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ANALYSIS OF A DAMAGED BEAM WITH VARIATION OF CRACK GEOMETRIC PARAMETERS

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Abstract. *Structures are subject to degenerative effects throughout their working cycle, or they can cause external failures, such as cracks, which can lead to catastrophic failures or collapse of the structure after a considerable period. For this reason, the dynamic behavior of a structural element is influenced by cracks, discontinuities, or imperfections, which modify the stiffness and damping quality of the structure. As a result, the structure's natural frequencies and modal shapes can convey important information about the location and proportions of the damage. In the present work, a mild steel cantilever beam is considered, in which modal analysis was used to study the natural frequency of the first five transverse vibration modes of a cantilever beam with a transverse crack. Several scenarios were evaluated for the modal analysis, considering the same base beam, that is, the same length and same rectangular cross-section but with different crack configurations: located in various positions along the length of the cantilevered beam, with different depths and openings. To obtain numerical results, the Finite Element Method was used with hexahedral and tetrahedral finite element meshes, and the simulations were carried out with the aid of commercial software. The results of the natural frequencies of the healthy beam are compared with those obtained theoretically in the literature. The numerical natural frequency values of the healthy beam are then validated to ensure the accuracy of simulation parameters such as a number of elements and local refinements. As a result, the presence of cracks reduces the natural frequency, and the magnitude of this reduction varies depending on the position of the crack. Another observation is that the natural frequency changes as the crack moves from the fixed end to the free end of the cantilever beam. Discussions and consideration of the influence of each of the parameters separately are detailed.*

Keywords: *Modal Analysis, Computational Mechanics, Cracked Beam, Failure, Structure Dynamics*.

1. INTRODUCTION

Several failures of structures such as buildings, bridges, turbines, and aircraft wings have been associated with the phenomenon of resonance (RAO, 2011). This phenomenon is the result of the superposition of the frequency of the external force with some of the natural frequencies of the system causing the amplitudes of the oscillations to increase dangerously (RAO, 2011). Thus, in the design of mechanical elements or structures subject to dynamic/cyclic loads, it is important to protect them from operation within resonance zones.

Otherwise, there are factors that can modify the natural frequency of an element, leading it to operate within a resonance zone, as is the case of the presence of geometric discontinuities of the crack type in the element (MIA; ISLAM AND GHOSH, 2017).

According to Ahiwale (2022), all structures undergo degradation during operation. A crack-like discontinuity nucleates and propagates from regions subject to cyclic plastic deformation (BUDYNAS; NISBETT, 2011). Thus, it is essential to monitor the structural integrity of the elements that operate under dynamic loads, this monitoring also serves for the implementation of effective maintenance programs, based on the actual condition of the equipment and not only on its time of use (GOPALAKRISHNAN, RUZZENE AND HANAGUD 2011).

Many studies have been carried out to develop means for detecting, locating, and measuring the percentage of structural damage in a simple, fast, efficient, and inexpensive way because the presence of damage in components compromises their functioning, which can give rise to events that endanger the safety of people and property (DOYLE, 1997; KRAWCZUK et al., 2006).

With the focus on this development, many studies have been carried out to analyze the dynamic behavior of structures and create methods of monitoring the structural integrity of mechanical elements. These works dealt with both basic elements such as beams, bars, shafts, and plates (DOYLE, 1997; KRAWCZUK et al., 2006), as well as more advanced elements, such as: composite structures and intelligent materials (PARK AND LEE, 2012; PARK AND LEE, 2015; LEE, 2004; GOPALAKRISHNAN ET AL., 2005).

The results of these investigations showed that by knowing the dynamic response of an element, one can identify the changes when inserting one, which creates possibilities for characterizing the behavior of a damaged element (KRAWCZUK et al., 2003). This type of knowledge facilitates the creation of faster and more accurate diagnostic methods (Krawczuk and Ostachowicz, 1996; Klikowicz et al., 2016), reducing the likelihood of abrupt failures occurring.

Many of the methods used to identify structural damage make use of the natural frequencies and modes of vibration of the component. In this type of analysis, the damage is indicated by the variation of the natural frequencies of the structure since the presence of the damage causes a change in its mechanical and/or physical properties (CURY; BORGES; BARBOSA, 2011).

The determination of the natural frequencies and vibration modes associated with the structure occurs through the Modal Analysis (RAO, 2011). This technique is quite widespread, appearing in experimental, numerical, and analytical studies (usually only for basic elements such as beams, bars, shafts, and plates).

As the presence of a discontinuity in the structure introduces complexity in its mathematical model, it is common to use numerical methods, modeling, and computer simulation. In this scenario, the Finite Element Method (FEM) is one of the most widely applied methods, being powerful and popular.

The FEM excels in engineering applications and applied sciences (KIM and SANKAR, 2011; FISH; BELYTSCHKO, 2009). Its matrix formulation favored its implementation in several commercial programs such as: ANSYS, CONSOL, ABAQUS, NASTRAN, ADINA, and MIDAS NFX among others.

In this study, we analyzed the natural frequencies and vibration modes of a beam in healthy and cracked conditions. For the numerical solution, the FEM was used, where a cracked beam model was implemented and simulated in the ANSYS© 2023 R1 software. The models were constructed by the variation of three parameters, namely: the opening, the location, and the depth of the crack. The effect of the modification of the studied parameters on the dynamic response of the beam was studied.

2. METODOLOGY

2.1 Presentation of the models

In this work a beam with the following dimensions was analyzed: length (*L*) of 1000 mm, width (*b*) equal to 25 mm, height (*h*) equal to 50 mm with variable depth (*p*), opening (*a*) and location (*x*) The mechanical properties of the beam are presented in Table 1. The boundary conditions are of the type: one end is fixed and the other is free. And the beam was modeled and analyzed under the following conditions: a healthy beam resolved analytically and numerically and a damaged beam (Figure 1) that was solved numerically via FEM.

Table 1. Mechanical properties of the material.

Properties	Value
Material	Light steel
Young Modulus (GPa)	210
Density (kg/m^3)	7.850
Poisson ratio	0,3

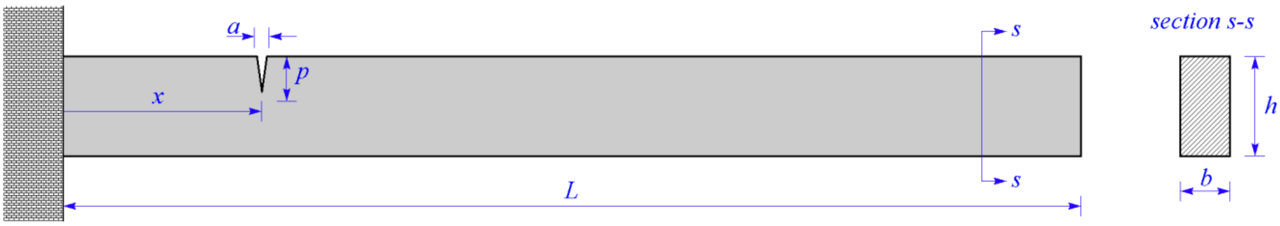

Figure 1. Cantilever beam with open edge crack.

2.2 Modelagem computacional

For analysis via FEM, the ANSYS Academic Research software, version 2023 R1, was used. The analysis followed the steps shown in Figure 2.

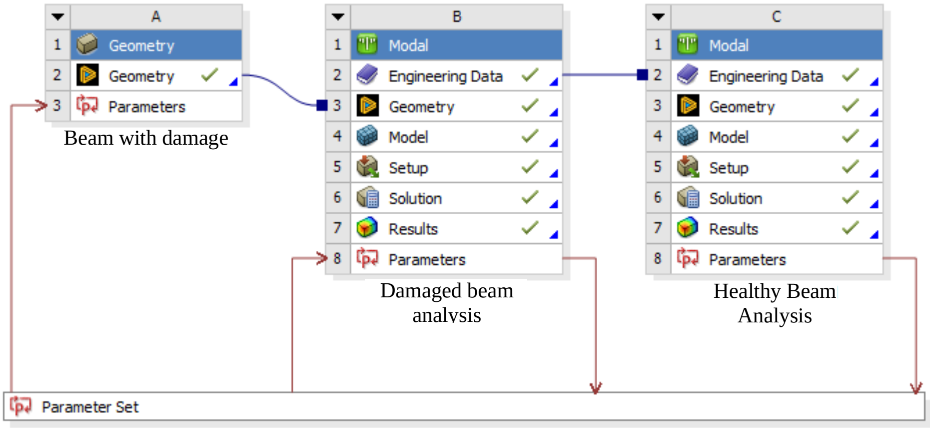

Figure 2. Free-set beam with open edge crack.

The models were designed using Ansys SpaceClaim and were discretized as a hexahedral Solid186 element. Solid186 is a high-order element that has 20 nodes, and each node has 3 degrees of freedom (GDL): translations in the nodal directions x, y, and z. This element was chosen due to its good characteristic for application in irregular meshes which is required in the vicinity of the defect.

As the crack introduces a distortion in the geometry, the modeling was performed differently for each condition of the beam as follows:

a) Modeling and simulation of the healthy beam:
- Global mesh was built with elements with a maximum size of 10 mm (Figure 3).
- Support condition for a cantilevered beam was applied.
- Simulation was performed to obtain the first 5 natural frequencies and associated modes of vibration.

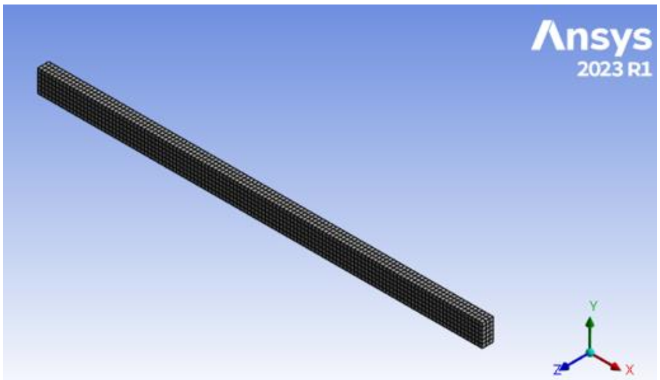

Figure 3. Discretization of the healthy beam.

b) Modeling and simulation of the damaged beam:
- An edge crack was introduced in the healthy beam and the following variables associated with the crack were parameterized: opening, depth, and location.
- The global mesh was made with elements with a maximum size of 10 mm and a local mesh was constructed.
- Figure 4a) in the crack region, with the following characteristics.
  - In the transverse edges of the crack.
  - Figure 4b) 150 divisions were made.
  - In the lateral edges of the crack.
  - Figure 4b) 50 divisions were made.
- Support condition for a cantilevered beam was applied.
- 150 models were simulated. Each model resulted from the unique combination of the values assumed by the five aperture values, six depth values, and five crack location values (Table 2).
- First 5 natural frequencies and modes of vibration associated with each simulated model were calculated.

Table 2. Parameterization of the crack.

Opening (mm)	Depth (mm)	Location (mm)
0,1	0,5	100
0,5	1	300
1	5	500
1,5	10	700
2	15	900

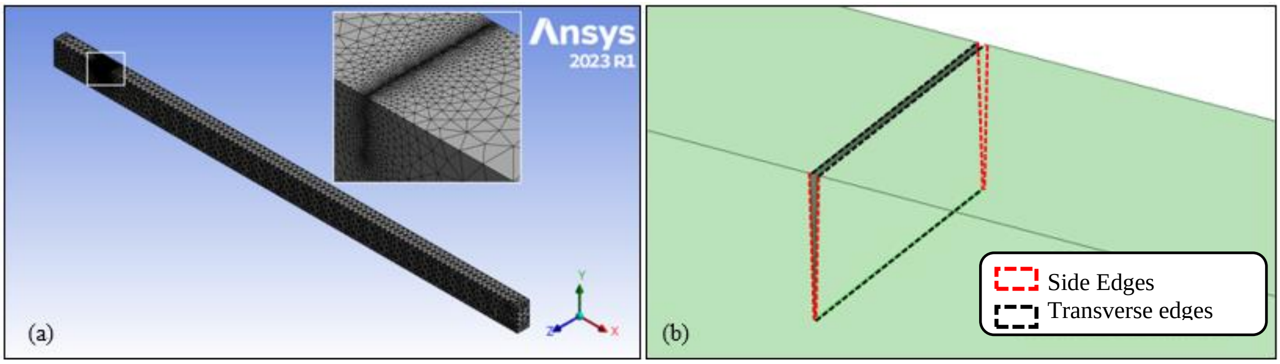

Figure 4. (a) Discretization of the damaged beam, (b) Division of the edges by number of elements.

3. NUMERICAL RESULTS

3.1 Validation

For the validation of the computational modeling, the results calculated analytically by means of the Eq. (1) were compared with those obtained through the application of the FEM for the first five natural frequencies associated with the transverse vibration modes of the healthy beam.

$$\omega_n = \frac{\lambda^2}{L^2}\sqrt{\frac{EI}{\rho A}}. \tag{1}$$

Where E, I e ρ are Young's module of the material (Pa), The moment of inertia (m^4) and the specific mass of the material (kg/m^3), respectively.

Based on the verification of the relative error between the theoretical and numerical natural frequencies (Table 3), it was verified that the configurations used for the computational modeling are suitable for the simulation since the relative errors of the approximations are less than 5%. In this way, the modeling settings and configurations will be replicated for analysis of the damaged beam.

Table 3. The first five natural frequencies for the healthy beam.

Mode	Natural Frequency (Hz)		Error%
	Theoretical	Numerical	
1º	20,875	20,424	2,16%
2º	41,749	40,755	2,38%
3º	130,816	127,624	2,44%
4º	261,632	252,491	3,49%
5º	366,296	355,733	2,88%

In addition, Figure 5 shows the first five vibration modes associated with the structure. Where, it turns out that modes 1, 3 and 5 are bending modes in the x-z plane while modes 2 and 4 are bending modes in the x-y plane which agrees with the theory.

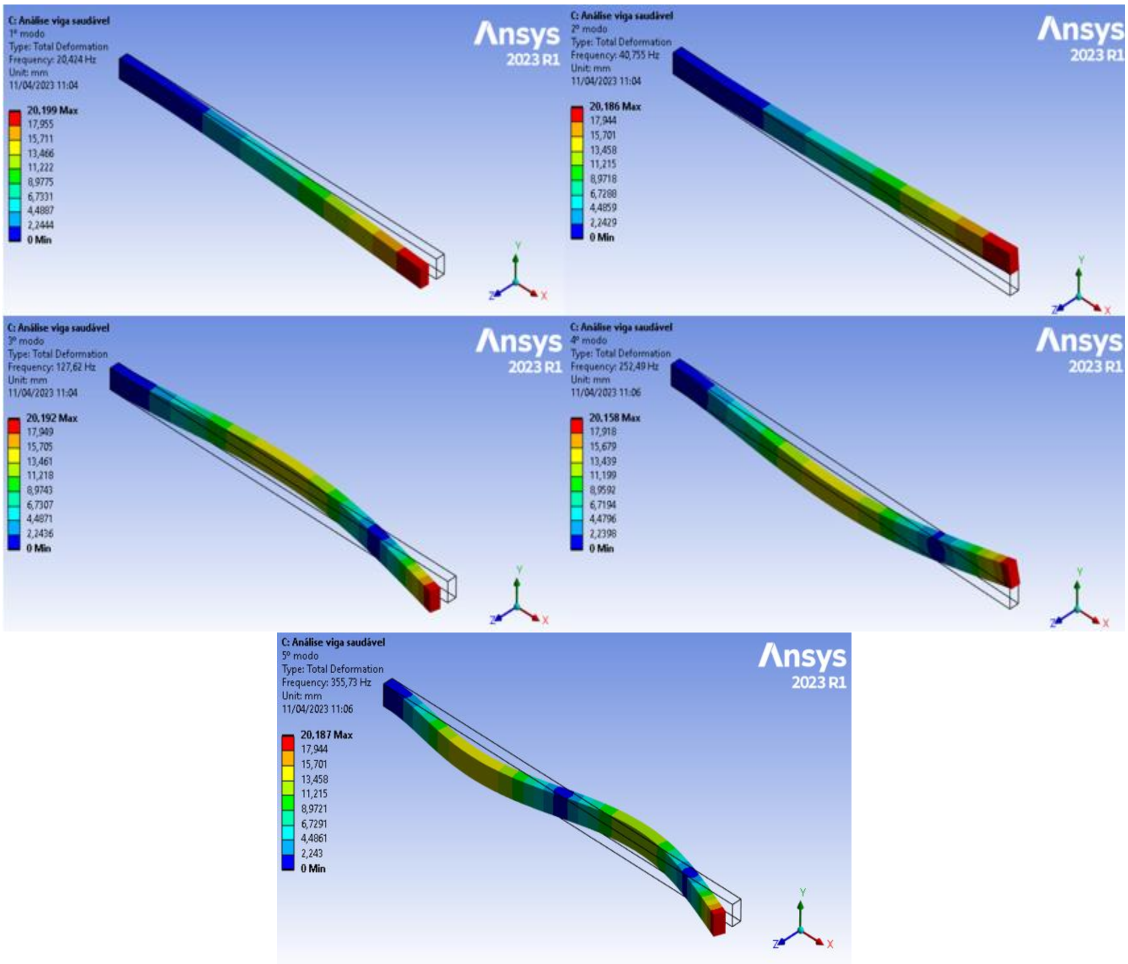

Figure 5. Vibration modes 1, 2, 3, 4 and 5 for healthy beam.

3.2 Parametric analysis for the damaged beam

In the following items, some simulation results for the damaged beam are presented and discussed. 150 models were simulated, but only part of these simulations is presented.

3.2.1 Crack opening size effect

In the graphs presented in Figure 6 and Figure 7, the behavior of the natural frequencies is shown with the variation of the crack opening for the depths fixed at 0.5 mm, 5 mm, 15 mm, and 20 mm. Each line represents a fixed position so that the level of distance between the lines demonstrates the influence of location on the natural frequency.

Observing Figure 6 it is noted that a crack with a depth of 0.5 mm in the aperture does not have a significant effect on the natural frequencies, it is verified that the variation of the frequency as a function of the opening occurs only after the second decimal place. Likewise, it is noticed that the curves remain very close, portraying the low influence of location on the modification of modal properties for this model.

In Figure 6 a, when the depth of the crack is varied to 5 mm the change in frequency as a function of the opening is still minimal, the curves are practically constant, but it is noted a certain distance from them, this demonstrates that as the depth increases, the position influences more in the dynamic response.

For modes 1 and 2 the cracks located in the vicinity of the set end of the beam have a greater influence on the dynamic response. For modes 3 and 4 the greatest decrease in frequency occurs for cracks located near the middle of the beam. For mode 5 the effect is most noticeable for cracks at 1/4 L and 3/4L. These locations correspond to the regions of greatest mechanical demand of the beam, as shown in the shapes of the vibration modes (Figure 5).

As the depth is increased to 15 mm and 20 mm (Figure 7), it is observed that the lines remain practically constant with the variation of the aperture. However, a greater influence of the position on the natural frequencies is noted since the level of distance between the curves of the frequencies for the healthy beam and damaged beam increased.

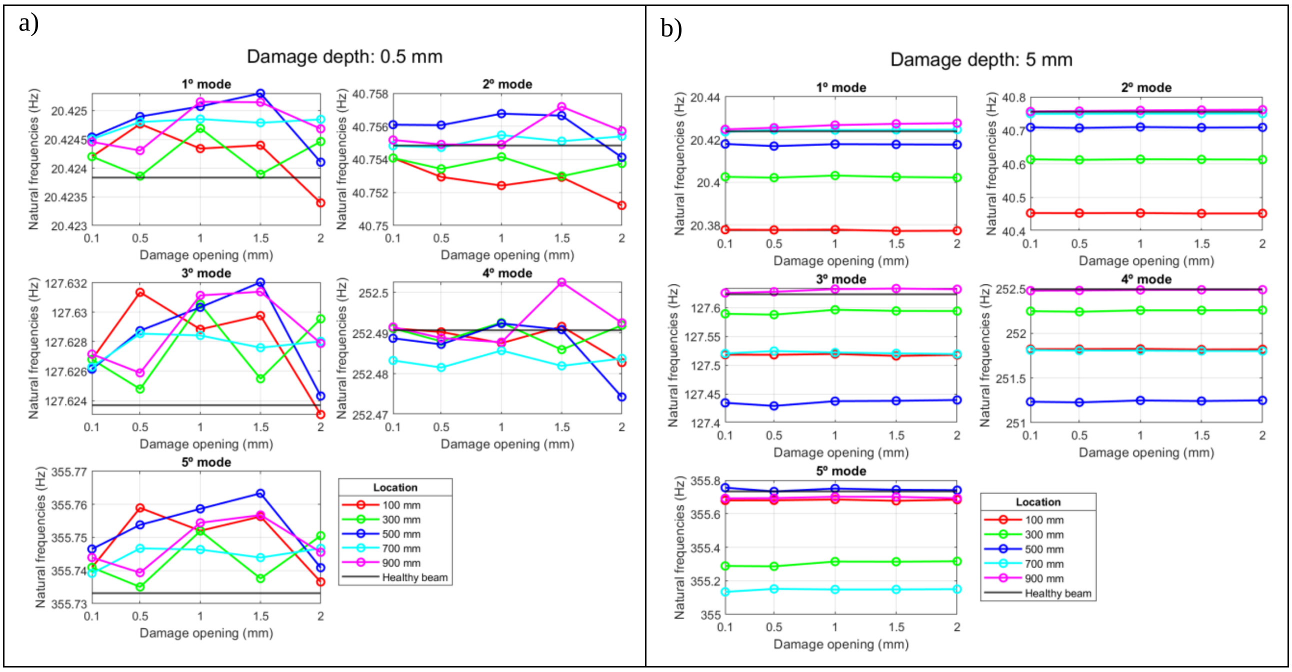

Figure 6. (a) Frequency versus aperture for depth equal to 0.5 mm, (b) frequency versus aperture for depth equal to 5 mm.

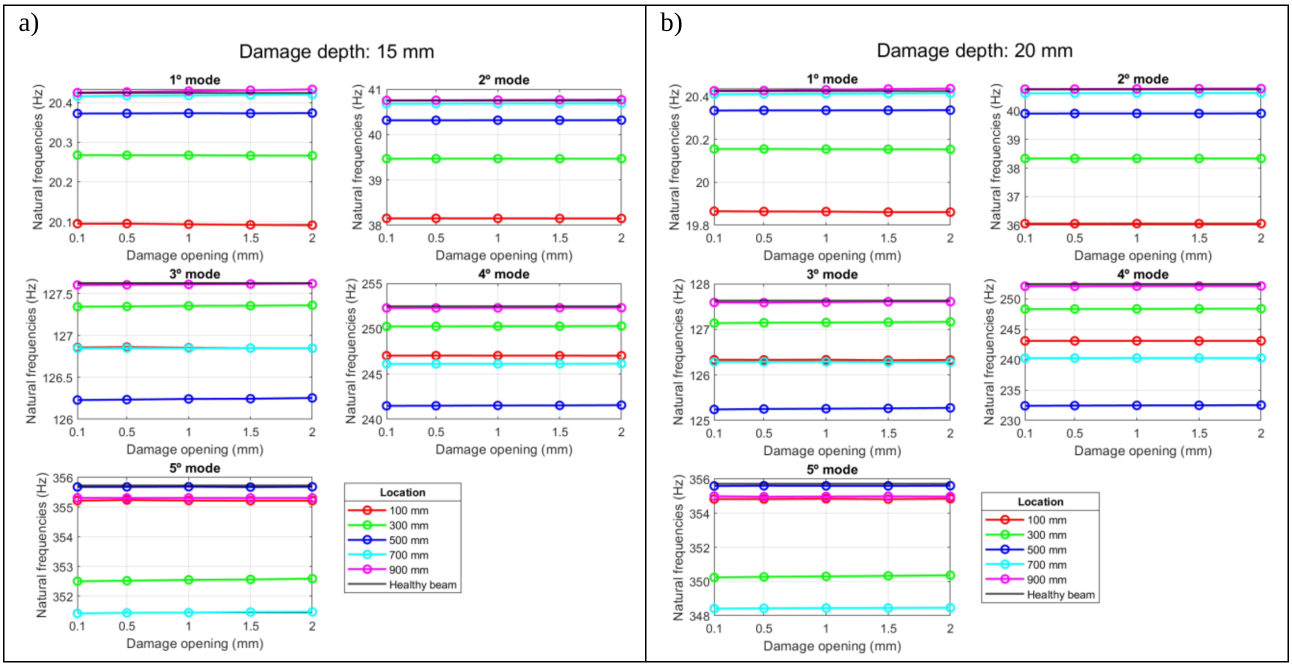

Figure 7. (a) Frequency versus aperture for depth equal to 15 mm, (b) frequency versus aperture for depth equal to 20 mm.

3.2.2 Effect of crack position

In Figure 8 and Figure 9 is shown behavior of natural frequencies as a function of the location (x) of the crack for an opening (a) and depth (p) fixed. Where, for each model are plotted the first five modes of vibrating, each line represents a fixed depth, and each point a location. Thus, each graph presents the behavior of 30 models (6 depths and 5 locations).

As observed in item 3.2.1 the opening does not significantly influence the dynamic response. Thus, a similar behavior is observed between the graphs shown in Figure 9. As follows:

- For modes 1 and 2 the behavior of the $\omega_n(x)$ function is analogous. Note that the effect of the crack on the natural frequency is more noticeable when the discontinuity is in the vicinity of the fixed end (region of great mechanical demand) and the value tends to the calculated for the healthy beam as $x \to L$. This shows that cracks located near the free end (region of low mechanical demand) do not significantly affect the dynamic response of the beam for vibration modes 1 and 2.

- For modes 3 and 4 the behavior of the OMEGA function is also analogous. Note that the effect of the crack on the natural frequency is quite noticeable in the vicinity of the fixed end and decreases as $x \to 1/4\,L$ and is more evident for cracks located near the 1/2 L position (region of greater mechanical demand for modes 3 and 4), decreasing again monotonously as $x \to L$. This shows that cracks located in the $x = 1/4\,L$ and $x = L$ positions (low mechanical stress regions) do not significantly affect beam dynamic response for vibration modes 3 and 4.
- For mode 5, the behavior of the frequencies with the variation of the crack position differs significantly from the 1, 2, 3 and 4. In this case, the proximity of the crack to the fixed end does not significantly affect the vibration response, the effect begins to be more observed for cracks located 1/4 L and 3/4 L from the beam. In addition, for a crack located at 1/2 L, the natural ω_n frequency is similar to that calculated for the healthy beam.
- The distance between the curves of the damaged beam and the healthy beam shows that the depth (t) is inversely proportional to the natural frequency ω_n.

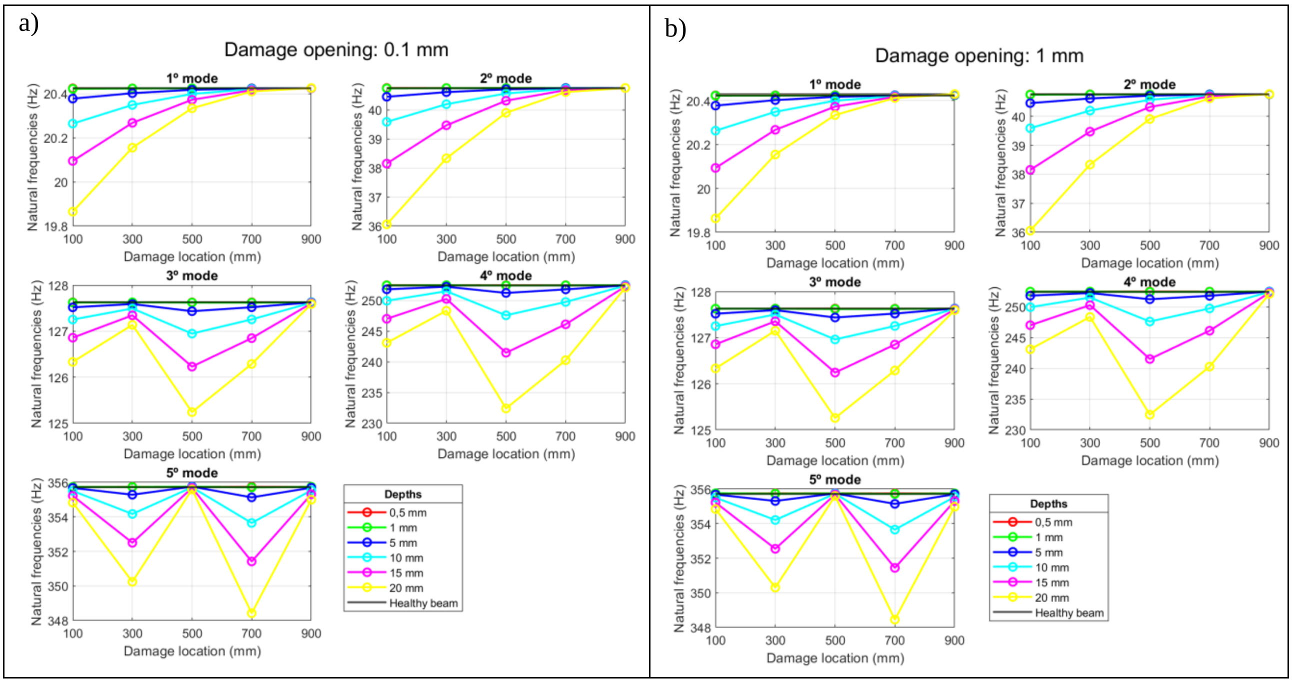

Figure 8. (a) Frequency versus location for aperture equal to 0.1 mm, (b) frequency versus location for aperture equal to 1 mm.

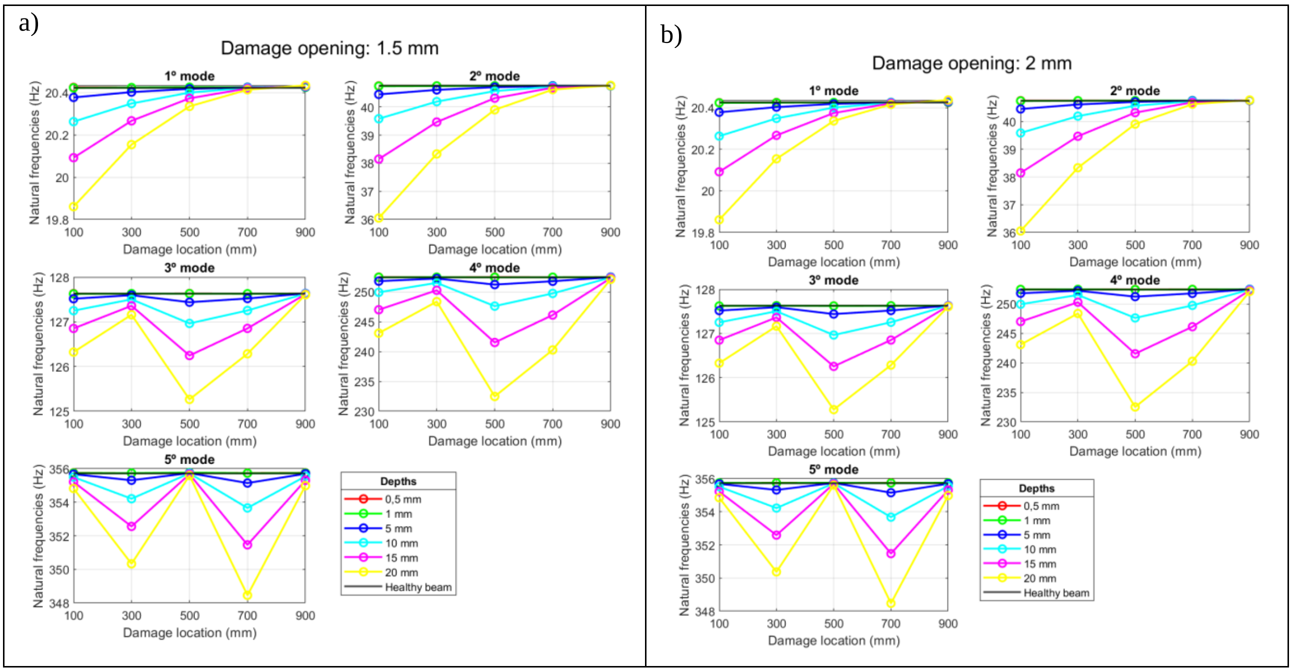

Figure 9. (a) Frequency versus location for aperture equal to 1.5 mm, (b) frequency versus location for aperture equal to 2 mm.

3.2.3 Effect of crack depth

The graphs presented in the figures: Figure 10 and Figure 11 show the behavior of the natural frequencies (ω_n) as a function of the depth (p) of the crack for a fixed opening (a) and location (x). Where, each line represents a fixed location, and each point a depth. Where a graph shows the behavior of 30 models. Since the aperture does not significantly influence the dynamic response, the behavior between the graphs shown is analogous.

For all 150 simulated models it is noted that the $\omega_n(t)$ function for the damaged beam has the same pattern, the depth (t) and the $\omega_n(t)$ function are inversely proportional. Observing the distance between the curves, it is noted that:

- For modes 1 and 2 the greatest decrease in the natural frequency of the beam occurs for the discontinuity located in the vicinity of the fixed end and the value tends to the calculated for the healthy beam as $x \to L$.
- For modes 3 and 4 the effect of the crack on the natural frequency is representative near the fixed end and decrease as $x \to 1/4\ L$ an, is more evident for cracks located near the 1/2 L position decreasing as $x \to L$.
- For mode 5, cracks located at 1/4 L and 3/4 L from the beam cause a greater reduction in natural frequency, whereas cracks located $\cong$ 1/2 L do not cause great variation in ω_n.

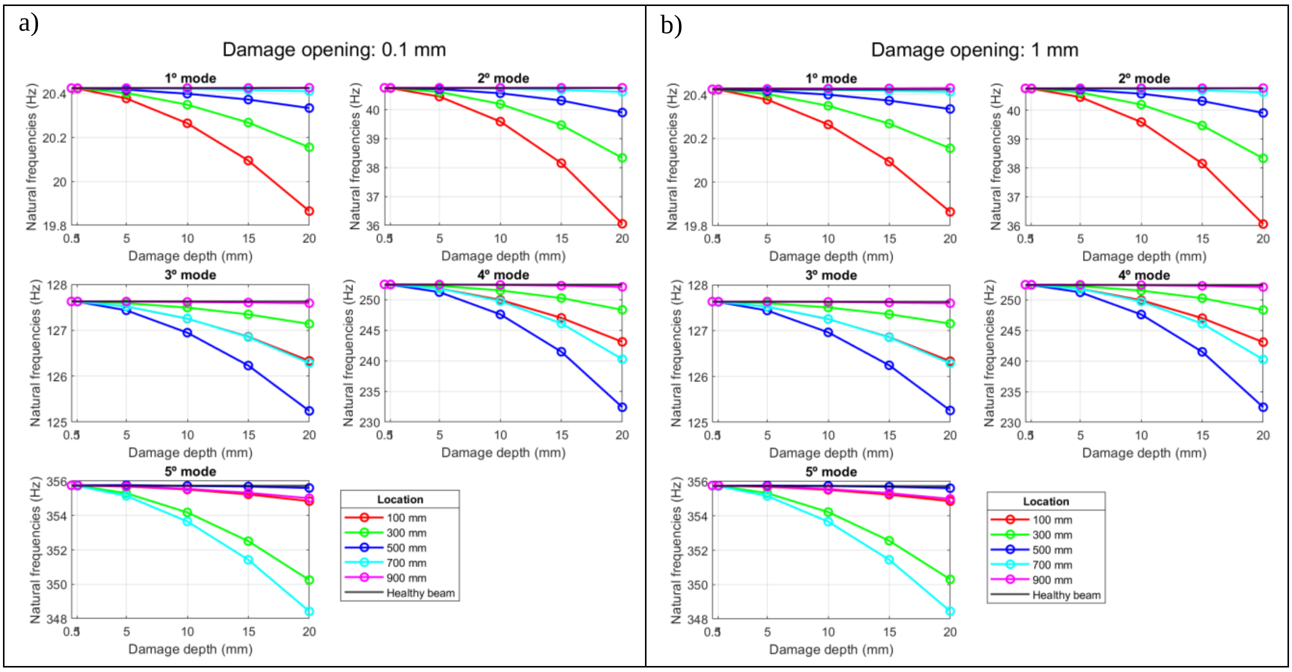

Figure 10. Frequency versus Depth for aperture equal to 0.1 mm, Frequency versus Depth for aperture equal to 1 mm.

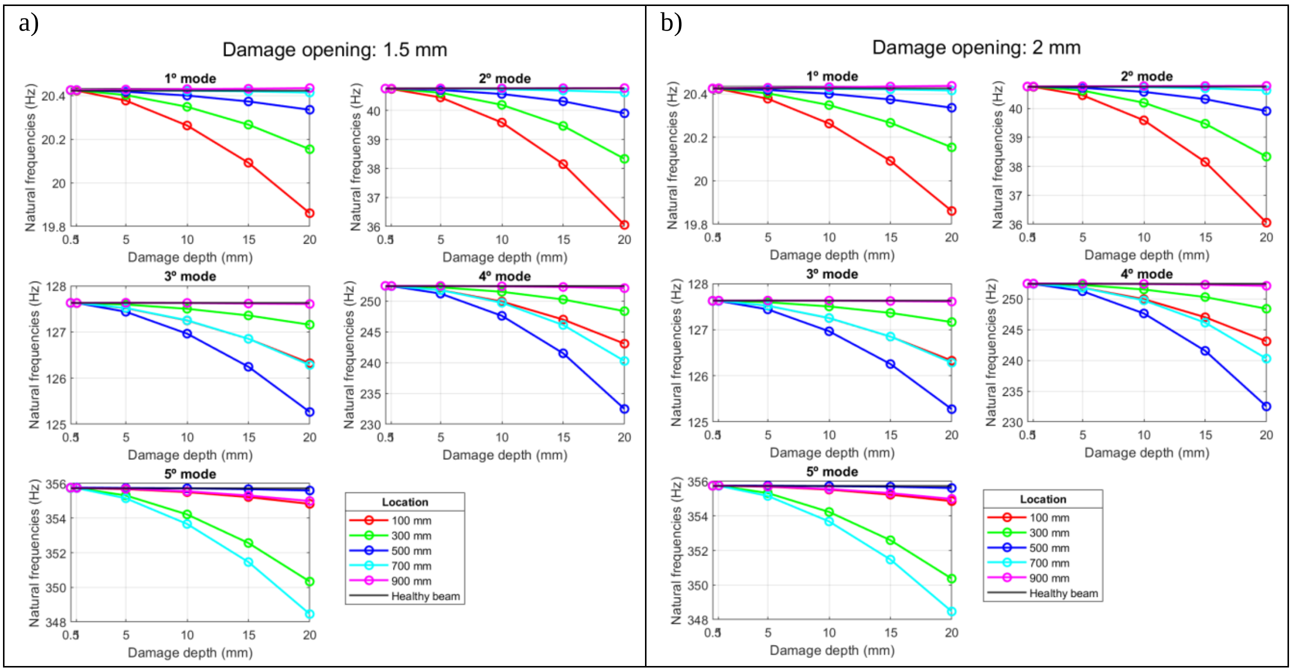

Figure 11. Frequency versus Depth for aperture equal to 1.5 mm, Frequency versus Depth for aperture equal to 2 mm.

4. CONCLUSION

The presence of the damage causes a change in its mechanical and/or physical properties, changes its modal parameters, such as its natural frequencies and the associated vibration modes. This change in frequencies can cause the component to operate in a resonance zone, which can lead to premature failure.

In the present work, we analyzed the natural frequencies and vibration modes of a beam in healthy and cracked conditions. For the numerical solution, the FEM was used via ANSYS© 2023 R1. The models were constructed by combining three parameters associated with the crack: opening, location, and depth.

In this study, we considered the first mode of rupture (opening mode) of the crack. From the analysis of the results, the following conclusions were reached:

- Openness does not significantly influence the dynamic response of the structure. For a fixed depth (*p*) and location (*x*), the variation in frequencies as a function of aperture is minimal.
- The variation of the natural frequency (ω_n) is inversely proportional to depth (*p*). The results show that the natural frequencies of a damaged beam decrease with increasing depth.
- Effect of the variation of the position of the crack (*x*) depends on the mode of vibrating of the structure, the greatest reduction in the natural frequency occurs for the regions of greater mechanical stress. It was observed that for modes 1 and 2 the greatest change in the natural frequency occurs for the discontinuity located in the vicinity of the fixed end x → L, for modes 3 and 4 the greatest change occurs for x → 1/2 L, for mode 5, cracks located at x → 1/4 L and x → 3/4 L cause a greater reduction in natural frequency, whereas localized cracks x = 1/2 L imply little variation of ω_n.

Based on the results obtained, it is concluded that it is very important to know the natural frequencies and the modes of vibration and the monitoring of these in components/structures subject to dynamic loading. In this sense, the measurement of the natural frequencies of the component provides an alternative for the identification of the presence of the damage.

5. ACKNOWLEDGEMENTS

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