COBEM 2023 27TH INTERNATIONAL CONGRESS OF MECHANICAL ENGENEERING
ABCM Brazilian Society of Mechanical Sciences and Engineering
27th ABCM International Congress of Mechanical Engineering
December 04-08, 2023, Florianópolis-SC, Brazil

COB-2023-1562
EXPANSION OF A NON-INTRUSIVE IMPLEMENTATION OF THE GENERALIZED FINITE ELEMENT METHOD - GLOBAL-LOCAL

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Abstract. *Due to cost and complexity, advances in the formulations to numerically simulate structural engineering problems may not yet be fully available to industry, such as, for example, the Generalized Finite Element Method with global-local enrichment (GFEM^{gl}). Its implementation in commercial software packages can contribute to understanding the behavior of complex problems, involving discontinuities, singularities, and localized nonlinearities. On the other hand, research programs that implement these new methods are restricted to their application range, as they do not prioritize computational efficiency. In recent years, several studies have been carried out to develop non-intrusive strategies capable of combining the resources of research software with existing simulation tools. In this context, the present work aimed to expand and investigate a strategy developed for a non-intrusive coupling between the commercial software Abaqus and the research computational platform INSANE (INteractive Structural ANalysis Environment). This so-called strategy, IGL-GFEM^{gl}, consists of a multiscale computational framework that combines an iterative global-local solution (IGL) with GFEM^{gl}. The structural problem is decomposed into three analysis scales with their respective models and type of discretizations: global scale, meso scale, and local scale. The global scale comprises the whole structure without the local features of interest. These local features are only described at the local scale. The meso scale is used as a bridge between the other two scales. It receives the information from the analysis of the global scale, performed by Abaqus, and transfers it to the local scale. The meso and the local scale represent the global and the local models, solved in INSANE, by GFEM^{gl}. An iterative procedure is performed to match the solutions of the global and meso scales. This strategy was first proposed by H. Li, P. O'Hara and C. A. Duarte in 2021. It is improved and evaluated here to simulate the presence of local features, such as holes in two-dimensional problems. The iterative algorithm is modified and relaxation techniques are used to improve the efficiency of the solution procedure. In the numerical example presented, the time computation cost, the number of iterations, the convergence behavior, and the quality of the solution are assessed to demonstrate the contribution of the modifications proposed.*

Keywords: *Generalized Finite Element Method with global-local enrichment, non-intrusive coupling, multiscale computational framework.*

1. INTRODUCTION

Academic programs, as the computational platform INSANE (INteractive Structural ANalysis Environment) (Fonseca and Pitangueira, 2007) (Alves *et al.*, 2013)), a free software project developed by the Department of Structural Engineering at the Federal University of Minas Gerais, implement advanced methods and formulations that, although well established in academia, still are not fully available to the industry, such as the Generalized Finite Element Method with global-local enrichment (GFEM^{gl}) (Duarte and Kim, 2008) (Kim *et al.*, 2010). However, the availability of such methods could be of great contribution to the treatment and understanding of complex problems, involving discontinuities, singularities and localized non-linearities.

Although robust, commercial software packages for modeling and simulating Engineering problems generally implement only the standard Finite Element Method (FEM), making the modeling and analysis of localized physical phenomena significantly more expensive in terms of human and computational resources. Additionally, implementing more advanced discretization methods, alternatives to FEM, in commercial program packages is often difficult and time-consuming (Filmore and Duarte, 2018). On the other hand, academic programs that implement these new methods, and that could strengthen the national software industry, have limitations in their scope of application, as they do not prioritize computational efficiency. One strategy to resolve this issue is to combine the robustness of commercial software with the capabilities of academic software to provide the end user with simulation and modeling methods not available in any single software.

In practice, the combination of computer simulations can be performed using coupling methods. In recent years, non-intrusive coupling methods have come to the fore (Duval *et al.*, 2014). These methods do not require changes in the software or in the solution strategies involved. Thus, one of the main consequences of non-intrusiveness is the possibility of easily combining commercial and academic programs without modifying the source code of any of the related applications. Among the non-intrusive coupling proposals we can highlight: coupling between global 2D and local 3D models (Guguin *et al.*, 2014); dynamic problems in transient regime (Bettinotti *et al.*, 2014); crack propagation study (Gupta *et al.*, 2012b); and thermal gradient analysis (Plews *et al.*, 2012). Also noteworthy are the works by Filmore and Duarte (2018) on non-intrusive hierarchical coupling algorithms for GFEM, Li *et al.* (2021) on non-intrusive implementation of GFEM^{gl} for multiscale problem analysis (IGL-GFEM^{gl}), and Li *et al.* (2022) on multiscale coupling of 3D solid and shell models.

In this context, the present work aimed to expand and investigate a non-intrusive coupling strategy between the commercial software Abaqus and the INSANE research computational platform. This strategy, initially proposed by Li *et al.* (2021), called IGL-GFEM^{gl}, consists of a multiscale computational structure that combines the global-local iterative algorithm (IGL) with GFEM^{gl}. The structural problem is decomposed into three analysis scales with their respective models and discretizations: global scale, meso scale and local scale. Thus, the global scale was solved by the commercial software Abaqus via FEM, the meso scale and the local scale were solved by GFEM^{gl} through INSANE. The coupling between the meso scale and the global scale was performed non-intrusively using an iterative procedure.

The non-intrusive coupling strategy IGL-GFEM^{gl} was used here to investigate two-dimensional problems, with the inclusion of local features such as holes. The iterative algorithm was modified and relaxation techniques were used to improve the efficiency of the solution procedure. In the presented numerical example, the cost of computational time, the number of iterations, the convergence behavior and the solution quality were evaluated to demonstrate the contribution of the proposed modifications.

2. ITERATIVE GLOBAL LOCAL IMPLEMENTATION OF THE GENERALIZED FINITE ELEMENT METHOD WITH GLOBAL-LOCAL ENRICHMENT FUNCTIONS (IGL-GFEM$^{\mathbf{gl}}$)

2.1 Iterative Global Local: a non-intrusive coupling strategy

Initially proposed by Whitcomb (1991), the IGL strategy consists of a highly non-intrusive methodology for solving FEM problems through the global-local approach, and can therefore be interpreted as a variation of the zooming technique (Li *et al.*, 2021) . Recently the IGL strategy has been adapted and incorporated into other methods, as performed by Duval *et al.* (2014), Allix and Gosselet (2020) and Li *et al.* (2021), for example.

The strategy adopts two distinct models: a global model discretized by a coarse mesh that describes the general behavior of the structure, and a local model that adopts a refined discretization capturing local features of interest (geometric details, defects, cracks, localized non-linearities, etc.). Thus, the idea behind non-intrusive coupling is to use the global model to obtain an initial solution for the local domain, and then use it as a starting point for an iterative process. In the hypothesis of convergence, the final reference solution will be obtained without the need for any change in the stiffness matrix of the global model.

Let be the problem presented in Fig. 1. The global model is associated with the domain $\Omega = \Omega_L \cup \Omega_C$, where Ω_L is the local domain and Ω_C is your complement to the global domain. The global model, therefore, has the stiffness composed by two parts: stiffness K_{GL} referring to the region of the global model over the local domain, and stiffness K_{GC} corresponding to the complementary region. The local model, in turn, is restricted to the Ω_L domain, and is defined by the region close to the hole, where a more complex behavior is expected for the displacement and stress fields, due to the geometric detail. The local model has stiffness K_L. The region $\Gamma_I = \partial\Omega_L \cap \partial\Omega_C$ is called the interface.

The global and local models represent Ω_L differently, since, in general, $K_L \neq K_{GL}$. Thus, when the displacements u_G – calculated from K_{GL} – are used as a boundary condition for the solution of u_L – calculated from K_L, there is an imbalance in the interface Γ_I between the global and local models (Gendre *et al.*, 2009). Based on this observation, Whitcomb (1991) proposed an iterative procedure that imposes equilibrium and continuity conditions along the Γ_I interface. The IGL strategy can be described by the following steps:

1. Global analysis: solve the global model and obtain an initial solution u_G^0;
2. Local analysis: solve the local model considering as boundary conditions along the interface Γ_I the solution obtained in the previous step (u_G^0 in the first iteration, and u_G^i in the other i iterations);
3. Calculation of residuals: use the solutions u_G^i and u_L^i to calculate , along the interface Γ_I, the reactions from Ω_L ($f_L^{\Gamma_i}$) and the internal forces from Ω_C ($f_C^{\Gamma_i}$), and obtain a residue vector defined by the difference between these forces. When equilibrium is not satisfied, there are residual forces to be considered. The vector of residual forces (f_R) is calculated as follows:

$$f_R = -(f_L^{\Gamma_I} + f_C^{\Gamma_I}) = -[(K_L \cdot u_L^i - f_L)\,|_{\Gamma_I} + (K_{GC} \cdot u_{GC}^i - f_{GC})\,|_{\Gamma_I}] \quad (1)$$

4. Verification of convergence: after calculating the residual forces, the convergence of the analysis is verified. Whitcomb (1991) used as a convergence criterion the maximum absolute value of the vector of residual forces f_R and indicated that tolerances lower than 10^{-7} increase the number of iterations without increasing the accuracy of the analysis. In this work, the adopted tolerance was 10^{-5}, but in terms of the ratio between the f_R and the first internal force $f_C^{\Gamma_i}$, named here by "relative residual force". Also, as the convergence of the IGL strategy is affected by the stiffness difference between K_{GL} and K_L, static relaxation techniques or dynamic relaxation were used to guarantee or accelerate the convergence;

5. Correction and iterations: if the convergence criterion is satisfied, the analysis is completed. Otherwise, the global model is updated by adding f_R and the algorithm goes back to step 1.

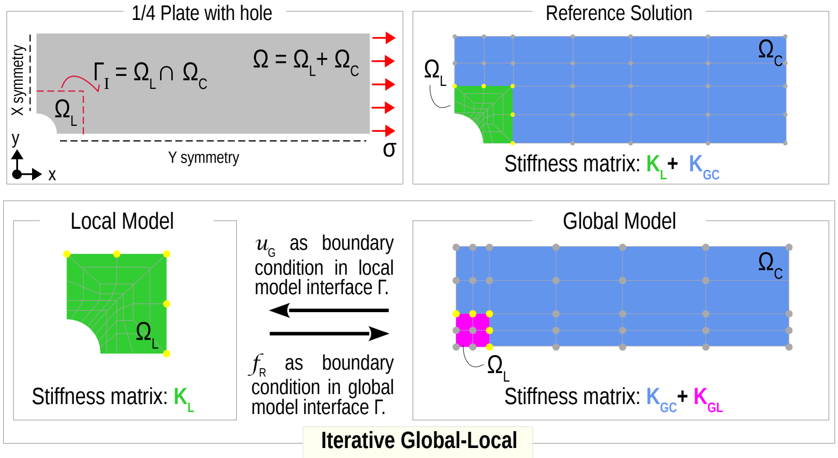

Figure 1. Iterative Global Local (IGL) approach.

The final solution of the IGL strategy is, therefore, the combination of the solutions of the global and local problems: the solution u_G is considered over the domain Ω_C, and the solution u_L about the domain Ω_L. Thus, since the IGL strategy is convergent, the combined solutions u_G and u_L represent the reference model solution, which explicitly discretizes the local model in the global model.

2.2 Generalized Finite Element Method with global-local enrichment functions (GFEM$^{\text{gl}}$)

For the treatment of problems with non-smooth solutions, which involve discontinuities, singularities and high gradients, the already consolidated FEM has limitations. The Generalized Finite Element Method (GFEM) emerges, therefore, as an alternative method to the conventional FEM, using more adequate approaches to deal with this class of problems.

The fundamental concept of the GFEM approach is the enrichment of the Partition of Unity (PU) with functions defined from the a priori knowledge of the solution of the problem of interest (Kim *et al.*, 2010). The use of enriching functions based on prior knowledge of the solution to the problem of interest, can be used to, among other applications, incorporate fields of discontinuity and singularity to the approximation.

The Heaviside function, for example, corresponds to a step-like enrichment, and introduces a strong discontinuity to the domain, that is, a jump in the (da Silva, 2016) displacement field. Still, in the region of the crack tip, the singularity of the stress field can be described through asymptotic functions, proposed by Szabo and Babuška (1991).

On the other hand, previously known enrichment functions are not always available and, therefore, are feasible only for a small range of relevant engineering problems. This context leads to the development of the GFEM^{gl} (Duarte and Kim, 2008), which combines the global strategy with the enrichment of the GFEM PU. This method provides a framework for enriching the solution space of the global problem with enrichment functions constructed numerically from the solution of a local boundary value problem (Gupta *et al.*, 2012a).

From the point of view of approximation, the strategy is based on decomposing the solution into two analysis scales: a coarse scale, capable of representing the smooth portion of the solution, and a refined scale, aiming at the description of local characteristics of the problem of interest. The solution of a problem through GFEM^{gl} is represented in Fig. 2, and can be described in three steps, as per Duarte and Kim (2008):

1. Initially, via conventional FEM or GFEM, the initial solution of the global problem ($\tilde{u}_G^0$) is obtained. This solution refers to the global domain of the problem, denoted by Ω_G, and does not consider possible localized phenomena, such as singularities, discontinuities, and high gradients;

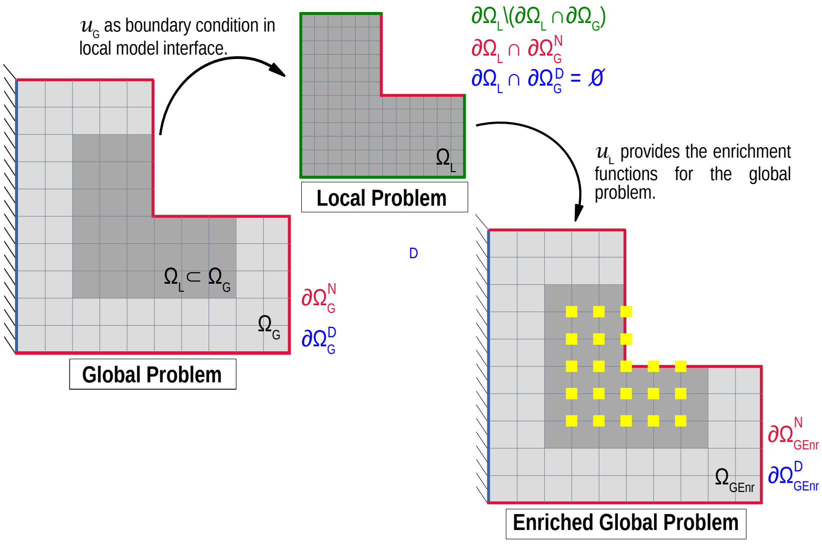

Figure 2. GFEM^{gl}: Global-local strategy for the GFEM.

2. A local domain is then defined, Ω_L, contained in the global domain, that is, $\Omega_L \subset \Omega_G$, covering the entire region in which the phenomena of interest occur. The solution of this domain is performed using $\tilde{u}_G^0$ as a boundary condition. The boundary of the local domain ($\partial\Omega_L$) is composed of three parts: the portion of the boundary of the local domain that intersects with the boundary of the global domain and that has the essential boundary conditions (conditions of Dirichlet contour), ($\partial\Omega_L^D = \partial\Omega_L \cap \partial\Omega_G^D$); the portion of the boundary of the local domain that intersects the boundary of the global domain and that has natural boundary conditions (Neumann boundary conditions) ($\partial\Omega_L^N = \partial\Omega_L \cap \partial\Omega_G^N$); and, the portion of the boundary of the local domain that does not intersect with the boundary of the global domain ($\partial\Omega_L = \partial\Omega_L \setminus (\partial\Omega_L \cap \partial\Omega_G)$);

3. Finally, the solution $\tilde{u}_L$ obtained from the local problem is used as enrichment in a new analysis of the global problem. The phenomenon of interest is represented through $\tilde{u}_L$ that enriches the PU of the global problem. The solution of the enriched global problem $\tilde{u}_G^E$ is obtained by solving the updated global problem with the global-local enrichment functions. The enriched global problem is solved with the same coarse mesh used in the initial global problem.

2.3 IGL-GFEM^{gl} for the solution of multi-scale problems

Li *et al.* (2021) proposed a multiscale computational framework, which combines the IGL strategy, described in Section 2.1, with the GFEM^{gl}, mentioned in Section 2.2. Such a framework, illustrated by Fig. 3, allows the non-intrusive coupling of different solvers to perform multiscale simulations. In IGL-GFEM^{gl}, the IGL algorithm couples a global model with a meso scale model, while GFEM^{gl} performs the global-local enrichment of the meso scale from the solution of a local model. The combination of these methods divides, therefore, the problem into three scales and their respective models and discretizations: a global model that does not contain the phenomena of interest and is discretized with a coarse mesh; a meso scale model that, in general, has the same discretization of the global model and serves as a transitional model between the global and local models; and one, or several, local models that are discretized with a finer mesh and represent the localized phenomena of interest.

According to Li *et al.* (2021) and Silveira Filho (2023), the main steps of the IGL-GFEM^{gl} strategy are therefore:

1. Analysis of the global model: solve the global model and obtain the initial solution u_G^0;

2. Analysis of the initial meso scale model: solve the meso scale model using the solution of the global model as a boundary condition (u_G^0 in the first iteration, and u_G in the others);

3. Analysis of the local model(s): solve the various existing local models using the meso scale model solution as a boundary condition. The obtained solution u_L is used in the construction of the meso scale global-local enrichment functions;

4. Analysis of the enriched meso scale model: resolve the meso scale model, which now has nodes enriched by functions built from the solution of the local problem. GFEM^{gl} global-local cycles, i.e. iterations between steps 3 and 4, can occur in this step to address the effects of low precision boundary conditions. In this work, two global-local cycles were adopted;

5. Calculation of the residual vector: obtain the reactions at the interface(s) between the enriched meso scale model and the global model and calculate the residual vector according to Eq (1). Update the force vector of the global model from the residual vector and obtain a new solution u_G;

6. Repeat steps 2 through 5 until some convergence criterion is met.

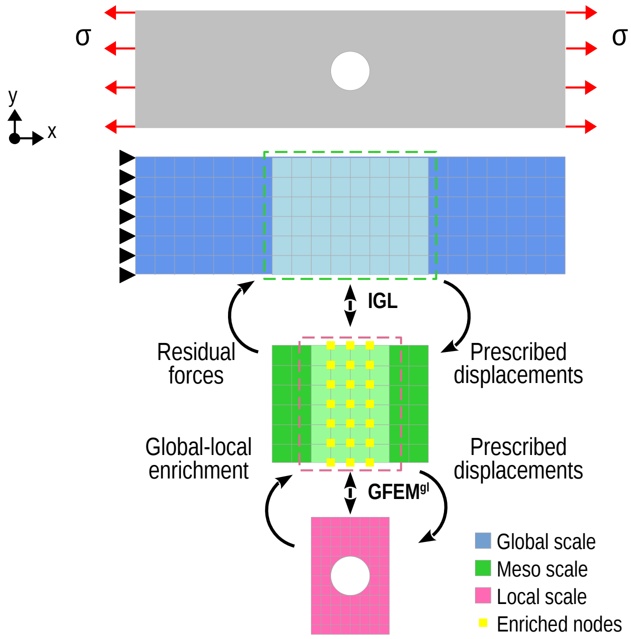

Figure 3. IGL-GFEM^{gl}: Iterative Global Local with global-local strategy for the GFEM.

The aforementioned steps describe a so-called monolithic algorithm as defined by Li *et al.* (2021). Li *et al.* (2022) proposed a different approach, named "staggered algorithm", which also combines IGL with GFEM^{gl}. The difference is that, in this case, the global-local enrichments provided by the solution of the local problem are updated only after the IGL iterations between the global model and the meso scale have reached some convergence criterion. Thus, the algorithm proposed in Li *et al.* (2022) requires less GFEM^{gl} simulations than the strategy proposed in Li *et al.* (2021), thus being computationally more efficient. It will be called here IGLSt-GFEM^{gl} to differenciate from the original IGL-GFEM^{gl}.

In the present work, the staggered algorithm was implemented in the computational platform INSANE, in Java programming language, according to the Object Oriented Programming (OOP) paradigm. The global model was solved using the commercial program Abaqus (Abaqus, 2014) and the meso scale enrichment was performed via GFEM^{gl} through a pre-existing solver on the INSANE platform. The results of the proposed implementation are shown in the numerical example below.

3. NUMERICAL EXAMPLE

The numerical example was taken from Silveira Filho (2023) and is shown in Fig. 4. It consists of a plate with a square hole under plane stress conditions. The only source of localized effects is due to the hole located in the central region of the plate. The material is homogeneous and isotropic, with modulus of elasticity $E = 2.0 \times 10^{11}$ and Poisson coefficient $\nu = 0.30$. Consistent units are adopted.

Three different computational models were built, all discretized with Q4 finite elements and 2x2 order of integration. The meshes of the global scale and meso scale models have square elements with size $l = 5.0$. The mesh of the local model, the only model that represents the hole, is built with elements with size $l = 2.5$. The three scales resulted in 690 degrees of freedom. A reference model was also constructed, obtained by Abaqus via standard FEM. In this case, elements of type Q4 were also used, with size $l = 2.5$ (the same size used in the local model), resulting in 1,064 degrees of freedom.

The results obtained by implementing the staggered algorithm (IGLSt-GFEM^{gl}) in terms of the displacement component in the y direction (u_y) are shown in Fig. 5. It is possible to see that the solution reflects the symmetry of the problem, a good indication of the coherence of the transfer of forces and displacements between the models. It is also noticed the stiffness loss in the central region of the meso scale, due to the presence of the hole in the local scale that is taken to the intermediate scale in the form of global-local enrichment functions.

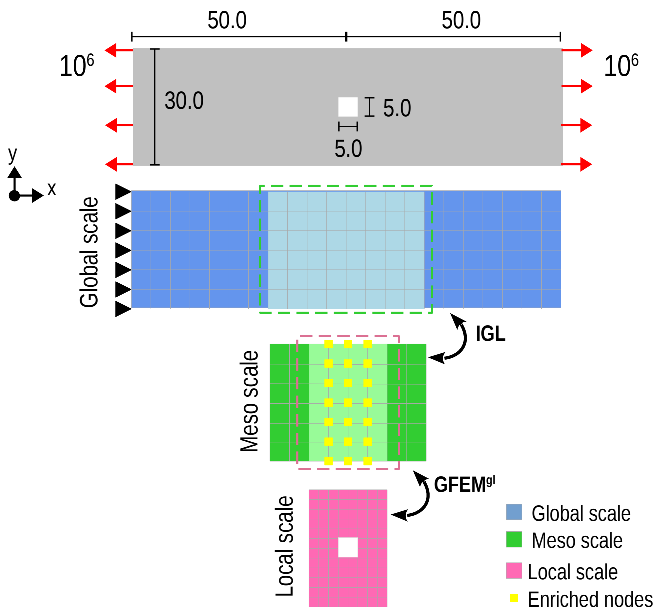

Figure 4. IGL-GFEM^{gl} Example: Tensile Plate (in consistent units).

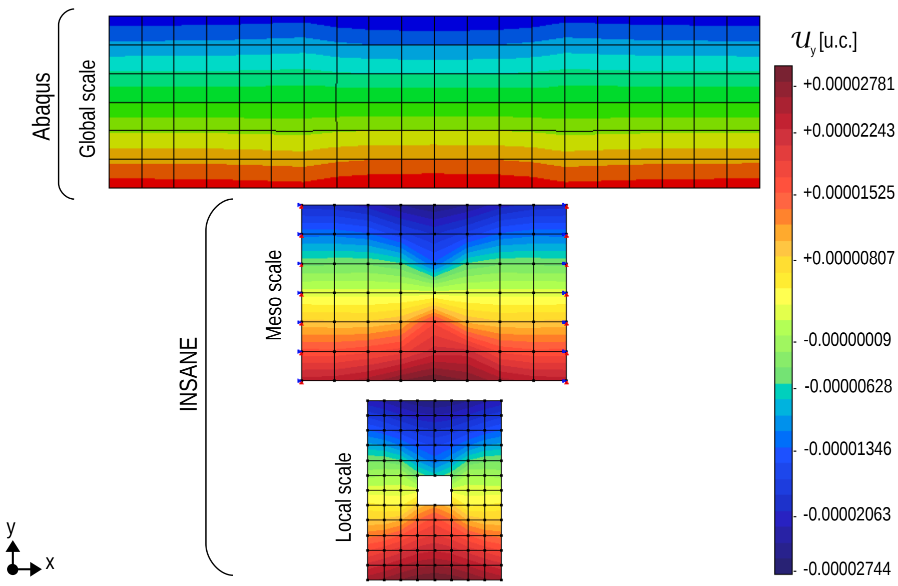

Figure 5. Vertical displacements by staggered algorithm.

Similar to Fig. 5, the results obtained in terms of the stress component in the direction x (Sxx) are presented in Fig. 6. In this case, it is essential to note that the color scales for representing results are differently defined by Abaqus and INSANE, which implies greater caution when analyzing those graphics. Additionally, it must be remembered that the hole is located only on the local scale. Its position covers 1/4 of the four central elements of the meso scale. Thus, as no hole is geometrically represented on the meso scale, its presence is indirectly considered by the enrichment provided by the numerical local solution. Furthermore, in the four central elements, the stiffness matrix is calculated by the contribution of the numerical integration points of the local elements. Hence, no integration is performed over the region of the hole. Even so, the approximation associated with the central node of the meso scale, coincident with the hole, contributes to the solution. The consequence is smoothing the discontinuous local solution in the meso scale approximation. The graphical representation aggravates this fact. A recovered stress field is used to extrapolate the results associated with the nodes of each element. Afterward, the nodal stress components are obtained by the average values resulting from the extrapolation in the elements that share the respective node. These two smoothing procedures, provided by the global-local solution and the graphical representation, explain the difference between the extreme values shown in Figure 6 for the reference model/Abaqus and the meso scale/INSANE. Without the smoothing imposed by the graphical representation, the maximum and minimum values obtained directly from the meso scale solution are, respectively, 2.065×10^6 and

-7.038×10^4. Indeed, they are very close to the ones from the reference mode where the hole is geometrically represented. It was also possible to observe that the onset of disturbances in the stress field due to the hole on the global scale/Abaqus coincides with the behavior observed in the reference model.

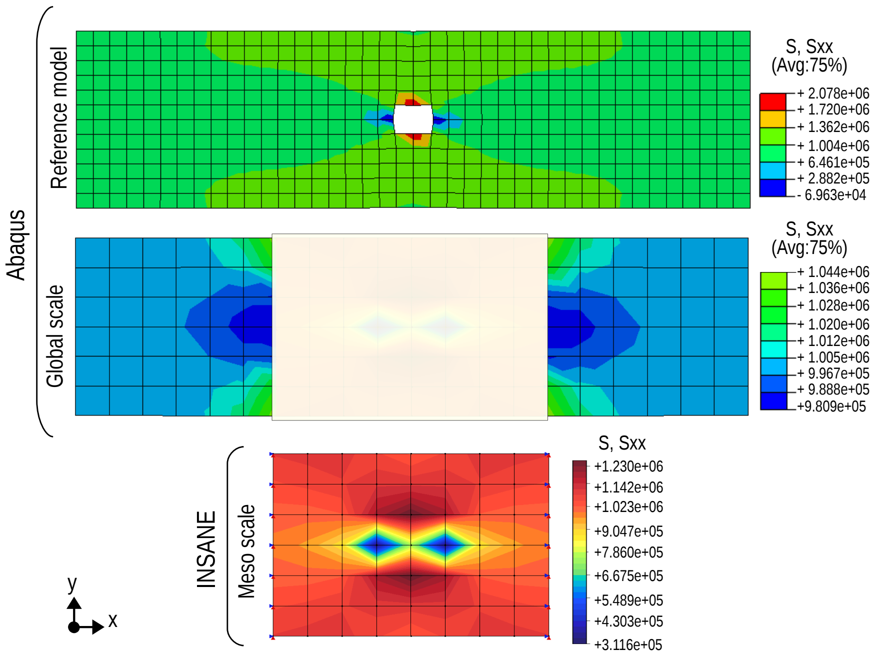

Figure 6. Stress in horizontal direction by staggered algorithm.

In order to validate the implementation and evaluate the quality of the results obtained, the results of the implementation of the staggered algorithm (IGLSt-GFEM^{gl}), in terms of u_y, obtained along the lower edge of the plate, were compared in Fig. 7, with the results obtained by the refence model and the implementation of Silveira Filho (2023) on INSANE platform. This is the implementation of the monolithic algorithm and it is exactly the same one proposed by Li *et al.* (2021) and it will named here IGL^{M1}-GFEM^{gl}. It is observed that the results obtained through the implementation of the staggered algorithm are in accordance with the results obtained by the other solutions analyzed, and may therefore be presented as an alternative solution to the solution obtained by the reference model, since it uses fewer degrees of freedom. In terms of computational cost, the implemented algorithm is also advantageous, since it requires less computational time than the solution obtained by implementing the IGL^{M1}-GFEM^{gl}.

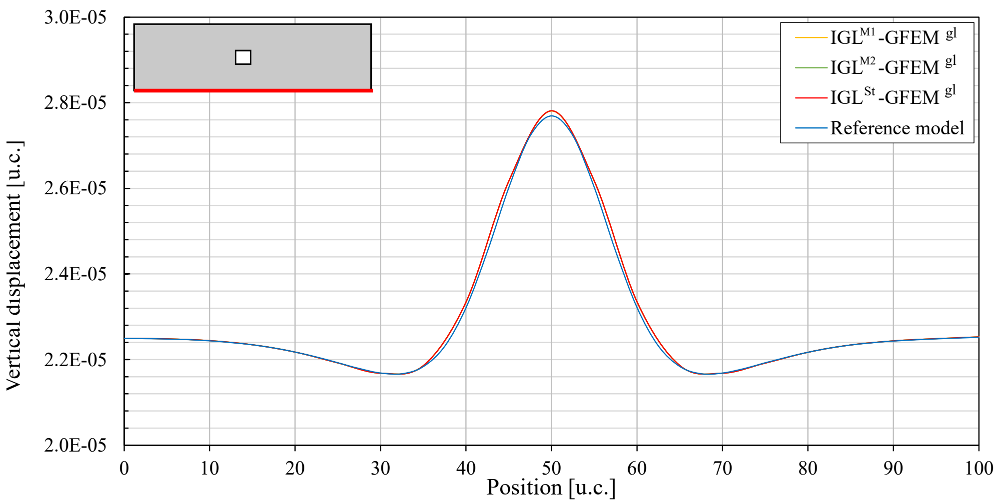

Figure 7. Comparison of u_y on the lower edge of the plate obtained by IGL^{M1}-GFEM^{gl}, IGL^{M2}-GFEM^{gl}, IGLSt-GFEM^{gl} and reference model.

In the implementation of Silveira Filho (2023), the controlling of the solution by Abaqus and INSANE is in charge of an external application. Differently, in IGLSt-GFEM^{gl}, INSANE becomes the manager of the solution process. This feature makes difficult to compare both solutions in terms of time consuming. Aiming a fair comparison, IGL^{M1}-GFEM^{gl} was adpated to be totally controlled by INSANE, named here IGL^{M2}-GFEM^{gl}. In this new implementation, there is a small difference between the two strategies. In IGL^{M1}-GFEM^{gl}, at each IGL iteration the meso scale model corresponds to the initial model, without enrichment. On the other hand, in IGL^{M2}-GFEM^{gl}, after the first iteration of the strategy, the meso scale model becomes an enriched model. Such modification does not significantly interfere with the results obtained by the two solutions, which was verified in Fig. 7.

Thus, taking the computational models according to Fig. 4, assuming for the meso scale and local scale models stiffness three times higher than the stiffness of the global model, and applying the dynamic relaxation technique (Li *et al.*, 2021), it was observed that, although IGL^{M2}-GFEM^{gl} converge in a smaller number of IGL iterations (5 for IGL^{M1}-GFEM^{gl} versus 7 for IGLSt-GFEM^{gl}), Fig. 8, it requires more computational time (205.509 seconds for IGL^{M1}-GFEM^{gl} versus 203.926 seconds for IGLSt-GFEM^{gl}), when compared to the IGLSt-GFEM^{gl}. There was, therefore, a reduction in computational time, mainly due to the smaller number of GFEM^{gl} iterations required by the staggered algorithm. While in IGL^{M2}-GFEM^{gl}, GFEM^{gl} iterations occur at each IGL iteration of the strategy, in IGLSt-GFEM^{gl}, the GFEM^{gl} iterations occur only in iteration 3 and iteration 7. It was also verified that when increasing the number of integration points of the meso and local scale elements from 2x2 to 4x4, for example, the use of the strategy IGLSt-GFEM^{gl} if makes it more relevant, leading to an almost 30% reduction in computational time compared to IGL ^{M2}-GFEM^{gl}.

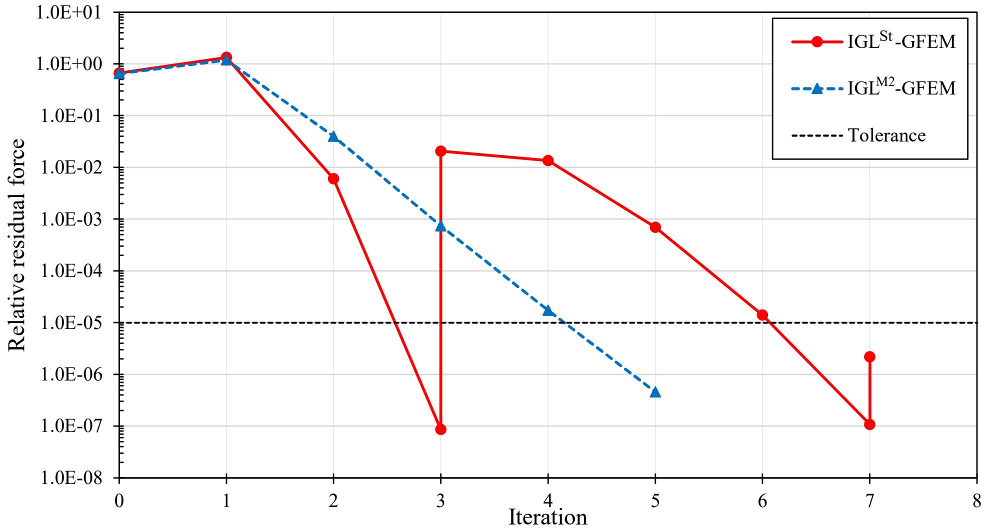

Figure 8. Convergence of IGLSt-GFEM^{gl} and IGL^{M2}-GFEM^{gl}.

As previously mentioned, the convergence of the IGL strategy depends on the difference in rigidity between the global model and the meso scale model. As the meso scale model represents a sub domain of interest of the global model, when It is stiffener, the IGL strategy may not converge (Silveira Filho, 2023). Therefore, we sought to evaluate the efficiency of static and dynamic relaxation techniques based on the problem presented in Fig. 4, increasing the stiffness of the meso scale and local scale models and keeping the global model stiffness given by $E = 2.0 \times 10^{11}$.

To evaluate the static relaxation technique (Xu, 1992), the modulus of elasticity of the meso scale and local scale elements were changed so that they were three times greater than that used in the global scale. Several relaxation factors ω were used in search of a solution to the problem, and the convergence evolution for each of the factors is shown in Fig. 9, considering IGLSt-GFEM^{gl}.

As expected, without the static relaxation technique ($\omega = 1.00$), the problem diverges. It is also observed that, in the evaluated problem, the convergence rate strongly depends on the relaxation factor applied, and the factors $\omega = 0.30$ and $\omega = 0.35$ produced better convergence rates. Silveira Filho (2023) verified in his work the inversely proportional relationship between the ratio E_M/E_G and the values of coefficients that lead to the best convergence rates, for problems where there is no distortion of the finite elements.

To investigate the behavior of the dynamic relaxation technique (Li *et al.*, 2021), the modulus of elasticity of the meso scale model and the local scale were make 3, 4, 6 and 10 times higher than the modulus of elasticity of the global model. Fig. 10 presents the results obtained. It was possible to verify that, in the first three iterations, the dynamic relaxation technique led to similar convergence rates, and that the models with smaller difference in stiffness between the scales presented lower absolute residual force. In iteration 3, the first GFEM^{gl} iteration occurs, and there is an inversion of the position of the graphics in terms of residual force. Now models with a greater difference in stiffness between scales lead to lower values of absolute residual force. This behavior was maintained, and the analyses continued to show the same convergence rate. In iteration 6, the second GFEM^{gl} iteration takes place, and once again the relation to the residual force is changed. The meso and local scale models with stiffness 4 and 10 times greater than the global model converged with

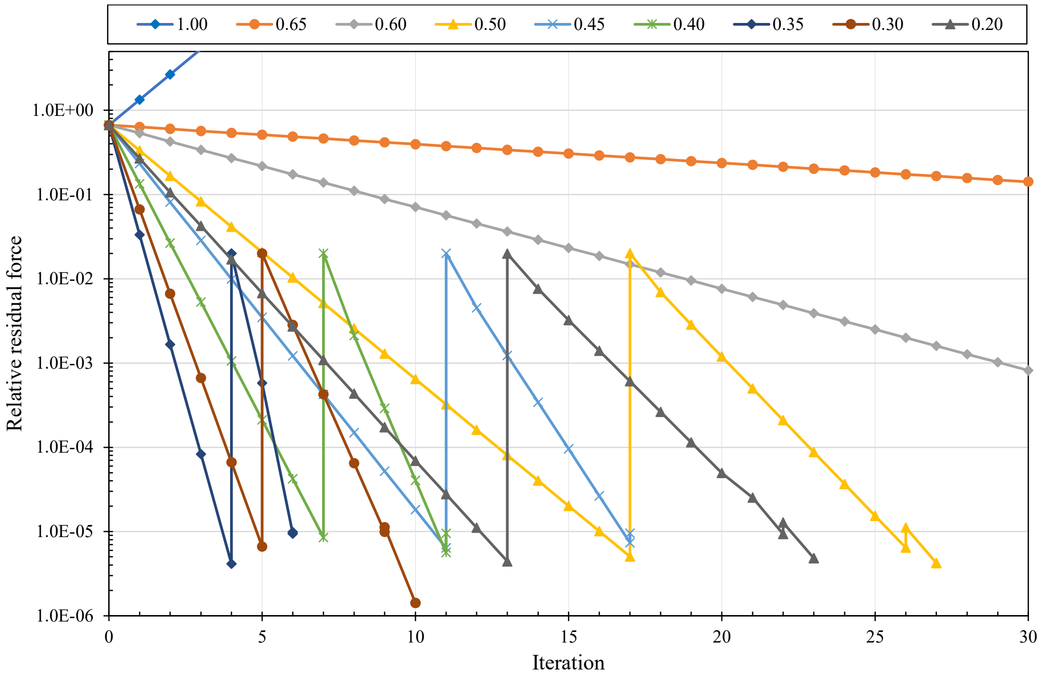

Figure 9. Convergence using static relaxation with different values of ω.

6 interactions, while the other ones, converged with 7 iterations.

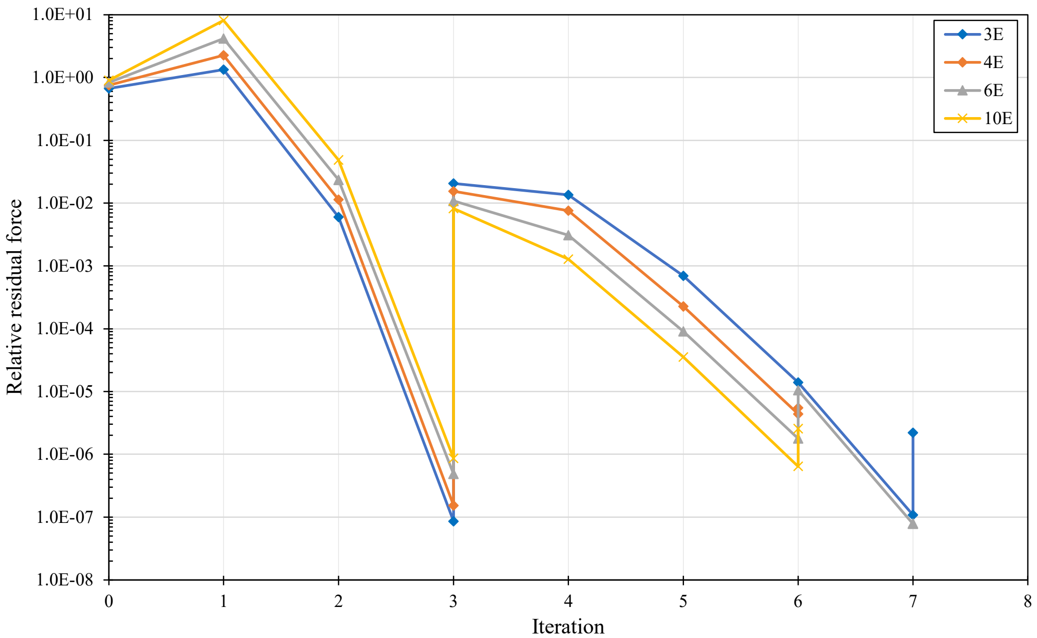

Figure 10. Convergence using dynamic relaxation for different meso scale stiffnesses.

4. CONCLUSIONS

The implementation of the staggered algorithm approach in the IGL-GFEM^{gl} strategy was successfully performed in the INSANE computational platform, allowing the non-intrusive coupling of the commercial software Abaqus with the academic program. The implementation was validated by reference solutions obtained in a previous work and by a reference model resulting from standard FEM analysis by commercial software.

The results obtained showed that the use of the staggered algorithm approach brings computational advantages to the GFEM^{gl} strategy, since it reduces the computational time without compromising the accuracy of the results. The application of relaxation techniques was also satisfactory within this approach, allowing the convergence of the strategy even when the meso scale model and the global model present a significant difference in stiffness.

5. ACKNOWLEDGEMENTS

The authors gratefully acknowledge the important support of the Brazilian research agencies FAPEMIG (in Portuguese "Fundação de Amparo à Pesquisa de Minas Gerais" - Grant APQ-01656-18), CNPq (in Portuguese "Conselho Nacional de Desenvolvimento Científico e Tecnológico" - Grant 308444/2022-1) and CAPES (Coordination for the Improvement of Higher Education Personnel).

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7. RESPONSIBILITY NOTICE