27th ABCM International Congress of Mechanical Engineering
December 04-08, 2023, Florianópolis-SC, Brazil

COB-2023-1766

INTEGRAL TRANSFORM SOLUTION OF INCOMPRESSIBLE FLOW WITH TRANSIENT PRESSURE GRADIENT

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Abstract. *In the past decades, the Generalized Integral Transform Technique (GITT), has been employed toward the solution of many engineering problems, including heat and mass transfer, fluid dynamics, chemical reaction systems, among others. However, most of the studies in that area concern regular geometries due to the complications which arise with the solution of the associated eigenvalues problem for irregular domains in this class of methods. Thus, to consider more practical applications, solutions for the transient Navier-Stokes equation are obtained for one-directional, incompressible, laminar flow in a hexagonal irregular domain using the hybrid analytical-numerical integral transforms technique. When employing it for irregular geometries, two methodologies are compared: the Coincident Domain Approach (CDA), where the mathematical domain and the analyzed irregular geometry are the same, and the Fictitious Domain Approach (FDA), which consists of solving the problem for a regular geometry that encompasses the irregular one. The focus of this study is to compare both approaches using an error estimation and computational time evaluation, and to analyze other parameters associated with the accuracy of each particular method. Numerical results show a slight advantage of the FDA regarding processing time, and better error estimate convergence for the CDA.*

Keywords: *Generalized Integral Transform Technique, irregular geometries, computational fluid dynamics.*

1. INTRODUCTION

The Generalized Integral Transform Technique (GITT) is a method for solving Partial Differential Equations (PDEs) as a series expansion (Cotta,1993), broadly used in different physical systems, such as heat transfer in microchannels with slip flow (Naveira-Cotta et al., 2010), transient natural convection (Lisboa et al., 2018), conjugated conduction-radiation heat transfer (Pinheiro et al., 2018), heat convection in porous channels (Sphaier and Barletta, 2014), to name a few examples of many applications of the method. However, in these studies, the technique is implemented towards regular geometries. The natural step in the advancement of Integral Transforms analyses is to employ it to irregular geometries, to contemplate more practical engineering situations.

The main difficulty in solving the technique for irregular geometries is establishing and solving an eigenvalue problem that will provide the basis for the series expansion of the solution, given that most traditional solutions for multi-dimensional problems, such as separation of variables, are not available for this type of problems. Some strategies to deal with this issue were noted in the literature, such as the work of Aparecido et al. (1989), where a solution of diffusion problems in arbitrary geometries by employing traditional concepts of Integral Transforms was obtained. Guerrero et al. (2000) sought the solution of the laminar flow in an expanding duct by means of a fourth order eigenvalue problem. Knupp et al. (2014) used a single domain approach to solve the conjugated heat transfer in irregularly shaped channels.

Here, the problem at hand is a laminar, one-directional flow in a hexagonal duct, such as found in rotatory desiccant wheels (Zhang et al., 2003), as a primary investigation. In determining which approach better suits this problem, one can move on to, for instance, model the convective heat transfer within the duct, and other more complex situations. The goal of this study is to compare two different methodologies for dealing with the irregular nature of the problem, the Coincident Domain Approach (CDA) and Fictitious Domain Approach (FDA). The first is based on the solution of an eigenvalue problem with boundary conditions defined on the original irregular domain, while the latter creates a regular domain containing the irregular one, and strategically defines the viscosity outside the irregular region to reflect the boundary conditions of the real problem. Pinheiro et al. (2019) compared both approaches for solving a two-dimensional eigenvalue problem for a triangular and an arch of circle geometry and, for these test cases, found that the CDA outperforms the FDA when dealing with Dirichlet boundary conditions, and the other way around for the Neumann case.

2. PROBLEM DESCRIPTION

The hexagonal irregular domain in which the problem is to be solved is presented in Figure 1, representing the duct cross-section. The no-slip Dirichlet boundary condition is imposed on the walls of said duct, i.e., homogeneous conditions, uniform along the contour. That being the case, given the symmetric nature of the geometry, the velocity profile will be symmetric along the x and y axis as well, allowing for the solution to be obtained solely on one quarter of the domain (right trapeze).

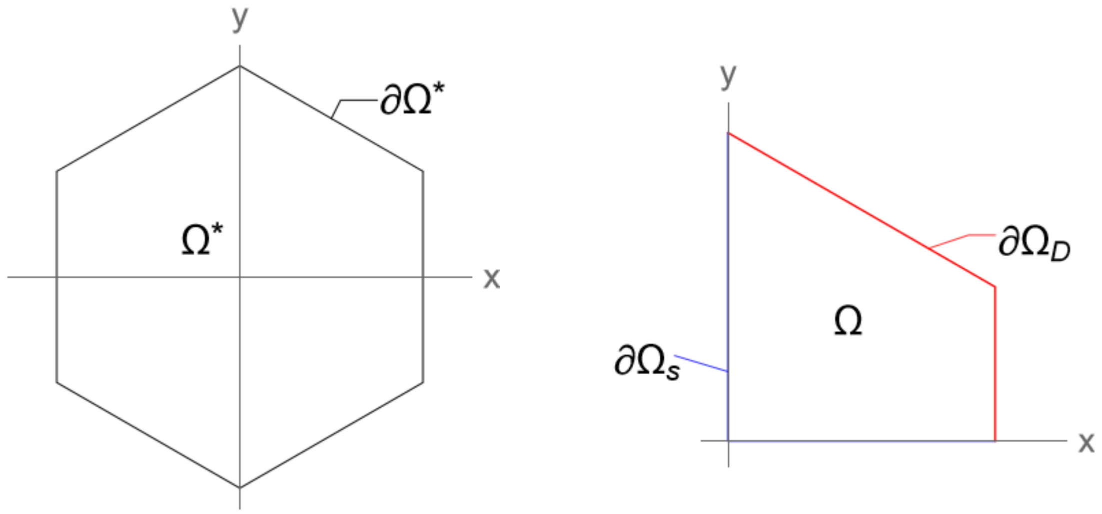

Figure 1: Hexagonal domain (a) Complete (b) Partitioned by symmetry.

Note that the problem could be solved for one sixth of the hexagon (equilateral triangle), however, the partial domain Ω would have two irregular boundaries, which could be problematic in terms of convergence, both for the CDA and FDA, or even for one-twelfth of the hexagon (a right triangle), in which case, the irregular geometry would receive a Neumann boundary condition, that would hinder the analytical procedure, and so, the proposed partition of the domain is suitable for the problem at hand.

2.1 Mathematical formulation

The Navier-Stokes equations for one-directional, incompressible laminar flow is presented in Eq. (1). The Neumann boundary conditions at $\partial\Omega_S$ represent the axial symmetry of the problem. In this study, the focus is to analyze the methodology, rather than to model any particular situation. Therefore, the physical properties of the fluid will be assigned as unitary, and the geometry will be treated as dimensionless.

$$\frac{\partial u}{\partial t} - \nu \nabla \cdot (\nabla \mathrm{u}) = -\frac{1}{\rho}\frac{dp}{dz} = g(t), \qquad \boldsymbol{x} \in \Omega \tag{1a}$$

$$u = u_0(x, y) = 0, \qquad t = 0 \tag{1b}$$

$$\nabla u \cdot \boldsymbol{n} = 0, \qquad \boldsymbol{x} \in \partial\Omega_S \tag{1c}$$

$$u = 0, \qquad \boldsymbol{x} \in \partial\Omega_D \tag{1d}$$

Where ν is the kinematic viscosity, ρ is the density of the fluid, and u(x, y, t) is the velocity profile in the z direction. By the assumption of fully developed flow (reasonable for low Reynolds numbers), it is known that the pressure decreases linearly along z, in other words, dp/dz is constant regarding the spatial coordinates. Moreover, the pressure gradient is directly proportional to the mass flow in the duct, that may vary in a crescent manner from zero to a steady value representing, for instance, the start of a pump (Figure 2). Therefore, the generation term in the equation can be modeled, rather generically, as shown in Eq. (2).

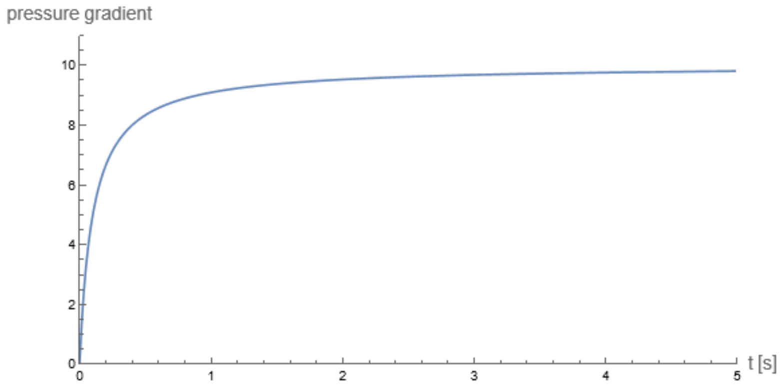

Figure 2: Transient pressure gradient

$$g(t) = \frac{1}{\alpha\, t + Q} + Q \tag{2}$$

Where Q is the mass flow, and the parameter α determines the abruptness of the variation from zero to the steady mass flow. The procedure to solve this problem via Integral Transforms is discussed further.

2.2 Integral Transforms Procedure

The GITT consists of writing the solution of a PDE as a series expansion in terms of orthogonal eigenfunctions, as in Eq. (3). The basis of such expansion are eigenfunctions ψ_i, solutions to the Stum-Liouville problem. The eigenvalue problem is defined differently for the CDA and FDA, however, the problem transformation is similar.

$$u(x,y,t) = \sum_{i=1}^{\infty} \tilde{\psi}_i(x,y)\, \bar{u}_i(t) \tag{3}$$

With the intention of determining the transformed velocity profile $\bar{u}(t)$, it is necessary to define the integral transform relation, as shown in Eq. (4). It is worth noting that this procedure is possible due to the orthogonality property of the eigenfunctions with the domain (Eq. (5)), and, in addition, the tilde over the eigenfunction denotes a normalization by $\sqrt{N_i}$.

$$\bar{u}_i(t) = \int_{\Omega} \tilde{\psi}_i(x,y)\, u(x,y,t)\, dV \tag{4}$$

$$\int_{\Omega} \tilde{\psi}_i \tilde{\psi}_j \, dV = \begin{cases} N_i, & i = j \\ 0, & i \neq j \end{cases} \tag{5}$$

Multiplying Eq. (1) by the eigenfunctions, and taking the volumetric integral over the domain, results in Eq. (6). The time derivative term can promptly be transformed by the definition in Eq. (4).

$$\frac{d\bar{u}_i}{dt} - \nu \int_{\Omega} \tilde{\psi}_i \nabla \cdot (\nabla u)\, dV = \int_{\Omega} \tilde{\psi}_i\, g(t)\, dV \tag{6}$$

This equation can be further manipulated using Green's first identity shown in Eq. (7). The second one is usually preferable for convergence reasons; however, Green's first identity should be used when there are discontinuities in the physical properties, for instance, in the FDA, as it will be demonstrated further.

$$\int_{\Omega} \tilde{\psi}_i \nabla \cdot (\nabla u) dV = \int_{\partial\Omega} (\tilde{\psi}_i \nabla u) \cdot \boldsymbol{n}\, dA - \int_{\Omega} \left(\nabla \tilde{\psi}_i\right)(\nabla u)\, dV \tag{7}$$

If the eigenvalue problem conforms to the boundary conditions of the original problem, the surface integrals in Eq. (7) will go to zero. In sequence, u is replaced using the inverse relation in Eq. (3) with a different index, resulting in the transformed problem, presented in Eqs. (8),

$$\frac{d\bar{u}_i}{dt} - \nu \sum_{j=1}^{\infty} \bar{u}_i A_{ij} = \bar{g}_i(t) \quad (8a)$$

$$\bar{u}_i(0) = \bar{u}_{0,i} \quad (8b)$$

in which,

$$A_{ij} = \int_{\partial\Omega} \tilde{\psi}_i \nabla\tilde{\psi}_j \cdot \boldsymbol{n}\, dA + \int_{\Omega} \nabla\tilde{\psi}_i \cdot \nabla\tilde{\psi}_j\, dV \quad (8c)$$

$$\bar{u}_{0,i} = \int_{\Omega} \tilde{\psi}_i u_0(x,y) dV \quad (8d)$$

$$\bar{g}_i(t) = \int_{\Omega} \tilde{\psi}_i\, g(t)\, dV \quad (8e)$$

Equation (8a) is, in fact, a coupled system of Ordinary Differential Equations (ODEs). Given that the problem is linear, it advantageous to write this system in its vectorial form, and solving via the diagonalization of the matrix **A**.

$$\frac{d\bar{\boldsymbol{u}}}{dt} - \nu \boldsymbol{A}\bar{\boldsymbol{u}} = \bar{\boldsymbol{g}}(t) \quad (9)$$

By obtaining the eigenvalues λ_i and the eigenvectors $\boldsymbol{\varphi}_i$ of the Matrix **A** and ordering the eigenvectors in a matrix $\boldsymbol{\Phi}$, the solution is readily obtained, as shown.

$$\boldsymbol{\Phi} = [\boldsymbol{\varphi_1} \quad \boldsymbol{\varphi_2} \quad \cdots] \quad (10)$$

In such a way that $\boldsymbol{\Phi}^{-1}\boldsymbol{A}\boldsymbol{\Phi}$ yield a diagonal matrix whose entries are the eigenvalues of **A**. Furthermore, assuming a vector $\boldsymbol{\theta}$**,** such that,

$$\bar{\boldsymbol{u}} = \boldsymbol{\Phi\theta} \quad (11)$$

one can substitute Eq. (11) in Eq. (9), and apply the inverse of the matrix $\boldsymbol{\Phi}$, resulting in Eq. (12). Note that the values of θ_i are uncoupled, which is favorable for obtaining the solution by means of the relation in Eq. (11).

$$\frac{d\theta_i}{dt} - \lambda_i\theta_i = \hat{g}_i \quad (12a)$$

$$\theta_i(0) = \hat{u}_{0,i} \quad (12b)$$

In which,

$$\hat{\boldsymbol{g}} = \boldsymbol{\Phi}^{-1}\bar{\boldsymbol{g}}; \quad \hat{\boldsymbol{u}}_{\boldsymbol{0}} = \boldsymbol{\Phi}^{-1}\bar{\boldsymbol{u}}_{\boldsymbol{0}} \quad (12c)$$

This procedure is far more efficient in terms of computational implementation, however, if the problem at hand were not to be a linear one, Eq. (9) would necessarily need to be solved by direct application of some numerical procedure, such as the Backwards Differentiation Formula.

2.3 Error estimate

The solution of the problem given by Eq. (3) establishes a summation to infinity, and so, it should be truncated to enable the implementation of the method. Naturally, an error rises in such truncation, that will diminish the more terms are incorporated in the series expansion. Among the different methods to estimate the error, here it was used the classical Root Mean Square (RMS) Error, by pondering over the volume the square of the difference between the sought solution u and the truncated solution u^T.

$$\bar{\varepsilon}(t) = \sqrt{\frac{\int_{\Omega}\left(u(x,y,t) - u^T(x,y,t)\right)^2 dV}{\int_{\Omega} 1\, dV}} \quad (13)$$

Substituting the inverse relation of Eq. (3) in the last equation, results in Eq. (14). The exact solution is a summation from one to infinity, and the truncated solution is from one to the truncation order N, therefore, the subtraction in Eq. (13) yields a summation from N to infinity. By strategically taking advantage of the orthogonality property of the eigenfunctions, the error estimate is reduced to Eq. (15).

$$\bar{\varepsilon}(t) = \sqrt{\frac{\int_{\Omega}\left(\sum_{i=N}^{\infty}\tilde{\psi}_i(x,y)\,\bar{u}_i(t)\right)^2 dV}{V_{\Omega}}} = \sqrt{\frac{\sum_{i=N}^{\infty}\sum_{j=N}^{\infty}\bar{u}_i\,\bar{u}_j\int_{\Omega}\tilde{\psi}_i\tilde{\psi}_j dV}{V_{\Omega}}} \tag{14}$$

$$\bar{\varepsilon}(t) = \sqrt{\frac{\sum_{i=N}^{\infty}\bar{u}_i^2}{V_{\Omega}}} \tag{15}$$

The issue of the summation to infinity remains in Eq. (15), and so, the error estimate used considers only the last twenty terms. More complex problems may require a more conservative analyses, and more terms should be considered. In contrast, simpler problems, such as the ones solved by the Classical Integral Transform Technique, may use simply the first term of Eq. (14).

$$\sqrt{V_{\Omega}}\bar{\varepsilon}(t) = \sqrt{\bar{u}_{N-20}^2 + \bar{u}_{N-19}^2 + \cdots + \bar{u}_{N-1}^2 + \bar{u}_N^2} \tag{16}$$

This is a reasonable approximation due to a property of the problem. The transformed potentials $\bar{u}_i$ are ordered by relevance to the solution, for the eigenvalues of the problem are ordered that way (From smaller to greatest), in such way that if the estimate of Eq. (16) is within a given tolerance, that's insurance enough that the total error is tolerable as well.

2.4 Reordering scheme

Regardless of the Sturm-Liouville problem proposed, within the context of this study, the eigenfunctions $\tilde{\psi}$ will always be composed of the product of two other functions, solution to auxiliary eigenproblems, each with its own set of eigenvalues. Assuming that each of those functions are also truncated at N terms, every paring of their terms will be contemplated in a series expansion of the solution, that is, N^2 terms. This denotes that the computational complexity of the problem escalates rather quickly the higher the truncation other. However, as mentioned, the eigenvalues are ordered in relevance to the solution. In other words, it is possible to establish a criterion to determine which N pairs of terms of the auxiliary eigenfunction will be more important to the solution.

$$\mu_i^2 = \lambda_{x,p}^2 + \lambda_{y,q}^2, \qquad p = q = 1, 2, \dots, N \tag{17a}$$

$$\{p, q\} \rightarrow i \tag{17b}$$

Where $\lambda_{x,p}$ and $\lambda_{y,q}$ are the eigenvalues of the auxiliary problems. The reordering criterion consists of taking the N pairs of eigenvalues that results in the smallest μ_i^2. This criterion is inspired by the two-dimensional Sturm-Liouville problem in cartesian coordinates for regular domains, and although it's not the case here, it is reasonably effective. From heron, the index i will represent a reordering of the index p and q.

3. DOMAIN TRANSFORMATION

For each of the approaches proposed, an eigenvalue problem must be defined, followed by the obtaining of the transformed problem, that is to be solved by the diagonalization procedure described.

3.1 Coincident Domain Approach

In the coincident domain approach, the domain in which the problem is to be solved coincides with the original one. For clarity, the boundary is analyzed by parts, as in Figure 3.

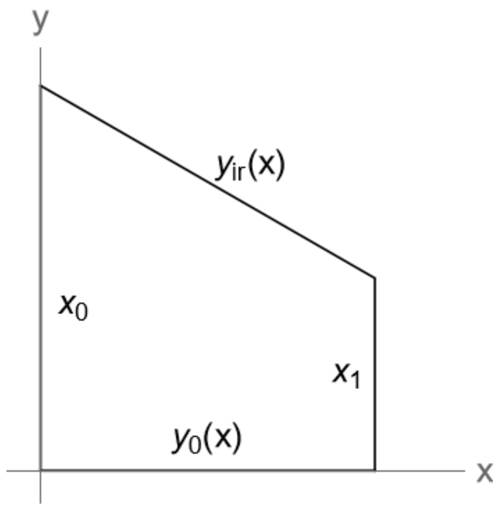

Figure 3: Sub boundaries for the CDA analysis

When establishing the eigenvalue problem, one can define a classic two-dimensional Sturm-Liouville problem and solve it via Integral Transform by defining two auxiliary problems (given that separation of variables wouldn't be a viable solution for an eigenvalue problem in intricate domains), or, as in this study, apply these auxiliary problems directly to the original potential. First, the eigenfunction used in the series expansion is defined.

$$\tilde{\psi}_i(x,y) = \tilde{X}_p(x)\tilde{Y}_q(x,y) \tag{18}$$

Where the functions X_p and Y_q are solutions of the problems in Eqs. (19) and (20). Remembering that the index i represents the reordering of the indexes p and q.

$$\frac{\partial^2 \tilde{X}_p}{\partial x^2} + \lambda_{x,p}^2 \tilde{X}_p = 0, \qquad x_0 \le x \le x_1 \tag{19a}$$

$$\frac{\partial \tilde{X}_p}{\partial x} = 0, \qquad x = x_0\,; \qquad \tilde{X}_p = 0, \qquad x = x_1 \tag{19b}$$

$$\frac{\partial^2 \tilde{Y}_q}{\partial y^2} + \lambda_{y,q}(x)^2 \tilde{Y}_q = 0, \qquad y_0(x) \le y \le y_{ir}(x) \tag{20a}$$

$$\frac{\partial \tilde{Y}_q}{\partial y} = 0, \qquad y = y_0(x)\,; \qquad \tilde{Y}_q = 0, \qquad y = y_{ir}(x) \tag{20b}$$

Note that the proposed form for the $\tilde{\psi}$ function is not a solution of an eigenproblem, but, in fact, the composition of the solution of two other problems; however, it can be shown that Eq. (18) holds the orthogonality property. Furthermore, it is also important that the functions in Eq. (18) have a progressive dependance on the spatial variables. For instance, if the problem were to be in three dimensions, The eigenfunctions would be of the form $\tilde{\psi}_i(x,y,z) = \tilde{X}_p(x)\tilde{Y}_q(x,y)\tilde{Z}(x,y,z)$. For most of the boundary, the proposed eigenfunction conforms with the boundary conditions of the problem in Eq. (1); nevertheless, that's not the case in $x = x_0$, which will certainly compromise convergence. The elements of matrix **A** are as follows:

$$A_{ij} = \int_{x_0}^{x1} \int_{y_0(x)}^{y_1(x)} \nabla\tilde{\psi}_i \cdot \nabla\tilde{\psi}_j dydx \tag{21}$$

Given the irregular nature of the approach, these integrals will be quite complex, which can get in the way of computational efficiency, but, in contrast, it may have better convergence then the FDA. As for the reordering scheme, the eigenvalues $\lambda_{y,q}(x)$ have a dependance of x, and the convergence was compared when using $\lambda_{y,q}(1.0)$ for the reordering criterion.

3.2 Fictitious Domain Approach

For the FDA, the eigenvalue problem is described for a regular geometry (a rectangle in cartesian coordinates) containing the irregular one, as shown in Figure 4. In the contours $y = y_1$ and $x = x_1$, the homogeneous Dirichlet conditions are defined, and in the remaining contours a symmetry Neumann condition is implemented.

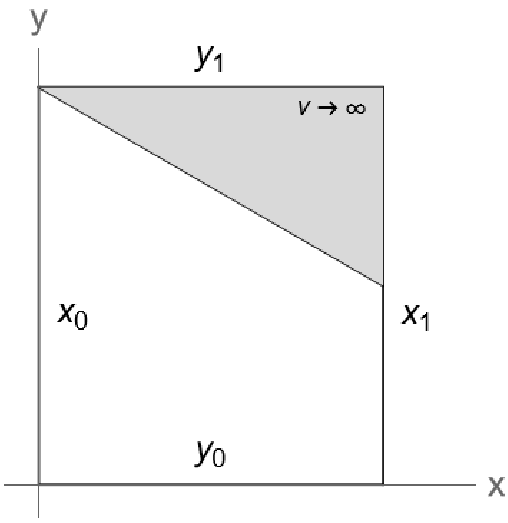

Figure 4: Fictitious regular domain encompassing the irregular one.

By defining the viscosity outside the irregular region (gray area in Figure 3) as a very high, the no-slip condition is assured in the irregular contour.

$$\nu = \begin{cases} \nu_{real}, & in\ y_0 \le y \le y_{ir}(x) \\ \nu_{fic} \to \infty, & in\ y_{ir}(x) \ge y \ge y_1 \end{cases} \tag{22a}$$

The eigenvalue problem for this approach can be defined as the classic two-dimensional Sturm-Liouville problem, with the solution given by traditional separation of variables:

$$\nu_{real}\left(\frac{\partial^2\tilde{\psi}_i}{\partial x^2} + \frac{\partial^2\tilde{\psi}_i}{\partial y^2}\right) + \mu_i^2\tilde{\psi}_i = 0 \tag{23a}$$

$$\frac{\partial\tilde{\psi}_i}{\partial x} = 0, \qquad x = x_0\ ; \qquad \tilde{\psi}_i = 0, \qquad x = x_1 \tag{23b}$$

$$\frac{\partial\tilde{\psi}_i}{\partial y} = 0, \qquad y = y_0\ ; \qquad \tilde{\psi}_i = 0, \qquad y = y_1 \tag{23c}$$

The convergence of the method may increase the more physical information is contemplated in the eigenvalue problem, and so, ideally, Eq. (23) would consider the abrupt change in the physical property of the problem, however, the solution of the problem wouldn't be obtained by separation of variables. That being the case, the problem was defined with a constant viscosity equal to ν_{real}. When constructing the matrix **A**, the integrals can be separated into two, one from y_0 to $y_{ir}(x)$ and another one from $y_{ir}(x)$ to y_1.

$$A_{ij} = \nu_{real}\int_{x_0}^{x_1}\int_{y_0}^{y_{ir}(x)} \nabla\tilde{\psi}_i \cdot \nabla\tilde{\psi}_j\, dydx + \nu_{fic}\int_{x_0}^{x_1}\int_{y_{ir}(x)}^{y_1} \nabla\tilde{\psi}_i \cdot \nabla\tilde{\psi}_j\, dydx \tag{24}$$

To further simplify Eq. (24), one can separate the first integral into the subtraction of an integral in the regular domain and another one in the fictitious one. The integral in the regular domain can be fully transformed by the relation of the eigenvalue problem of Eq. (23), and subsequentially by the relation in Eq. (4), and the integral in the fictitious domain can be merged with the second integral in Eq. (24) as follows.

$$A_{ij} = \nu_{real}\, \mu_i^2 \delta_{ij} + (\nu_{fic} - \nu_{real}) \int_{x_0}^{x_1} \int_{y_{ir}(x)}^{y_1} \nabla\tilde{\psi}_i \cdot \nabla\tilde{\psi}_j \, dydx \tag{25}$$

Where δ_{pq} is the Kronecker delta.

4. RESULTS

As mentioned, the aim of this work is to compare numerical convergence with processing time for both approaches. Figure 5 depicts the analyses of the processing time for $t = 1.0\ s$, considering the same computer hardware, and Figure 6 for the error estimate, both versus the truncation order (N).

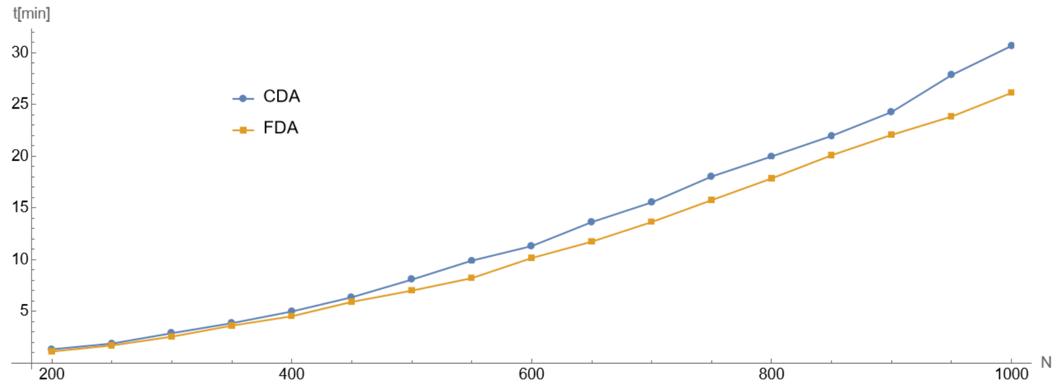

Figure 5: Computational time versus truncation order.

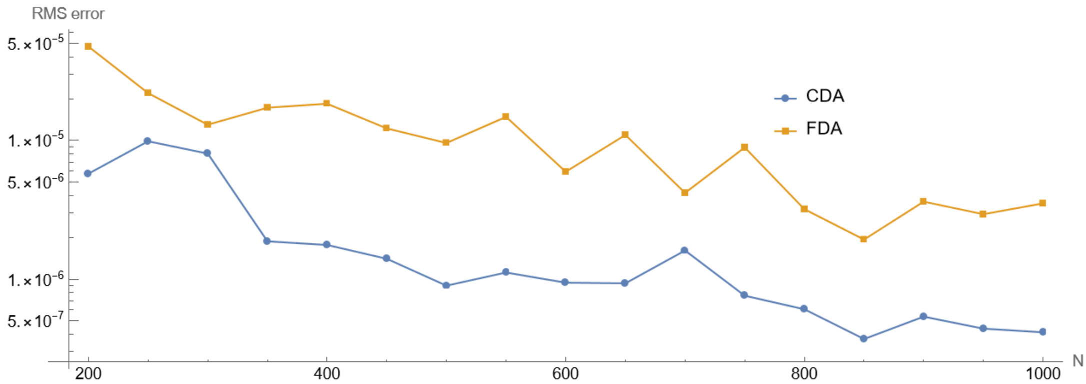

Figure 6: Error estimate of the solution versus truncation order for different reordering criterion.

The fluctuation in the error is possibly due to the reordering scheme proposed. As a higher truncation order adds more terms to matrix **A**, the reordering "shuffles" the terms of the matrix for the new truncation. In addition, the error contemplates only the last twenty terms of the vector $\overline{\boldsymbol{u}}(t)$, and it can be inferred that, if more terms were considered, the error analysis would be more stable. In addition to the RMS error, there are other parameters that should be monitored to ensure convergence.

As mentioned in section 3.1, the Neumann boundary condition for the CDA eigenvalue problem in $x = x_0$ does not conform with the condition of the original problem. For that reason, it is worth verifying the maximum value of the derivative of the result function in that contour, shown in Figure 7, as a parameter of convergence. The behavior is as expected, as the derivative diminishes the higher the truncation orders up to N = 850, after which it starts to increase, again, possibly due to the reordering scheme proposed.

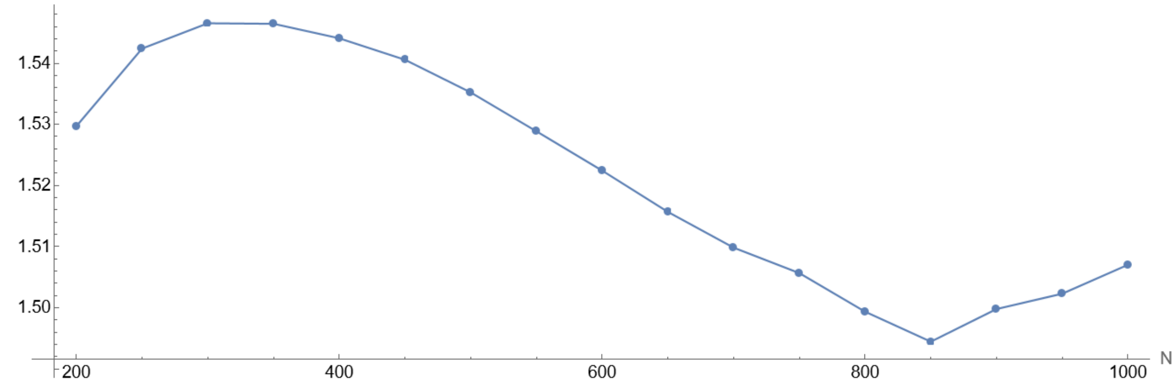

Figure 7: Maximum value of the derivative in the x direction at $x = x_0$ versus truncation order for the CDA.

For the FDA, the maximum value of the function at the irregular contour is analyzed, considering that no boundary condition was imposed in that contour, and the only assurance that the result will be null there is the high ν_{fic}. However, preliminary examinations show that the higher the value of the fictitious viscosity, the poorer is the convergence. So, ν_{fic} should be high enough to cause the no slip condition at the irregular boundary, and low enough to maintain a reasonable convergence. Figure 8 show the behavior of the maximum value of the result function at $y_{ir}(x)$, as the truncation increases.

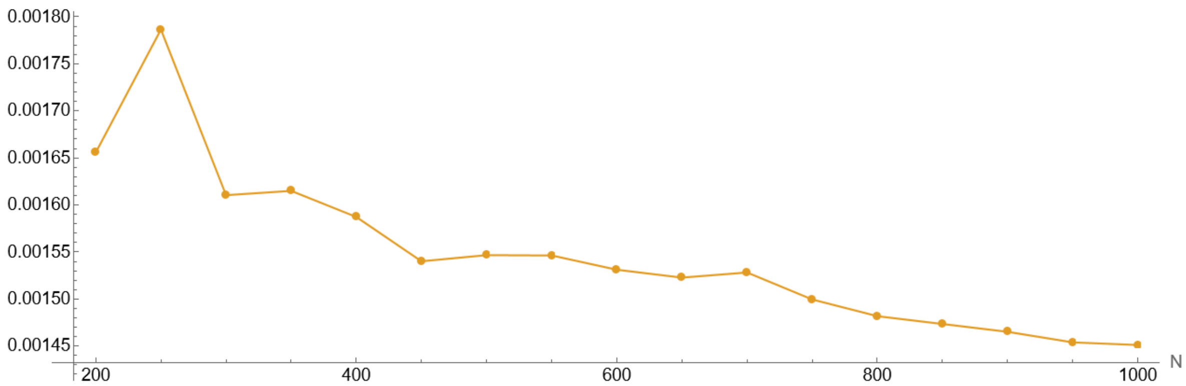

Figure 8: Maximum value of the velocity at the irregular boundary versus truncation order for the FDA.

Figure 9 shows the velocity profile obtained with N = 700 for one quarter of the geometry at t = 1.0 s by means of the CDA.

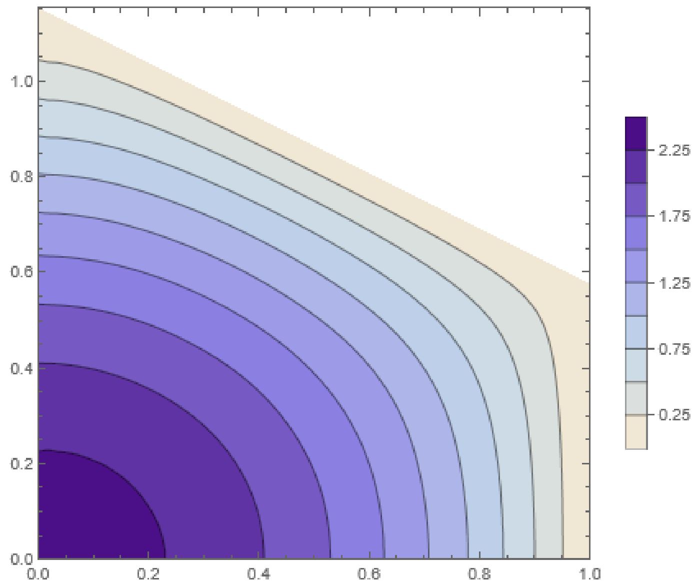

Figure 9: Velocity profile in a quarter of the hexagon at t = 1.0s

5. CONCLUSION

For the CDA, the key parameter of analysis that determines the accuracy of the approach is the value of the derivative in the x direction at the left contour, and although the approach shown good convergence of the RMS error, the derivative is still quite high, in addition, that parameter is converging very slowly to zero. In the FDA, the key parameter is the value of the velocity profile at the top contour, which is certainly more aggravating than the derivative of the velocity as in the CDA, and so, one should be more conservative for the acceptable values in that regard. The FDA shown worst convergence of the RMS error, however, even though the velocity at the irregular boundary is high for the truncation analyzed, it is showing better convergence to zero. The conclusion that can be drawn is that both methods would require a very high truncation order so the results could be considered reasonable, which is the onus of dealing with irregular geometries with spectral methods, but even so, given the results obtained, at a higher truncation order, the FDA should have a more trustworthy result.

One way to bypass the problem with the derivative in the CDA, is to solve the problem for one half of the hexagon, a five-side geometry as if the hexagon shown in Figure 1(a) was split at the x axis, where the symmetry exists only in the bottom contour, and the remaining ones receive a homogeneous Dirichlet condition. In that situation, the irregular boundary wouldn't be defined by a continuous function, and the integrals along x would forcefully be divided in two, which could greatly complicate the analytical process, and increase the process time (as the integral coefficients would carry more calculations), however, it is possible it would have better convergence. Another option to deal with the irregular geometry is to map it into another system of coordinates where the domain is regular, solve the problem, and then apply the inverse relation to recover the solution. This procedure would improve convergence, however, the problem in the new coordinate system may be quite intricate (even non-linear), and so, one should be careful when choosing the appropriate way to deal with irregular geometries.

6. AKNOWLEGDGMENTS

The authors would like to acknowledge the financial support provided by the Brazilian government funding agencies CAPES and FAPERJ (grants numbers E-26/200.261/2023 and E-26/203.114/2023).

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