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MODELING THE MULTI-LIFT UAV FORMATION

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Abstract. *The multirotor configuration of Unmanned Aerial Vehicles (UAVs) has increased relevance due to its simplicity to deploy for various applications. In particular for aerial transportation missions, the use of a single UAV is usually limited by its low endurance and small payload capability. On the other hand, using a formation flight with multiple vehicles collaboratively transporting a suspended payload, the weight is distributed such that each vehicle operates within adequate conditions. This work presents the dynamics of this multi-lift system in terms of the non-generalized coordinates used to describe the system. The differential and algebraic equations are transformed into an equivalent problem of solving a system of ordinary differential equations using an index-reduction procedure followed by a constraint stabilization procedure to result in a stable integration. A leader-follower controller using virtual constraints is implemented to control the formation. By modifying the follower outer-loop to receive the leader configuration as a reference and based on the desired formation geometry, the followers can define their attitude reference for their inner-loop. The presented results show that this is an interesting approach for modeling flight formations and allows one to design formation controllers that achieve the desired mission.*

Keywords: UAV, Formation, Multi-lift, Dynamics, Control

1. INTRODUCTION

Unmanned Aerial Vehicles (UAV) are a type of aerial robot used in different applications unsuited for tripulated missions. For instance, UAVs can be deployed in dangerous environments and transmit information to a ground station located in a safe region. Usually, in a mission involving a UAV, the task is normally achieved by the payload that is being transported (e.g., camera, crop-spraying system, medical supplies), in which the UAV is used as a tool for positioning the payload accurately in a specified position or through a specified trajectory in space. Particularly, the multirotor UAV configuration has gained attention due to its ingenious design using the variable angular velocity from its rotors for both propulsion and control, which simplifies the mechanical complexity and reduces overall costs in comparison with conventional configurations such as the helicopter. However, the requirement of changing the angular velocity from the multirotor propulsion system also limits their use to applications involving the transportation of small masses by low distances, since a heavier payload requires the UAV to provide more thrust which may saturate its propulsion system and destabilize the vehicle. An interesting alternative to address this limitation is the use of multiple UAVs flying in a formation.

In the application of transporting payloads using a formation flight, known as multi-lift system, the multiple UAVs are usually connected to the payload using a suspension system (Geng and Langelaan, 2020). By distributing the payload weight such that the constraint force balanced by each UAV is within its admissible envelope, then the transportation of heavier payloads is feasible. On the other hand, managing the flight of multiple vehicles simultaneously poses an additional challenge for both remotely piloting or autonomous controllers, since the multi-lift configuration requires a non-zero attitude for statically balance the UAVs depending on the geometry of the attachment (Berrios *et al.*, 2014). Thus, developing algorithms that extends the capabilities of the UAVs to include an autonomous flight mode is essential for operating a multi-lift formation. Therefore, developing models that accurately describes the dynamics of the multi-lift formation correspond to a first step to design multi-lift formation controllers.

However, obtaining the equation of motion from the multi-lift formation requires further considerations, since the suspension system couple the coordinates through constraint equations which must be satisfied during all instants of time. The model considered in Bisgaard *et al.* (2009) using cables to suspend the payload introduces unilateral constraints to the system which requires additionally modeling of the impact when the cables transitions between the slack and

tightened states (Stewart, 2000). Usually, a mechanical system is described in a higher-dimensional space and constraints (e.g., rotational, sliding, spherical joints) are included for each specific problem (Marsden and Ratiu, 1999). Then, the system dynamics is projected onto a lower-dimensional space using generalized coordinates that identically satisfy the constraint equations (Goldstein, 2002). This process is common for systems with simple parameterization such as the single-lift system where one can choose spherical coordinates to parameterize the payload position (Sadr *et al.*, 2014; Silva *et al.*, 2021). However, there is a non-trivial task involved for finding a suitable transformation if more than one vehicle is considered. A different approach using non-generalized coordinates is employed in this work, where the constraint equations are included through Lagrange undetermined multipliers in a multi-body system approach.

Additionally, if the payload orientation during the flight is important for a particular mission, such as in the assembly of structures using UAVs as aerial-crane, then, it is necessary to model the payload as a body, and the simplest representation that account for this feature is the rigid-body model. The assumption of a rigid-body consists that the relative distance between any two of its particles is constant. By prescribing the position from three attachment points, the payload configuration is fully determined, which implies that if three vehicles at least are considered to define a multi-lift formation, then it is possible to move the payload to arbitrary configurations in space.

For managing the flight of the UAVs simultaneously, a formation controller based on a leader-follower strategy with virtual-constraints defining an attractive hyper-surface in the configuration space is designed. The controller stabilize the followers in a desired position with respect to the leader position (Ihle *et al.*, 2006). The leader have its control strategy unaltered and it is responsible of communicating with ground station and tracking the desired mission trajectory while simultaneously contributing to transport the suspended payload.

The remaining from this work is described by: Section 2 details the system under consideration; Section 3 introduces the equation of motion from the multi-lift system; Section 4 presents the proposed controller for this formation; Section 5 presents the numerical results of a multi-lift system transporting a heavy payload and Section 6 introduces the final conclusions from the multi-lift system.

2. SYSTEM DESCRIPTION

The multi-lift formation is composed by n UAVs, $\mathcal{V}_i, i = 1, \dots, n$, and a payload \mathcal{P} . Each UAV is connected to the payload by a rigid rod with one end attached to the UAV center-of-mass and the other one connected to the payload at an arbitrary position. The attachment consists of ideal spherical joints which constraints the attachment position of each body to be at the same position and with arbitrary direction. Figure 1 shows four illustrative UAVs transporting a rigid-body payload connected to the rigid rods. The inertial coordinate system is defined by the origin and the orthonormal basis $\mathcal{I} : (O, i, j, k)$, and attached to the center of mass of the i th UAV, $\mathbf{r}_i(t) = \{x_i(t) \ y_i(t) \ z_i(t)\}^\top$, with $i = 1, \dots, n$, there is a body coordinate frame \mathcal{B}_i used to describe the UAV attitude in terms of the Euler angles $\boldsymbol{\eta}_i(t) = \{\phi_i(t) \ \theta_i(t) \ \psi_i(t)\}^\top$ with respect to the inertial coordinate system. The coordinate vector for the i -th UAV is $\mathbf{q}_i(t) = \{\mathbf{r}_i^\top(t) \ \boldsymbol{\eta}_i^\top(t)\}^\top$.

The payload consists of a rigid-body with the trajectory of the center of mass parameterized by the time t and denoted by $\mathbf{r}_p(t)$. There is a payload coordinate system \mathcal{B} attached to the center of mass which rotates solidarity with the body, whose orientation with respect to the inertial coordinate system is parameterized using Euler angles $\boldsymbol{\eta}_p(t) = \{\phi_p(t) \ \theta_p(t) \ \psi_p(t)\}^\top$, in which $\phi_p(t)$, $\theta_p(t)$, and $\psi_p(t)$ are the roll, pitch, and yaw angles from the payload, respectively. The coordinate vector of the payload is $\mathbf{q}_p(t) = \{\mathbf{r}_p^\top(t) \ \boldsymbol{\eta}_p^\top(t)\}^\top$.

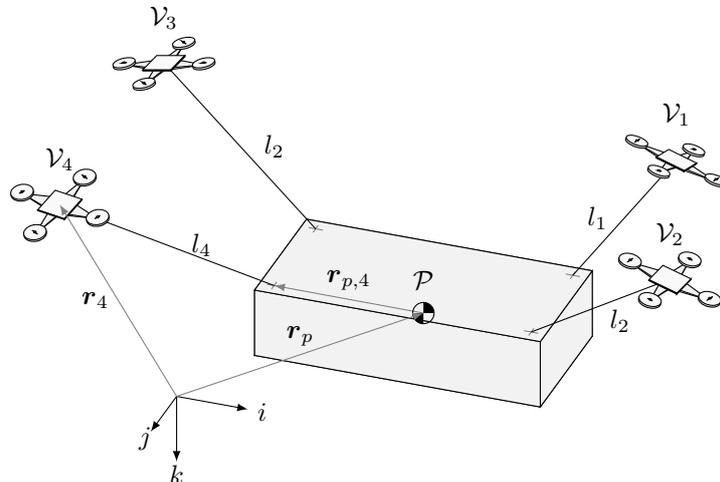


Figure 1. Four illustrative UAVs transporting the body payload using inelastic suspension rods attached to arbitrary positions on the body.

The multi-lift formation is then described in terms of the coordinates from the n UAVs and the payload, such that the following vector of $7n$ coordinates defines the configuration of the system:

$$\mathbf{q}(t) = \{\mathbf{q}_1^\top(t) \dots \mathbf{q}_n^\top(t) \mathbf{q}_p^\top(t)\}^\top. \quad (1)$$

The formation coordinates in Eq. (1) does not constitute a set of generalized coordinates, since they are related by the constraints equations from the suspension system; hence, there is the additional complexity of computing the trajectories satisfying the constraint equations. Nonetheless, the description and physical interpretation of the coordinates from the multi-lift system are easier using non-generalized coordinates.

2.1 Suspension System

The suspension system is composed by a set of n inelastic rods connecting each UAV to the payload. Each suspension rod is attached to an arbitrary point on the payload according to the position vector ${}_{\mathcal{B}}\mathbf{r}_{p,i} = \{l_{x,i} \ l_{y,i} \ l_{z,i}\}^\top$, which is defined in the payload coordinate system. The position of the attachment point in the inertial frame of reference is described in terms of the position from the payload center of mass :

$$\mathbf{r}_{p,i} = \mathbf{r}_p + \mathbf{R}_p {}_{\mathcal{B}}\mathbf{r}_{p,i}, \quad (2)$$

where \mathbf{R}_p denotes the transformation matrix that maps a vector defined in the body frame to the inertial frame of reference. The transformation matrix is computed in terms of the Euler angles as:

$$\mathbf{R}_p = \mathbf{T}_{\psi_p} \mathbf{T}_{\theta_p} \mathbf{T}_{\phi_p}. \quad (3)$$

2.2 Constraint Equations

The i -th suspension rod constraint the distance from the center of mass of the i -th UAV to the corresponding attachment point on the payload such that it is equal to length l_i of the rigid-rod, as described by the following equation:

$$\Theta_i(\mathbf{q}) \equiv (\mathbf{r}_i - \mathbf{r}_{p,i})^\top (\mathbf{r}_i - \mathbf{r}_{p,i}) - l_i^2 = 0. \quad (4)$$

There is a total of n constraints defined in the multi-lift formation, one from each connected UAV, such that the vector of algebraic constraints is defined by

$$\Theta \equiv \{\Theta_1 \dots \Theta_n\}^\top. \quad (5)$$

The constraint acceleration is required to obtain the trajectories from the multi-lift formation (cf. Sec. 3). Thus, each algebraic constraint (Eq. 4) is differentiated two times with respect to the time t , such that the following expressions for the velocity and acceleration are obtained, respectively:

$$\dot{\Theta}_i = 2(\mathbf{r}_i - \mathbf{r}_{p,i})^\top (\dot{\mathbf{r}}_i - \dot{\mathbf{r}}_{p,i}) = 0, \quad (6)$$

$$\ddot{\Theta}_i = 2(\mathbf{r}_i - \mathbf{r}_{p,i})^\top (\ddot{\mathbf{r}}_i - \ddot{\mathbf{r}}_{p,i}) + 2(\dot{\mathbf{r}}_i - \dot{\mathbf{r}}_{p,i})^\top (\dot{\mathbf{r}}_i - \dot{\mathbf{r}}_{p,i}) = 0. \quad (7)$$

Equation (6) is expressed in terms of the multi-lift coordinates through the following equation

$$\dot{\Theta}_i = \frac{\partial \Theta_i}{\partial \mathbf{q}} \dot{\mathbf{q}}, \quad (8)$$

and a further differentiation which respect to the time leads to the equation for the constraint acceleration:

$$\ddot{\Theta}_i = \frac{\partial \Theta_i}{\partial \mathbf{q}} \ddot{\mathbf{q}} + \dot{\mathbf{q}}^\top \mathbf{H}_i \dot{\mathbf{q}}, \quad (9)$$

where \mathbf{H}_i is the Hessian matrix from the constraint equation Θ_i , with its components computed by $H_{ij} = \frac{\partial^2 \Theta_i}{\partial q_i \partial q_j}$. Considering the n constraint equations from each UAV suspension, the following vector of constraints in acceleration level is obtained:

$$\ddot{\Theta} = \Theta_q \ddot{\mathbf{q}} + \mathbf{h}, \quad (10)$$

with $\Theta_q = \frac{\partial \Theta(\mathbf{q})}{\partial \mathbf{q}}$ denoting the constraint Jacobian, and $\mathbf{h} = \{\dot{\mathbf{q}}^\top \mathbf{H}_1 \dot{\mathbf{q}} \dots \dot{\mathbf{q}}^\top \mathbf{H}_n \dot{\mathbf{q}}\}^\top$ is the vector containing the velocity dependent terms from Eq. (9).

3. EQUATION OF MOTION FROM THE MULTI-LIFT FORMATION

Hamilton's principle states that a system configuration evolves following the trajectory $q(t)$ such that action functional is stationary (Goldstein, 2002). To account for the constraints of the suspension system, this functional is extended to include the constraints using the Lagrange multipliers (Simeon, 2006).

$$\delta \int_{t_1}^{t_2} [\mathcal{L}(q(\tau), \dot{q}(\tau)) - \Theta(q(\tau))^\top \Lambda] d\tau = 0 \quad (11)$$

where $\Lambda = \{\lambda_1 \dots \lambda_n\}$ is a vector of Lagrange multipliers representing each constraint from the multi-lift system and δ is the variational operator. The trajectories that satisfy the principle of stationary action are the Euler-Lagrange equations, which for a constrained system assumes the following form (Saletan and Cromer, 1970; Leyendecker *et al.*, 2008):

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) - \left(\frac{\partial \mathcal{L}}{\partial q_i} \right) + \Theta_{q_i}^\top \Lambda = f_i, \quad i = 1, \dots, 7n \quad (12)$$

where f_i denote the generalized force applied to the i -th coordinate. The equations of motion from the multi-lift system are obtained by solving Eq.(12) for each coordinate of $q_i \in q$. Then, in addition with the n constraint equations from the suspension system, the following set of differential and algebraic equations (DAE) represents the dynamics of the multi-lift system:

$$M\ddot{q} + \Theta_q^\top \Lambda = f, \quad (13)$$

$$\Theta(q) = 0. \quad (14)$$

Due to the involved difficulties for solving the system of differential and algebraic equations simultaneously (Braun and Goldfarb, 2009), the trajectories of the multi-lift formation are obtained from the equivalent problem (i.e., a system of ordinary differential equations) by considering the constraints in acceleration level Eq. 10 (Pogorelov, 1998). However, the numerical residue obtained from the numerical solution is integrated twice with respect to the time t , resulting in the problem drift of the trajectories from the constraint manifold. The method of numerical stabilizing terms is used to transform the constraint hyper-surface to an attractor (Baumgarte, 1972) such that the constraint error, i.e., the right-hand side of Eq. (4), is controlled and the obtained trajectories approximately satisfy the constraint. This process is done by including stabilizing terms $\alpha_{bg} > 0$ and $\beta_{bg} > 0$, to Eq.(10). Then, the trajectory of the multi-lift formation system is obtained by integrating the stabilized system of ordinary differential equations:

$$\begin{bmatrix} M & \Theta_q^\top \\ \Theta_q & 0 \end{bmatrix} \begin{Bmatrix} \ddot{q} \\ \Lambda \end{Bmatrix} = \begin{Bmatrix} f \\ -h - 2\alpha_{bg}\dot{\Theta} - \beta_{bg}^2 \Theta \end{Bmatrix}, \quad (15)$$

where the Coriolis forces, control, and potential forces define the formation force vector f as follows:

$$f = -C\dot{q} + Bu - f_g. \quad (16)$$

The block-diagonal inertia matrix $M = \text{diag}(M_1 M_2 \dots M_n M_p)$, and M_i , $i = 1, \dots, n$ is the inertia matrix for each i -th UAV, and M_p is the inertia matrix from the payload. Similarly, $C = \text{diag}(C_1 \dots C_n C_p)$ is the block diagonal assembled Coriolis matrix composed by the Coriolis matrix from the UAVs and payload. The gravitational force vector from the assembly is computed by $f_g = \{f_{g,1}^\top f_{g,2}^\top \dots f_{g,n}^\top f_{g,p}^\top\}^\top$. The control matrix $B = \text{diag}(B_1 B_2 \dots B_n B_p)$ is similarly assembled, however, the payload has no direct input term, i.e., the matrix $B_p = 0$.

The Lagrange multipliers and the acceleration from the multi-lift formation are computed by following equations, respectively:

$$\begin{aligned} \Lambda &= \left(\Theta_q M^{-1} \Theta_q^\top \right)^{-1} \left(\dot{\Theta}_q \dot{q} + 2\alpha_{bg} \beta_{bg} \dot{\Theta} + \beta_{bg}^2 \Theta + \Theta_q M^{-1} f \right) \\ \ddot{q} &= M^{-1} \left(f - \Theta_q^\top \left(\Theta_q M^{-1} \Theta_q^\top \right)^{-1} \left(h + 2\alpha_{bg} \beta_{bg} \dot{\Theta} + \beta_{bg}^2 \Theta + \Theta_q M^{-1} f \right) \right). \end{aligned} \quad (17)$$

If one is interested on obtaining the constraint forces, it can be readily computed as:

$$f_c = \Theta_q^\top \Lambda \quad (18)$$

4. MULTI-LIFT FORMATION CONTROLLER

The UAVs have their controller based on a cascade structure with an inner-loop responsible to stabilize the actuated degrees of freedom, whereas the outer-loop is responsible to compute the attitude references (Raffo *et al.*, 2011). The formation controller consists of a leader-follower architecture, with the leader responsible of tracking the mission trajectory whereas the followers maintain an adequate distance with respect to the leader. The followers UAVs have their outer-loop altered to use the leader position to compute virtual constraint forces and the corresponding attitude reference for the inner-loop stabilization. The formation controller consists of maintaining a prescribe distance with respect to the leader. The method of virtual constraint is presented in Ihle *et al.* (2005) for a fleet of marine vehicles. Zhao *et al.* (2018) obtain the forces of control to manage a formation of mobile robots.

4.1 Propulsion Saturation

The multirotor configuration has its thrust and control provided by balancing the angular velocity from each of its rotors. The propulsion characteristics from the multirotor are parameterized by the collocation matrix \mathbf{K}_r (Kotarski *et al.*, 2021), arm length l_r , the maximum angular velocity from the rotors $[\omega_r]$, and the linear coefficients of thrust and moment k_T, k_D , respectively. The propulsion system limitation is implemented by checking if the control signal obtained in the inner-loop is within the operational limits, as described as follows.

For each i th UAV, the control input, \mathbf{u}_i , is mapped to the resulting angular velocity from each rotor using $\omega_{r_i} = \mathbf{K}_r^{-1} \mathbf{u}_i$. If the angular velocity from each rotor is within its operational range, $\omega_r \leq [\omega_r]$, no change is applied. If the control is outside the operational range, then this control signal saturate the UAV rotors. Based on the collocation matrix, it is possible to maintain the rotational control while exerting the maximum possible thrust. Consider the following partition from the UAV propulsion model:

$$\begin{Bmatrix} u_T \\ \mathbf{u}_\eta \end{Bmatrix} = \begin{bmatrix} \mathbf{K}_u \\ \mathbf{K}_\eta \end{bmatrix} \omega_r \quad (19)$$

where \mathbf{K}_u is the $1 \times n_r$ matrix mapping the squared angular velocity from each rotor to the thrust, and \mathbf{K}_η is the $3 \times n_r$ matrix mapping to the attitude control. The angular velocity required to control the rotational coordinates is obtained by:

$$\omega_\eta = \mathbf{K}_\eta^\dagger \mathbf{u}_\eta, \quad (20)$$

where the symbol $(\cdot)^\dagger$ denotes the generalized inverse. and the thrust component is obtained considering an additional angular rotor velocity from the null-space of \mathbf{K}_η . If ω_u is a basis from the null-space of \mathbf{K}_η , i.e., $\omega_u \in \mathcal{N}(\mathbf{K}_\eta)$, then $a\mathbf{K}_\eta\omega_u = \mathbf{0}$ for any value of coefficient a . This strategy allows one to preserve the rotational control and computes the maximum possible thrust without saturating by writing the angular velocity vector as $\omega_r = \omega_\eta + a\omega_u$. Then, the following equation can be written:

$$\mathbf{u}_i = \mathbf{K}_\eta\omega_\eta + a\mathbf{K}_u(\omega_\eta + \omega_u), \quad (21)$$

for which the value a is computed such that the following equation is satisfied for each rotor angular velocity

$$\omega_\eta + a\omega_u \leq [\omega_r]. \quad (22)$$

Equations (21) and (22) allow one to obtain the maximum possible thrust while preserving rotational control authority.

4.2 Follower Controller

The followers UAVs consider the leader configuration to compute their control action through the virtual constraints. Consider the following equation for the virtual constraints representing the desired position from the follower with respect to the leader:

$$\Theta_{vc_i} = \mathbf{r}_i - \mathbf{r}_1 - \mathbf{v}_c, \quad (23)$$

such that $\Theta_{vc_i} \equiv \mathbf{0}$, and \mathbf{v}_c denotes the desired relative distance. This equation is differentiated two times with respect to the time t . By considering the leader in uniform motion, i.e., $\ddot{\mathbf{r}}_1 = \mathbf{0}$, the following equations for the virtual constraint velocity and acceleration are obtained

$$\dot{\Theta}_{vc_i} = \dot{\mathbf{r}}_i - \dot{\mathbf{r}}_1 = \mathbf{0} \quad (24)$$

$$\ddot{\Theta}_{vc_i} = \ddot{\mathbf{r}}_i = \mathbf{0} \quad (25)$$

The trajectories from Eq. (25) are stabilized by considering two positive definite feedback terms, $\alpha_{vc} > 0$ and $\beta_{vc} > 0$, such that:

$$\ddot{\Theta}_{vc_i} = \ddot{r}_i + 2\alpha_{vc}\dot{\Theta}_{vc_i} + \beta_{vc}^2\Theta_{vc_i} \quad (26)$$

Considering the virtual constraint, the dynamic from the i -th follower is:

$$M_i\ddot{q}_i + (\Theta_{vc_i})_{q_i}^\top \mu_i = f_i \quad (27)$$

where μ_i is the Lagrange multiplier associated to the i -th virtual constraint and f_i is the gravitational, Coriolis, and input forces from the i th follower. Note that Eq. (27) is dependent only on the parameters from the corresponding follower, and the leader parameters enters as an input term through the constraint derivatives, i.e., Eqs.(23) and (24).

Since the constraint force does not exist, the follower must artificially introduce it through a redefinition of its attitude reference. Defining the components related to the translational coordinates $u_{vc,i} = -(\Theta_{vc})_{r_i}^\top \mu_i$ as the virtual force required by the follower to keep in the formation, the projection from the thrust to result in such forces are computed as $u_{vc,i} = \frac{1}{u_{T_i}} \{u_x \ u_y \ u_z\}^\top$, and the attitude references are updated through:

$$\begin{aligned} \phi_d &= \arcsin(u_x s_\psi - u_y c_\psi), \\ \theta_d &= \arcsin\left(\frac{u_x c_\psi + u_y s_\psi}{c_{\phi_d}}\right). \end{aligned} \quad (28)$$

Then, the follower stays in this designated position with respect to the formation if the adequate stabilization in their inner-loop is considered. Note that the altitude reference is updated directly (Eq. 23, since the altitude is an actuated coordinate and it is stabilized in the inner-loop).

5. RESULTS AND DISCUSSION

The multi-lift mission herein consists of four identical UAVs transporting a payload through the path informed to the leader, which is shown in the Figs (2) and (6) by dashed lines. The mission trajectory consists of the UAVs lifting the payload and moving in the x - and y - directions. After the UAVs reach the desired position in space, they hover in place. The payload is assumed to be a rigid rectangle with mass density ρ_p and dimensions of length l_p , width w_p , and height h_p . The positioning from the UAVs and the suspensions are symmetric such that the length from the suspension rods are equal to each other. The parameters from this multi-lift system are presented in Tab. 1.

Table 1. Parameters from the UAV, payload, and suspension.

Parameter	Value	Unit
m_i	2.5	kg
J_i	diag(0.957, 0.485, 0.355)	kg m ²
k_T	1.83×10^{-6}	N/rad ²
k_D	1.85×10^{-7}	N m/rad ²
$[\omega_{r,i}]$	0	rad/s
$[\omega_{r,i}]$	2100	rad/s
ρ_p	1200	kg/m ³
l_p	0.8	m
w_p	0.1	m
h_p	0.1	m
l_i	9.95	m

The mission is assigned to the leader and it consists of four phases typically defined for UAVs (Raymer, 1989):(i) take-off with the leader climbing with a constant climb-rate for 5s, (ii) cruise in the i direction for 5s, (iii) cruise in the j direction, and (iv) the UAVs hovers in the place for the remaining of the time. The trajectories are obtained by solving Eq.(17) using the 5th order Runge-Kutta algorithm and considering the numerical stabilizing parameters of $\alpha_{bg} = 20$ and $\beta_{bg} = 20$. The resulting trajectories satisfy the constraint equation for each suspension, i.e., Eq. (4), within 10^{-13} .

Figure 2 presents the leader trajectory in contrast with the desired mission trajectory, shown using dashed lines. The leader accurately tracks its designed path in all directions. In addition, the virtual constraint equations defined to maintain the formation allows the followers to contribute to the lifting force required to transport the payload. Figure 3 shows the thrust force from each UAV during the mission.

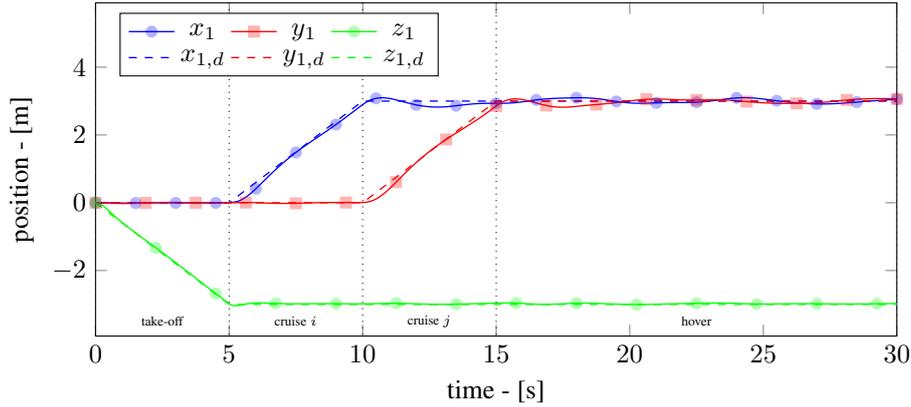


Figure 2. Trajectory from the leader in the multi-lift system. The mission trajectory is shown in dashed lines.

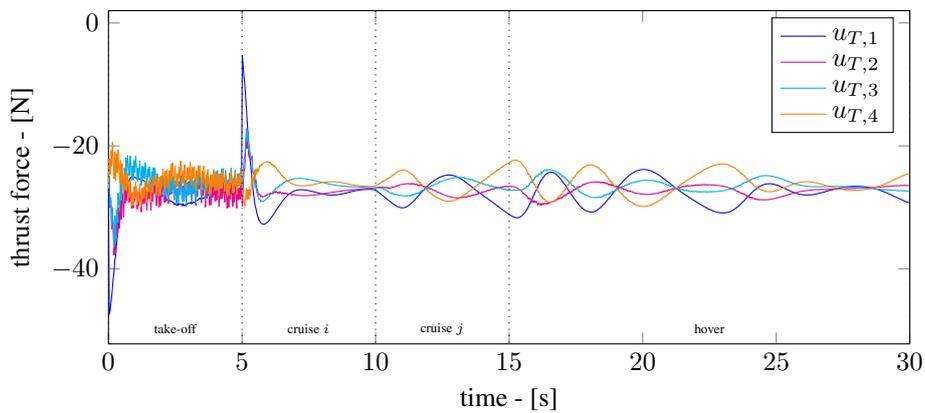


Figure 3. Thrust force exerted by each UAV.

Figure 4 presents the payload trajectory during the mission. Note that during the phase of take-off, no significant motion is observed in the i and j directions. However, as the UAVs pull the payload, they also introduce motion which persists during the remaining phases. This oscillation is further transmitted to the UAVs through the constraint forces, which require continuous attitude adjustments to remain in their designated positions during the mission provided by the followers outer-loop, as shown in Fig. 5.

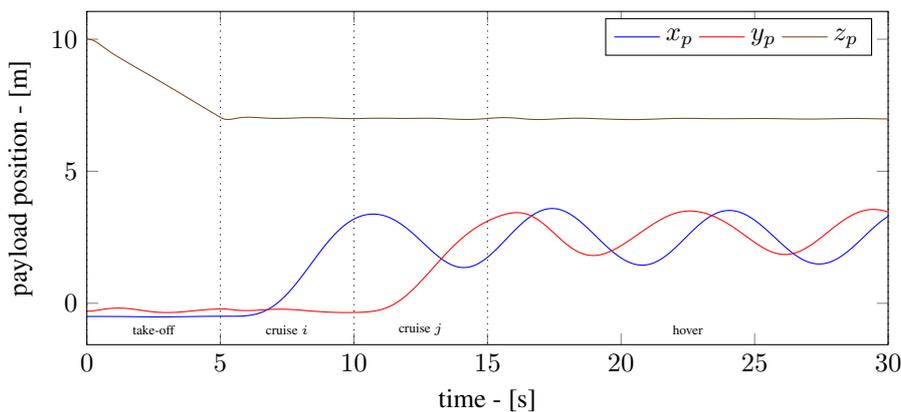


Figure 4. Payload trajectory.

The trajectories in the perspective view from the multi-lift mission are presented in Fig. 6. Note the circular trajectory of the payload during the hover phase. This motion persists since there is no damping included in the model. Two snapshots of the system configuration are highlighted: (1) the initial system configuration, indicated by $t = 0$ s, which is arbitrary defined such that the system configuration is compatible with the constraint equations, i.e., on the constraint manifold (Braun and Goldfarb, 2009), and (2) the system configuration in the final instant $t = 30$ s.

Figure 7 shows the payload orientation during the mission. It can be seen that the roll, ϕ_p , and yaw, ψ_p , angles have a greater value when comparing with the pitch angle θ_p . This dynamic of varying attitude is detrimental for practical

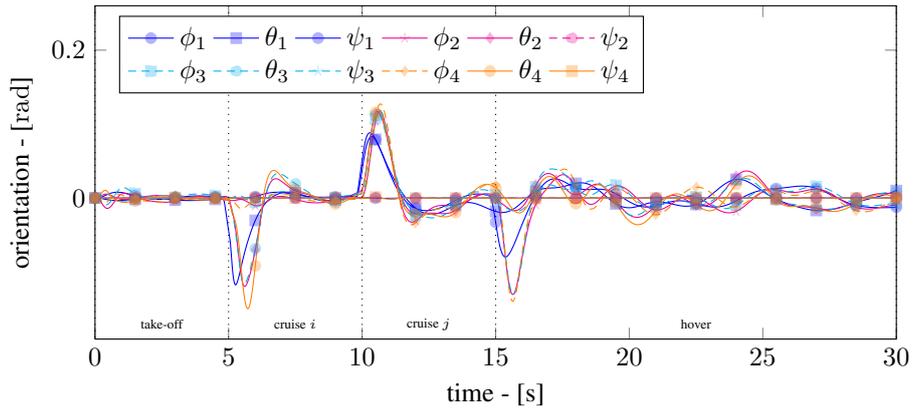


Figure 5. Attitude from the UAVs during the mission.

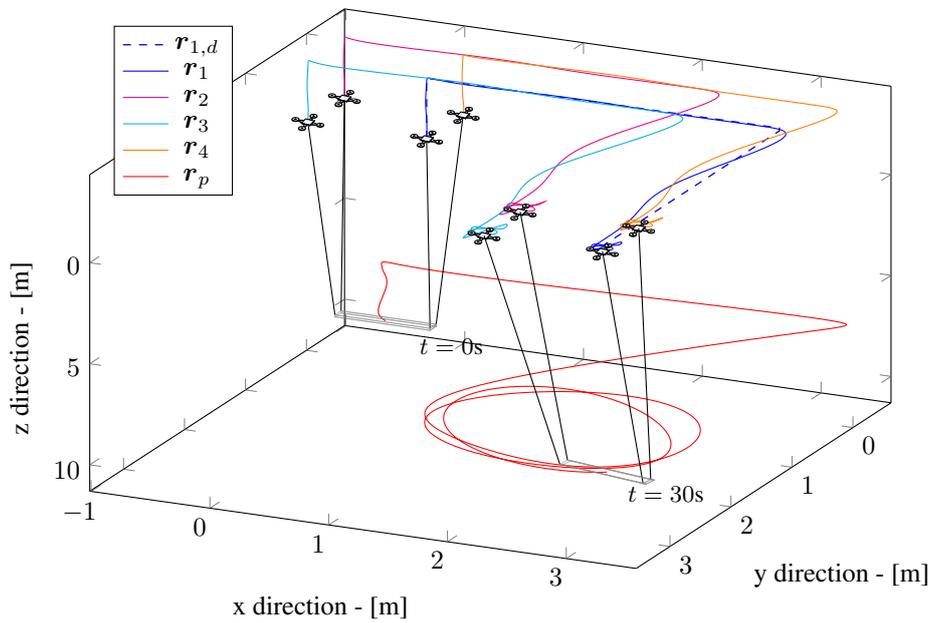


Figure 6. Three dimensional visualization from the multi-lift system trajectories and snapshots from the system configuration at the initial and final positions.

applications, since the precise positioning from the payload requires damping this motion, which may be provided by the wind drag force does not included in the present modeling. However, further investigations can be carried out to evaluate its influence.

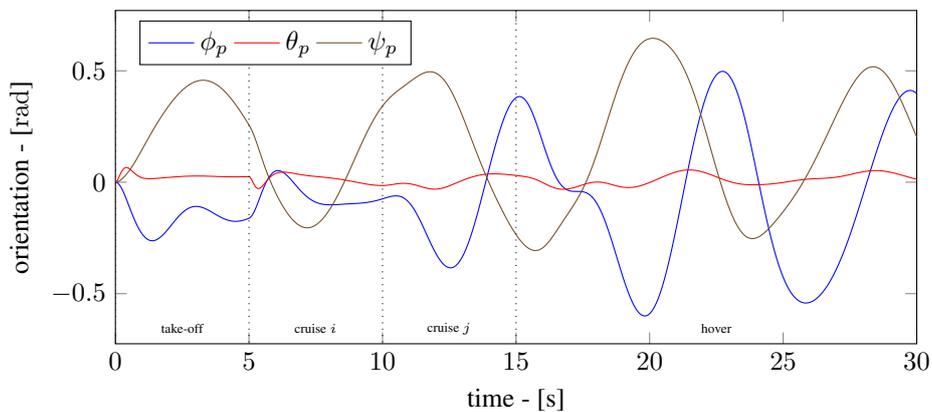


Figure 7. Orientation from the payload during the mission.

6. CONCLUSIONS

This work presents the dynamic modeling of a multi-lift formation transporting a rigid payload. The suspension is modeled using rigid rods attached to arbitrary points on the payload and the equation of motion from this multi-lift system is obtained using the Euler-Lagrange formulation with the holonomic constraints from the suspension system included using Lagrange multipliers. Advantages from this approach involve to obtain the model for formations with an arbitrary number of vehicles whereas preserving the geometric interpretation from the coordinates used to describe the system.

We implement a formation controller based on leader-follower architecture with virtual constraints, which consists of defining a set of equations based on the desired coordinates between vehicles. Then, by informing each follower from its position with respect to the leader, and attributing the mission path to the leader, it is possible to coordinate the motion of the formation and simultaneously lift the payload.

A system with four UAVs transporting a rigid payload with mass beyond the propulsive limits from a single UAV is defined. The results demonstrate that the multi-lift formation can successfully perform the mission, which is shown by comparing the leader trajectory with the mission trajectory. However, an undesired pendulum oscillation mode from the suspension of the rigid-body is introduced to the system. This motion adversely is detrimental for the mission since the positioning accuracy is compromised.

Further studies for defining the mission trajectory based on the payload position and attitude can be carried out in future investigations, and it is of practical interest for the assembly of large structures using UAVs formation as aerial cranes, since the suspended payload oscillations in a multi-lift transportation mission is detrimental for accurate positioning. The stabilizing properties of the designed formation controller is also subject of future investigations.

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