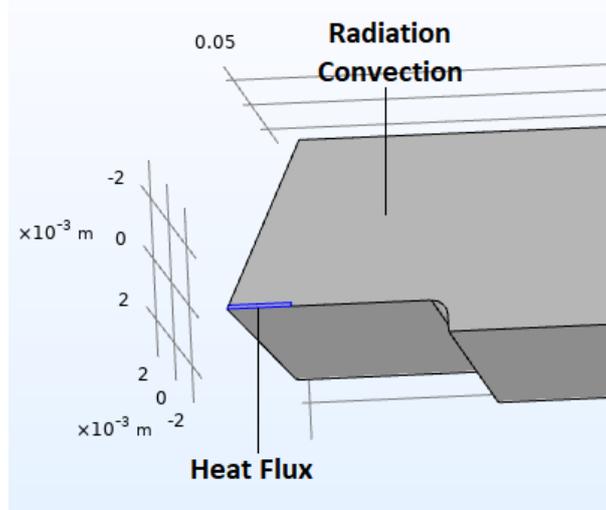
Figure 1: Geometry modeled using SolidWorks imported to the COMSOL software.

For surfaces exposed to the environment, Eq. (2) is applied, representing the heat loss by convection, where $\delta T/\delta\eta$ is the derivative along the outward drawn normal to the surface, $\mathbf{h}$ is the convection coefficient and $\mathbf{T}_\infty$ the medium temperature. The Equation (3) represents the radiation effect, where α is the thermal diffusivity coefficient and ϵ is the emissivity:

$$-k_i * \frac{\delta T}{\delta\eta} = h(T - T_\infty) \tag{2}$$

$$-k_i * \frac{\delta T}{\delta\eta}(x, y, z, t) = \sigma\varepsilon(T)(T^4 - T_\infty^4) \tag{3}$$

The research described in Figueiredo *et al.* (2022) analyzes the temperature distribution in the tool support, governed by the Eq. (4):

$$\nabla(k\nabla T) = \rho c_p \frac{\delta T}{\delta t} \tag{4}$$

Where $\mathbf{T} = \mathbf{T}(\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{t})$ is the temperature function, $\mathbf{k}$ is the thermal conductivity coefficient, ρ is the density, $\mathbf{C_p}$ is the specific heat, and $\mathbf{t}$ is the time.

2.2 Numerical solution

Several inverse numerical techniques can be coupled to thermal models to optimize the solution, such as Simulated Annealing, Golden Section and Sequential Specified Function (Santos *et al.*, 2014). The criteria that differentiate the methods are, mainly, the difficulty of implementing the inverse techniques, the computational time and the error rate of the comparison between the mathematical and the experimental result.

In Custódio (2016) the inverse technique of the Golden Section is used to estimate the heat flux of the problem. This method stands out among the others due to the ease of implementing inverse techniques and parameter adjustment, as shown by Santos *et al.* (2014). In addition, this study of 2016 author highlights other advantages of the technique, such as the possibility of solving functions with more than one variable and the known convergence rate. However, it also points out the high computational cost of the process.

Therefore, one of the most used inverse techniques is the Sequential Specified Function (Evensen, 1994), due to the relation of computational time and favorable average residue in many cases. In addition, compared to the others, it is an easy to implement technique of the Inverse Method and easy to adjust parameters, facilitating investigative purposes through its use.

For this method is defined a "future time steps (r)" concept, that analyzes the temperature distribution in future moments to improve the precision of the heat flux parameter convergence. In each iteration, the momentary and future times are analyzed, bringing, each one of these, an adaptation for the heat flux parameter. This adaptation is calculated by the product of the difference between the reference temperature and the calculated temperature by a sensibility coefficient (ϕ). This coefficient is determined by the calculation of the temperature, using a multiphysical software, in the analyzed point, using a unitary heat flux.

The inverse method used refers to a procedure that seeks to determine the causes or unknown parameters of a system from observed or measured information. In order to obtain this observed or measured information, it was necessary to carry out a laboratory experiment with the objective of collecting temperature data from certain points of the tool to estimate the heat flux in the region of contact between the part and the cutting tool. For this, thermocouples were used to measure the temperature at points close to the contact zone, due to the impossibility of coupling thermocouples in areas of high contact influence. Therefore, an unknown heat flux was generated in the contact zone between the workpiece and the cutting tool, due to frictional forces, and the temperature close to the contact zone was collected. The inverse method was used to estimate this unknown heat flux, from numerical methods that use temperature data obtained through thermocouples Farzan *et al.* (2017).

This temperature obtained by thermocouples in the referenced study of Kaminise *et al.* (2012) is represented by Y in the Eq. (5). The parameter T is the Temperature obtained by the algorithm, M is the current moment, r the future time step, i is the step of r iteration ($1 <= i < r$) and ϕ the sensibility coefficient.

$$q_m = \frac{\sum\limits_{i=1}^{r} (Y_{M+i-1} - T_{M+i-1})\phi_i}{\sum\limits_{i=1}^{r} \phi_i^2} \tag{5}$$

This equation is solved for each iteration of the algorithm for current moment and future times steps, being the heat flux adapted in each one of these steps, before the algorithm skips to the next moment M Usually, using a higher value of time steps r improves the precision of the numerical method, however increasing also the limitation of the method in the lasts moments of the software iterations.

Initially, a bibliographic review was carried out, building the theoretical basis for the correct elaboration of the methodology of this work. The specifications of the turning process were revised, regarding the parameters and boundary conditions. It was necessary to define the boundary conditions that would be necessary for numerical reproduction of a specific event analyzed in the laboratory.

The choice of the Inverse Method used was careful, according to the specifications of the process, taking into account the level of complexity of each one, the computational time required for each study and the acceptable residual error for this analysis. For the correct adaptation of the heat flux parameter, the Inverse Method uses the sensibility coefficient.

By the product of the difference in the temperature and the sensibility coefficient correspondent to that coordinate and that moment, the algorithm adapts the heat flux to the next iteration. If the temperature retrieved from the software is higher than the one collected from the thermocouple, the heat flux is adjusted upwards. The same thing happens if, in the comparison, the temperature in the software is lower than that obtained by the thermocouple, correcting the downward heat flow. In order to improve the accuracy of this adjustment in the heat flux, the concept of "future times (r)" was used.

The utilization of high values for the future time steps parameter in the Sequential Function Specification Method does have limitations. When 'r' is set to a high value, it can lead to increased computational complexity and resource consumption. This is primarily because a larger 'r' entails more iterations and calculations to estimate the heat flux. Another constraint arises in the final phases of the analysis when there are insufficient future time steps available for examination. In such cases, the iterations must conclude at the point where M(i), representing the moment in iteration i, becomes equal to M(N) - r, with N denoting the total number of steps. To solve this problem, an adaptation of the SFSM was developed, as illustrated in Fig. 2.
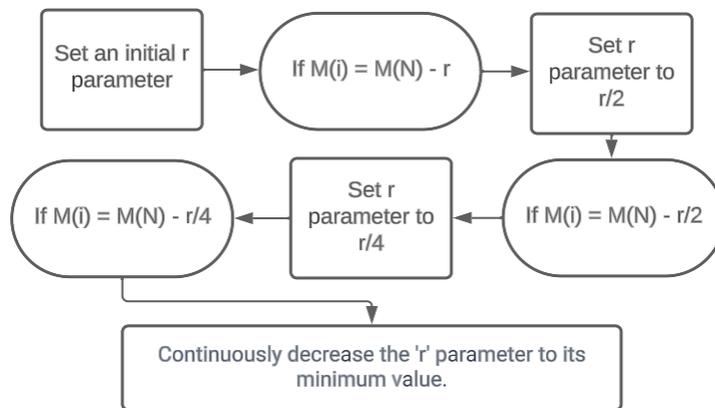

Figure 2: Adaptation scheme of future time steps in the SFSM

Finally, the results obtained were analyzed, tracing the profile and specifications of the studied case. The temperature of numerical experiment was compared with the temperature obtained by thermocouple in the compared study.

2.3 Variables and indicators

In this study, was computationally simulated and estimated heat flux for an experiment previously tested in the laboratory (as described in Kaminise et al., 2012). The procedure began with comprehending the relevant physical phenomena, mathematical methods, and utilizing COMSOL and MATLAB resources. It was necessary a clear understanding of the original system's physical model and heat transfer equations to parametrize the software.

Input data into the software are the ambient temperature, the initial temperature, geometry of the cutting tool, material properties (thermal conductivity, density and specific heat), heat transfer by convection coefficient, mesh structure, total time and frequency of study iterations. To determine the contact area between the cutting tool and the part in the software, a curve was created from the interpolation of points in the coordinates of the contact region boundary, collected in the laboratory, through a microscope. In addition to all these parameters, for the execution of the Finite Element Method by COMSOL, an initial heat flux is required as input data.

An initial heat flux derived from prior algorithm tests is established at a value of $5.6x10^7$ and is distributed across the tool's contact area. This area's dimensions are determined by the cutting parameters of feed rate ($f = 0.121mm/turn$) and depth ($a_p = mm$), which were employed in the process examined in the study by Kaminise et al. (2012). The area is calculated as the product of both parameters ($A = f * a_p$).

Among the various existing inverse methods, the SFSM was chosen due to the availability of information about the method programming, the lower amount of computational resources required and reasonable complexity (Santos *et al.*, 2014). This iterative method was chosen, which searches for a function that best fits the data by minimizing a measure of discrepancy between the observed values and the values estimated by the function.

Result convergence involved comparing the thermal field generated by the Finite Element Method in COMSOL with thermocouple-measured temperatures from a practical experiment used as a reference. Achieving convergence required calculating the sensitivity coefficient for the Sequential Specified Function. This coefficient quantified how changes in heat flux during iterations affected the temperature at specific experiment points, ultimately aligning numerical temperatures with thermocouple measurements.

2.4 Referenced experiments and results

In Kaminise *et al.* (2012) study, was developed a machining laboratory experiment with the goal of collecting temperature data from certain points of the tool. An unknown heat flux was generated in the contact zone between the workpiece and the cutting tool, due to frictional forces, and the temperature in coordinates close to the contact zone was collected using Thermocouples, because of the impossibility to couple this equipment in the contact region. It was chosen the collected data from thermocouple 3 (TC3), on stainless steel support-tool experiment, as reference for the inverse calculation. With this temperature data, shown on Fig. 3a, it was possible to estimate the heat flux in the region of contact between the part and the cutting tool, a parameter that the experimental study used as reference could not measure.
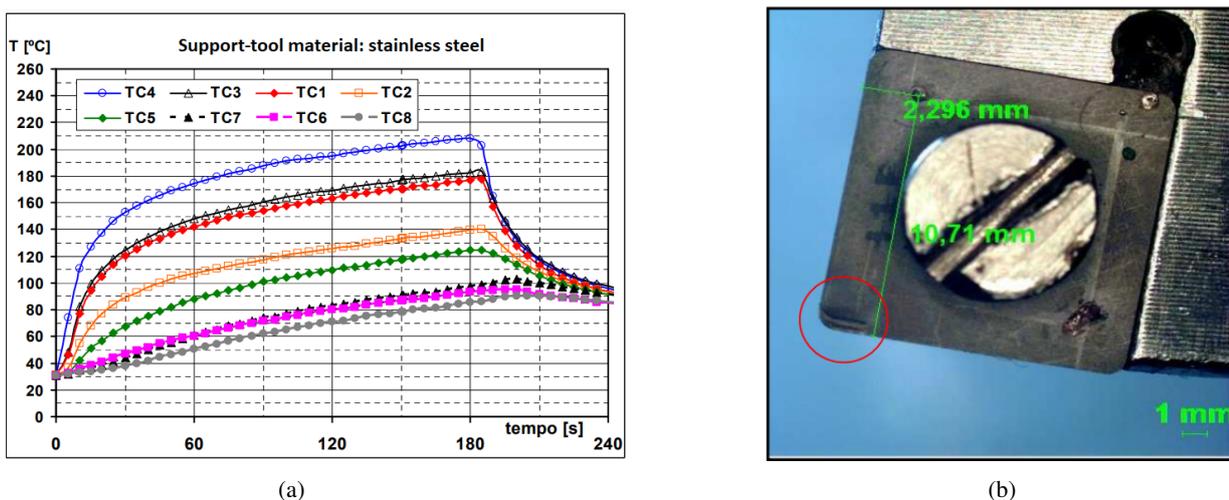

Figure 3: (a) Temperature measured by thermocouple on stainless steel support-tool experiment and (b) position of TC3 , collected and adapted from (Kaminise *et al.*, 2012) study

The thermocouple 3 was installed in the position of the cutting tool specified on Fig. 3b, marked by the meeting of the green lines. This same distance of the cutting region shown on Fig. 3b was used in software COMSOL to extract temperature information and the sensibility coefficients, for the comparison (experimental and numerical temperature results) at the same point, performed by Sequential Procedure.

2.5 COMSOL software modeling

To simulate the temperature result from a given heat flow applied at a given point, the COMSOL software was programmed. As shown in Fig. 4, a geometry was imported (imp 1), representing the cutting tool in the turning process and an area determined by a polygon was considered as the contact region of the part with the tool. In this simulation, only the cutting tool and the heat flux generated from the contact are considered, so the part being machined is not necessary for the calculations.
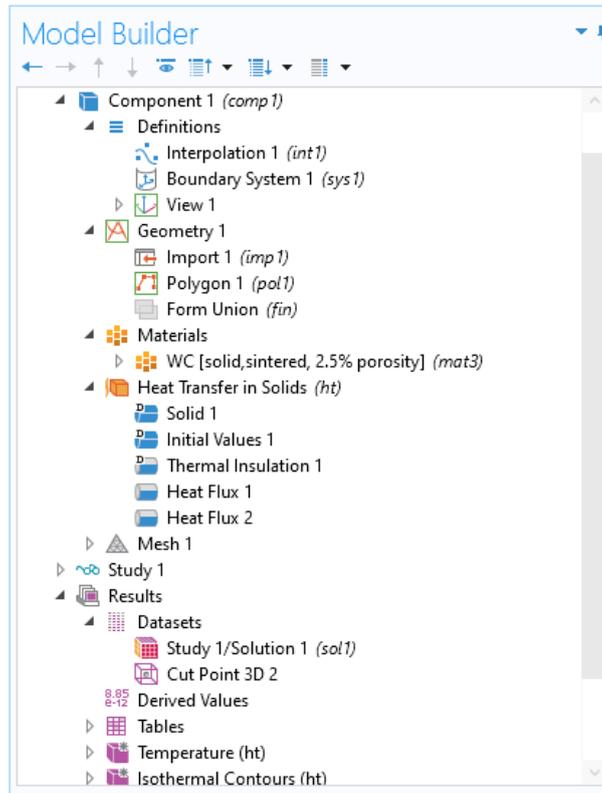

Figure 4: COMSOL software programming to generate the temperature as a result of the simulation

The study of heat transfer in solids was selected and in its scope, two heat flows are programmed, one being the heat input into the system, applied under the area created from the polygon, representing the heat flow generated in the region of contact between the workpiece and the cutting tool, and the other the loss of heat to the environment by convection. The study is set up to have 1 second iterations over 60 seconds.

In "Datasets", inside "Model Builder" in Fig. 4, a 3D Cut Point was created, to recover the temperature generated in a selected position, which in this case is the position in which the thermocouple was positioned.

The temperature resulted from COMSOL simulation was exported to a local destination and loaded into the MATLAB script for comparison with the temperature measured by the thermocouple in the laboratory test. The temperature difference resulting from the two processes is considered to recalculate the heat flux at a given time by the algorithm. This difference is multiplied by a sensitivity matrix, which was also generated using the COMSOL software.

To generate the sensitivity matrix, a unitary-fractional heat flow was considered, which varies from 0 to 1. This flow was calculated by creating an "Analytic" in "Definitions" and formulating the heat flow in its expression (in this case, inserted from the interpolation "int 1") divided by the maximum value reached by the heat flux (if it is in a transient regime, as the flux in this study).

In addition, the initial temperatures within the ht field and the external temperature considered in the convection flow were reset to zero (in Celsius degrees).

3. RESULTS

The coordinates of TC3, where the temperature was collected as reference for this study, are described on Fig. 3b. In this same point was extracted the temperature information of the software COMSOL and used in the inverse method, that refers to a procedure that seeks to determine the causes or unknown parameters of a system from observed or measured information. The heat flux was estimated by the Sequential Function Specification Method using 8 future time steps, and plotted in the graphic shown in the Fig. 5a, representing an approximation of the heat flux generated in the experimental

procedure developed on Kaminise *et al.* (2012) work. This heat flux was calculated with a sequence of converging adaptations in its value, caused by each iteration of the algorithm.

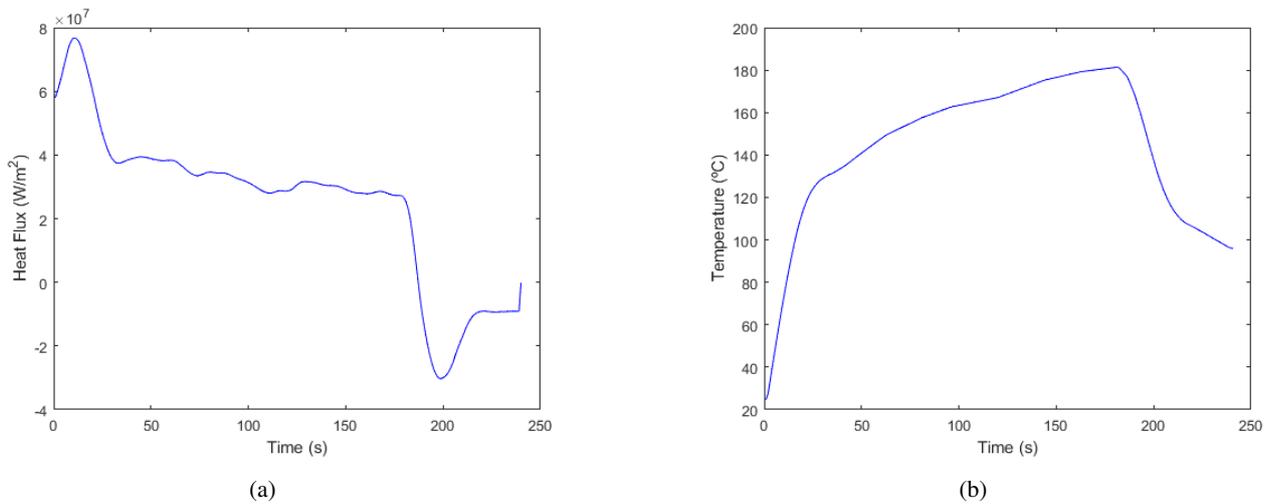(a)   (b)

Figure 5: Heat Flux calculated by the MATLAB algorithm (a) and Temperature obtained using TC3 coordinates on COMSOL (b)

This heat flux was, once again, imported to COMSOL to generate the temperature field. The temperature generated by the estimated heat flux in the coordinates correspondent to the thermocouple 3 (used in referenced experimental procedure), was collected and plotted, as illustrated by Fig. 5b.

3.1 Numerical results and accuracy

This iterative approach aims to find an optimal function that closely aligns with the data by reducing the disparity between the observed values and the function's predicted values. For the formulation of the method, the temperature results calculated from the COMSOL software are used. The temperature calculated by the software at the selected point, with the same coordinates as a thermocouple used in the laboratory, is compared to the temperature collected by the thermocouple. If the temperature retrieved from the software is higher than the one collected from the thermocouple, the heat flux is adjusted upwards. The same thing happens if, in the comparison, the temperature in the software is lower than that obtained by the thermocouple, correcting the downward heat flow.

The comparison graph of Fig. 6a was obtained by plotting the temperature used as reference, measured by Kaminise *et al.* (2012) in the thermocouple 3, and the temperature calculated using the heat flux estimated by the Sequential Function Specified algorithm, considering 8 future steps **r**. The residual error generated by the percentage of the difference between both mentioned temperatures is shown on Fig. 6b.

(a)   (b)
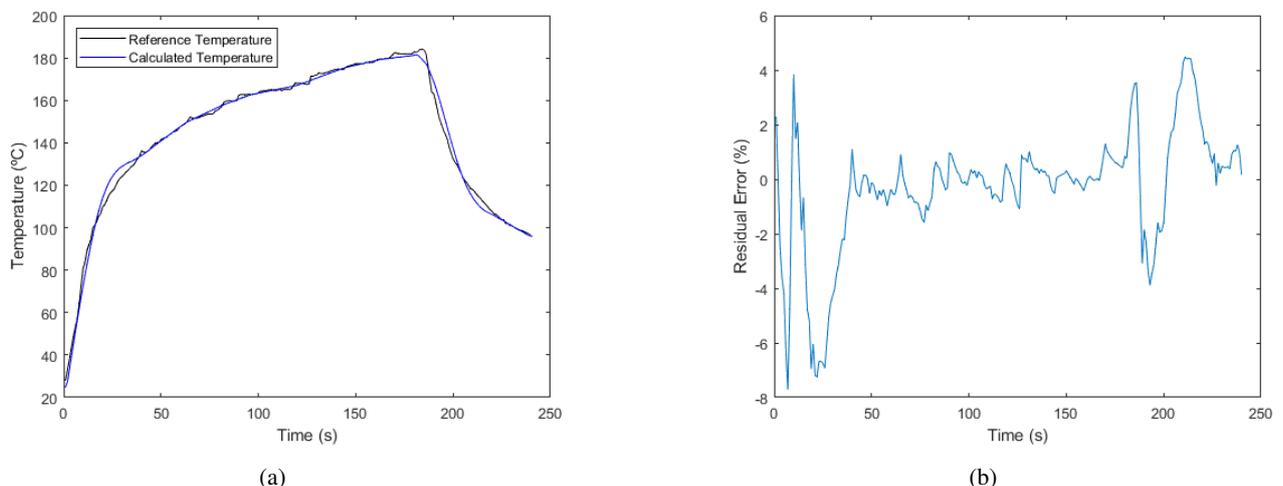
Figure 6: Comparison graph between the temperature calculated and the temperature of the article used as a reference (a) and the residual error (b)

Prior to commencing the analysis using the Sequential Procedure, an initial heat flux value of $5.6 * 10^7$ was established. This step is crucial in ensuring the accuracy of the subsequent analysis, as the initial heat flux significantly influences the error in the initial stages of the process. In machining operations, the initial moments of work typically experience the highest heat flux peaks, primarily due to static friction forces. Since the Sequential Procedure faces challenges when adapting the heat flux parameter when it deviates significantly from the reference, it becomes imperative to set the initial heat flux value as close as possible to the laboratory experiment's actual conditions. Achieving this requires preliminary testing using the Sequential Procedure to identify an initial heat flux value that closely approximates the real-world conditions.

In order to improve the accuracy of this adjustment in the heat flow, the concept of "future times (r)" was used. Using this tool, trends of non-compliance with the result or oscillation were considered in comparison between the temperatures resulting from simulations (in COMSOL) programmed with the heat flow generated by the algorithm and the temperatures measured in the thermocouple.

The analysis of the behavior of the heat flux estimated and temperature calculated leads to emphasize the precision and the limitation of the method used. The residual error tends to be more expressive when the material is suffering bigger changes of temperature, depending in this point of the future time steps parameter r.

3.2 Future Time Steps Study

The influence of future time steps in this computational estimation method is an important aspect to consider when modeling and simulating dynamic systems. In such simulations, the state of the system at any given time step depends not only on its current state but also on the previous state and potentially the future state. To prove that incorporating a high number of future time steps can improve predictive accuracy in some cases, some adaptations of the algorithm were tested, firstly increasing the future time steps parameter. An important limitation of this increase is that the same amount of time steps used to foresee the future results must be reduced in the final moments of the analysis, because of the impossibility of analyzing future time steps after the end of the analysis.

This limitation was worked with other adaptation of the Sequential Specified Function algorithm, elaborated by the authors of this study. This adaptation consists of a chain of conditioning structures that reduces the future time steps used in certain moments when it becomes closer to the end of the analysis. As an example, if you configure the parameter to use 20 future time steps of analysis (r=20) when it lasts 20 steps to finish the Sequential Procedure, the algorithm adapts the parameter to use 10 future time steps (r=10), until it becomes the 10 lasts steps of the Procedure, when this parameter is reduced again, becoming possible to simulate the final moments using future time steps resource, even being reduced.

Considering these adaptations of the algorithm, different parameters of future time steps were tested to analyze the influence of the increase of this parameter in the error measured between the temperature calculated using the heat flux estimated by the Sequential Specified Function and the temperature measured in Kaminise *et al.* (2012) study.

Analyzing the impact of increasing the parameter for future time steps, it becomes evident that the Sequential Procedure exhibits improved accuracy. This enhancement is attributed to the greater potential for a finer adjustment of heat flux, allowing for a more comprehensive examination of future moments. This phenomenon is visually demonstrated in Fig. 7, which illustrates a reduction in errors as higher values for the future time steps parameter are employed.
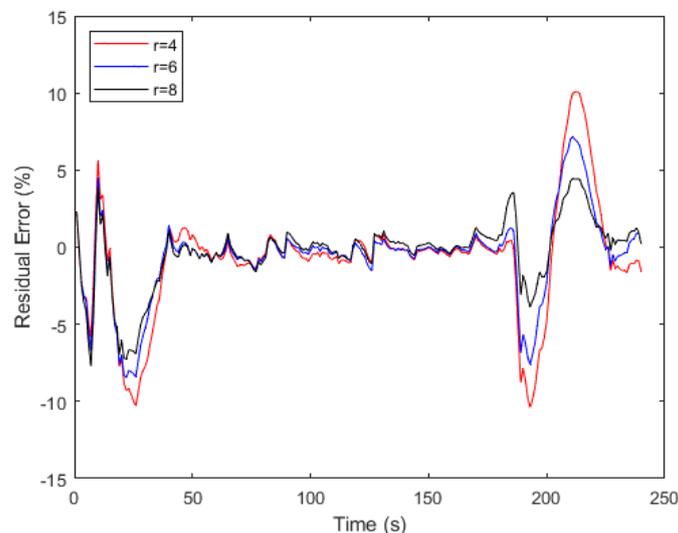

Figure 7: Comparison graph of error between the referenced temperature and the calculated by different future time steps (r) values.

In Figure 7, it is feasible to examine the error variation while systematically altering the value of 'r' within the Sequential Procedure. By doing so, we can gain insights into how the adjustment of the heat flux becomes increasingly refined over subsequent time steps. This refinement takes place as the algorithm iteratively compiles a series of 'r' iterations for adapting the heat flux.

The parameters were tested with an initial heat flux set at $5.6x10^7$. This choice was made because, during the initial moments of a turning process, the heat flux tends to rapidly increase to its peak values. However, the Sequential Procedure requires some time to adapt the heat flux to the desired value, especially when the initial heat flux is set to 0. Therefore, initiating with an initial value that closely approximates the real heat flux value is crucial to minimize errors in the process.

4. CONCLUSION

Similar to the use of future times, several software programming parameters that can influence the accuracy of the generated results were observed, such as the number of iterations, the mesh configuration, regarding the number of elements and admission tolerance. These parameters can be optimized from tests, so it is not necessarily necessary to increase refinement or the number of iterations to improve the accuracy of the results.

At the same time, high accuracy of the method was observed after the correct configuration of the programming parameters of COMSOL and MATLAB, with an error lower than 10% of the solution generated by the algorithm being calculated, in the form of heat flux, in comparison with the heat flux used as a reference base.

The persistence of a small percentage of errors can still be justified by a limitation of the method used. The Inverse Methods, including the Sequential Specified Function, have some limitations, such as the loss of precision in the final moments of the analysis, due to the impossibility of calculating the correct amount of future times from the final moments of the analysis.

In addition, the method works from discrepancies between results. Therefore, it is necessary to first detect the temperature difference in the results, and then convert the solution to the desired one.

Finally, considering the low error rate of the method, it can be seen that the numerical function presents a viable solution for the analysis of the phenomenon of turning, since it can achieve relative accuracy with low use of computational resources, drastically reducing the cost of resources such as equipment, workers and time spent on physicists.

The method, however, can be improved, considering more specific details of the turning process, such as a greater specification of the contact area between the chip and the tool. In addition, greater accuracy can be obtained by performing a complete mesh refinement study and analysis of iteration configuration parameters and admitted tolerance in the COMSOL software.

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