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NUMERICAL ANALYSIS OF ROTORDYNAMIC SYSTEMS USING HARMONIC BALANCE METHOD

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Abstract. *Hence the investigation of techniques to reduce the computational cost for numerical solutions of dynamical systems is in constant development. This research aims to solve the differential equations that describe the behavior of a rotating machine using Harmonic Balance Method (HBM), which determines steady-state vibration amplitudes and phase angles directly by solving nonlinear equations of dynamic systems in the frequency domain. Traditional numerical integration methods in the time domain possess limitations due to the required longer integration times. Thus, depending on the complexity of the model, it may be necessary to simulate the system for an extended period to ensure that we obtain a steady-state solution. The finite element method is used to model the behavior of a rotor system, considering the effects of rotor inertia and gyroscopic moments. This rotary machine under investigation has a flexible horizontal shaft, two rigid discs, and two rolling element bearings. An external force is introduced into the system to simulate non linearity. The initial step of the procedure involves defining an approximate solution for displacements, velocity, and acceleration as a sum of harmonics using the Fourier Series for all degrees of freedom. The unknown coefficients of the harmonic functions are determined by substituting these approximations into the differential equation. Hence, the unknown coefficients constitute a set of nonlinear equations that can be solved using numerical techniques. Once this system of equations has been solved, it is possible to determine the vibration responses in the frequency and time domains. The number of equations that the method needs to solve corresponds to double the number of degrees of freedom of the system, and the Levenberg-Marquardt numerical method is used to solve the resulting nonlinear algebraic equations. The results obtained through the HBM demonstrate that this method solves the differential equations of the movement with similar accuracy, but less computation cost when compared to the conventional time-domain method.*

Keywords: *Harmonic Balance, rotor, computational cost, numerical solutions, finite elements.*

1. INTRODUCTION

The study and improvement of mathematical and experimental models that simulate the dynamic behavior of rotating machines are continuously advancing, being used to predict critical speeds and even fault diagnosis. According to Tanaka (2011), these rotors have the potential to generate significant vibrations, leading to issues such as noise, cracks, fatigue, and bearing failures. However, due to the complexity of the mathematical and numerical models of the rotor components, the solution using the traditional integration in the time domain demands a high computational cost. The possible approach to circumvent this cost is to simplify the models of mechanical components, although this may result in deviations from the actual behavior of the machine. Linearizing a real system can lead to the loss of crucial information regarding its dynamic behavior, thereby causing substantial deviations between theoretical and experimental models (Thomson, 1996).

The differential equation of motion to determine the rotor vibration responses can be solved using analytical, numerical, or as both methodologies. To compute the steady-state response of differential equations several approaches such Harmonic Balance method, Perturbation method Nataraj and Nelson (1989), and Multi-Scale method are applied. These techniques exhibit comparable computational costs. In accordance with the research conducted by Zhang *et al.* (2022), Harmonic Balance method (HBM) exhibits notable advantages over alternative approaches. One primary advantage is attributed to the widespread applicability of the algebraic equations derived from HBM in dynamic systems. Furthermore, the method surpasses the limitation of being applicable solely to weakly nonlinear systems. Notably, among the approximate analytical methods, HBM stands out for its ability to offer the clearest and most intuitive physical interpretation.

Von Groll and Ewins (2001) present a numerical technique, based on Harmonic Balance method, for computing the periodic response of a non-linear system subject to periodic excitation, considering the dynamics of rotor/stator interactions. In their study, Zhang *et al.* (2022) employed Harmonic Balance method and Rung-Kutta integration to solve the dynamic response of a double disc rotor system, considering the influence of pedestal looseness. They compared the results obtained using these methods. Zheng *et al.* (2022) proposed a modified incremental Harmonic Balance method (IHB) combined with Tikhnov regularization. This proposed approach aims to achieve a semi-analytical solution for periodic nonlinear systems.

The Harmonic Balance method is an efficient approach for obtaining approximate analytical solutions and evaluating nonlinear frequency responses. In this investigation, the application of HBM is employed to solve the governing differential equation that describes the motion of a horizontal rotor featuring two discs and two rolling bearings. The bearings are subjected to an external force. The HBM is combined with another numerical method to solve the system efficiently, robustly, and with a lower computational cost.

2. METHODOLOGY

2.1 Harmonic Balance Method

The Harmonic Balance method offers an advantage over traditional numerical techniques in terms of computational time efficiency. The HBM is dependent on the sampling rate or simulation. While traditional approaches focus on solving initial value problems, Harmonic Balance method solves periodic boundary value problems. Utilizing the frequency domain, the HBM determines the vibration amplitudes and phase angles of the rotor. (Krack and Gross, 2019). Figure 1 presents a flowchart with an overview of the application of HBM for evaluating responses in dynamic systems.

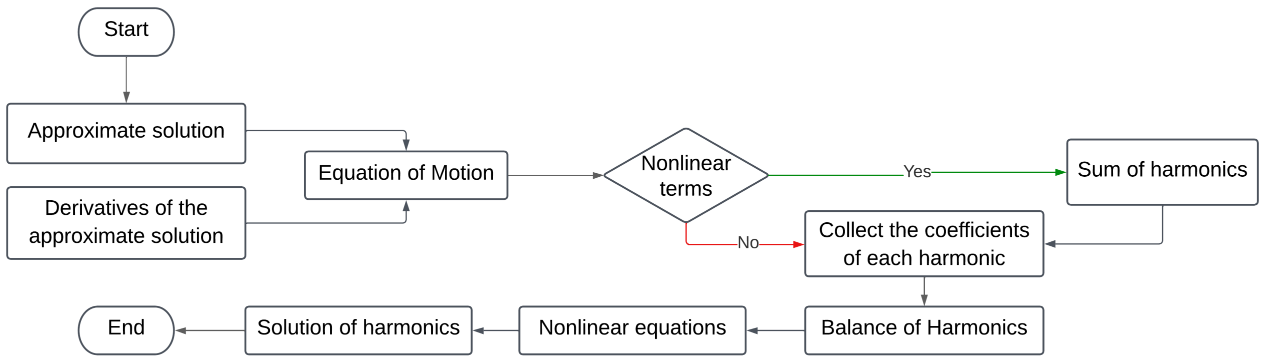

Figure 1: Flowchart illustrating Harmonic Balance method applied to the dynamics of rotation.

The first step of the procedure is to define an approximate solution for the displacements, velocities, and accelerations as a sum of harmonics using the Fourier Series for all degrees of freedom. The unknown coefficients of harmonic functions (displacements, velocity, and acceleration) are determined by substituting these approximations into the differential equation. Thus, the unknown coefficients form a system of nonlinear equations that can be solved using numerical methods. Once this system has been solved, it is possible to determine the vibration responses in the frequency and time domains. The number of equations that the method needs to solve corresponds to double the number of degrees of freedom of the system, and Levenberg-Marquardt numerical method is used to solve the resulting nonlinear algebraic equations.

2.2 Rotor Model

The finite element model of the flexible rotor is represented by a matrix differential equation that describes the dynamic behavior of the system. This model is represented mathematically according to Eq. (1) (Lalanne and Ferraris, 1998).

$$[\mathbf{M}]\,\ddot{\mathbf{q}}(\mathbf{t}) + [\mathbf{C} + \Omega\;\mathbf{C}_g]\,\dot{\mathbf{q}}(\mathbf{t}) + [\mathbf{K}]\,\mathbf{q}(\mathbf{t}) = \mathbf{W} + \mathbf{F}_u + \mathbf{F}_e \tag{1}$$

where $[\mathbf{M}]$ is the inertia matrix, $[\mathbf{C}]$ is the damping, $[\mathbf{C}_g]$ is the gyroscopic matrix, $[\mathbf{K}]$ is the stiffness matrix, $\mathbf{q}(t)$ is the generalized displacement, $\mathbf{W}$ is the gravity force, $\mathbf{F}_u$ is the unbalance force, $\mathbf{F}_e$ is the external force vector, and Ω is the angular velocity of the rotor.

The shaft was modeled by using Timoshenko's beam elements with two nodes and four degrees of freedom per node (two displacements and two rotations). The damping considered was the proportional damping, as show in Eq. (2)

$$[C] = \alpha\,[M] + \beta\,[K] + C_{Bearing} \tag{2}$$

where α and β are positive real constants.

2.3 Rotor model represented by finite element method

Figure 2 shows the general outline of the rotor test rig used in the numerical applications performed in the present work. It is composed of a flexible steel shaft with 780 mm length and 12 mm diameter ($E = 206\ GPa$, $\rho = 7850\ \frac{Kg}{m^3}$, and $\nu = 0.29$), two rigid steel discs, and two self-alignment ball bearings.

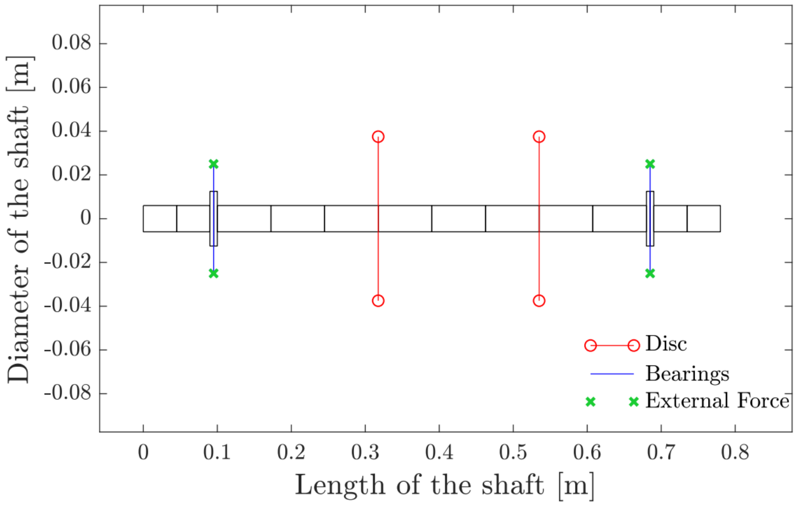

Figure 2: Rotor scheme

The unbalance force considered is 100 (g.mm). The bearing's dynamic properties correspond to stiffness in the x and z direction is $1.4 \cdot 10^5$ (N/m) $2.4 \cdot 10^5$, respectively. An the damping in both directions is 0.5 (Ns/m).

2.4 Harmonic Balance method applied in rotating machine

The approximation of the solution of HBM is done by Fourier series, which present excellent convergence rates, in particular for periodic functions. The displacement vector is described as indicated by Eq. (3)).

$$\mathrm{q(t)} = \frac{\Delta \mathrm{Q_0}}{2} + \sum_{\mathrm{n=1}}^{\mathrm{m}} \left[\left(\frac{\Delta \mathrm{Q_n}}{2} \right) \mathrm{e^{in\Omega t}} + \left(\frac{\Delta \mathrm{Q_{-n}}}{2} \right) \mathrm{e^{-in\Omega t}} \right] \tag{3}$$

where m represents the number of harmonics used to represent the solution vector.

The velocity and the acceleration is obtained by deriving the displacement vector. The unbalance force for the rotor with constant speed is showed according to Eq. (4), where $\mathrm{F_A}$ and $\mathrm{F_B}$ are written by Eq. (5).

$$\mathrm{F_u} = \left(\frac{\mathrm{F_A - iF_B}}{2} \right) \mathrm{e^{in\Omega t}} + \left(\frac{\mathrm{F_A + iF_B}}{2} \right) \mathrm{e^{-in\Omega t}} \tag{4}$$

$$\mathrm{F_A} = \mathrm{m_u\ d\ \Omega^2} \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \qquad \mathrm{F_B} = \mathrm{m_u\ d\ \Omega^2} \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \tag{5}$$

where $\mathrm{m_u}$ represents the unbalance mass [SI.]; d is the distance of the unbalance mass to the center of the disc [SI.] and θ represents the phase of the unbalance mass [SI.].

The external force is written according to Eq. (6).

$$\mathrm{F(t)} = \frac{\Delta \mathrm{F_0}}{2} + \sum_{\mathrm{n=1}}^{\mathrm{m}} \left[\left(\frac{\Delta \mathrm{F_n}}{2} \right) \mathrm{e^{in\Omega t}} + \left(\frac{\Delta \mathrm{F_{-n}}}{2} \right) \mathrm{e^{-in\Omega t}} \right] \tag{6}$$

The other parameters present in the problem formulation such as the mass matrix, damping, stiffness and the weight force are constants, therefore, they do not need to be described as a sum of harmonics. Substituting the terms in the motion equation, collecting the harmonics, we obtain the unknown terms: $\mathrm{Q_0}$, $\Delta\mathrm{Q_1}$, $\Delta\mathrm{Q_{-1}}$, ..., $\Delta\mathrm{Q_m}$, $\Delta\mathrm{Q_{-m}}$.

The general solution is calculated as shown in Eq. (7).

$$\begin{bmatrix} \Delta Q_0 \\ \Delta Q_1 \\ \Delta Q_{-1} \\ \Delta Q_2 \\ \Delta Q_{-2} \\ \Delta Q_3 \\ \Delta Q_{-3} \\ \vdots \\ \Delta Q_m \\ \Delta Q_{-m} \end{bmatrix} = \begin{bmatrix} K & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & H_1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & H_{-1} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & H_2 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & H_{-2} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & H_3 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & H_{-3} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & H_m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & H_{-m} \end{bmatrix}^{-1} \begin{bmatrix} 2\mathbf{W} + \Delta F_0 \\ F_A - iF_B + \Delta F_1 \\ F_A + iF_B + \Delta F_{-1} \\ \Delta F_2 \\ \Delta F_{-2} \\ \Delta F_3 \\ \Delta F_{-3} \\ \vdots \\ \Delta F_m \\ \Delta F_{-m} \end{bmatrix} \quad (7)$$

where H_m and H_{-m} are calculated according to Eq. (8).

$$\begin{aligned} H_m &= -m^2\Omega^2\mathbf{M} + mi\Omega\mathbf{C} + mi\Omega^2\ \mathbf{C}_g + \mathbf{K} \\ H_{-m} &= -m^2\Omega^2\mathbf{M} - mi\Omega\mathbf{C} - mi\Omega^2\ \mathbf{C}_g + \mathbf{K} \end{aligned} \quad (8)$$

Once the values of ΔQ_0, ΔQ_1, ΔQ_{-1} ..., ΔQ_m and ΔQ_{-m} are obtained it is possible to obtain the solution in the time domain using Eq. (3).

3. RESULTS

This section presents an analysis of results obtained from numerical approaches. The finite element model of the rotor is represented by 16 elements that were implemented in Matlab ® of this present contribution. This paper presents the implementation of Harmonic Balance method to obtain the vibration responses of a rotor subject to gravity force, unbalance and an external force. The rotational speed is 1200 rpm.

Figure 3 shows the comparison between the original external force (reference) with the same one expressed in Fourier series (sum of harmonics) in the frequency spectrum in the semi-log scale in the degrees of freedom corresponding. In this case DOF represent the x and z directions of the two bearings.

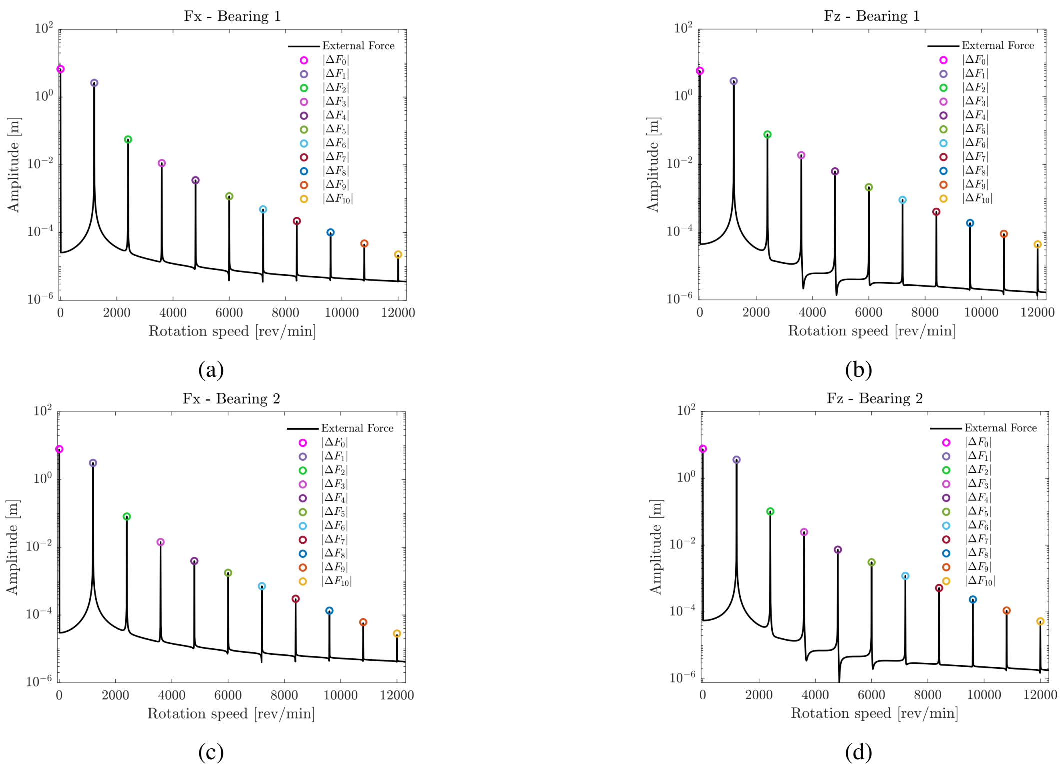

Figure 3: External force validation. (a) Bearing 1 - direction x. (b) Bearing 1 - direction z. (c) Bearing 2 - direction x. (d) Bearing 2 - direction z.

Analyzing the frequency spectrum, it is noted that the values of $|\Delta F_n|$ correspond to the FFT peak amplitude of the external force signal in the frequency spectrum.

Figure 4 illustrate the comparison of the discs and bearings responses obtained through Harmonic Balance method and the traditional integration in the frequency domain in the semi-log scale.

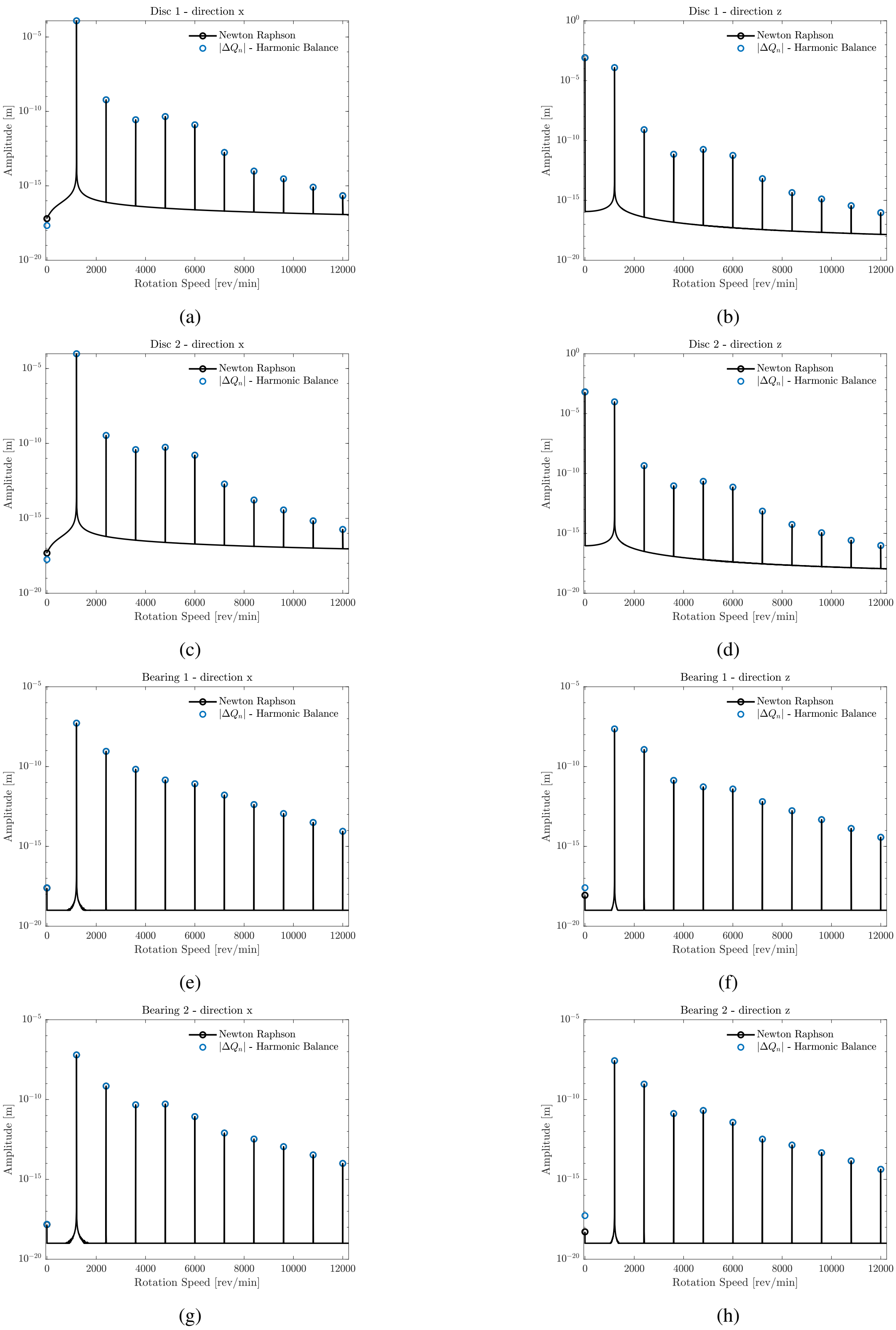

Figure 4: Displacements obtained via Harmonic Balance method in the frequency spectrum.(a) Disc 1 - direction x. (b) Disc 1 - direction z. (c) Disc 2 - direction x. (d) Disc 2 - direction z. (e) Bearing 1 - direction x. (f) Bearing 1 - direction z. (g) Bearing 2 - direction x. (h) Bearing 2 - direction z.

The number of harmonics considered in this modeling were $m = 10$. However, for this problem specifically, only the first harmonic would be enough to represent the solution, since the values of $|\Delta Q_n|$ and $|\Delta Q_{-n}|$ of the other harmonics are negligible when compared with the static component and the first harmonics (1X). Note that the main harmonics present in the displacements of both disc 1 and disc 2 represent the harmonic (1X) and the static component (present only in the degrees of freedom referring to the z direction). This is due to the nature of the forces the rotor is subjected to. In this example specifically, it corresponds to the unbalance force and the external force that represent the excitation forces present in the system.

Figures 5 illustrates the displacements in steady-state along the x and z directions, in the disc and bearings from the rotor. The Harmonic Balance results were compared with the results using the traditional integration system in the time domain, in this case using Newton–Raphson method.

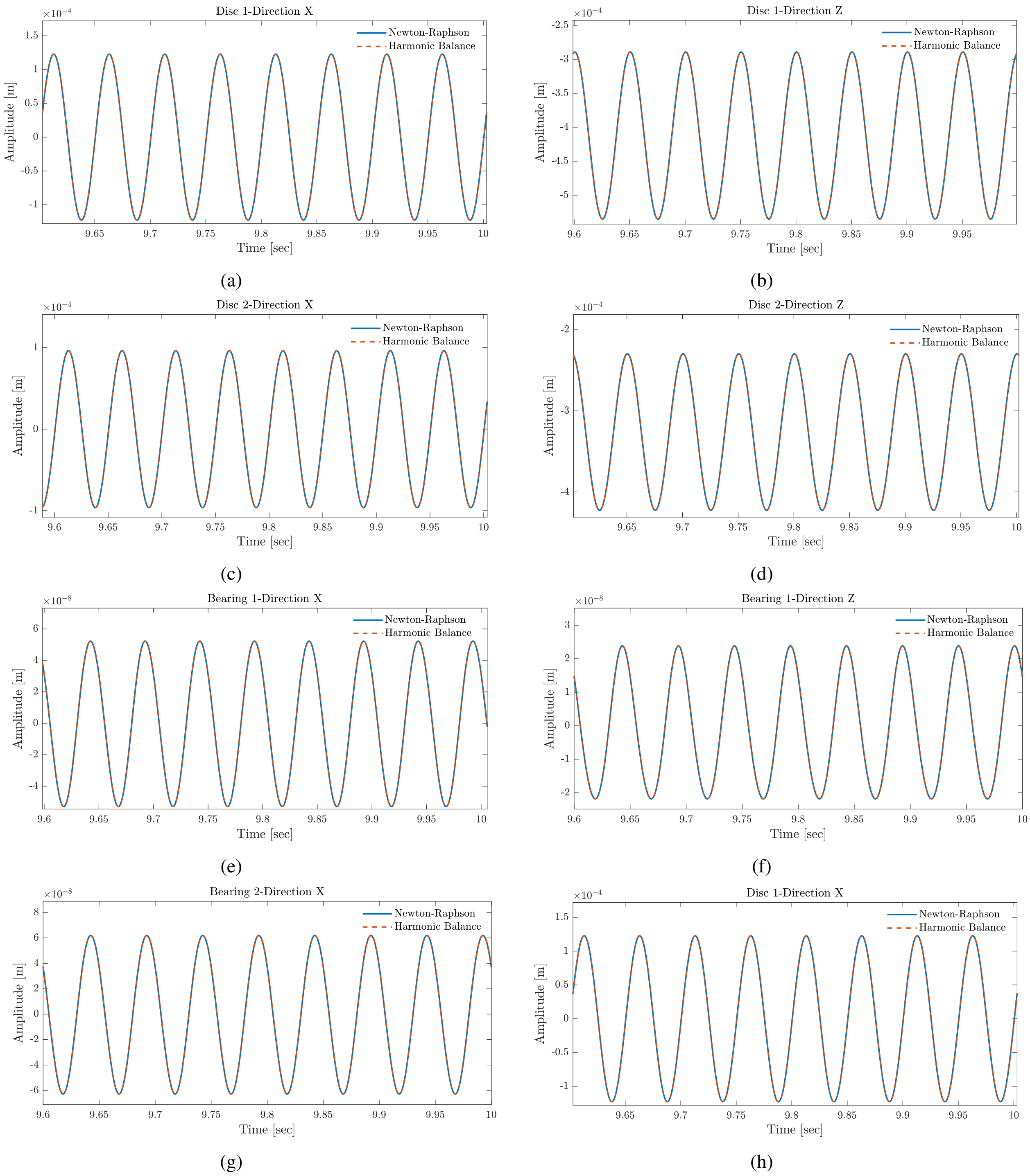

Figure 5: Steady-state (a) Disc 1 direction x. (b) Disc 1 direction z. (c) Disc 2 direction x. (d) Disc 2 direction z. (e) Bearing 1 direction z. (f) Bearing 2 direction x. (g) Bearing 2 direction z.

Analyzing the temporal responses and the frequency spectrum, it is possible to say that the response obtained by Harmonic Balance method was coherent with the response obtained by traditional method of integration. It is worth noting that there is no right method, however the vibration responses need to be close or equal in the comparison of methods.

4. DISCUSSIONS AND CONCLUSIONS

This paper solves the vibrations in a rotating machine using Harmonic Balance method. The results show that the method obtains a similar answer that the traditional integration method solved in the time domain.

The proposed method was able to solve the equation of motion containing nonlinear effects in a robust, fast and efficient way. The advantage of Harmonic Balance method is related to the integration time when compared to traditional integration methods. The processing time for the solution of the equation of motion using Harmonic Balance method is much smaller than the traditional solution.

Therefore, it can be concluded that Harmonic Balance method obtained the same vibration responses as the traditional method. The difference is that Harmonic Balance method has a lower computational cost than the traditional integration method.

5. ACKNOWLEDGMENTS

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