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An implementation of the TOBS method for topology optimization of large-scale 3D structures

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Abstract. *This paper presents an implementation of the Topology Optimization of Binary Structures (TOBS) method for the design of large-scale, 3D structures. TOBS is a well-established method of topology optimization, whose binary-density formulation makes it more suitable to address the design of large-scale buildings than continuous-density variation methods, because intermediate densities yield unfeasible designs for engineering practice, and are known for resulting in numerical instability. The implementation consists of a finite element solver that uses classical, hexahedral, linear-elastic finite elements with eight nodes with three degrees of freedom per node. Material interpolation is implemented according to the classical Solid Isotropic Material with Penalisation (SIMP) method. Elemental and nodal sensitivity and filtering schemes are implemented according to an extension of the classical BESO to three dimensions. TOBS' integer linear programming algorithm has shown to be able to navigate the added complexity of the 3D problems with no difficulty. The results show selected applications of the model to the design of large-scale 3D problems.*

Keywords: *Topology Optimization, TOBS, 3D problems*

1. INTRODUCTION

Topology optimization is a powerful approach in engineering design that aims to find the optimal distribution of material within a given design space to achieve desired performance objectives. In recent years, the Topology Optimization of Binary Structures (TOBS) method has gained significant attention due to its ability to handle binary-density formulations and its suitability for addressing the design of large-scale structures Sivapuram and Picelli (2018). In this paper, we present an implementation of the TOBS method for the design of large-scale, three-dimensional (3D) structures.

TOBS is particularly well-suited for designing large-scale buildings and structures when compared to continuous-density variation methods. Intermediate densities in continuous-density methods can lead to unfeasible designs in engineering practice and are known to introduce numerical instability in certain cases Sigmund and Petersson (1998). TOBS' binary-density formulation addresses these challenges, making it a robust and practical approach for large-scale structural design.

The implementation of TOBS in this study employs a finite element solver using classical, hexahedral, linear-elastic finite elements with eight nodes and three degrees of freedom per node. Material interpolation follows the Solid Isotropic Material with Penalisation (SIMP) method, a widely adopted technique in topology optimization. Furthermore, elemental and nodal sensitivity and filtering schemes are implemented based on an extension of the classical Bi-directional Evolutionary Structural Optimization (BESO) method to three dimensions (Huang and Xie, 2010).

To address the added complexity of 3D problems, TOBS utilizes an integer linear programming algorithm, which has proven effective in navigating larger-scale problems. In this paper, we showcase the application of the TOBS implementation to the design of two large-scale 3D problems. The results demonstrate the efficacy of the proposed approach in generating optimized designs that meet the desired performance criteria.

2. FORMULATION OF THE TOBS METHOD

The TOBS method has been proposed by (Sivapuram and Picelli, 2018). In this section, the basis of the method will be presented. The TOBS method performs topology optimization of structures according to a binary elemental state scheme, where each element of the mesh is assigned to a binary variable that can assume the values 0 or 1 that represents void and solid elements, respectively. The general optimization problem consists in obtaining a final geometry that minimizes an objective function $f(x)$ subjected to one or more constraints $g_i(x)$. This problem can be summarized in the following form:

$$\begin{aligned} \text{Minimize} \quad & f(x) && \text{(1a)} \\ \text{Subjected to} \quad & g_i(x) \leq \bar{g}_i(x), i \in [1, N_g] && \text{(1b)} \\ & x_j \in \{0, 1\}, j \in [1, N_d], && \text{(1c)} \end{aligned}$$

in which $\boldsymbol{x}$ is the vector of design variables of size N_d, and corresponds to the binary variables of the structure,that is, the element densities, f is the objective function, which corresponds to the main parameter of interest for the design of the structure. Typically, f is the total volume of the structure, the equivalent compliance of the structure, or the displacements of the elements. In Eq. (1), g_i are the constraint functions, $\bar{g}_i$ are the maximum allowable values of these functions and N_g is the number of constraints. Both the objective function and constraints are function of the design variable x.

One can rewrite the objective and constraint functions using first-order Taylor approximations:

$$\begin{aligned} \text{Minimize} \quad & \frac{\partial f(x^k)}{\partial x} \cdot \Delta x^k, && \text{(2a)} \\ \text{Subjected to} \quad & \frac{\partial g_i(x^k)}{\partial x} \cdot \Delta x^k \leq \bar{g}_i - g_i(x^k) := \Delta g_i^k, i \in [1, N_g], && \text{(2b)} \\ & ||\Delta x^k||_1 \leq \beta N_d, && \text{(2c)} \\ & \Delta x_j^k \in \{-x_j^k, 1 - x_j^k\}, j \in [1, N_d], && \text{(2d)} \end{aligned}$$

in which the superindex k represents the kth iteration of the optimization loop, Δx^k is the change in the design variables and can assume the values $\{0, 1\}$ for void elements and $\{0, -1\}$ for solid elements, and β is the maximum fraction of the elements that can change their design variables value in a single iteration. The first-order Taylor approximation is only valid if the change in the objective and constraint function is small enough. In order to respect this limitation, the change in the constraint function Δg_i^k can be altered using a relaxation parameter ϵ_i, which forces the solution to converge to the value $\bar{g}_i$:

$$\Delta g_i^k = \begin{cases} \epsilon_i g_i(x^k): & \bar{g}_i < (1 - \epsilon_i) g_i(x^k), \\ \bar{g}_i - g_i(x^k): & \bar{g}_i \in [(1 - \epsilon_i) g_i(x^k), (1 + \epsilon_i) g_i(x^k)], \\ \epsilon_i g_i(x^k): & \bar{g}_i > (1 + \epsilon_i) g_i(x^k), \end{cases} \quad (3)$$

The topology optimization problem approached in this paper is the minimum compliance problem, which consists in achieving the minimum compliance of a structure given a volume constraint. In other words, the main goal of the optimization procedure is to reduce the initial volume of the structure in such a way that the optimized geometry reaches at least a local maximum for its stiffness. This optimization problem is posed over a three dimensional domain that is discretized by classical 8-noded solid finite elements with three degrees of freedom per node, the nodal displacements and forces of which are given by $\mathbf{u}$ and $\mathbf{F}$. This problem can be stated as

$$\begin{aligned} \text{Minimize} \quad & C(x) = \mathbf{u}^T \mathbf{K} \mathbf{u}, && \text{(4a)} \\ \text{Subjected to} \quad & \frac{V(x)}{V_0} \leq \bar{V}, && \text{(4b)} \\ & \mathbf{K}\mathbf{u} = \mathbf{F}, && \text{(4c)} \\ & x_j \in \{0, 1\}, j \in [1, N_d], && \text{(4d)} \end{aligned}$$

in which $\mathbf{K}$ is the stiffness matrix of the domain, V_0 is the initial volume of the structure, $\bar{V}$ is the imposed final volume that the structure must achieve at the end of the optimization process, and $C(x)$ is the scalar compliance of the structure, which can be seen as the inverse of the stiffness of the structure. The optimization algorithm consists in solving iteratively the linear balance problem, $\sigma_{ji,j} = 0$, where σ_{ji} is the stress tensor, through the finite element method in Eq. (4c). The next step consists in removing or adding elements of the mesh until the imposed volume restriction is reached.

The contribution of each element for the objective function must be computed according to some rule, to decide upon whether it should be kept or removed in a given iteration. This contribution is assessed by computing how sensitive the objective function is to the removal or addition of the element.

2.1 Sensitivity analysis

Applying the first-order Taylor approximation to the minimum compliance problem (Eq. (4)) leads to:

$$\begin{cases} \text{Minimize} & \dfrac{\partial C(x^k)}{\partial x} \cdot \Delta x^k, \\ \text{Subjected to} & \dfrac{\partial V(x^k)}{\partial x} \cdot \Delta x^k \leq \bar{V} - V(x^k) := \Delta V^k, i \in [1, N_g], \\ & ||\Delta x^k||_1 \leq \beta N_d, \\ & \Delta x_j^k \in \{-x_j^k, 1 - x_j^k\}, j \in [1, N_d]. \end{cases} \tag{5}$$

The sensitivity of the compliance function can be obtained using the adjoint method presented by Haftka and Gürdal (2012), which leads to

$$\frac{\partial C(x)}{\partial x_j} = \mathbf{u}^T \frac{\partial \mathbf{K}}{\partial x_j} \mathbf{u}. \tag{6}$$

The global stiffness matrix of Eq. (6) can be approximated as the sum of all the elemental stiffness matrices that constitute the mesh. Assuming an isotropic and homogeneous material, the expression of $\mathbf{K}$ is given by:

$$\mathbf{K} = \sum_{j=1}^{N_d} (E_{min} + x_j^p (E_0 - E_{min}))\mathbf{k_0}, \tag{7}$$

in which $\mathbf{k_0}$ is the stiffness matrix of an arbitrary element of the mesh, E_0 is the Young's modulus of an element j that remains in the mesh in a given iteration ($x_j = 1$), and E_{min} is the Young's modulus of an element that is to be removed from the mesh in a given interation ($x_j = 0$). Instead of $E_{min} = 0$, in this paper a very small value is used for E_{min} in order to avoid numerical problems during the solution of the finite element part of the problem. Substituting Eq. (7) into Eq. (6) yields

$$\frac{\partial C(x_j)}{\partial x_j} = -\frac{1}{2} p x_j^{p-1} (E_0 - E_{min}) \mathbf{u}_j^T \mathbf{k}_0 \mathbf{u}_j, \tag{8}$$

in which p is a penalization factor. Even though TOBS doesn't work with intermediate densities, the material interpolation scheme is used to aid in the sensitivity derivation (Picelli *et al.*, 2021). The value $p = 3$ was set throughout the implementation so that the element densities x_j in Eq. 8 don't cancel out. The same analysis can be carried for the sensitivity of the constraint function for the jth element of the mesh,

$$\frac{\partial g(x)}{\partial x_j} = \frac{V_j}{V_0}. \tag{9}$$

2.2 Filtering scheme

A numerical filter was adopted in this implementation in order to avoid the well-known checkerboard effect (Huang and Xie, 2010). In this filter, the sensitivity calculated at each iteration is replaced by (Andreassen *et al.*, 2011; Picelli *et al.*, 2021):

$$\frac{\partial \tilde{C}}{\partial x_j} = \frac{1}{\sum_{m \in N_m} H_{jm}} \sum_{m \in N_m} H_{jm} \frac{\partial C}{\partial x_m}, \tag{10}$$

in which N_m represents the number of neighbouring elements to element j that have influence over the new value of sensitivity of the element j. These are the elements contained in a sphere of prescribed radius r_{min} centered at the center of element j. The scalar H_{jm} in Eq. 10 represents a weighting factor, which is included to cause elements that are closer to j to have a larger influence in its sensitivity than elements that are more distant. This weighting factor is given by

$$H_{jm} = \max(0, r_{min} - \text{dist}(\boldsymbol{x_e} - \boldsymbol{x_m})), \tag{11}$$

in which x_e corresponds to the element whose sensitivity in being calculated and x_m are the neighbour elements inside the sphere of radius equal to r_{min} and the operation dist$(\boldsymbol{x_e} - \boldsymbol{x_m})$ corresponds to the distance between the center of element $\boldsymbol{x_e}$ and the center of element $\boldsymbol{x_m}$ Liu and Tovar (2014), Picelli *et al.* (2021).

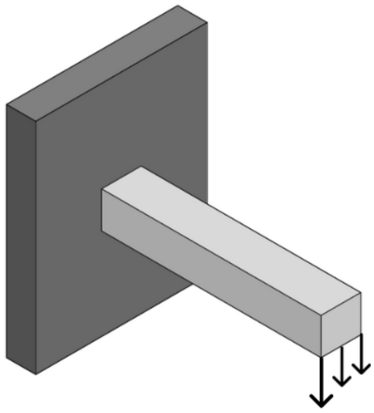
3. Results

The validation of the TOBS implementation was carried out through the comparison with selected problems. These problems had different boundary conditions and initial geometries. This included classical problems such as clamped-free beams and bars and three-point beams. Various combinations of optimization parameters were used as well. The optimized topologies were compared with the results obtained with other methods of topology optimization, such as BESO Cavalcante *et al.* (2022), Zuo and Xie (2015) and SIMP Liu and Tovar (2014) . Neither the optimized topologies nor the quantitative compliance of the optimized topology were the same as those obtained with the reference methods in any case. This is expected, due to differences intrinsic to each optimization method, and to the fact that even small variations in the optimization procedure leads to various local minima of the objective function. However, the results show physical consistency with what is known of optimized topologies for these problems.

3.1 Cantilever Beam

We illustrate the optimization of classical problems with the case of a cantilever beam (Fig. 1a). The initial domain was discretized with a mesh of $60 \times 20 \times 10$ elements. A uniformly-distributed load is applied to the bottom 11 elements of the free end of the beam. The constraint for the final value was $\bar{V} = 0.3$. The optimization parameters were: $r_{min} = 1.5$, $\beta = 0.05$ and $\epsilon = 0.01$ which was chosen based in a similar example implemented in Liu and Tovar (2014).

The results are presented in terms of the normalized volume $\bar{V} = V_k/V_0$, in which V_k is the total volume of the structure at iteration k, and of the normalized compliance $\bar{C} = C_k/C_0$, in which C_k is the compliance of the structure at iteration k (Fig. 2).



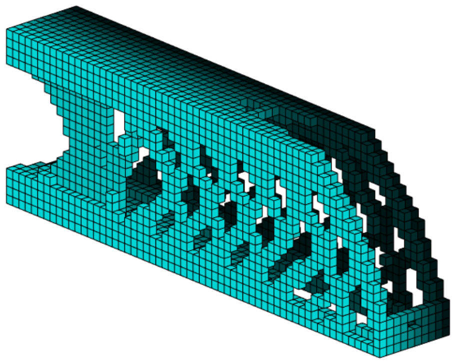


Figure 1: (a) Initial domain for the cantilever beam problem and (b) optimized topology for the cantilever beam.

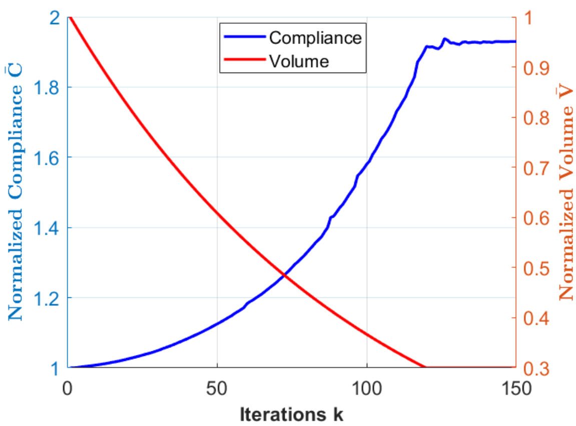


Figure 2: Normalized compliance and volume for each iteration of the optimization algorithm.

The optimized topology shown in Fig. 1b is comparable to classical results from the literature, such as Liu and Tovar (2014), which solved this problem with the classical SIMP method. In both results, the basic shapes of the structures are similar: flat surfaces are observed in the bottom and top parts of the beam, and a criss-cross pattern of reinforcements is observed in the side panes of the beam. More material is allocated around the clamped part of the beam, which is understandable due to the increased bending moment in that region, and almost all material is removed from the center of the beam, so that more material is kept in the outer portions, where it has an increased contribution to making the beam stiffer by increasing its moment of inertia. All these aspects are physically consistent, and can be explained from a general knowledge on the behavior of beams.

3.2 Tower problem

This problem consists of a tower of sides $a \times a$ and height $5a$, with uniformly-distributed load on its top surface over an area of sides $a/5$. Boundary conditions of zero displacement in all directions are imposed in each of the four corners of the tower (Fig. 3). Vertical (Fig. 3a) and horizontal (Fig. 3b) loads are considered. The towers were discretized by a mesh of $15 \times 60 \times 15$ elements. The constraint for the final value was $\bar{V} = 0.25$. The optimization parameters were: $r_{min} = 1.1$, $\beta = 0.05$ and $\epsilon = 0.01$ which was chosen based in the examples in Liu and Tovar (2014).

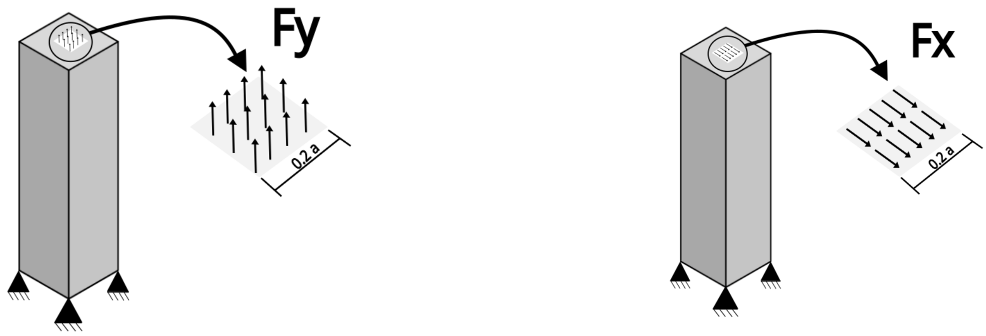


Figure 3: Tower problem under (a) vertical and (b) horizontal load.

Figures 4 and 5 show the evolution of the topology optimization at different iterations of the optimization process for the cases of tower under vertical and horizontal load, respectively. The optimized topologies for each case are shown in Fig. 6. These results are comparable to the ones obtained by Cavalcante *et al.* (2022), which used the classical BESO method to solve the same problem.

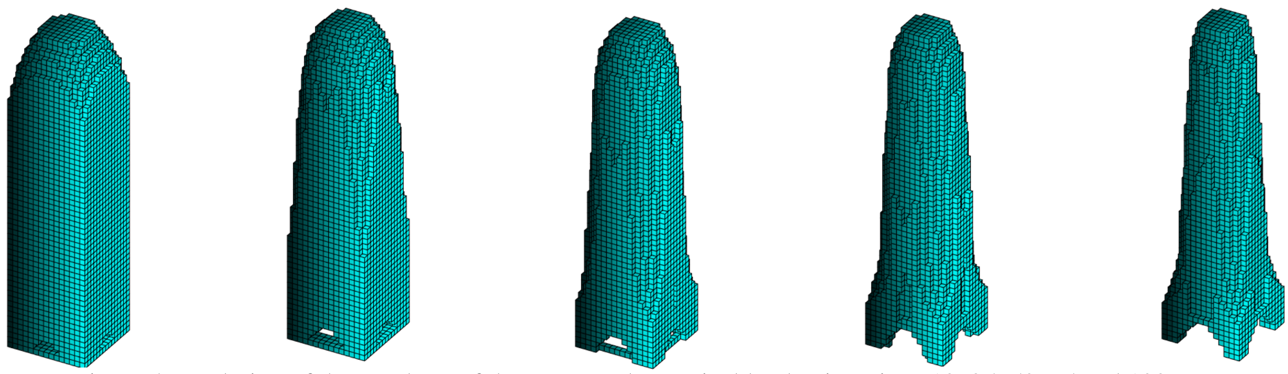


Figure 4: Evolution of the topology of the tower under vertical load at iterations 10, 25, 50, 75 and 100.

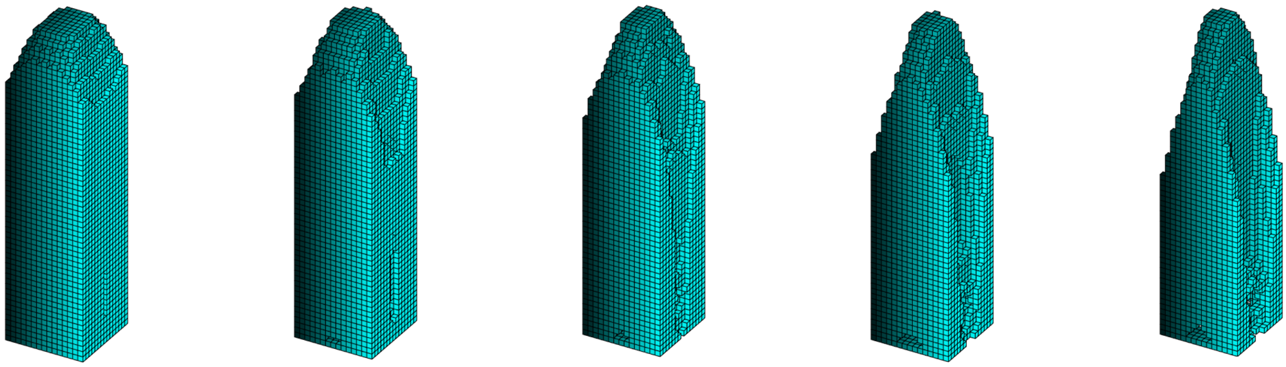


Figure 5: Evolution of the topology of the tower under horizontal load at iterations 10, 25, 50, 75 and 100.

Quantitative results for these analyses are shown in Fig. 7. These results show a stable convergence of the objective function towards the mininum compliance for the prescribed volume reduction. The number of iterations required to achieve the optimized topology were 140 and 138 for the tower under vertical and horizontal load, respectively, while the BESO method used by Cavalcante *et al.* (2022) took 79 and 100 iterations. The compliance of the optimized tower was 1.3664 and 1.1837 for the tower under vertical and horizontal load, respectively, while the compliance obtained by Cavalcante *et al.* (2022) for these problems were 1.3 and 1.18.

Figure 6: Optimized topology for the tower under (a) vertical and (b) horizontal load.

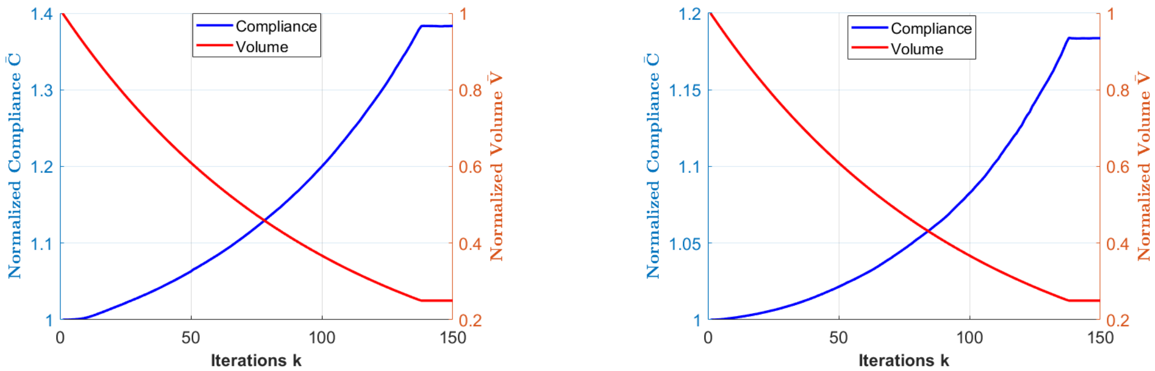


Figure 7: Normalized compliance and volume along iterations for the tower under (a) vertical and (b) horizontal load.

3.3 Bridge problem

The bridge problem consists of an initially non-optimized prismatic domain of sides $a \times a$ and length $5a$, the four bottom corners of which are restricted in all directions (Fig. 8). Uniformly-distributed vertical loads are applied on a patch of sides $a/5 \times 3a$ of the top surface of the domain. Figures 9 and 10b shows qualitatively and quantitatively the evolution of the topology of the bridge along the iterations of the optimization algorithm. The final optimized topology is shown in Fig. 10a. The algorithm took 138 iterations to reach the final optimized topology for this problem. The normalized compliance of the optimal topology is 1.5539.

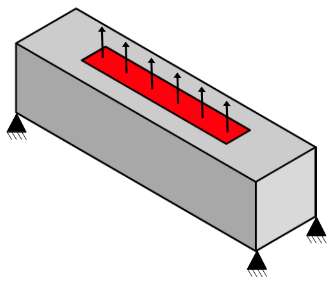


Figure 8: Bridge under vertical loads problem

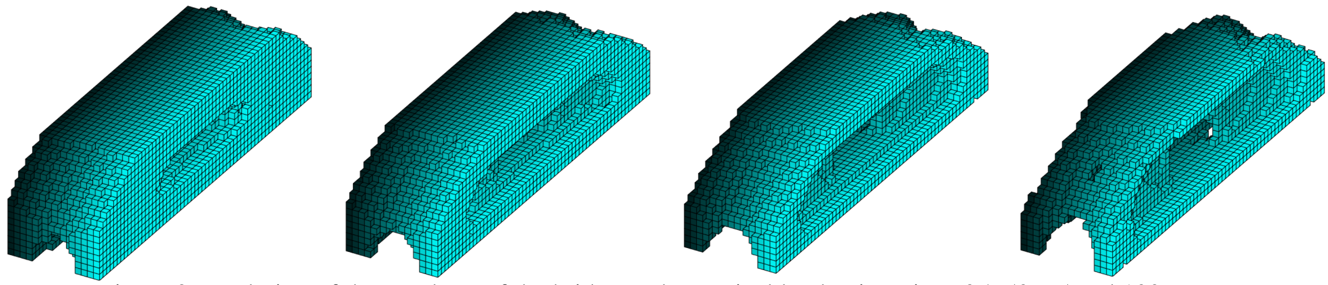


Figure 9: Evolution of the topology of the bridge under vertical load at iterations 25, 50, 75 and 100.

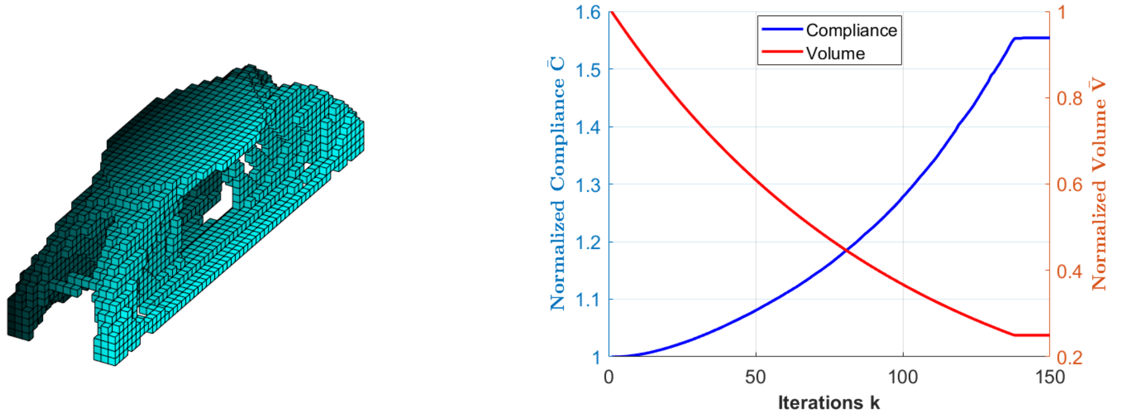


Figure 10: (a) optimized topology for the bridge under vertical loads (b) normalized compliance and volume along iterations for the bridge under vertical load.

4. Conclusions

In this paper, we have presented an implementation of the TOBS method for the design three-dimensional structures. The implementation utilizes a classical finite element solver, together with material interpolation based on the SIMP method. Elemental and nodal sensitivity and filtering schemes are implemented using an extension of the classical BESO method to three dimensions. Through the application of TOBS to three representative problems we have demonstrated the effectiveness of the method in generating optimized designs that meet prescribed requirements. The results illustrate the ability of TOBS to navigate the added complexity of 3D problems, with the integer linear programming algorithm providing robust optimization solutions. Overall, the implementation of TOBS for large-scale, 3D structural design presents a valuable contribution to the field of topology optimization, which can be applied directly to the design of optimized topologies in engineering practice.

5. ACKNOWLEDGEMENTS

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