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Fatigue life calculation using incremental damage

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Abstract. *Fatigue failure is a phenomenon that can be verified in many different types of structures. The first records of the study of fatigue date back to 1827, performed by the mining official, Wilhelm Albert. Since then the study of fatigue has been essential to design all types of equipment in order to avoid premature failure. The work therefore aims to numerically estimate fatigue life through incremental damage. To this end, it will be necessary to implement the 1D Isotropic and Kinematic constitutive model in MATLAB language, validate the implementation by reproducing examples presented in the literature for constant amplitude cyclic, variable amplitude cyclic, and random amplitude loading histories, implement a damage indicator, and study the cases in mechanical components subjected to cyclic loading. The approach proves advantageous when used for fatigue life calculation under random loading.*

Keywords: *Fatigue Life, Fracture Indicators, Ductile Fracture . . .*

1. INTRODUCTION

Fatigue is a common problem in equipment subjected to sequential loads, requiring periodic maintenance and replacement of components. Its study dates back to 1827, when Wilhelm Albert observed failures in conveyor chains, performing the first known fatigue test. Gutierrez de Lima *et al.* (2018)

Figure 1: Conveyor chain in a mine. joh

In the early 19th century, frequent failures in railway wagon axles led to the recognition of fatigue as an important factor. August Wöhler studied railway axles and found that failure occurred due to the number of stress cycles over time. He established a fatigue resistance limit for steels, resulting in the S x N (stress versus number of cycles) diagram, also known as the Wöhler curve. Gutierrez de Lima *et al.* (2018), Gutierrez de Lima *et al.* (2018)

The study of material fatigue is crucial as it affects the service life and safety of various equipment. A famous example of fatigue failure is the crash of the De Havilland Comet aircraft, which disintegrated in flight due to the failure of the fuselage after pressurization and depressurization cycles. Walker and Henderson (2000)

It is necessary to consider fatigue life and the maximum number of cycles when designing components in order to ensure their durability. The study of fatigue also allows to establish maintenance plans, avoiding sudden failures by analyzing and replacing components near the end of their estimated useful life, requiring a study of the material to determine the appropriate replacement frequency.

2. Metodology

After implementing the 1D Isotropic and Kinematic constitutive model in MATLAB, it was tested under three different loading conditions: a) strain history with constant amplitude over time; b) strain history with variable amplitude over time; c) random strain history over time. In both cases, an attempt was made to determine the stress versus strain curve, given the activation of isotropic, linear kinematic, isotropic and non-linear kinematic and kinematic hardening only.

In order to validate the implementation of the numerical model, the data of the first strain history under constant amplitude were taken from the literature, Besson *et al.* (2010). The results obtained for the hysteresis loops are in line with those observed in the literature, Besson.

3. Results

In order to verify the evolution of the damage generated by different loading histories and types of hardening, the Vaz Jr damage indicator was applied, verifying the number of cycles necessary for the accumulated damage to reach 1 and the consequent failure of the material.

3.1 Influence of hardening on fatigue life

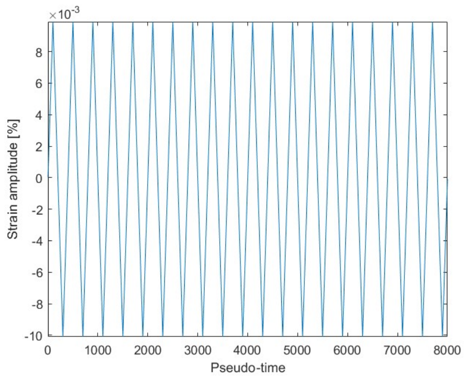


Figure 2: Cyclic history with constant amplitude and zero mean strain

1. Linear Isotropic Hardening

   When applied to the Von Mises model, isotropic hardening corresponds to an increase in the radius of the Von Mises cylinder in the principal stress space. The Prager model allows for describing the plastic response of materials subjected to cyclic loading.

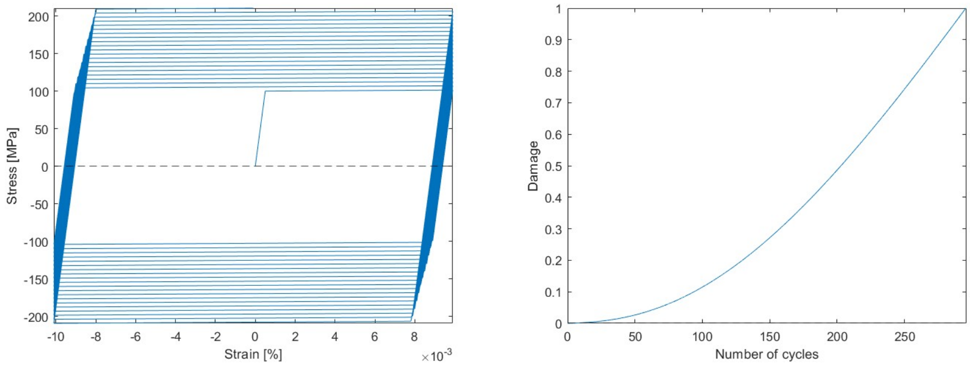
$$\dot{\beta} = \frac{2}{3} H^k \dot{\varepsilon}^p \tag{1}$$

   where:

   - β corresponds to the kinematic stress tensor;
   - H^k is the kinematic hardening modulus;
   - $\dot{\varepsilon}^p$ is the plastic strain rate.

E	200000[MPa]
σ_y	100[MPa]
H^I	150[MPa]
H^K	0[MPa]
b	0

Table 1: Linear isotropic hardening data



(a) Hysteresis curve (b) Damage evolution curve

Figure 3: Linear Isotropic Hardening

For isotropic hardening, 296 cycles were required for the damage to reach 1 and fatigue failure to occur.

2. Linear Kinematic Hardening

Kinematic hardening occurs when the yield limit only translates in the strain space. This effect is common in cyclic loading, due to loading reversal and is called the Bauschinger effect. In the Armstrong-Frederick model, a nonlinear saturation term is introduced to the Prager law, which allows for a better description of the stress-strain curve for cyclic loadings.

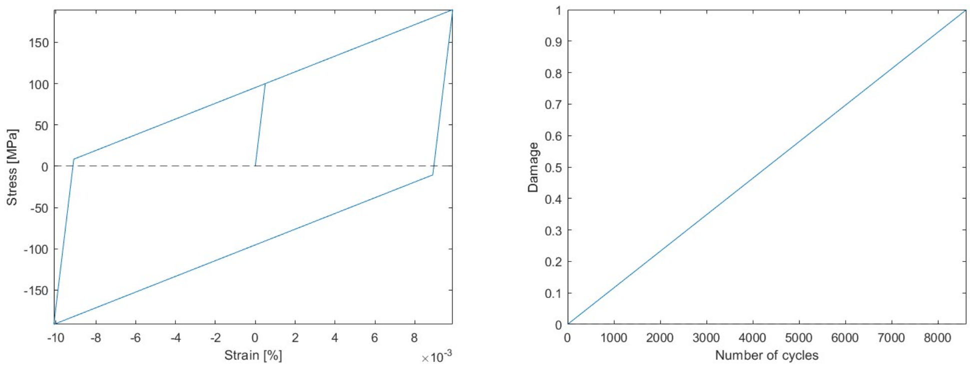
$$\dot{\beta} = \frac{2}{3} H^k \dot{\varepsilon}^p - \dot{\bar{\varepsilon}}^p b \beta \tag{2}$$

where:

- $\dot{\bar{\varepsilon}}^p$ is the equivalent plastic strain rate;
- b is a material constant;
- $-\dot{\bar{\varepsilon}}^p b \beta$ introduces the saturation effect in the kinematic hardening rule.

E	200000[MPa]
σ_y	100[MPa]
H^I	0[MPa]
H^K	10000[MPa]
b	0

Table 2: Linear kinematic hardening data



(a) Hysteresis curve (b) Damage evolution curve

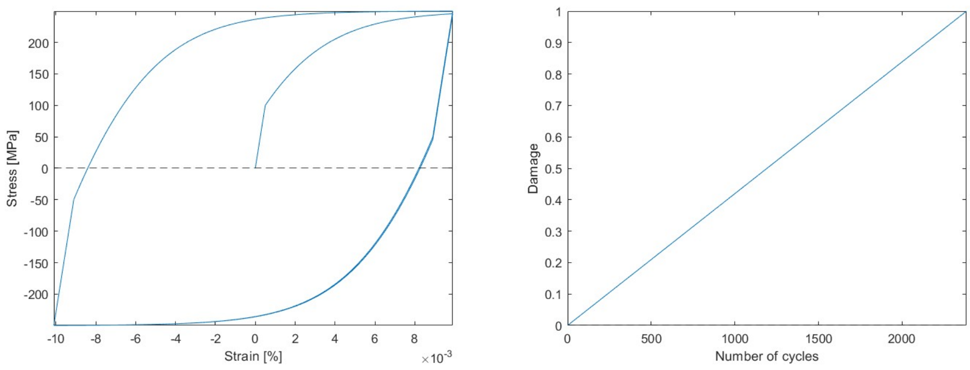
Figure 4: Linear kinematic hardening

For kinematic hardening it took 8618 cycles for the damage to reach 1 and fatigue failure to occur, showing a higher estimated life than in the case of isotropic hardening.

3. Non-linear Kinematic Hardening

E	200000[MPa]
σ_y	100[MPa]
H^I	0[MPa]
H^K	60000[MPa]
b	400

Table 3: Non-linear kinematic hardening data



(a) Hysteresis curve (b) Damage evolution curve

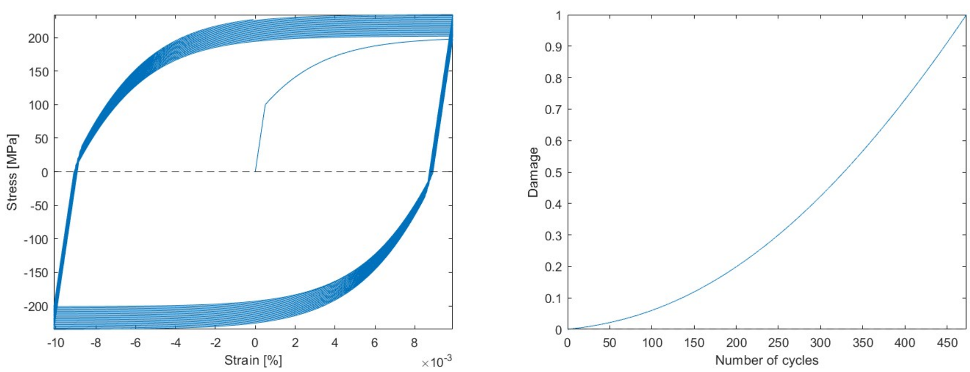
Figure 5: Non-linear kinematic hardening

With the introduction of non-linearity to kinematic hardening, 2385 cycles are required for fatigue failure to occur, representing a reduction in estimated life compared to linear isotropic hardening.

4. Non-linear Kinematic and Isotropic Hardening

E	200000[MPa]
σ_y	100[MPa]
H^I	50[MPa]
H^K	40000[MPa]
b	400

Table 4: Non-linear kinematic and isotropic hardening data



(a) Hysteresis curve (b) Damage evolution curve

Figure 6: Non-linear kinematic and isotropic hardening

For non-linear kinematic and isotropic hardening, 473 cycles are required for fatigue failure to occur, which exceeds the expected life for linear isotropic hardening but is still less than the expected life for linear and non-linear kinematic hardening.

3.2 Influence of average strain on fatigue life for linear kinematic hardening

E	200000[MPa]
σ_y	100[MPa]
H^I	0[MPa]
H^K	15000[MPa]
b	0

Table 5: Linear kinematic hardening data

1. Strain history under constant amplitude and zero mean strain

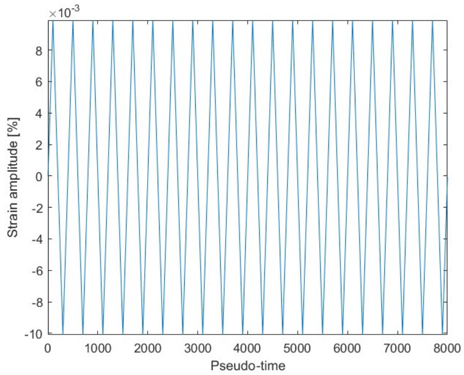
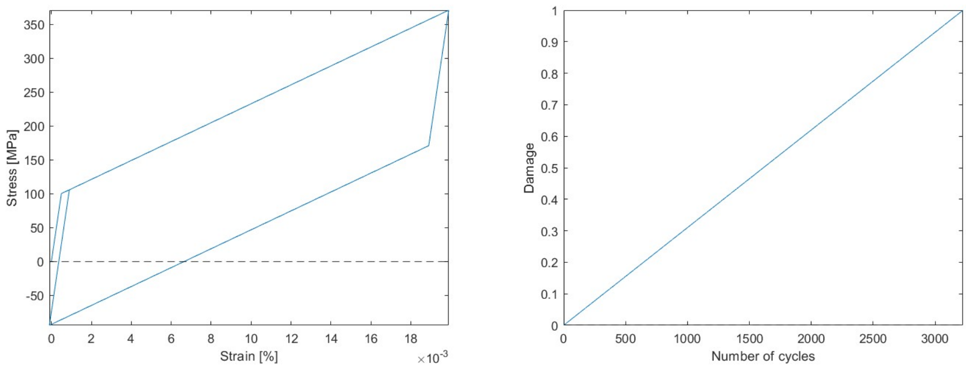


Figure 7: Strain history with constant amplitude and average strain equal to zero



(a) Hysteresis curve (b) Damage evolution curve

Figure 8: Linear kinematic hardening under constant amplitude loading and zero mean strain

For a loading under constant amplitude and mean strain equal to 0, 3225 cycles are required for the damage to reach 1 and fatigue failure to occur.

2. Strain history with constant amplitude and non-zero average strain

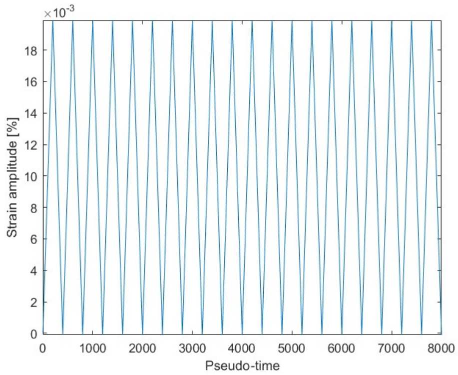
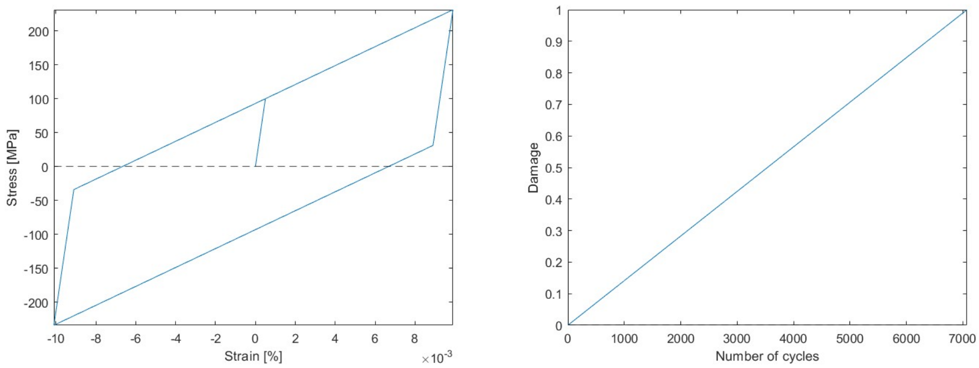


Figure 9: Cyclic history with constant amplitude and non-zero mean strain



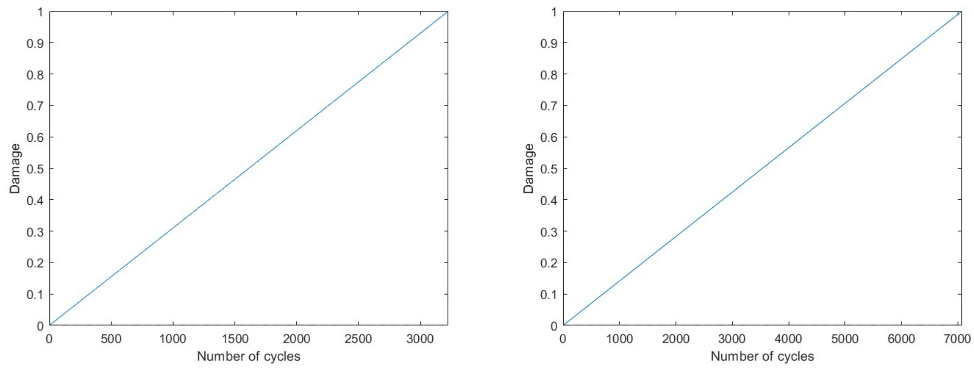
(a) Hysteresis curve (b) Damage evolution curve

Figure 10: Linear kinematic hardening under constant amplitude loading and non-zero mean strain

For a loading of constant amplitude and mean strain different from 0, 7073 cycles are required for the damage to reach 1 and fatigue failure to occur.

3. Comparison of damages

- 3225 cycles until fatigue failure occurs. - 7073 cycles until fatigue failure occurs.



(a) Zero mean strain. (b) Non-zero mean strain.

Figure 11: Damage evolution curve for linear kinematic hardening under constant amplitude loading

We can see that for a mean amplitude equal to zero the material resists a larger number of cycles while for a mean amplitude different from zero the material fails for a fewer number of cycles. This result is consistent with expectations since positive stress values are detrimental to the fatigue life of the material.

3.3 Influence of loading sequence on fatigue life for linear kinematic hardening

E	200000[MPa]
σ_y	100[MPa]
H^I	0[MPa]
H^K	10000[MPa]
b	0

Table 6: Linear kinematic hardening data

1. Strain history under increasing variable amplitude and zero mean strain

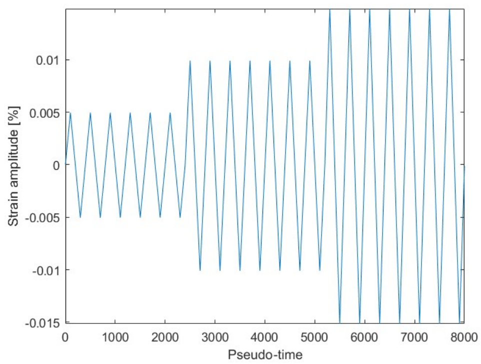
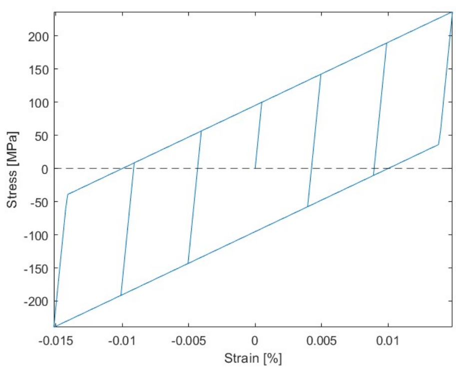
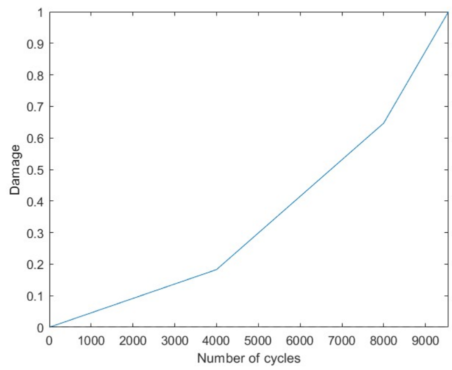


Figure 12: Cyclical history with increasing variable amplitude



(a) Hysteresis curve



(b) Damage evolution curve

Figure 13: Linear kinematic hardening under increasing variable amplitude loading

For a variable loading, increasing amplitude and mean strain equal to 0, 9550 cycles are required for the damage to reach 1 and fatigue failure to occur.

2. Strain history under decreasing variable amplitude and zero mean strain

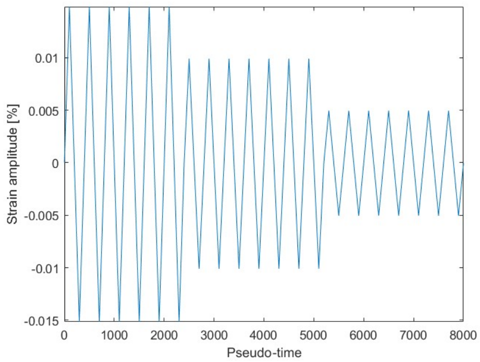
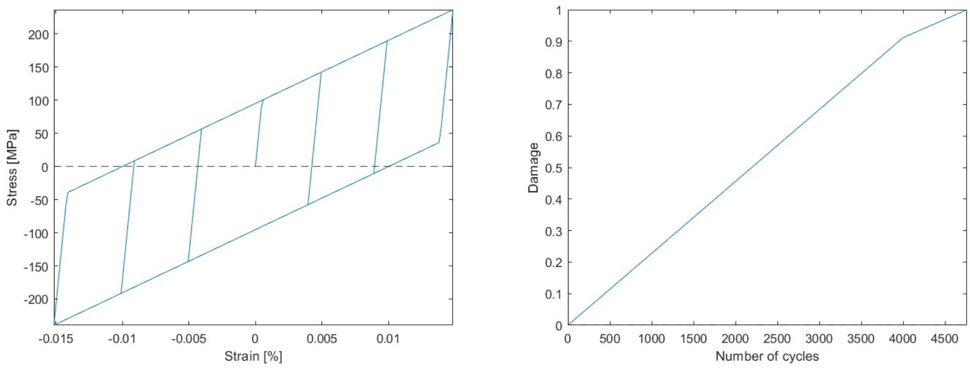


Figure 14: Cyclical history with decreasing variable amplitude



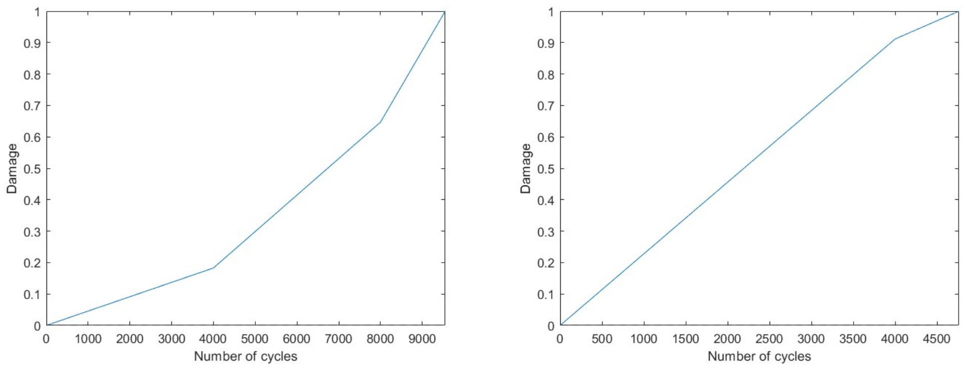
(a) Hysteresis curve (b) Damage evolution curve

Figure 15: Linear kinematic hardening under decreasing variable amplitude loading

For a variable loading, decreasing amplitude and mean strain equal to 0, 4762 cycles are required for the damage to reach 1 and fatigue failure to occur.

3. Comparison of damages

- 9550 cycles until fatigue failure occurs - 4762 cycles until fatigue failure occurs



(a) Increasing variable amplitude loading (b) Decreasing variable amplitude loading

Figure 16: Damage evolution curve for linear kinematic hardening

We can see that for an increasing variable strain amplitude the material withstands a higher number of loading cycles, while for a decreasing variable strain amplitude the material withstands a lower number of loading cycles, demonstrating the effect of the loading sequence.

3.4 Influence of random amplitude loading on fatigue life

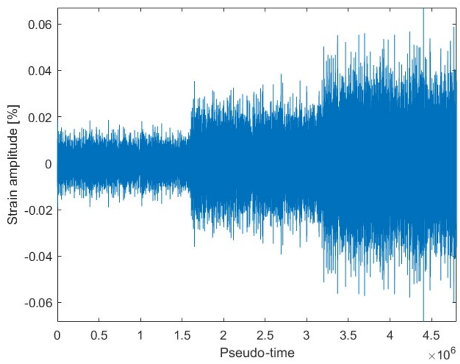
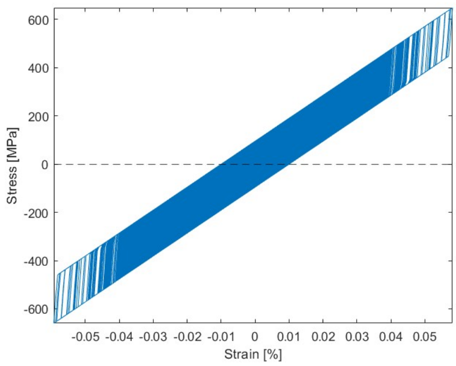


Figure 17: Cyclic history with random amplitude

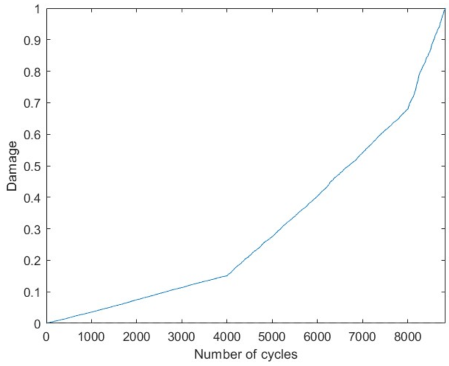
1. Linear kinematic hardening

E	200000[MPa]
σ_y	100[MPa]
H^I	0[MPa]
H^K	10000[MPa]
b	0

Table 7: Linear kinematic hardening data



(a) Hysteresis curve



(b) Damage evolution curve

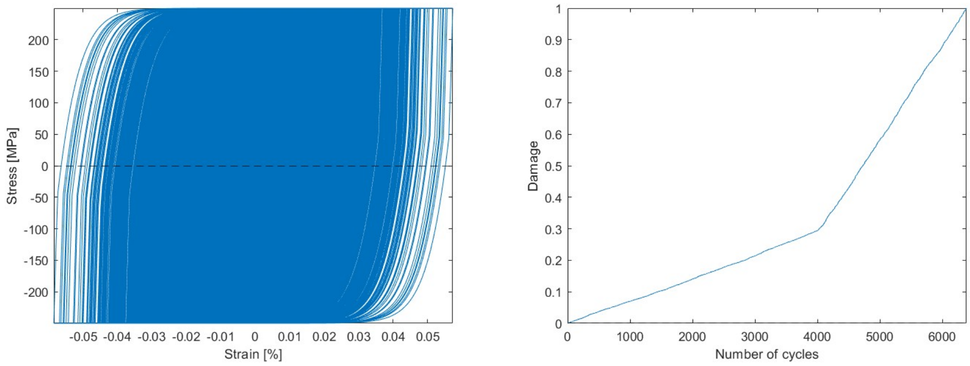
Figure 18: Linear kinematic hardening under random amplitude loading

For a linear kinematic hardening case under random amplitude loading, 8876 cycles are required for the damage to reach 1 and fatigue failure to occur.

2. Non-linear kinematic hardening

E	200000[MPa]
σ_y	100[MPa]
H^I	0
H^K	30000[MPa]
b	200

Table 8: Non-linear kinematic hardening data



(a) Hysteresis curve (b) Damage evolution curve

Figure 19: Non-linear kinematic hardening under random amplitude loading

For a case of non-linear kinematic hardening under random amplitude loading, 6334 cycles are required for the damage to reach 1 and fatigue failure to occur, presenting a shorter life than in the case of linear kinematic hardening.

4. Conclusion

In this work, a 1D algorithm with isotropic hardening and nonlinear kinematics was implemented to determine fatigue life through incremental damage, based on the Armstrong-Frederick approach. The model was validated by comparing the results with the literature, where the cyclic stress versus strain curves calculated by the model were able to reproduce the expected behavior even under non-zero mean stress loading histories. The robustness of the model was verified when considering loadings with variable and random amplitude, obtaining expected stress versus strain curve shapes. In addition, the Vaz Jr damage indicator was applied for different loadings and hardenings, being able to predict fatigue life through incremental damage and obtaining results compatible with the expected.

5. REFERENCES

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