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EPISTEMIC UNCERTAINTIES AND THEIR ROLE IN OPTIMAL DESIGN OF ENGINEERING SYSTEMS

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Abstract. *It is nowadays well accepted that engineering systems need to be designed considering the inherent uncertainties in demands and capacities. Robust, reliability-based and risk-based optimization are three modern approaches to address the optimal design problem under objective aleatory uncertainty, that is: uncertainty which can be quantified using probability theory. Yet, subjective epistemic uncertainties, related to limitations of knowledge, social, political, financial, organizational, and behavioral factors, human errors, etc., cannot be quantified probabilistically, but also need to be considered in design optimization. In this manuscript a procedure is proposed to address epistemic uncertainties in optimal design of structural systems. The optimal redundancy allocation problem is addressed as a case study.*

Keywords: *Robust optimization, risk-based optimization, reliability-based optimization, epistemic uncertainty, latent failure probability*

1. INTRODUCTION

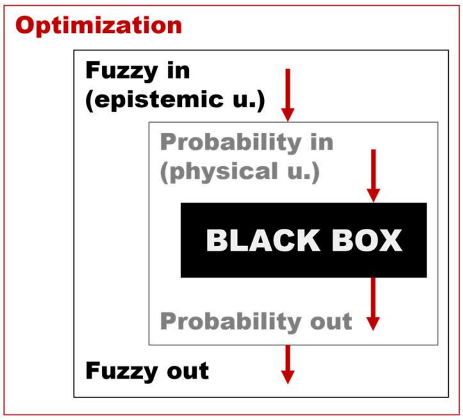
Structural design must be robust with respect to aleatory uncertainties in loads and material strengths, as well as to epistemic uncertainty arising from non-structural factors (Der Kiureghian and Ditlevsen, 2009). This has led to the development of Reliability-Based Design Optimization (RBDO), where minimum reliability is a design constraint; to life-cycle cost or Risk Optimization (RO), where reliability is included in the objective function, targeting the balance between the costs of fabrication, inspection and maintenance, and the expected costs of failure; and robust optimizations, where compromise solutions are sought between minimizing the mean and the variance of an objective function. Overview reviews on RBDO, RO and robust optimizations are given in (Beyer and Sendhoff 2007; Schuëller and Jensen 2008; Aoues and Chateauneuf 2010; Valdebenito and Schuëller 2010; Lopez and Beck 2012).

Typically, the above formulations address objective, aleatory uncertainties in structural loads, material strengths and model errors, which can be modelled using probability theory, with limited subjectivity. Yet, the lifetime performance of engineering structures may also be affected by disturbance events significantly characterized by epistemic or subjective uncertainties, arising from ignorance, vagueness and subjective judgement, gross errors in design and manufacturing, human errors, operational abuse, abnormal loading and so on (Ditlevsen 1983; Faber et al. 2017a,b, 2018; Melchers 2001, 2002, 2007). Optimal designs are intrinsically less robust to disturbance events not objectively addressed in the optimization. The history of structural design contains many examples of structural collapses which occurred due to unanticipated failure modes and loading conditions (phenomenological uncertainty), bad management decisions, and human errors in design and execution (Petroski 1992, 2006, 2012).

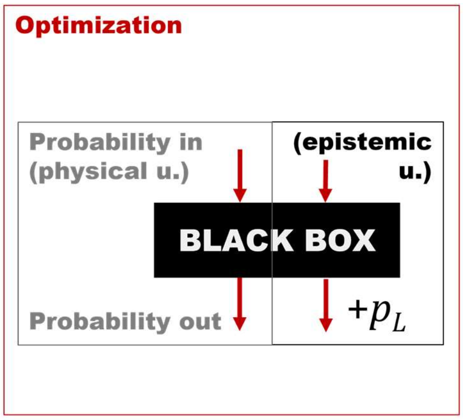
In the reliability-based design optimization literature, subjective uncertainty of epistemic origin has been typically handled using set theory, intervals, fuzzy sets and imprecise probabilities (Möller et al. 2003; Möller and Beer 2004, 2008; Jiang et al. 2018; Acar et al. 2021). Ellishakoff and co-authors (Elishakoff et al. 1994; Tonon et al. 2001; Qiu et al. 2008; Ben-Haim and Elishakoff 1990) developed the concept of optimization with anti-optimization to address uncertain-but-bounded uncertainty in loads (worst case approach). Approaches addressing information insufficiency have led to compromise solutions between reducing objective functions and increasing confidence on constraint feasibility (Gunawan and Papalambros 2006; Srivastava and Deb 2013; Cho et al. 2016; Ito et al. 2018). Other approached have led to conservative failure probability estimates, like in (Rockafellar and Royset 2010) and (Saad et al. 2018).

RBDO approaches combining random variables to model aleatory uncertainty, with interval/fuzzy variables to model epistemic uncertainties lead to triple-nested loops (Jiang et al. 2018; Acar et al. 2021). The outer loop is typically the design optimization; the inner loop is the reliability analysis, and the intermediate loop handles the interval/fuzzy variables (Figure 1). As triple-nested loops pose a severe burden on computation, most of the papers in this class address the efficient de-coupling of the nested loops (Nannapaneni and Mahadevan 2016; Zaman and Mahadevan 2017). See (Jiang et al. 2018) for a comprehensive review on decoupling schemes.

The above formulations to handle mixed aleatory and epistemic uncertainties in optimal design of structural systems are considered of limited usefulness because:



A) Usual nested approach



B) proposed *latent failure probability* approach

Figure 1. Nesting of optimization and uncertainty quantification loops (u. = uncertainty).

A. Interval/fuzzy uncertainty representations are appropriate to handle subjective uncertainties arising from ignorance, vagueness and subjective judgement. Yet, they may not be ideal to consider disturbance events like gross errors in design and manufacturing, unaccounted load cases or failure modes, human errors, and operational abuse.
B. Nesting of optimization, interval/fuzzy and reliability analysis loops, and/or complex or approximate de-coupling schemes (Figure 1).
C. Complex handling of interval, convex, fuzzy and random sets within an optimization framework.

In this paper, we employ the latent failure probability concept (Beck 2020; Beck et al. 2020a,b; Beck et al. 2022a,b; Beck et al. 2023; Rodrigues da Silva et al. 2023) to address the possible impacts of epistemic uncertainties related mainly to gross errors in design and manufacturing, unaccounted load cases or failure modes, human errors, and operational abuse. The formulation avoids the difficulties associated with points B and C above (see Figure 1). We address the problem by way of a fundamental RBDO problem.

2. RELIABILITY-BASED DESIGN OPTIMIZATION OF A PARALLEL SYSTEM WITH IDENTICAL COMPONENTS

Let $n_c \in \mathbb{N}$, or $n_c \in \{1,2,3,..\}$ be the number of identical components of a parallel system (Figure 2), each component with a reliability $r_i = r \in (0,1]$. Parameter $\rho \in [0,1)$ is the correlation between component failures, and $p_L \in [0,1)$ is a factor to account for epistemic uncertainties, following (Beck 2020; Beck et al. 2020a,b; Beck et al. 2022a,b; Beck et al. 2023; Rodrigues da Silva et al. 2023). The system reliability is given by (Fiondella and Xing 2015):

$$\mathcal{R}_{SYS}(r, p_L, \rho, n_c) = \frac{1-(1-(r-p_L)\eta)^{n_c}}{\eta}, \qquad \eta = 1 + \rho\frac{\big(1-(r-p_L)\big)}{r-p_L}. \tag{1}$$

We want to find the optimal reliability of each component, and the minimal number of components, with a constraint on system reliability, by solving:

$$\begin{aligned} &\text{given: } \mathcal{R}_T,\ p_L \\ &\text{find: } \mathbf{d}^* = \{r, n_c\} \\ &\text{which minimizes: } f_{obj}(\mathbf{d}) = n_c\,(-Log_{10}[1-r])^q \\ &\text{subject to: } \mathcal{R}_T - \mathcal{R}_{SYS}(r, p_L, \rho, n_c) \le 0,\ n_c \in \mathbb{N}, \end{aligned} \tag{2}$$

where $\mathbf{d}$ is the vector of design variables, $f_{obj}(\cdot)$ is an objective function representing the cost of reliability, $\mathcal{R}_{SYS}(r, p_L, \rho, n_c)$ is the system reliability and $\mathcal{R}_T$ is the target system reliability. In Eq. (2), component reliabilities $r_i = r$ reflect objective aleatory uncertainty, and parameter p_L considers the effect of subjective epistemic uncertainty on component reliabilities.

Analytical solution of Eq. (1) is not simple, because it involves discrete variable n_c. However, since the problem involves only two design variables, objective function and constraints can be observed in two-dimensional plots. Moreover, candidate solutions can be easily tested for discrete n_c values.

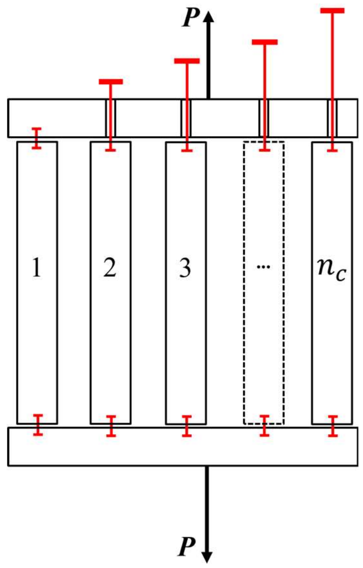


Figure 2. Parallel system with n_c identical redundant elements.

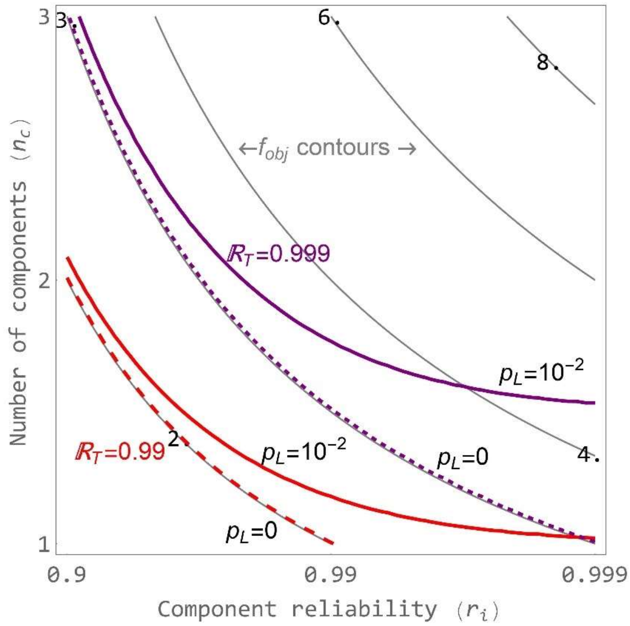


Figure 3. Objective function and constrains for the optimization problem in Eq. (9), $\rho = 0$.

Table 1. Optimal component reliabilities for $q = 1$ and $\rho = 0$.

	$p_L = 0$		$p_L = 10^{-2}$	
n_c	$\mathcal{R}_T = 0.99$	$\mathcal{R}_T = 0.999$	$\mathcal{R}_T = 0.99$	$\mathcal{R}_T = 0.999$
1	0.99	0.999	-	-
2	0.9	0.96838	0.90442	0.97287
3	0.78456	0.9	0.80182	0.93338

Figure 3 illustrates the problem for $q = 1$, $\rho = 0$, and two values $p_L = 0$ and $p_L = 10^{-2}$. The figure shows contour curves of the objective function in Eq. (2), for $q = 1$, and the system reliability constraints $\mathcal{R}_T = 0.99$ and $\mathcal{R}_T = 0.999$. It can be observed that the objective function is convex, and that it decreases by reducing both n_c and r. Intuitively, the reliability constraint will be active at the optimal points. This is confirmed by writing the Lagrangian function, taking the derivatives w.r.t. design variables, Lagrange multiplier and slack variable, and equating them to zero. By assuming the constraint to be active, and solving the resulting system of equations, we confirm that Lagrange multipliers are positive; hence, the constraint is active at the minima. The lengthy derivations are omitted but can be easily verified by the reader.

As shown in Figure 3, for $p_L = 0$ the reliability constraints are parallel to the objective function. In this case Eq. (2) has several optimal solutions, which are *indifferent* to specific combinations of r and n_c: the solution for $\mathcal{R}_T = 0.99$ can be two components with $r = 0.9$, or a single component with $r = 0.99$ (or the solutions in between, if $n_c \in \mathbb{R}$); the solution for $\mathcal{R}_T = 0.999$ can be three bars with $r = 0.9$, or a single bar with $r_i = 0.999$ (and solutions in between, like two bars with $r_i = 0.96838$). For $p_L = 10^{-2}$ the single-component solutions become unfeasible, but two components with $r > 0.9$ (actually, $r = 0.90442$) still lead to $\mathcal{R}_T = 0.99$; and three components with $r > 0.9$ (actually, $r = 0.93338$) yield $\mathcal{R}_T = 0.999$. Hence, we see that $p_L > 0$ makes the system more redundant, also increasing robustness of optimal solutions w.r.t. epistemic uncertainties. Table 1 summarizes the results for $q = 1$ and $\rho = 0$.

For $q = 1$, the objective function in Eq. (2) assumes a linear relationship between the cost of one component and the base-ten logarithm of its failure probability, as shown in Figure 4. With a reference unitary cost for $r = 0.9$, the cost doubles for $r = 0.99$ and triples for $r = 0.999$. This cost function leads to the equivalent optimal solutions appearing in Figure 3 for $p_L = 0$. The cost of a structural component can be assumed to be proportional to its reliability index: Figure 4 shows this cost is also nearly proportional to the base-ten logarithm of the failure probability, at least in the practical range $0.9 < r_i < 0.9999$. Figure 4 also identifies two regions called "faster cost growth" and "slower cost growth", which refers to the growth of cost of component reliabilities with respect to the linear relationship in Figure 4. If component reliability cost grows "faster" than linear ($q > 1$), multi-bar solutions with smaller reliability are preferred, as shown in Table 2. If component reliability grows "slower" than linear ($q < 1$), then the solution of RBDO problems like Eq. (2) is a single-component system, for $p_L = 0$ (see Table 2). Hence, for any reliability cost function that grows slower than for $q = 1$ in Eq. (2), the optimal RBDO solution is a single component system. This is likely the case in practical structural reliability problems. This is the case for cost function proportional to cross-section area (a_i, next section). In this scenario, $p_L > 0$ can be employed to impose minimal redundancy, increasing robustness of optimal solutions w.r.t. epistemic uncertainties.

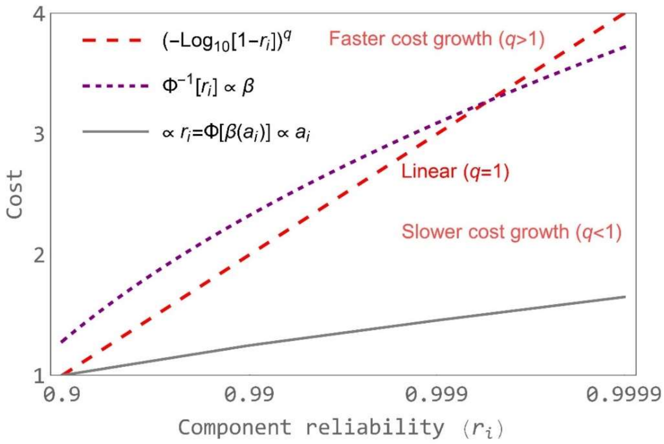


Figure 4. Cost of reliability curves considered in Eqs. (2) and (4).

Table 2: Objective function values (and component reliabilities) for $\rho = 0$, $p_L = 0$ and $\mathcal{R}_T = 0.999$.

n_c	$q = 1$	$q = 2$	$q = 1/2$	(r)
1	3	9	1.73	(0.999)
2	3	4.5	2.45	(0.96838)
3	3	3	3	(0.9)
4	3	2.25	3.43	(0.82217)

Table 3: Objective function values (and component reliabilities) for $\rho = 0.01$, $p_L = 0$ and $\mathcal{R}_T = 0.999$.

n_c	$q = 1$	$q = 2$	$q = 1/2$	(r)
1	3	9	1.73205	(0.999)
2	3.13309	4.90812	2.50323	(0.97287)
3	3.52913	4.15158	3.25383	(0.93338)

Finally, in order to illustrate the impact of correlation between failures of different components, Figure 5 shows the objective function and constraints in Eq. (2) for $\rho = 0.01$. As observed in Figure 5, the slight shift of the constraints, produced by $\rho > 0$, means that the 2 and 3 component solutions are no longer optimal, for $q = 1$. This is confirmed by the results in Table 3.

Following Figure 5 for $q = 1$, in order to achieve $\mathcal{R}_T = 0.99$ for $p_L = 0$ with $n_c = 2$, the individual component reliabilities need to be larger than 0.9, to "compensate" for correlation (actually, $r = 0.91$). The impact of correlation is larger for $\mathcal{R}_T = 0.999$, as observed: to achieve this system reliability, it is required that $n_c = 2$ with $r = 0.97837$, or $n_c = 3$ with $r = 0.91$. Fiondella and Xing (2015) show that reliability of the parallel system is very sensitive to correlation ρ, and that system reliability is limited for large ρ, independently of the number of components. Herein we show that correlation favors single component optimal solutions for $q \leq 1$. Correlation between component failures are expected in structural reliability, due to common materials and load effects. Hence, again, $p_L > 0$ is a tool to achieve robustness of optimal systems w.r.t. epistemic uncertainties. These results hold for systems with non-identical components, as shown in (Beck 2020; Beck et al. 2020a,b; Beck et al. 2022a,b; Beck et al. 2023; Rodrigues da Silva et al. 2023).

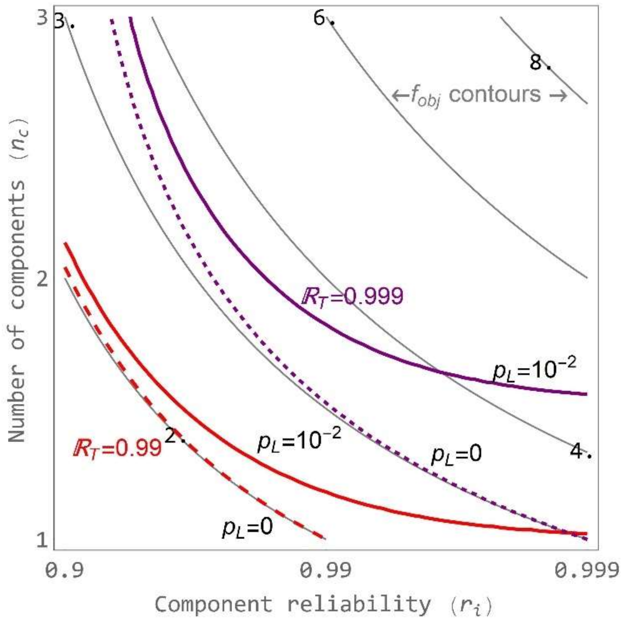


Figure 5. Objective function and constrains for the optimization problem in Eq. (1), $\rho = 0.01$.

3. STRUCTURAL RELIABILITY VARIANT

A structural reliability variant of the above optimization problem is obtained by considering the optimal design of the cross-section areas of the bars in Figure 2. The bars are arranged in a passive redundancy scheme[1], such that any bar can hold load P by itself, and bars $2, 3, \dots, n_c$ only come into service in case of failure of bars $1,2, \dots, n_c - 1$, respectively.

Objective aleatory uncertainties are given by load P and material strength S, both with normal distributions: $P \sim N(\mu_P, \sigma_P)$, $S \sim N(\mu_S, \sigma_S)$; such that the nominal element failure probability becomes:

$$p_{N,i} = \Phi(-\beta_{N,i}), \quad \beta_{N,i} = \frac{a_i\, \mu_S - \mu_P}{\sqrt{a_i^2\, \sigma_S^2 + \sigma_P^2}}, \tag{3}$$

where $p_{N,i}$ is the nominal failure probability, $\beta_{N,i}$ is the nominal reliability index, $\mathbf{d} = \{n_c, a_i\}$, $i \in (1,2, \dots, n_c)$ is the vector of design variables and $\Phi(\cdot)$ is the standard Gaussian cumulative distribution function. For simplicity, there is no dynamic load amplification upon progressive failure of one or more bars. Material strength of all bars is identically distributed and is likely to introduce correlation in the probability of failure of individual bars. This correlation is controlled via independent parameter ρ.

A problem of identically distributed correlated components is obtained by requiring all components to have same cross-section ($a_i = a$, $i \in (1,2, \dots, n_c)$), such that all components have same nominal failure probability $p_{N,i} = p_N$; and same latent failure probability ($p_{L,i} = p_L$, $i \in (1,2, \dots, n_c)$); hence, also same failure probability: $p_f = \Phi(-\beta_N) + p_L$.

Writing the constraint in terms of reliability indexes, the RBDO problem becomes:

$$\begin{aligned} \text{given: } & \beta_T,\ p_L \\ \text{find: } & \mathbf{d}^* = \{(\beta_N^* \text{ or } a^*), n_c\} \\ \text{which minimizes: } & f_{obj}(\mathbf{d}) \\ \text{subject to: } & \beta_T - \beta_{SYS}((\beta_N \text{ or } a), p_L, \rho, n_c) \leq 0,\ n_c \in \mathbb{N}. \end{aligned} \tag{4}$$

where $\mathbf{d}$ is the vector of design variables, $f_{obj}(\cdot)$ is an objective function, $\beta_{SYS}((\beta_N \text{ or } a), p_L, \rho, n_c)$ is the system reliability index and β_T is the target system reliability index. As stated in Eq. (3), the design variables can be the nominal reliability index, β_N, or the cross-section areas of the bars, a. The objective functions for these two cases are:

[1] Active-passive type of redundancy is more common in structural mechanics, as illustrated in Beck (2020). Yet, the best simple model for progressive collapse analysis is with passive redundancy, since the passive strength of nearby elements is mobilized, in case of failure of one or more structural elements.

$$f_{obj}(\mathbf{d}) = \frac{\beta_N(a) \times n_c}{\beta_T} \text{ or } f_{obj}(\mathbf{d}) = \frac{a \times n_c}{a_{ref}} \tag{5}$$

These functions are normalized by the target system reliability index, β_T; or by a reference cross-section area, obtained by solving $\beta_N(a_{ref}) = \beta_T$ for a_{ref}.

Figure 6 illustrates solution of the System RBDO problem in Eq. (3) in terms of β_N, with $\beta_T = 4$ and $\rho = 10^{-3}$, for increasing values of $p_{L,i} = p_L$. As observed, for $p_L < \Phi(-\beta_T) = 3.167 \times 10^{-5}$, the optimal system has a single component, as obtained for $p_L = 0$. If p_L increases past the point $p_L = \Phi(-\beta_T)$, the optimal solution becomes a two-component system, and β_N^* drops accordingly (less reliable components are required for the two-component system). If p_L keeps increasing, the optimal system eventually becomes a three or four bar solution, with corresponding drops in optimal component reliabilities β_N^*. The increase in p_L "within" a fixed number of components is compensated by an increase in β_N^*. Eventually, the increase in β_N^* is no longer cost-effective, and the optimal solution requires adding another component. These trends reflect the system adjusting itself for increasing effect of epistemic uncertainties. Following Figure 6, the greater p_L, the larger the optimum number of identically distributed components in the parallel system. Note that for $\beta_T = 4$, the reliability cost function is below the diagonal ($q = 1$) in Figure 3. For $\beta_T = 3$ or $\beta_T = 2$, the cost function is above the diagonal, and the optimal solutions have more than one bar, as in the case for $q > 1$ (Tables 2 and 3).

Figure 7 shows that essentially the same results are obtained for minimization of the cross-section areas. In this case, the reliability cost function is always below the $q = 1$ diagonal in Figure 3, and the optimal solutions are single-bar systems for $p_L = 0$.

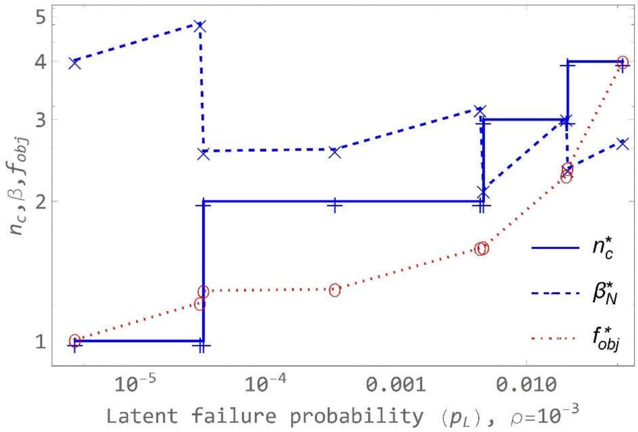


Figure 6. Optimal component reliabilities (β_N^*) and optimal number of components (n_c^*) of parallel system with identically distributed correlated components, with $\beta_T = 4$ and $\rho = 10^{-3}$.

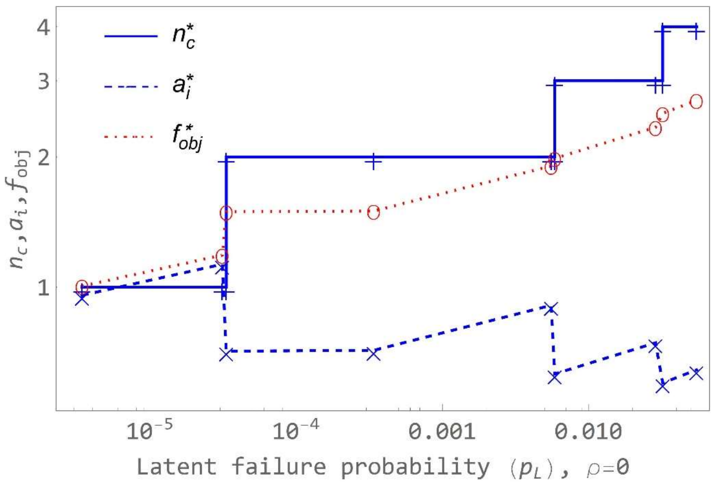


Figure 7. Optimal bar cross-section areas (a_i^*) and optimal number of components (n_c^*) of parallel system with identically distributed correlated components, with $\beta_T = 4$ and $\rho = 0$.

4. CONCLUDING REMARKS

In this manuscript we addressed the Reliability-Based Design Optimization (RBDO) problem considering the effects of epistemic uncertainties. Specifically, we addressed optimal design of a parallel system with identical elements. We looked for the optimal number of elements, and their optimal reliabilities, for the system to attain a given minimum system reliability target. We have shown that epistemic uncertainties should be accounted for by an independent term, called latent failure probability or p_L. We have shown that, for usual cost of reliability functions, if epistemic uncertainties are neglected ($p_L = 0$), optimal solutions are single-bar (non-redundant) systems. We have shown that as the effect of epistemic uncertainties ($p_L > 0$) increases, optimal solutions present two, three and more bars. Hence, the optimal system becomes redundant not because of aleatory uncertainties in load and strength parameters, but to cope with the effect of epistemic uncertainties. We have discussed other forms of dealing with epistemic uncertainties, like intervals, fuzzy variables, fuzzy probabilities, etc.; but we have not directly compared our formulation to these alternative approaches. In the presentation, we will show that very similar results are obtained when the reliability of individual elements, or the latent failure probability p_L, are modelled as intervals, fuzzy variables or fuzzy probabilities.

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6. REFERENCES

Acar, Erdem, Gamze Bayrak, Yongsu Jung, Ikjin Lee, Palaniappan Ramu, and Suja Shree Ravichandran. 2021. "Modeling, Analysis, and Optimization under Uncertainties: A Review." Structural and Multidisciplinary Optimization 64 (5): 2909–45. https://doi.org/10.1007/s00158-021-03026-7.

Aoues, Younes, and Alaa Chateauneuf. 2010. "Benchmark Study of Numerical Methods for Reliability-Based Design Optimization." Structural and Multidisciplinary Optimization 41 (2): 277–94.

Beck, A. T. 2020. "Optimal Design of Redundant Structural Systems: Fundamentals." Engineering Structures 219: 110542. https://doi.org/10.1016/j.engstruct.2020.110542.

Beck, A. T., L. R. Ribeiro, M. Valdebenito. 2020a. "Risk-Based Cost-Benefit Analysis of Frame Structures Considering Progressive Collapse under Column Removal Scenarios." Engineering Structures 225 (May). https://doi.org/10.1016/j.engstruct.2020.111295.

Beck, A. T., L. R. Ribeiro, M. Valdebenito. 2020b. "Risk-Based Cost-Benefit Analysis of Frame Structures Considering Progressive Collapse under Column Removal Scenarios." Engineering Structures 225 (May). https://doi.org/10.1016/j.engstruct.2020.111295.

Beck, A. T., L. R. Ribeiro, M. Valdebenito, H. Jensen. 2022a. "Risk-Based Design of Regular Plane Frames Subject to Damage by Abnormal Events: A Conceptual Study." Journal of Structural Engineering 148 (1). https://doi.org/10.1061/(asce)st.1943-541x.0003196.

Beck, A.T., L. R. Ribeiro, M. A. Valdebenito. 2022b. "Cost-Benefit Analysis of Design for Progressive Collapse under Accidental or Malevolent Extreme Events." In Engineering for Extremes: Decision-Making in an Uncertain World, edited by M. G. Stewart and D. v. Rosowsky. New York: Springer. https://doi.org/10.1007/978-3-030-85018-0_15.

Beck, A.T., L.A. Rodrigues da Silva, L.F.F. Miguel. 2023. "The Latent Failure Probability: A Conceptual Basis for Robust, Reliability-Based and Risk-Based Design Optimization." Reliability Engineering & System Safety 233, 109127, https://doi.org/10.1016/j.ress.2023.109127

Ben-Haim, Y., and I. Elishakoff. 1990. Convex Models of Uncertainty in Applied Mechanics. Elsevier.

Beyer, Hans-Georg, and Bernhard Sendhoff. 2007. "Robust Optimization – A Comprehensive Survey." Computer Methods in Applied Mechanics and Engineering 196 (33–34): 3190–3218. https://doi.org/10.1016/j.cma.2007.03.003.

Cho, H., K. K. Choi, N. J. Gaul, I. Lee, D. Lamb, D. Gorsich. 2016. "Conservative Reliability-Based Design Optimization Method with Insufficient Input Data." Structural and Multidisciplinary Optimization 54 (6): 1609–30. https://doi.org/10.1007/s00158-016-1492-4.

Der Kiureghian, A., O. Ditlevsen. 2009. "Aleatory or Epistemic? Does It Matter?" Structural Safety 31 (2): 105–12. https://doi.org/10.1016/j.strusafe.2008.06.020.

Ditlevsen, Ove. 1983. "Fundamental Postulate in Structural Safety." Journal of Engineering Mechanics 109 (4): 1096–1102. https://doi.org/10.1061/(asce)0733-9399(1983)109:4(1096).

Elishakoff, I., R.T. Haftka, and J. Fang. 1994. "Structural Design under Bounded Uncertainty—Optimization with Anti-Optimization." Computers & Structures 53 (6): 1401–5. https://doi.org/10.1016/0045-7949(94)90405-7.

Elishakoff, I. 2021. "Review of Uncertainty Quantification, Associated Parables, and Raphael Tuvia Haftka's Contributions." Structural and Multidisciplinary Optimization 64 (5): 2947–57.

Faber, M. H., J. Qin, S. Miraglia. 2017a. "On Decision Support for Sustainability and Resilience of Infrastructure." 12th Int. Conf. on Structural Safety and Reliability, no. August: 1646–55.

Faber, M. H., J. Qin, S. Miraglia, and S. Thöns. 2017b. "On the Probabilistic Characterization of Robustness and Resilience." Procedia Engineering 198 (September 2016): 1070–83. https://doi.org/10.1016/j.proeng.2017.07.151.

Faber, Michael H., Simona Miraglia, Jianjun Qin, and Mark G. Stewart. 2018. "Bridging Resilience and Sustainability - Decision Analysis for Design and Management of Infrastructure Systems*." Sustainable and Resilient Infrastructure 5 (1–2): 102–24. https://doi.org/10.1080/23789689.2017.1417348.

Fiondella L., Xing L., 2015: Discrete and continuous reliability models for systems with identically distributed correlated components, Reliability Engineering and System Safety 133, pp. 1–10, 2015, doi: 10.1016/j.ress.2014.08.004.

Gunawan, Subroto, and Panos Y. Papalambros. 2006. "A Bayesian Approach to Reliability-Based Optimization With Incomplete Information." Journal of Mechanical Design 128 (4): 909–18. https://doi.org/10.1115/1.2204969.

Ito, M., N.H. Kim, N. Kogiso. 2018. "Conservative Reliability Index for Epistemic Uncertainty in Reliability-Based Design Optimization." Structural and Multidisciplinary Optimization 57 (5): 1919–35. https://doi.org/10.1007/s00158-018-1903-9.

Jiang, C., J. Zheng, and X. Han. 2018. "Probability-Interval Hybrid Uncertainty Analysis for Structures with Both Aleatory and Epistemic Uncertainties: A Review." Structural and Multidisciplinary Optimization 57 (6): 2485–2502..

Lopez, Rafael Holdorf, and André Teófilo Beck. 2012. "Reliability-Based Design Optimization Strategies Based on FORM : A Review." Journal of the Brazilian Society of Mechanical Sciences and Engineering 34 (4): 506–14.

Melchers, R. E. 2001. "Rational Optimization of Reliability and Safety Policies." Reliability Engineering and System Safety 73 (3): 263–68. https://doi.org/10.1016/S0951-8320(01)00051-5.

Melchers, R. E. 2002. "Safety and Risk in Structural Engineering." Progress in Structural Engineering and Materials 4 (2): 193–202. https://doi.org/10.1002/pse.110.

Melchers, R. E. 2007. "Structural Reliability Theory in the Context of Structural Safety." Civil Engineering and Environmental Systems 24 (1): 55–69. https://doi.org/10.1080/10286600601025191.

Möller, B., and M. Beer. 2004. Fuzzy Randomness – Uncertainty in Civil Engineering and Computational Mechanics. Berlin Heidelberg New York: Springer.

Möller, B., M. Beer. 2008. "Engineering Computation under Uncertainty – Capabilities of Non-Traditional Models." Computers & Structures 86 (10): 1024–41. https://doi.org/10.1016/j.compstruc.2007.05.041.

Möller, B., W. Graf, M. Beer. 2003. "Safety Assessment of Structures in View of Fuzzy Randomness." Computers and Structures 81 (15): 1567–82. https://doi.org/10.1016/S0045-7949(03)00147-0.

Nannapaneni, S., S. Mahadevan. 2016. "Reliability Analysis under Epistemic Uncertainty." Reliability Engineering & System Safety 155 (November): 9–20. https://doi.org/10.1016/j.ress.2016.06.005.

Petroski, H. 1992. To Engineer Is Human: The Role of Failure in Successful Design. 1st ed. Vintage Books.

Petroski, H. 2006. Success through Failure: The Paradox of Design. Princeton University Press, UK.

Petroski, H. 2012. To Forgive Design: Understanding Failure. Harvard University Press.

Qiu, Z.. Di Yang, I. Elishakoff. 2008. "Probabilistic Interval Reliability of Structural Systems." International Journal of Solids and Structures 45 (10): 2850–60. https://doi.org/10.1016/j.ijsolstr.2008.01.005.

Rockafellar, R. T., and J. O. Royset. 2010. "On Buffered Failure Probability in Design and Optimization of Structures." Reliability Engineering and System Safety 95 (5): 499–510. https://doi.org/10.1016/j.ress.2010.01.001.

Rodrigues da Silva, L.A., A. J. Torii, A. T. Beck. 2023. "Hyperstatic and Redundancy Thresholds in Truss Topology Optimization Considering Progressive Collapse Due to Aleatory and Epistemic Uncertainties." Probabililistic Engineering Mechanics 71, 103384, https://doi.org/10.1016/j.probengmech.2022.103384.

Saad, L., A. Chateauneuf, and W. Raphael. 2018. "Robust Formulation for Reliability-Based Design Optimization of Structures." Structural and Multidisciplinary Optimization 57 (6): 2233–48.

Schuëller, G.I., and H.a. Jensen. 2008. "Computational Methods in Optimization Considering Uncertainties – An Overview." Computer Methods in Applied Mechanics and Engineering 198 (1): 2–13.

Srivastava, R., K. Deb. 2013. "An Evolutionary Based Bayesian Design Optimization Approach under Incomplete Information." Engineering Optimization 45 (2): 141–65. https://doi.org/10.1080/0305215X.2012.661730.

Tonon, F., A. Bernardini, I. Elishakoff. 2001. "Hybrid Analysis of Uncertainty: Probability, Fuzziness and Anti-Optimization." Chaos, Solitons & Fractals 12 (8): 1403–14. https://doi.org/10.1016/S0960-0779(00)00103-X.

Valdebenito, M. A., G. I. Schuëller. 2010. "A Survey on Approaches for Reliability-Based Optimization." Structural and Multidisciplinary Optimization 42 (5): 645–63. https://doi.org/10.1007/s00158-010-0518-6.

Zaman, K., S. Mahadevan. 2017. "Reliability-Based Design Optimization of Multidisciplinary System under Aleatory and Epistemic Uncertainty." Structural and Multidisciplinary Optimization 55 (2): 681–99.

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