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PASSIVE FAULT TOLERANT CONTROL ON QUADROTORS USING A ROBUST SLIDING MODE CONTROL

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Abstract. *Quadrotor UAVs are subject to different technical and operational constraints, thus, it is important to increase its technical reliability in critical areas to ensure a safe flight despite faults. Quadrotor hardware redundancy (in actuators and sensors) has been used extensively in order to solve reliability shortcomings last years but software improvement, especially, that associated to fault tolerant control laws needs more attention and research. This paper leads with the stabilization problem of an under-actuated quadrotor during the occurrence of actuator partial fault. The main idea of the proposed solution is to verify the robustness of a Sliding Mode Control (SMC) based control law called Conditional Integrator SMC (CISMC) in order to stabilize a quadrotor and allow a safe trajectory tracking by controlling quadrotor attitude and position. The effectiveness of the proposed Passive Fault Tolerant Control (PFTC) was tested by numerical simulations in the presence of partial loss of effectiveness (hard-over) of one actuator and compared to classical PID controller. Results demonstrated a better performance of CISMC when compared to PID in the presence of partial faults, with an improvement of at least* 28% *of more capacity to withstand failures.*

Keywords: *Quadrotor, Actuator Fault, Conditional Integrator Sliding Mode Control, Passive Fault Tolerant Control*

1. INTRODUCTION

Quadrotors are an emerging rotorcraft concept for unmanned aerial vehicle (UAV) platforms. Due to its specific capabilities, as a high flying flexibility, a small size, an agile maneuverability and an easy take-off and landing (Benrezki *et al.*, 2015), they have experienced dramatic developments in the last decades. Its potential applications are not limited only as individual vehicles but also as its use in multiple vehicle teams, including surveillance, search and rescue and mobile sensor networks.

Quadrotors present some challenges in terms of stability and control. They are inherently unstable and are subject to control design constraints, since its dynamic behavior is highly nonlinear, multi-variable, strongly coupled and is an under actuated system (less control variables than states to be controlled). Then, it is important to increase their technical reliability to ensure a safe flight in critical areas (urban zones) in the presence of disturbances and faults.

Due to a lack of motor redundancy in quadrotors, the complete loss of a rotor or even a partial failure of a motor results in a vehicle that is not fully controllable (Hamadi, 2020). One solution, adopted by multirotor manufacturers is to consider multirotors with additional motors like hexarotors and octorotors, as a result, the cost of the redundancy actuators is increased. In terms of control software, Fault Tolerant Control (FTC) strategies can be used to increase reliability and safety in multirotors helping to maintain the system's stability even when a partial or a complete fault occurs.

The actuator faults can be produced by a propeller or motor damage and can be manifested through the time in several ways, permanent or intermittent. The loss of effectiveness, if not corrected properly may have catastrophic consequences regarding human lives or economical impact. Its failure could occur after a progressive degradation of the motor caused by prolonged use or particles in the motor housing, Electronic Speed Controller (ESC) overheating or a sudden contact with part of the environment (Hamadi, 2020).

One solution to tackle with actuator faults without hardware redundancy is implementing control loops based on Fault-tolerant control laws. FTC systems may be regrouped into two main families: passive fault-tolerant controllers and active fault-tolerant controllers. An Active Fault-Tolerant Controller (AFTC) usually contains a separate module, a FDI (Fault Detection and Isolation) system that monitors the health of the aircraft, the FDI system informs a supervision module of the seriousness of the fault/failure or damage and based thereon, the supervision module may decide to reconfigure the flight controllers (Ducard, 2009), this systems are expensive and require the use of modern techniques in order to identify quickly the fault presence aiming an efficient controller correction. On the other hand, Passive FTC (PFTC) system can tolerate a predefined set of faults by using a specially-designed fixed controller (Sharifi *et al.*, 2010), that is, wait until a fault occurs and based on the deviations of the plant parameters compensate that deviations, in a cheaper way, using a fixed robust feedback controller.

Quadrotors using PFTC control systems need robust controllers to overcome all the previously mentioned difficulties and ensure a safe flight. In this paper, an Sliding Mode Controller (SMC) based technique was chosen in order to keep the quadrotor stability after the occurrence of an actuator fault. According to Merheb *et al.* (2015) robustness to modeling errors, parametric uncertainties, and external disturbances are inherent properties of sliding mode controllers and match with PFTC needs specially, in the presence of partial loss of an actuator because it can be treated as an external disturbance.

SMC has been used in the last decade as a PFTC control technique demonstrating good results. In Merheb *et al.* (2015) both regular and cascaded SMCs were used in order to stabilize the vehicle after the occurrence of two different types of faults: partial loss of motors' speed resulted from rusted shaft, and a partial loss in supply voltage of motors caused by power problems, the strategy demonstrated good fault tolerant properties. In Besnard *et al.* (2012) a combination of SMC controller and Sliding Mode Observer (SMO) is designed to provide robust position and attitude control of a quadrotor during mechanical failure and treated as external disturbance. Results demonstrated that the position and the attitude of the quadrotor are clearly affected by the disturbance but the controller estimated the disturbance and compensated it quickly. Other works also showed the ability of SMC control law to tackle with quadrotors faults like in Nguyen and Hong (2018), Khebbache *et al.* (2012) and Sharifi *et al.* (2010).

To the best of this paper authors' knowledge, as observed in the literature review, in most cases some authors do not consider control input position and/or rate saturation or simply do not show the control inputs generated by controllers which difficulties a fair comparison between the real performance of two or more of them. Quadcopter delays in actuators inputs can have a significant impact on the performance of control laws, particularly when using Sliding Mode Control (SMC) based controllers, the delayed information can cause the controller to respond to outdated data, resulting in an slow down of the system's response and exhibiting slower settling times. As a consequence, the controller may not be able to quickly adjust to changes as disturbances oscillations or faults in the quadcopter. Even in SMC control law, the control law designer can overestimate or underestimate the controller gains, due to the qualitative affirmations as: the gains must be taken "large enough" or "sufficiently small" used in stability demonstration theories leading to chattering or excess of control command.

In the present paper, the performance of a PFTC control law based in SMC called CISMC (Conditional Integrator Sliding Mode Control) will be used in order to guarantee the reference tracking of a quadrotor during a motor fault and its performance will be compared with the classical PID (Proportional Integral Derivative) controller, widely used in the most famous commercial quadrotor drones. The comparison will be done considering position and rate control inputs saturation, that it is crucial to consider the realistic operating conditions and actuator constraints in order to guarantee a fair comparison between the control laws. Some features that make the CISMC control law ideal for dealing with failures are: *(i)* due to its nonlinear nature, its design provides a convenient way to design controllers without gain scheduling (there is no need of linearization) (Diaz-Mendez *et al.*, 2022); *(ii)* when compared with ideal SMC, CISMC can guarantee that tracking error can be reduced to zero; *(iii)* The used of a saturation function instead sign (switching) function reduces chattering in control input and *(iv)* the transient response can be enhanced due to its anti-windup action, specially, during abrupt changes during a faults scenario. A more detail of the CISMC structure will be done later.

Among the main contributions of this work are: *(i)* the use of the CISMC control law and its advantages in a PFTC strategy, with scarce or even non-existent previous works approaching it, and *(ii)* the fair comparison with PID, through of the saturation deployment in position and rates of the control input.

2. QUADROTOR MODEL

Quadrotor unmanned aerial vehicles are rotary wing aircraft equipped with four rotors to maneuver in three-dimensional space. With its set of four actuators, a quadcopter has the capability to ascend or descend (along the z-axis), execute roll maneuvers (rotation around the x-axis), pitch maneuvers (rotation around the y-axis), and yaw maneuvers (rotation around the z-axis). They are classified as under-actuated control systems due to have less control input than states to be controlled. Figure 1 (extracted from Freddi *et al.* (2011)) shows the schematic of the quadrotor along with its reference coordinate systems called body reference frame ($R_B = [x_B, y_B, z_B]$) whose origin coincides with the drone center of gravity and Earth reference frame fixed on the ground ($R = [x, y, z]$). As shown in Figure 1, the front and rear motors (M_1 and M_3) spin in the counter-clockwise direction with angular velocities ω_1 and ω_3, while the other two rotors (M_2 and M_4) spin in the clockwise direction with angular velocities ω_2 and ω_4.

The mathematical model adopted in this work is based on the following assumptions (Freddi *et al.*, 2011): *(i)* Quadrotor frame is symmetrical; *(ii)* Quadrotor body and propeller are rigid; *(iii)* The flexibility of the blade is relatively small and can be neglected and *(iv)* Drag is supposed to be linear, thus obeying Stoke's law. It is important to highlight that in some papers is assumed that the motors dynamic is relatively fast and generally is neglected, in this work the actuators position and rates are limited in order to make a more realistic and fair comparison between controllers response.

The quadcopter model was extracted from Labbadi *et al.* (2018) and Labbadi and Cherkaoui (2019) and previously adopted by one of the authors of this paper in Diaz-Mendez *et al.* (2022). It consists of six second order differential equations as presented in Eq. (1).

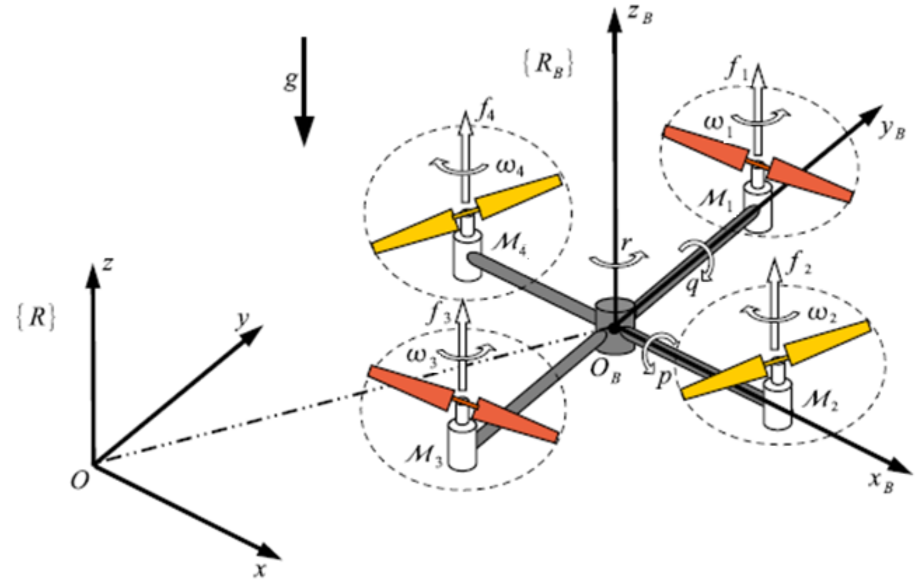

Figure 1. Quadrotor scheme (Freddi *et al.*, 2011).

$$\begin{cases} \ddot{\phi} = \dot{\theta}\dot{\psi}\frac{(I_y - I_z)}{I_x} - \frac{J_r}{I_x}\Omega_r\dot{\theta} - \frac{k_{fax}}{I_x}\dot{\phi}^2 + \frac{1}{I_x}\tau_\phi \\ \ddot{\theta} = \dot{\phi}\dot{\psi}\frac{(I_z - I_x)}{I_y} + \frac{J_r}{I_y}\Omega_r\dot{\phi} - \frac{k_{fay}}{I_y}\dot{\theta}^2 + \frac{1}{I_y}\tau_\theta \\ \ddot{\psi} = \dot{\phi}\dot{\theta}\frac{(I_x - I_y)}{I_z} - \frac{k_{faz}}{I_z}\dot{\psi}^2 + \frac{1}{I_z}\tau_\psi \\ \ddot{x} = -\frac{k_x}{m}\dot{x} + (cos\phi sin\theta cos\psi + sin\phi sin\psi)\frac{F}{m} \\ \ddot{y} = -\frac{k_y}{m}\dot{y} + (cos\phi sin\theta sin\psi - sin\phi cos\psi)\frac{F}{m} \\ \ddot{z} = -\frac{k_z}{m}\dot{z} - g + (cos\phi cos\theta)\frac{F}{m} \end{cases} \tag{1}$$

Where m is the total mass of the quadrotor, J_r is the rotor inertia, g the earth gravity acceleration and $[Ixx, Iyy, Izz]$ are the quadrotor inertia moments around its body reference axis x_B, y_B, z_B respectively. $\Omega_r = \omega_1 - \omega_2 + \omega_3 - \omega_4$ indicates yaw imbalance when $\Omega_r \neq 0$ and $[K_{fax}, K_{fay}, K_{faz}]$ and $[K_x, K_y, K_z]$ represent rotation and translation aerodynamic drag coefficients. The system states are defined as: $[\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, x, \dot{x}, y, \dot{y}, z, \dot{z}]$ being $[\phi, \theta, \psi]$ the rotation (Euler) angles and $[x, y, z]$ the quadrotor position in relation to the earth's reference system. The control inputs are: total force F and the torques applied to the quadrotor τ_ϕ, τ_θ and τ_ψ. Due to the plus-type frame quadrotor configuration, forces and moments can be written according to the motor speeds as follows:

$$\begin{bmatrix} F \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & -lb & 0 & lb \\ -lb & 0 & lb & 0 \\ d & -d & d & -d \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \tag{2}$$

Where ω_i is the angular speed of motors $i = [1, 2, 3, 4]$, b and d are the thrust and torque constants of the rotors that depend on the propeller geometry and air density.

Doing the change of variables: $X = [\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, x, \dot{x}, y, \dot{y}, z, \dot{z}]$ where X is the state vector $X \in R^n$ with $n = 12$ and $u = [F, \tau_\phi, \tau_\theta, \tau_\psi]$ as the control input vector $u \in R^n$ with $n = 4$ the original second order of six equations in Eq. (1) is transformed to the first order, twelve equations system in Eq. (3).

$$\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = h_1 x_4 x_6 + h_2 x_4 + h_3 x_2^2 + \overline{h}_1 u_2 \\ \dot{x_3} = x_4 \\ \dot{x_4} = h_4 x_2 x_6 + h_5 x_2 + h_6 x_4^2 + \overline{h}_2 u_3 \\ \dot{x_5} = x_6 \\ \dot{x_6} = h_7 x_2 x_4 + h_8 x_6^2 + \overline{h}_3 u_4 \\ \dot{x_7} = x_8 \\ \dot{x_8} = h_9 x_8 + \frac{1}{m} u_x u_1 \\ \dot{x_9} = x_{10} \\ \dot{x_{10}} = h_{10} x_{10} + \frac{1}{m} u_y u_1 \\ \dot{x_{11}} = x_{12} \\ \dot{x_{12}} = h_{11} x_{12} - g + \frac{1}{m}(cos x_1 cos x_3) u_1 \end{cases} \tag{3}$$

Where the coefficients $h_i, (i = 1, ..., 11)$ and $\bar{h_j}, (j = 1, ..., 3)$ are as described in Table 1.

Table 1. h_i and $\bar{h_j}$ coefficients definition.

$h_1 = \frac{I_y - I_z}{I_x}$	$h_2 = -\frac{\Omega_r J_r}{I_x}$	$h_3 = -\frac{k_{fax}}{I_x}$
$h_4 = \frac{I_z - I_x}{I_y}$	$h_5 = \frac{\Omega_r J_r}{I_y}$	$h_6 = -\frac{k_{fay}}{I_y}$
$h_7 = \frac{I_x - I_y}{I_z}$	$h_8 = -\frac{k_{faz}}{I_z}$	$h_9 = -\frac{k_x}{m}$
$h_{10} = -\frac{k_y}{m}$	$h_{11} = -\frac{k_z}{m}$	$\overline{h}_1 = \frac{1}{I_x}$
$\overline{h}_2 = \frac{1}{I_y}$	$\overline{h}_3 = \frac{1}{I_z}$	

The type of fault injected on the quadrotor actuator is of the type "Hard-Over" (see Fig. 2) which is manifested when the actuator reaches a saturation position (in our case, not necessarily will stay at that position but is limited to that value). Mechanical failures as partial loss of propellers or some motor failures may be represented by hard-over. In terms of numerical simulations, the control effectiveness matrix (Eq. 2) was inverted (as in Merheb (2016)) and a maximum ω_i^2 limit was defined in one motor. The corresponding $\omega_{i_{max}}$ (δ_{max} in Fig. 2) is defined according to the maximum rotor thrust F_i given by $F_{i_{max}} = b\omega_{i_{max}}^2$.

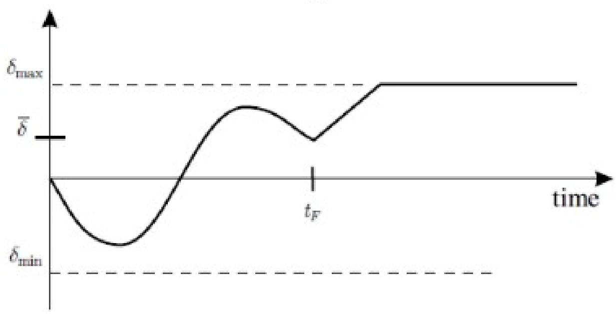

Figure 2. Hard-Over Fault (Merheb, 2016).

3. CISMC CONTROLLER DESIGN

In order to solve the under-actuated control problem, two virtual control inputs u_x and u_y (see Eq. 3) are independently designed. According to Diaz-Mendez *et al.* (2022), the virtual control inputs u_x and u_y are the control input result of the position $[x, y]$ CISMC controllers and are used to generate the desired Euler angles (ϕ_d and θ_d) for attitude controllers. To do this, the control inputs $u_x = (cos\phi sin\theta cos\psi + sin\phi sin\psi)$ and $u_y = (cos\phi sin\theta sin\psi - sin\phi cos\psi)$ need to be inverted to obtain the Euler angle commands that will be used as reference for the attitude subsystem allowing roll and pitch channels to be decoupled. The inverted system is defined as follows:

$$\begin{cases} x_{1d} = arcsin(u_x sin x_{5d} - u_y cos x_{5d}) \\ x_{3d} = arcsin(\frac{u_x cos x_{5d} + u_y sin x_{5d}}{cos x_{1d}}) \end{cases} \tag{4}$$

The Conditional Integrator Sliding Mode Control (CISMC), as mentioned before, is robust, and easier to implement when compared to other nonlinear techniques. It was created by Seshagiri and Khalil (2005) and presented in a simple form called Universal Integral Regulator whose design is similar to PID with some improvements but keeping the SMC advantages. CISMC robustness has been demonstrated in flight control systems by authors of the present work (Díaz-Méndez *et al.*, 2019; Diaz-Mendez *et al.*, 2022), and by other researchers like Nguyen and Damm (2013); Burger *et al.* (2009); Seshagiri and Promtun (2008). Despite that efforts, the use of CISMC in quadrotors in the presence of actuator faults is scarce or inexistent. The present work novelty aims to demonstrate its potential use in FTC strategies.

The CISMC will be implemented, basically, in two control loops, the outer one controls the position and altitude for the quadrotor while the inner control loop is responsible of stabilizing the attitude. One independent CISMC sliding surface is used for each control variable (state) to be controlled. The sliding surface of the CISMC is defined in Eq. (5).

$$s_i = k_0^i \sigma_i + \sum_{j=1}^{\rho_i - 1} k_j^i e_j^i + e_{\rho_i}^i \tag{5}$$

Where e_j^i is the tracking error for the i-th output, ρ^i the relative degree of the system (number of the necessary derivatives of the output to find a direct relation with the corresponding input). The positive constants $k_1^i, ..., k_{\rho_i - 1}^i$ are chosen such that the polynomial $\lambda^{\rho_i - 1} + k_{\rho_i - 1}^i \lambda^{\rho_i - 2} + ... + k_1^i$ is Hurwitz, which guarantees that in the sliding phase ($s_i = 0$) the tracking error and its derivatives will converge to zero. The σ_i state is the output of the conditional integrator defined in Eq. (6).

$$\sigma_i = -k_0^i \sigma_i + \mu_i sat(s_i/\mu_i) \tag{6}$$

With $k_0^i > 0$ and μ_i a sufficiently small positive parameter. μ_i is known as boundary layer and results from the continuous approximation of the sign function by using a saturation function. The conditional integrator defined in Eq. (6) provides integral action only inside the boundary layer. Finally, the CISMC control law is defined through the application of the equivalent control method (u_i) and the continuous approximation of ideal SMC given by v_i, the last one is designed to handle the uncertain terms in the resulting expression for $\dot{s_i}$. The CISMC control input can be written as in Eq. (7).

$$\begin{aligned} u_i &= g(e_1^i, e_2^i)^{-1}[-f(e_1^i, e_2^i) + v_i] \\ v_i &= -K_i sat(s_i/\mu_i) \end{aligned} \tag{7}$$

Where $e_2^i = \dot{e}_1^i$ and $f(\cdot)$ and $g(\cdot)$ are assumed to be smooth functions. It is important to highlight that the quadrotor governing equations (Eq. 3) can be written in the form: $\dot{x} = f(x) + g(x)u$ with $g(\cdot)$ invertible for all e_j^i and non-singular for all $x_i \in E$. At the specific case of quadrotor $g = [\bar{h}_1, \bar{h}_2, \bar{h}_3, u_1/m, u_1/m, cos(x_1)cos(x_3)/m]^T$, then, due to quadrotor mass and inertia are constants and different to zero and u_1 be zero only at rest, the only possible singularity appears when $cos(x_{1,3}) = 0$, that is, for $\phi = \pm\pi/2$ or $\theta = \pm\pi/2$. As consequence, in order to ensure the non-singularity condition for g is recommended to avoid aggressive flight maneuvers, in other words, $x_i \in E\{x_{1,3} \leq \pm\pi/2\}$.

The CISMC gains tuning, in this work is done by a simple "trial and error" process. As mentioned before, the constants k_0^i and K_j^i must be positive to guarantee conditional integrator output and sliding surface convergence. K_i can be chosen sufficiently large (limited by actuator maximum limit) and μ_i small enough (large values reduce tracking precision and small ones increase chattering). The PID controller used for comparison purposes is defined as follows (Benrezki *et al.*, 2015) (see Eq. 8).

$$u_i = k_P^i e_1^i + k_I^i \int e_1^i + k_D^i e_2^i \tag{8}$$

Where $e_2 = \dot{e_1}$ and k_P^i,k_I^i, k_D^i the PID gains.

4. SIMULATIONS AND RESULTS

In order to verify the performance of the proposed CISMC controller, a trajectory tracking problem is simulated. The physical properties, aerodynamic and inertia parameters of the quadrotor are presented in Table 2. The reference trajectory to be tracked is a closed square path of $1.5m$ long sides at $2m$ height as shown in Table 3 while $\psi_d = 0$ during the whole simulation time.

Table 2. Quadrotor Parameters. Extracted from Labbadi and Cherkaoui (2019).

Parameters	Value	Parameter	Value
g (m/s^2)	9.81	k$_y (N \cdot s/m)$	5.567 $\times 10^{-4}$
m (kg)	0.486	k$_z (N \cdot s/m)$	5.567 $\times 10^{-4}$
I$_x (kg \cdot m^2)$	3.827 $\times 10^{-3}$	k$_{fax} (N \cdot s/m)$	5.567 $\times 10^{-4}$
I$_y (kg \cdot m^2)$	3.827 $\times 10^{-3}$	k$_{fay} (N \cdot s/m)$	5.567 $\times 10^{-4}$
I$_z (kg \cdot m^2)$	7.656 $\times 10^{-3}$	k$_{faz} (N \cdot s/m)$	5.567 $\times 10^{-4}$
I$_r (kg \cdot m^2)$	2.838 $\times 10^{-5}$	b (N$\cdot s^2$)	2.984 $\times 10^{-3}$
k$_x (N \cdot s/m)$	5.567 $\times 10^{-4}$	d (N $\cdot$s^2)	3.232 $\times 10^{-2}$

Table 3. Quadrotor position reference trajectories .

Variables	Value	Time (s)
	$[0, 0, 0]$	0
	$[1.5, 0, 2]$	10
[x$_d, y_d, z_d](m)$	$[1.5, 1.5, 2]$	20
	$[0, 1.5, 2]$	30
	$[0, 0, 2]$	40 - end time

In order to do a fair and balance comparison between CISMC and PID controllers, the control inputs $u_i = [F, \tau_\phi, \tau_\theta, \tau_\psi]$ were passed by a filter that saturates their maximum positions and rates $\dot{u}_i^{sat} = -\tau u_i^{sat} + \tau u_i$. The pre-established limits were: $F^{max} = 2mg(N)$, $|\tau_{\phi,\theta}| \leq mg(Nm)$, $|\tau_\psi| \leq 2d[\omega_i^2]^{max}(Nm)$, $\dot{F}^{max} = 20F^{max}(N/s)$, $|\dot{\tau}_{\phi,\theta}| \leq 10mg(Nm/s)$ and $|\dot{\tau}_\psi| \leq 60d[\omega_i^2]^{max}(Nm/s)$ being $[\omega_i^2]^{max} = 800$.

In a first case, the controllers performance has been tested first in a "No fault" condition, the tuned controller gains are summarized at Table 4 and will be used in all simulations.

Table 4. CISMC and PID Control System Parameters and Gains .

CISMC	Value	PID	Value
k_0^x, k_1^x, K_x, μ_x	1, 2, -5, 15	k_P^x, k_I^x, k_D^x	2,0,1
k_0^y, k_1^y, K_y, μ_y	1, 2, -5, 15	k_P^y, k_I^y, k_D^y	2,0,1
k_0^z, k_1^z, K_z, μ_z	1, 1, 30, 1	k_P^z, k_I^z, k_D^z	40, 1, 20
k_0^ϕ, k_1^ϕ, K_ϕ, μ_ϕ	1, 3, 12, 1	k_P^ϕ, k_I^ϕ, k_D^ϕ	0.03, 0, 0.005
k_0^θ, k_1^θ, K_θ, μ_θ	1, 3, 12, 1	k_P^θ, k_I^θ, k_D^θ	20,12,2,0.003
k_0^ψ, k_1^ψ, K_ψ, μ_ψ	1, 20, 10, 1	k_P^ψ, k_I^ψ, k_D^ψ	5, 0.005, 1

Figure 3 shows the trajectory tracking response, it is possible to note that CISMC controller allows a more accurate tracking without large rotations. Figures 4 and 5 illustrate the control inputs necessary to execute the manuevers, demonstrating again, the less control amplitude demmanded by CISMC when compared to classical PID and less chattering tendency.

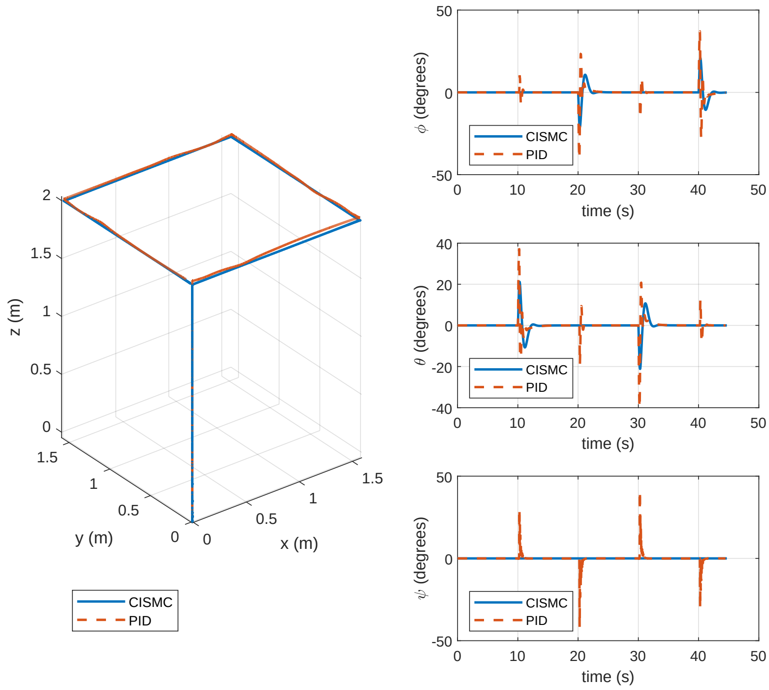

Figure 3. Quadrotor states response in "No Fault" condition.

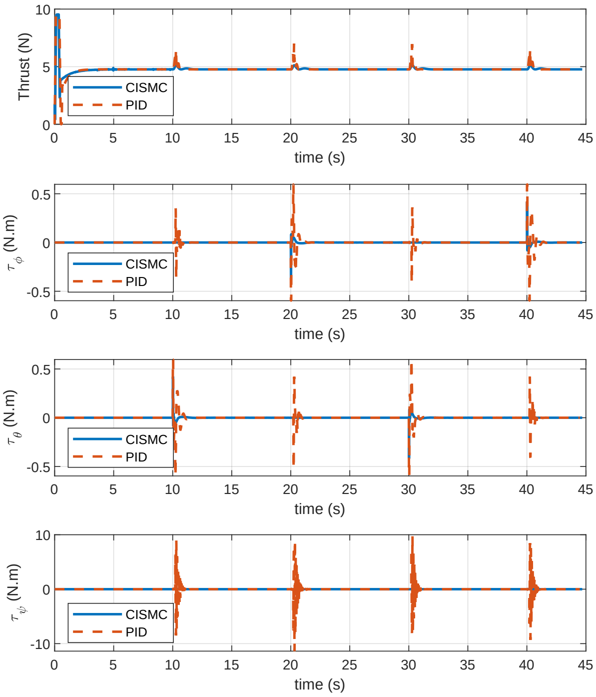

Figure 4. Quadrotor control input response in "No Fault" condition.

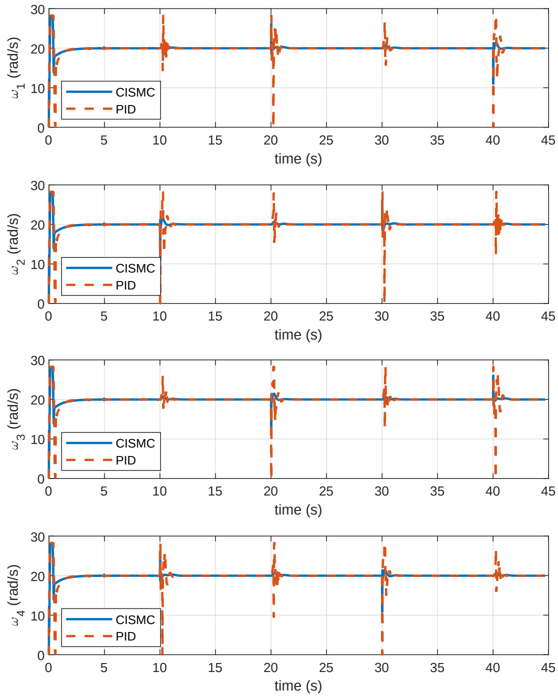

Figure 5. Quadrotor rpm saturation response in "No Fault" condition.

In a second case, a hard-over fault was injected in motor one limiting the maximum rotation to $\omega_i^2 = (1-\gamma)[\omega_i^2]^{max}$ where $\gamma = 0$ indicates "No Fault" and $\gamma = 1$ total rotor fault. It was simulated a partial fault $\gamma = 0.15$ starting at $t = 10s$. For reason of comparison, results of CISMC and PID controllers have been plotted. in Figure 6 it easy to see that PID controller was unable to ensure the stability of the drone leading it to crash. The cause was expected due to yaw controller saturation. Yaw imbalance is manifested by the continuous rotation of the quadrotor around its vertical axis. On the other hand, the CISMC was able to successfully complete the trajectory tracking. Figures 7 and 8 demonstrate the CISMC inherent capacity of rejecting disturbances, in this case, represented by the motor 1 hard-over fault (See red rectangle in Fig. 8 starting at $t = 10s$).

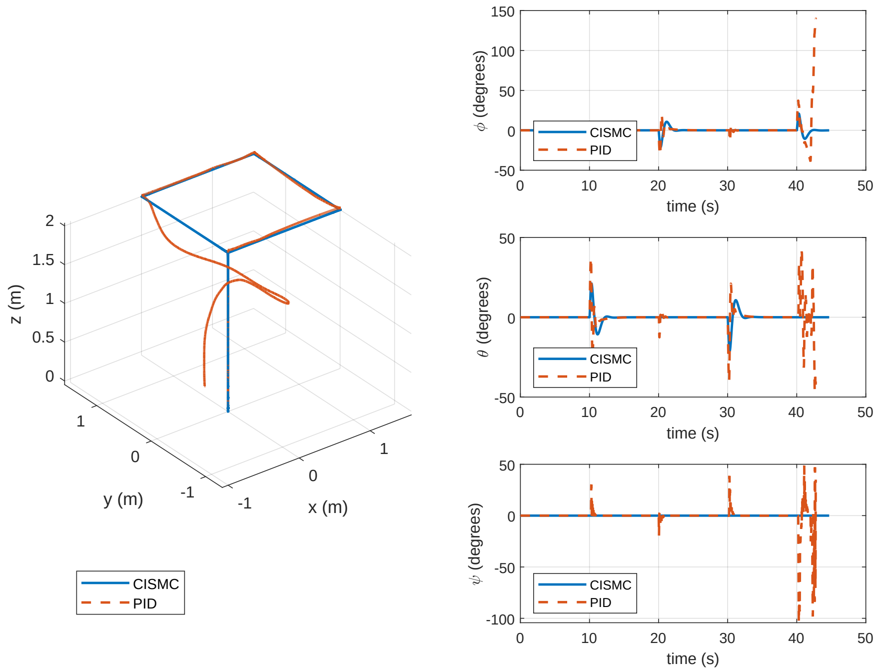

Figure 6. Quadrotor states response in "Partial Fault" condition.

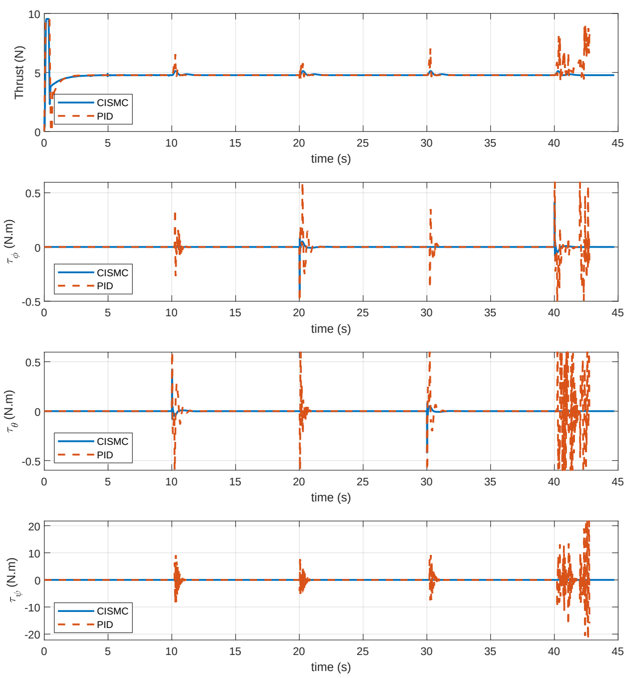

Figure 7. Quadrotor control input response in "Partial Fault" condition.

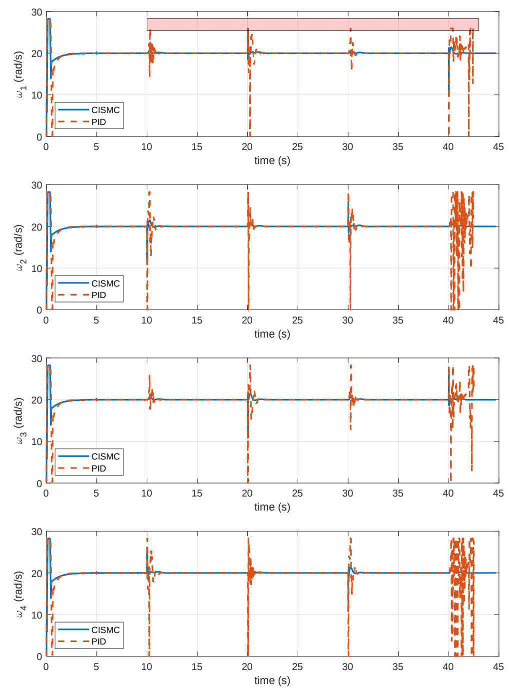

Figure 8. Quadrotor rpm saturation response in "Partial Fault" condition.

Finally, the partial fault percentage γ was increased in order to verify the maximum capacity of CISMC to tackle with fault in one motor. Figure 9 illustrates the result. It was verified that even with a reduction of 43% of maximum rpm of a quadrotor motor, the CISMC is able to guarantee stability and avoid a crash. With $\gamma = 0.44$ and after $15s$ of fault injection, Figure 11 reveals that motors 3 and 4 accelerated until reach its maximum value and motor 2 reduced its value to zero in order to compensate the partial fault of motor 1 and the unavoidable yaw inbalance. As a consequence, it is confirmed at Figure 10, that, almost at the same time, the total thrust of quadcopter saturated causing the drone drop.

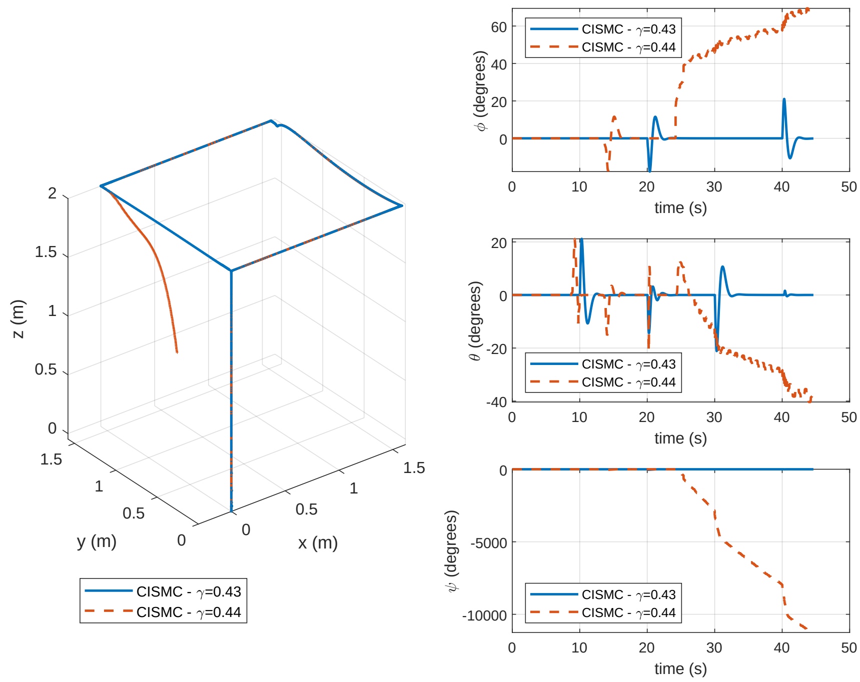

Figure 9. CISMC tracking response before and after maximum partial fault.

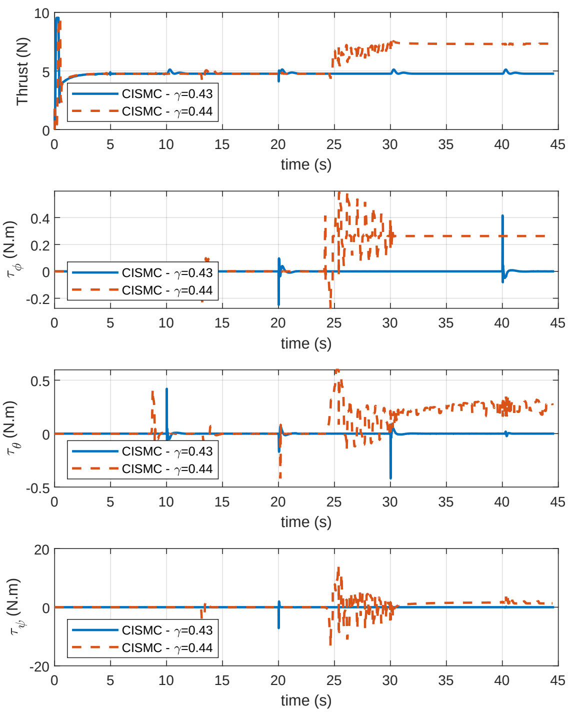

Figure 10. CISMC Quadrotor control input response before and after maximum partial fault.

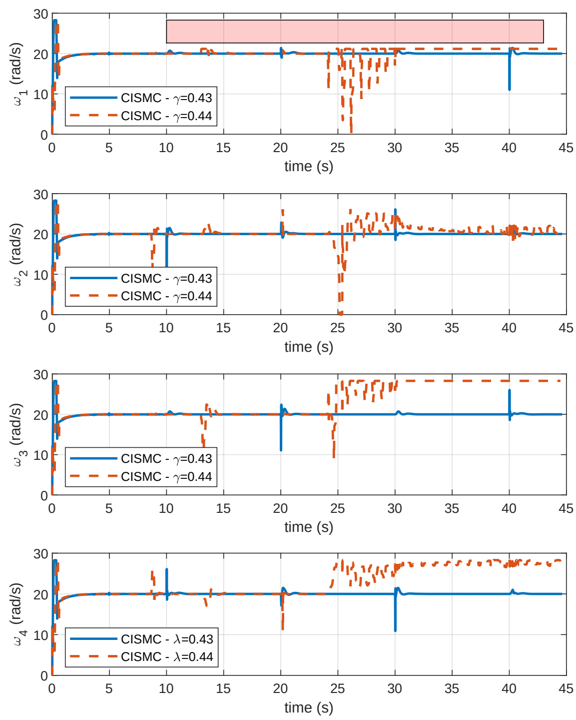

Figure 11. CISMC Quadrotor rpm saturation response before and after maximum partial fault.

5. CONCLUSIONS

In this paper, it was proposed the use of the robust CISMC control law as one of the first attempts of its use in PFTC strategy applied in quadcopters during actuator faults. The quadrotor dynamical model and the type of actuator fault injected were presented. Secondly, we presented the basic formulation of a CISMC control law. The simulation results have shown high efficiency of this control strategy when compared to classical PID. Incorporation of actuator limits and saturation into the evaluation of the control laws provided a more accurate representation of their performance in real-world scenarios and a fair comparison. CISMC showed to keep at same time the stability and the performance of quadrotor during a malfunction of one actuator and demonstrated to tackle well with at least 28% of more loss of effectiveness than PID controller. The numerical simulations showed that yaw controller saturates while trying to avoid yaw imbalance and causing the drone crash, it encourages the use of AFTC strategies in order to reconfigure the control laws and increase the fault rejection capacity.

6. ACKNOWLEDGEMENTS

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