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COB-2023-0921 SLIDING MODE CONTROL FOR SUPPRESSION OF STICK-SLIP VIBRATIONS IN OILWELL DRILLSTRINGS

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Abstract. *In drill strings, stick-slip vibrations are a particular case of torsional vibrations, which cause large fluctuations in the angular velocity at the bottom of the well, resulting in premature wear of the bit and other components of the drilling system. This work introduces a sliding mode controller designed to regulate the torque applied to the top of the column in order to mitigate or eliminate stick-slip vibrations in the dynamic behavior of the drillstring. The drillstring is modeled as a two-degree-of-freedom lumped parameter model, representing its torsional behavior at both the top of the column and the bottom hole. The controller stability analysis is performed from the Lyapunov stability theory, in order to guarantee the system convergence to the desired state. Initially, the behavior of the system without the action of the controller is investigated and later, its influence on the dynamic behavior of the system is analyzed. For the studied cases, the controller was able to eliminate stick-slip vibrations, keeping the column angular velocity at the desired value, presenting results in agreement with experimental and field data. The chattering behavior is also verified and its elimination through a signal saturation function, guaranteeing an adequate behavior to the developed controller.*

Keywords: *drill string, stick-slip, sliding mode control, non-linear dynamics.*

1. INTRODUCTION

The drilling phase represents a significant portion of the total cost of oil and gas exploration, accounting for approximately 40% of the overall exploration and production cost (Kriesels *et al.*, 1999). Due to its high cost, the drilling process has become one of the major challenges in oil and gas exploration, attracting the attention of researchers worldwide.

The drilling column is a long, slender tube with a variable wall thickness that comprises drill pipes, tools, the drill bit, and additional downhole equipment. Its main function is to transmit the torque generated at the top of the column by the rotary system, while also allowing for the circulation of drilling fluid, removal of cuttings, and application of weight on the drill bit to advance through the rock formation during the drilling process.

From a dynamic perspective, during the drilling process, the column is subjected to three types of vibration: axial, lateral, and torsional. These vibrations can have several causes, such as mass imbalances, misalignments and bending of the column, irregularities in the wellbore, friction between the drill bit and column with the wellbore and well wall, as well as the dynamics involved in the interaction of the drill bit with the formation, among others (Besaisow and Payne, 1988).

Stick-slip vibrations are defined by Kamel and Yigit (2014) as torsional vibrations where the drill bit periodically undergoes two phases: stick, where the drill bit completely ceases its rotational motion, and slip, where the drill bit is released from the rock formation and rotates at angular velocities considerably higher than its nominal angular speed. Studies focused on torsional vibration analysis indicate that stick-slip behavior occurs during 50% of the drilling time in a well, which can excite lateral and axial vibrations. These vibrations are responsible for approximately 80% of premature damages to drilling columns, representing a loss of approximately 5% of the total cost of well exploration (Spanos *et al.*, 2003). Therefore, finding effective ways to reduce torsional vibrations and eliminate stick-slip is highly valuable in terms of reducing process time and preventing premature drilling equipment failures.

The objective of this work is to numerically reproduce the torsional behavior of the column in the presence and absence of stick-slip vibrations using a well-established mathematical model simulation, considering the drill-bit-rock interaction, in order to identify operational conditions where such vibrations occur, leading to a better understanding of this phenomenon. Furthermore, based on this initial analysis, the aim of this work is to propose a sliding mode torque controller at the top of the column to prevent the column from operating in the presence of these vibrations.

2. LITERATURE REVIEW

The studies on drilling string dynamics began in the late 1950s. Bailey and Finnie (1960) conducted an analytical approach that starts with the wave equation to analyze the axial and torsional dynamics of the drilling string, aiming to calculate its natural frequencies. Deily *et al.* (1968) measured force and displacement at the bottom of the well, observing axial, lateral, torsional loads, and bending moments. Analyzing the data obtained by Deily *et al.* (1968), Cunningham (1968) observed an alternation between zero rotation values and rotations higher than the column's rotation, making the first observation of the stick-slip phenomenon in drilling strings.

Halsey *et al.* (1988) conducted an analytical study on the torsional dynamics of the drilling string using a mathematical model based on a simple torsional pendulum, considering friction between the string and the rock formation. This model allowed for the formulation of analytical expressions to represent the stick-slip phenomenon. In this study, a speed controller was also proposed with the aim of maintaining the rotation speed of the string as constant as possible. This was the first controller developed to prevent torsional vibrations in drilling strings.

Starting in the early 1990s, with significant advancements in computing power, investigations into the dynamics of drilling strings began to be conducted numerically. The pioneering numerical analysis on this subject was presented by Lin and Wang (1990) in the study of torsional vibrations, where it was found that the fluctuation of the angular velocity of the drill bit increased as the rotation at the top of the string was also increased. However, above a critical value, such fluctuation ceased to occur. In other words, it was observed that stick-slip vibrations were eliminated when the string rotated above a critical angular velocity.

In (Javanmardi and Gaspard, 1992), using a system developed by the Shell Research Institute, an enhancement to the controller developed in Halsey *et al.* (1988) was proposed. Based on angular velocity and torque information obtained from current and voltage measurements of drill rig motors, the authors present a system for torque control through modifications to the speed controller of the rotary table.

The authors Christoforou and Yigit initiated studies on the dynamics of a drilling string in Christoforou and Yigit (1996), using a model with and without coupling between axial and lateral vibrations. They observed that when the coupling between modes was considered, the system exhibited unstable behavior. In Yigit and Christoforou (2006) the authors presented a model where axial, torsional, and lateral vibrations are fully coupled. This model has become one of the most well-established models on the subject and has served as a basis for numerous subsequent works. Additionally, in this study, the authors proposed a linear controller for the elimination of stick-slip vibrations.

In Navarro-López and Cortés (2007a), the authors investigate the behavior of stick-slip torsional vibrations using dynamic analysis tools to obtain parameters for stable drilling operations. This analysis is performed through numerical simulation using a more generalized model than those previously presented in the literature. However, the model proposed by the author takes into account the variation of the string length as the drilling process progresses. In this study, the authors defined operating ranges for key parameters such as weight on bit, motor torque, and rotation speed to reduce oscillatory phenomena.

In Navarro-López and Licéaga-Castro (2009), the author presents a four-degree-of-freedom model for the torsional dynamics of the drilling string. The aim of this work was to conduct a more detailed investigation of the sliding mode controller developed in Navarro-López and Cortés (2007b). Based on the obtained results, the authors concluded that the studied sliding mode controller exhibits more attractive characteristics compared to proportional-integral (PI) controllers. The specific details and findings regarding the model and controller can be found in the referenced publication.

In Bayliss *et al.* (2012), the author develops an adaptive controller using a recursive least squares algorithm. This controller continuously monitors the input value for the rotation speed of the drilling string, aiming to minimize the fluctuation of the drill bit's angular velocity and eliminate stick-slip vibrations during the drilling process. Under the action of the developed controller, it was observed that after 50 seconds, the stick-slip vibrations are completely eliminated. The specific details and experimental findings regarding the controller's performance can be found in the referenced publication.

In Vromen *et al.* (2015), the author proposes a robust controller for the elimination of stick-slip torsional vibrations, resulting in a four-degree-of-freedom model similar to the one presented in Fig. 1. The controller takes into account the nonlinearity of the interaction between the drill bit and the rock formation. The aim of the developed controller in this work is to maintain a constant velocity throughout the length of the drilling string. The input variables of the controller are the rotation speed and the torque at the top of the string. One of the main advantages of the controller developed by the author is that only measured data at the top of the string are required. The specific details and performance evaluation of the proposed controller can be found in the referenced publication.

In Monteiro and Trindade (2017), the authors analyzed the performance of a proportional-integral (PI) controller to assess its efficiency in eliminating stick-slip vibrations in the torsional dynamics of a drilling string. The study investigated the combination of proportional and integral gains to generate contour plots that depict the standard deviation of the drill

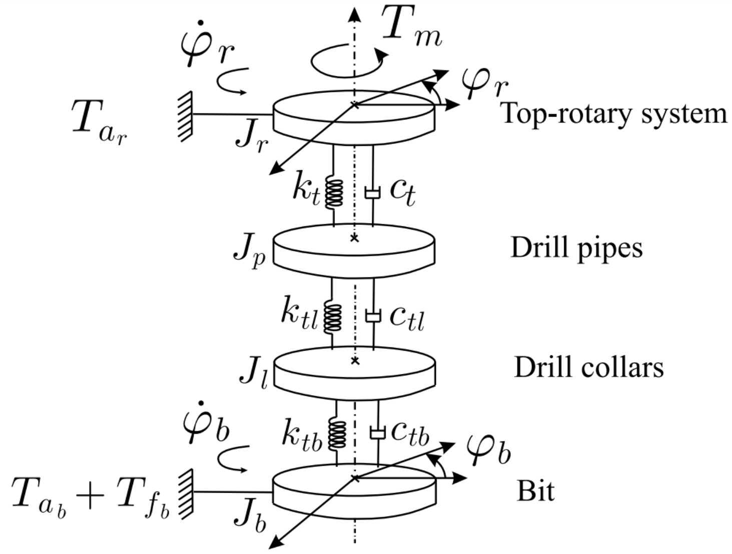


Figure 1: Mechanical model describing the torsional behaviour of a simplified drillstring (Navarro-López and Licéaga-Castro, 2009).

bit's angular velocity for different values of weight on bit. Based on this initial analysis, the authors examined the influence of the controller for combinations of gains that resulted in different standard deviation values for the drill bit's velocity. It was observed that the controller was able to eliminate stick-slip vibrations for standard deviation values below 69%. The specific methodology and findings of the study can be found in the referenced publication.

3. PROBLEM FORMULATION

The chosen mathematical model to represent the rotary system and the drill-bit-rock interaction is based on Navarro-López and Suárez (2004). This model is widely used in the field due to its ease of numerical implementation and its ability to accurately reproduce experimental and field results. The motion equations are derived based on the following assumptions: perfectly vertical well, no lateral movement of the drill bit, non-zero velocity at the top of the string, neglecting friction between the pipe connections and between the pipes and the walls, considering a viscous friction coefficient to represent drilling mud, assuming laminar drilling fluid flow, and that the drillstring operates under the assumption of the absence of bit-bounce, which refers to the loss of contact between the drill bit and the rock formation. These considerations form the basis of the mathematical model used in various works in the field of drilling dynamics.

The Fig. 2 presents the concentrated parameter model with two torsional degrees of freedom used in this study and proposed in Navarro-López and Suárez (2004).

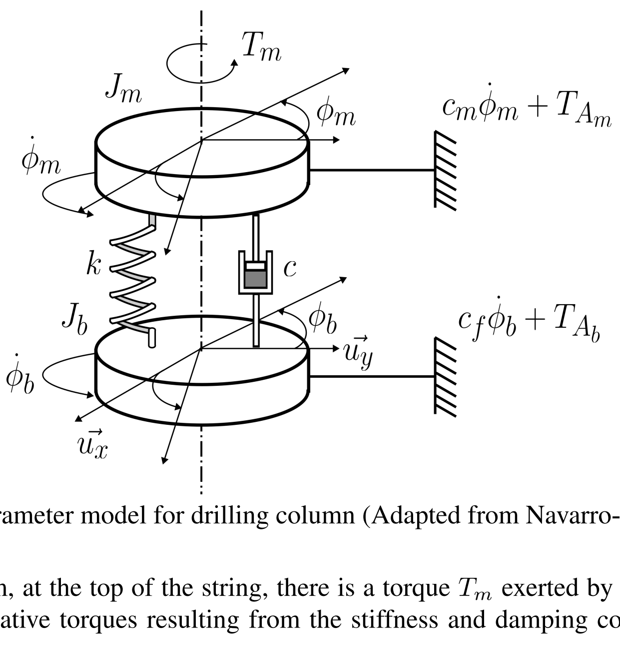


Figure 2: Concentrated parameter model for drilling column (Adapted from Navarro-López and Suárez (2004)).

Regarding the rotary system, at the top of the string, there is a torque T_m exerted by the electric motor. In addition, there are restorative and dissipative torques resulting from the stiffness and damping coefficient of the string, denoted by T_{res} and T_d respectively. The restorative torque at the top of the string is calculated based on the torsional stiffness coefficient k of the string and the difference between the angular displacements of the drill bit φ_b and at the top of the string φ_m, as follows:

$$T_{res} = k(\varphi_m - \varphi_b) \tag{1}$$

The dissipative torque is calculated considering a damping coefficient associated with the drilling string c and the difference between the angular velocities of the degrees of freedom of the system, as follows:

$$T_d = c(\dot{\varphi}_m - \dot{\varphi}_b) \tag{2}$$

where $\dot{\varphi}_b$ is the angular velocity of the drill bit, and $\dot{\varphi}_m$ is the angular velocity at the top of the string. Additionally, there is a variable torque acting on the rotary system resulting from dry friction T_{A_m} and viscous damping at the top of the string, given by:

$$T_r(\dot{\varphi}_m) = c_m\dot{\varphi}_m + T_{A_m} sign(\dot{\varphi}_m) \tag{3}$$

Thus, the equation of motion for the rotary system, considering J_m as the moment of inertia associated with the top of the string, is given by:

$$J_m\ddot{\varphi}_m + c(\dot{\varphi}_m - \dot{\varphi}_b) + k(\varphi_m - \varphi_b) = T_m - T_r(\dot{\varphi}_m) \tag{4}$$

The bit-rock interaction model adopted predicts that the external torque T_b exerted on the drill bit is composed of a term related to the viscous damping caused by the drilling fluid c_f, and another term related to the friction T_{A_b} between the drill bit and the rock formation, as follows:

$$T_b = c_f\dot{\varphi}_b + T_{A_b} \tag{5}$$

The friction torque exerted by the rock formation on the drill bit T_{A_b} is defined by a discontinuous system of equations characterized by three different states: stick, when the drill bit is stuck in the rock formation; slip, where there is relative angular velocity between them; and a transition phase that defines the onset of drill bit detachment. To differentiate between the three phases of the torque model, let's consider D_v as a reference value for the drill bit's angular velocity, representing a small region near $\dot{\varphi}_b = 0$, and the magnitude of the static friction torque T_{e_b} exerted by the rock formation defined as follows:

$$T_{e_b} = R_b P \mu_{e_b} \tag{6}$$

where R_b is the radius of the drill bit, P is the applied load at the top of the string to advance the drill bit through the rock formation, and μ_{e_b} is the coefficient of static friction between the drill bit and the rock formation. When the angular velocity of the drill bit is lower than the reference value D_v, the system can be in the stick phase or the transition phase. To distinguish between these phases, a comparison is made between the values of the friction torque T_{A_b} and the magnitude of the static friction. If the variable friction torque is smaller than the static friction torque, i.e., $T_{e_b} < T_{\text{stick}}$, the system will be in the stick phase. In this case, the torque is given by the variable static friction torque expression proposed by Leine *et al.* (1998):

$$T_{stick} = c(\dot{\varphi}_m - \dot{\varphi}_b) + k(\varphi_m - \varphi_b) - c_b\dot{\varphi}_b \tag{7}$$

When the angular velocity of the drill bit is below D_v and the value of the static friction torque is lower than the torque calculated by Eq. 7, the system will be in the transition from the stick phase to the slip phase. In this situation, the torque on the drill bit will be given by:

$$T_{tr} = T_{e_b} sign(T_{stick}) \tag{8}$$

If the magnitude of the drill bit's angular velocity is above the reference value D_v, the system enters the slip phase. In this condition, the torque on the drill bit is calculated as follows:

$$T_{slip} = R_b P \mu_b(\dot{\varphi}_b) sign(\dot{\varphi}_b) \tag{9}$$

where $\mu_b(\dot{\varphi}_b)$ is the variable friction coefficient as a function of velocity, described as:

$$\mu_{\dot{\varphi}_b} = \mu_{c_b} + (\mu_{e_b} + \mu_{c_b}) e^{-\gamma|\dot{\varphi}_b|} \tag{10}$$

where μ_{c_b} is the Coulomb friction coefficient and γ is a positive constant associated with the rate of friction coefficient decay with velocity. With the information provided, we can describe the friction torque exerted by the rock formation on the drill bit as a discontinuous system of equations, expressed as follows:

$$T_{A_b} = \begin{cases} T_{stick} & \text{se } |\dot{\varphi}_b| < D_v \text{ e } |T_{stick}| \leq T_{e_b} \\ T_{e_b} sign(T_{stick}) & \text{se } |\dot{\varphi}_b| < D_v \text{ e } |T_{stick}| > T_{e_b} \\ R_b P \mu_b(\dot{\varphi}_b) sign(\dot{\varphi}_b) & \text{se } |\dot{\varphi}_b| > D_v \end{cases} \tag{11}$$

Considering J_b as the moment of inertia of the drill bit, the equation of motion for the drill bit can be defined as:

$$J_b \ddot{\varphi}_b + c(\dot{\varphi}_b - \dot{\varphi}_m) + c_b \dot{\varphi}_b + k(\varphi_b - \varphi_m) = -T_b(\dot{\varphi}_m) \tag{12}$$

The system of equations obtained, characterized by Eq. 4 and Eq. 12, consists of a set of second-order coupled differential equations with a discontinuity arising from the adopted model for describing the bit-rock interaction.

4. IMPLEMENTATION OF CONTROL BY SLIDING MODES

The change of variables presented in Eq. 13 reduces the number of state variables in the system. However, for understanding its dynamics and for the development of the controller, it is sufficient to know the difference between the angular displacements of the table and the drill bit.

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \dot{\varphi}_m \\ \varphi_m - \varphi_b \\ \dot{\varphi}_b \end{pmatrix} \tag{13}$$

Re-writing the system in its explicit form, considering the change of variables, we obtain:

$$\dot{X} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{J_m}[T_m - T_r(x_1) - c(x_1 - x_3) - kx_2] \\ x_1 - x_3 \\ \frac{1}{J_b}[-T_b(x_3) - c(x_3 - x_1) + kx_2] \end{pmatrix} \tag{14}$$

In the present work, the goal of the controller is to eliminate stick-slip vibrations at the bottom of the well, keeping the drill bit rotating at a desired angular velocity Ω_d. For this purpose, a surface must be proposed in which the trajectories of the system converge to a state where the angular velocity of the drill bit tends to the angular velocity of the rotating system, which in turn tends to Ω_d. Thus, the vector of the tracking errors of the variables of interest can be written as:

$$\tilde{x} = \begin{pmatrix} x_1 - \Omega_d \\ x_1 - x_3 \end{pmatrix} \tag{15}$$

Let $S(t) \in \Re^2$ be the sliding surface defined by the equation $s(\tilde{x}, t) = 0$ given, for the presented second order system, by Eq. 16:

$$s = (x_1 - \Omega_d) + \lambda \int_0^t x_1 - \Omega_d \, d\tau \; + \lambda \int_0^t x_1 - x_3 \, d\tau \tag{16}$$

The foundation of the design of a sliding mode controller is to convert an n in $x^{th}-$ order path-following problem into a first-order stabilization problem in s. However, the sliding surface $s(x, t)$ defined by the designer, as well as the values of the tracking errors, must tend to zero in a finite time interval. That is, the sliding mode controller forces the states of the system to converge to the region defined by the sliding surface and after reaching it, the system error should converge to zero (Slotine *et al.*, 1991). The control law is obtained from the Fillipov construction for the equivalent dynamics of the system obtained in $\dot{s}(x,t) = 0$.

The time derivative of the sliding surface shown in Equation 16, substituting x_1 by its equivalent expression in Equation 14, is:

$$\dot{s} = \frac{1}{J_m}[T_m - T_r(x_1) - c(x_1 - x_3) - kx_2] + \lambda(x_1 - \Omega_d) + \lambda(x_1 - x_3) \tag{17}$$

The expression for the torque at the top of the column is given by the sum of the equivalent torque T_{eq} obtained by the control law and a switching torque T_g for the system to reach the slip surface from an initial state. Therefore, the torque obtained by the control law considering $\dot{s} = 0$ can be written as:

$$T_{eq} = T_r(x_1) + c(x_1 - x_3) + kx_2 - J_m\lambda(x_1 - \Omega_d) - J_m\lambda(x_1 - x_3) \tag{18}$$

With switching term defined as $T_g = -ksign(s)$ or, to avoid chattering issues, $T_g = -ksat(\frac{s}{\Phi})$, where Φ is the boundary layer width and describes the distance of the system response from the sliding surface $S(t)$.

When applying the sliding mode controller, the entire trajectory of the system reaches and remains in the region $s = 0$ in a finite time. This can be proved from the selection of a Lyapunov Function V and its derivative with respect to time given as per Liu (2015).

From the moment the system reaches the state $s = 0$, the system converges asymptotically to an equilibrium state, called the desired equilibrium state, defined here by:

$$\bar{X}_d = \begin{pmatrix} \bar{x}_{1_d} \\ \bar{x}_{2_d} \\ \bar{x}_{3_d} \end{pmatrix} = \begin{pmatrix} \Omega_d \\ \frac{c_m\Omega_d + T_b}{k} \\ \Omega_d \end{pmatrix} \tag{19}$$

This statement can be verified from the stability analysis of the controller. For this, consider the new Lyapunov Function proposed in (Vaziri *et al.*, 2018):

$$\begin{aligned} \bar{V} &= \tfrac{1}{2}[J_m(x_1 - \bar{x}_1) + k(x_2 - \bar{x}_2) + J_m(x_3 - \bar{x}_3)] \\ \dot{\bar{V}} &= J_m\dot{x}_1(x_1 - \bar{x}_1) + k\dot{x}_2(x_2 - \bar{x}_2) + J_b\dot{x}_3(x_3 - \bar{x}_3) \end{aligned} \tag{20}$$

When analyzing Equation 20, it is noted that for the equilibrium state, $\dot{\bar{V}} = 0$. If we now consider the desired equilibrium state presented in Equation 19, Equation 20 can then be defined as:

$$\dot{\bar{V}}(\bar{X}_d) = \Omega_d(-T_b - K) \tag{21}$$

With Ω_d and K being strictly positive constants, it can be said that $\dot{\bar{V}} \leq 0$. Thus, it can be stated that the system is asymptotically stable, therefore, when reaching the slip surface, the system will remain on it or in a small neighborhood.

5. RESULTS AND DISCUSSIONS

In this section, the results of the numerical simulation for the mathematical model of the drilling string obtained in the previous section are presented and discussed. The numerical integration of the system's equations of motion is performed using the fourth-order Runge-Kutta method. In all the results, initial conditions are assumed to be zero, except for the rotary system at the top of the string, which has an initial velocity equivalent to the desired angular velocity value Ω_d.

The parameters adopted for the characteristics of the drilling string and the controller parameters are taken from Vaziri *et al.* (2018), considering a reduced model for the drilling string with a length of 1 meter. The parameters for the torque model are taken from Navarro-López and Suárez (2004). These values are presented in Tab. 1.

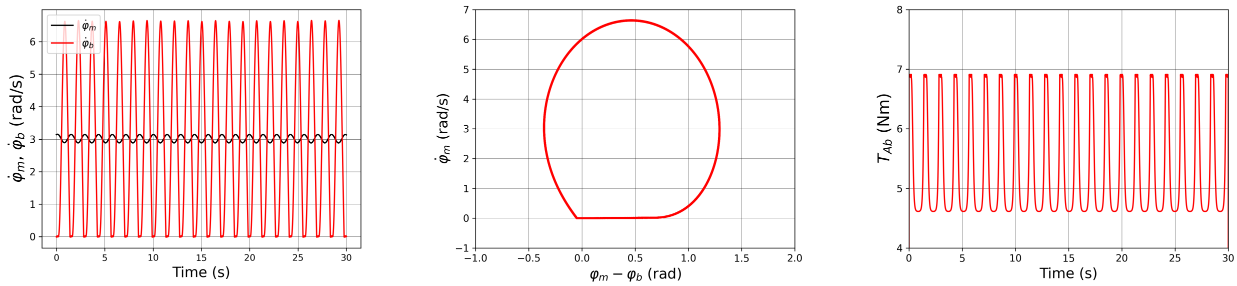
The results will be presented in three subsections. The first subsection aims to verify the torsional dynamics of the drilling string under stick-slip regime by comparing the obtained results with well-established results in the literature, in order to ensure the validity of the motion equations and the numerical integration code. The second subsection verifies the operation of the ideal controller obtained from the control law, disregarding the effect of the switching term on its operation. In the last subsection of this chapter, the influence of the switching term on the controller's operation is examined. In this last subsection, the influence of the saturation function on mitigating or eliminating excessive switching phenomenon or chattering is also investigated.

Table 1: Parameters used in simulations.

Parameters	Value
J_m[kg/m^2]	13.92
J_b[kg/m^2]	0.5
c_m[Nms/rad]	11.38
c_b[Nms/rad]	0.03
c[Nms/rad]	0.005
k[Nm]	10
T_{A_m}[Nm]	0.5
P[N]	1790
D_ρ	10^{-6}
γ	0.9
μ_{e_b}	0.3
μ_{c_b}	0.2

5.1 Analysis of drill string dynamics in stick-slip regime

Initially, the dynamic behavior of the system is examined without considering any action from the controller. For this purpose, it is assumed that the torque at the top of the string T_m has a constant value of 39.57 Nm. Figure 3a shows the angular velocities at the top of the string, the rotary system, and the drill bit, represented in black and red, respectively. It can be observed that while the rotary system exhibits an angular velocity that oscillates around the desired value of 3.1 rad/s, the drill bit alternates its angular velocity between periods of zero value, indicating the stick phase, and periods where it reaches magnitudes of more than twice the reference value of the rotary system at the top of the string, indicating the slip phase. The stick-slip behavior can also be easily observed in the phase space of the system, as shown in Fig. 3b. While the drill bit is in the stick phase, indicated by the horizontal line at $\dot{\phi}_b = 0$, the angular distortion increases until the energy accumulation is large enough to release the drill bit from the rock formation, causing it to enter the slip phase.



(a) Angular velocity at top of string and drill for constant torque.

(b) Trajectory in phase subspace for constant motor torque.

(c) Torque on drill for constant motor torque.

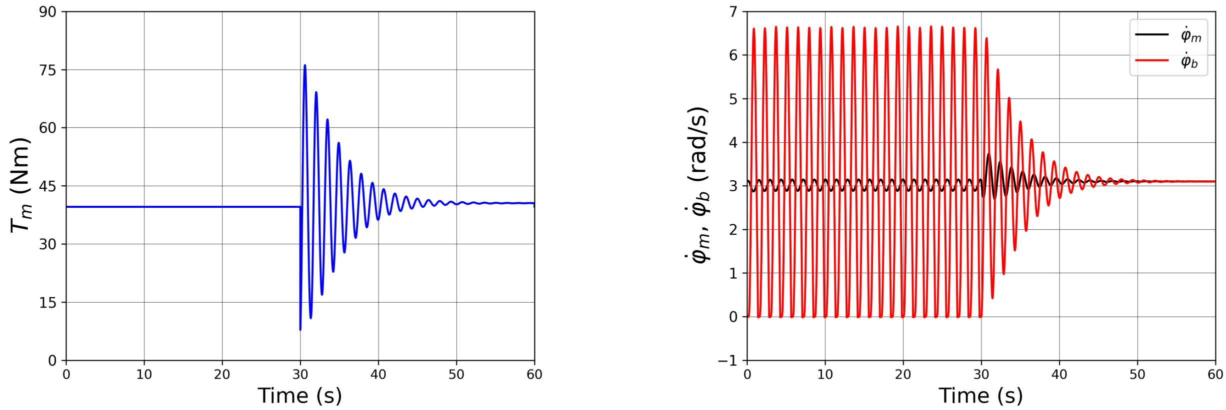
Figure 3: Drill string dynamics in stick-slip regime.

The behavior of the torque exerted by the rock formation on the drill bit is illustrated in Fig. 3c. Maximum and minimum values can be observed corresponding to the stick and slip phases, as indicated by the periods shown in Fig. 3a. Regions of instability can also be noticed in relation to the stick phases, which are related to the transition range in the adopted torque model.

It is worth noting that the results presented in Fig. 3a and Fig. 3b are consistent with those shown in Vaziri *et al.* (2018), while the behavior illustrated in Fig. 3c aligns with the torque behavior shown in Navarro-López and Suárez (2004), validating the obtained results.

5.2 Ideal controller

At this point, the behavior of the system under the effect of the ideal controller is analyzed. It is assumed that the control torque at the top of the string is given for a switching term gain of zero, i.e., $K = 0$. For an initial analysis, the behavior under the same conditions as in the previous section is studied, with a motor torque of 39.57 Nm and a desired angular velocity of 3.1 rad/s. However, after the initial twenty seconds, the motor torque at the top of the string is defined by the equivalent control law described in Eq. 18, with $\lambda = 0.8$. The value of the torque at the top of the string over time is illustrated in Fig. 4a. At the beginning of the controller's operation, a significant oscillation with large amplitude can be observed. As time progresses, the oscillation decays significantly until it converges to a constant torque value slightly higher than the initial torque value.

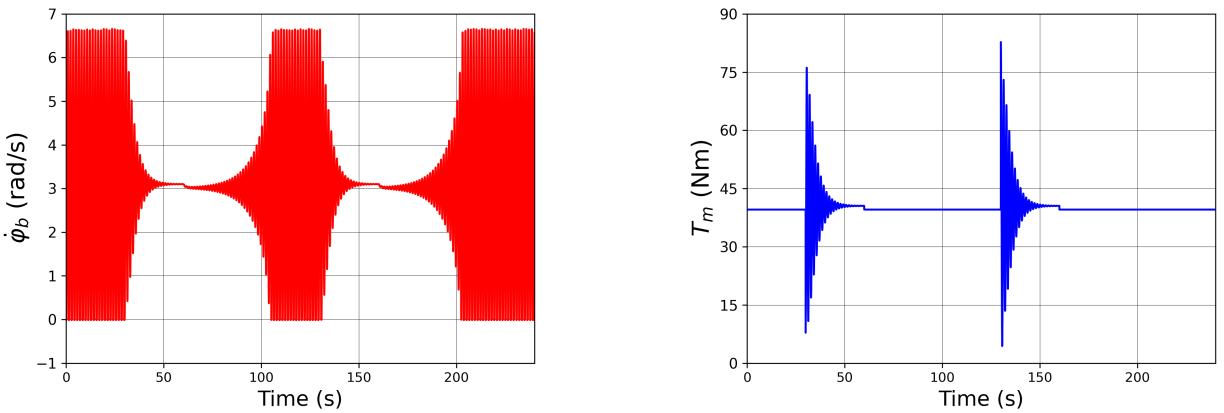


(a) Torque at top of column with equivalent controller for λ=0.8.

(b) Angular velocity at the top of the column and at the drill under the effect of the equivalent controller for λ =0.8.

Figure 4: Equivalent controller for $\lambda = 0.8$.

When the control torque value converges to an approximate value of the initial torque at the top of the string, the system converges to the desired angular velocity of 3.1 rad/s. This can be observed by analyzing Fig. 4b, where initially, when subjected to a constant torque of 39.57 Nm at the top of the string, the system operates under stick-slip regime. However, when the sliding mode controller starts to act, both the angular velocity at the top of the string and the angular velocity of the drill bit converge to the desired angular velocity value of 3.1 rad/s.



(a) Angular velocity at the top of the column with equivalent controller at different time intervals to $\Omega_d = 3.1 rad/s$.

(b) Torque at top of column with equivalent controller at different time intervals to $\Omega_d = 3.1 rad/s$.

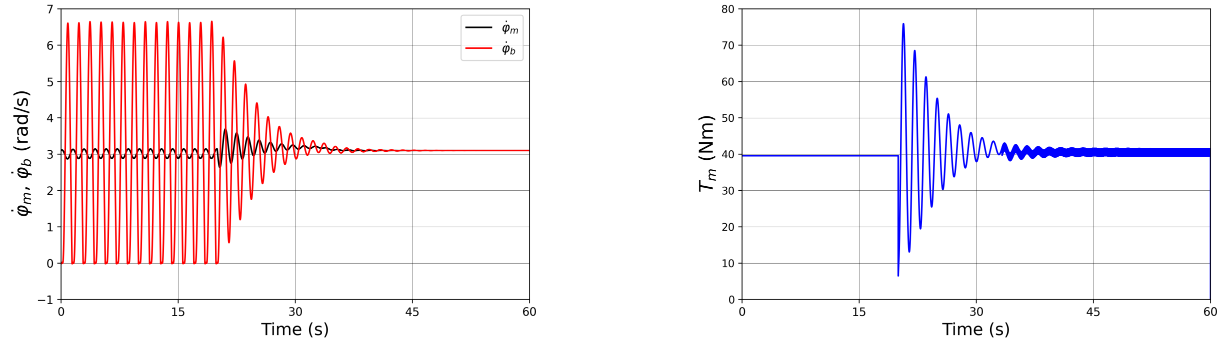
Figure 5: Controller effect when activated at different time intervals to $\Omega_d = 3.1 rad/s$.

By analyzing Fig. 5, where the controller is activated only during the time intervals of 30 to 60 and 130 to 160 seconds, it is possible to verify its effectiveness in eliminating stick-slip vibrations. When the controller is activated, the angular velocity of the system converges to the desired value, while during the moments when the torque at the top of the string is given by a constant value, the system returns to the stick-slip regime, highlighting the significant effect of the controller on the torsional dynamics of the string.

5.3 Controller with switching term

As seen in the previous section, the torque controller obtained from the control law, with $K = 0$, proved to be capable of eliminating stick-slip in the system. However, for real cases, the influence of uncertainties in parameter calculation and the influence of unknown phenomena can negatively affect the behavior of the system and the torque controller, which, in turn, may result in the system not converging to the desired operating condition. Therefore, the addition of the switching term in the controller, for real-world implementation, becomes necessary for the system to behave as desired.

Figure 6 illustrates the temporal behavior of the system's angular velocity and the control torque when considering the switching term with a unity gain. It can be observed that the angular velocity at the top of the string and the drill bit exhibit behavior identical to that shown in Fig. 4b for the same conditions. However, when comparing the control torque with that shown in Fig. 4a, it can be seen that for the case analyzed in this moment, the phenomenon of chattering appears in its behavior, characterized by high-frequency oscillations after the convergence of the system's angular velocities to the desired state.



(a) Torque at top of column with equivalent controller for K = 1.

(b) Angular velocity at the top of the column and at the drill under the effect of the equivalent controller for K =1.

Figure 6: Control considering switching term for K=1.

The amplitude of chattering is directly related to the intensity of the gain of the switching term in the control law. The higher the gain value, the larger the amplitude of this phenomenon. It is worth noting that for the gain values analyzed in this image, all of them were able to bring the system to the desired operating conditions. However, the improper selection of the gain can lead to premature wear of the components of the actual controller.

If we consider the switching term, according to $T_g = -ksat(\frac{s}{\Phi})$, the chattering phenomenon is completely eliminated for the analyzed gain values, providing the controller with the switching term a behavior similar to the ideal controller, as shown in Fig. 7.

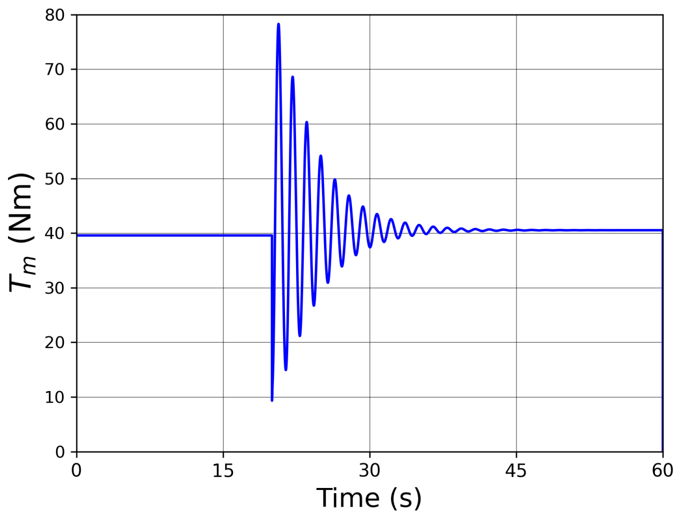


Figure 7: Control torque considering saturation function with $\Phi = 10^{-4}$.

6. CONCLUSIONS

In this work, the development and functionality verification of a sliding mode torque controller for the elimination of stick-slip vibrations in drilling strings have been studied. Based on the presented results, it was possible to verify that the mathematical model satisfactorily reproduced the angular behavior of the system under stick-slip regime, while the sliding mode torque controller proved to be fully capable of eliminating such vibrations in drilling strings, yielding results consistent with those found in the literature. It was observed that considering non-zero gain values for the control maintains the ability to drive the system to the desired state. However, the phenomenon of excessive switching or chattering arises in the behavior of the controller, which can lead to premature wear of control device components. Finally, it was found that considering a saturation function in the switching term ensures that the controller exhibits behavior similar to the ideal controller, mitigating the chattering phenomenon.

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