

COB-2023-0342

Uncertainty in rotordynamics: A SOBOL's sensitivity analysis approach

G.N. Lacerda

T.G. Ritto

Department of Mechanical Engineering, UFRJ - Universidade Federal do Rio de Janeiro
guilherme.lacerda@ufrj.br;tritto@mecanica.coppe.ufrj.br

Abstract. *In this work, a Global Sensitivity Analysis was applied in rotordynamics. For the numerical simulations using the finite element method and Timoshenko beam elements, a specific open-software called ROSS was utilized. Another specialized open-software, OpenTURNS, was employed to handle uncertainties and their propagation. The uncertainty applied to the rotor model is parametric, with the chosen parameters for uncertainty application being the stiffness and damping of both bearings. The rotor used in the simulations was selected from literature, and all four parameters were assumed to follow a truncated normal distribution. The results consist of a stochastic Frequency Response Function (FRF) and the SOBOL' indices for sensitivity, including both the first and total sensitivity indices for two cases; the first one was when the rotor is without rotation, and the second was when the rotor is rotating. The stiffness had a greater impact than damping, even with lower standard deviation; while the gyroscopic coupling effect is directly seen in the results of a rotating rotor; is also observed through the variation of the SOBOL' indices between the stiffness along the x-axis and y-axis.*

Keywords: *Rotordynamics, Global sensitivity analysis, stochastic dynamics, Parametric uncertainty, Monte Carlo simulation*

1. INTRODUCTION

The propagation of uncertainties in rotordynamic model parameters is an effective method to gain more comprehension into the behavior of complex systems. In the context of rotor systems, each parameter's uncertainty has a unique impact on the dynamic response of the system. While rotor parameters are commonly experimentally identified, the estimation process can be affected by sensor and measurement errors, resulting in intrinsic uncertainties.

To model the rotor numerically, the Rotordynamic Open-Source Software (ROSS), presented in Timbó *et al.* (2020), is used in conjunction with a finite element method. The model includes the rotor's shaft, bearings, and disks, along with their respective materials. Dealing with uncertainties in rotor parameters requires the use of specialized software such as OpenTURNS (Dutfoy *et al.*, 2009), which enables the definition of probabilistic distributions, stochastic processes, reliability, and sensitivity. The stochastic response of the rotor, including the frequency response function (FRF), is analyzed through Monte Carlo simulations (Rubinstein and Kroese, 2011). By varying parameters such as bearings damping and stiffness, materials, disk shapes, and shaft parameters, the sensitivity of each parameter is evaluated, and the impact of their variations is studied in the context of the rotor's dynamic response. An intire review of uncertainties analysis in rotordynamics is preseted by Fu *et al.* (2023), where presents the fields of studies such as propagation of parameters and matrices uncertainties; fault analysis; parameter identification; reliability.

Global sensitivity analysis provides insight into the complex interactions between various parameters and their uncertainties, allowing for a comprehensive understanding of the system's behavior. Then, the global sensitivity of the system is evaluated using SOBOL's indices (Sobol, 2001). This technique allows for the quantification of the contribution of each parameter to the variance of the model output, enabling to focus on the most significant parameters in the design optimization process. Lara-Molina *et al.* (2019) shows a sensitivity analysis in flexible rotordynamics systems, investigating the FRFs, Campbell diagram, and run-up responses. Furthermore, Denimal and Sinou (2021) presented a study applying a kriging metamodel and sensitivity analysis for rotordynamics system with uncertainties. The gyroscopic coupling is the modal coupling effect in rotating systems, Sawicki and Genta (2001) presented a method to uncoupling the modes.

The aim of this study is to gain a deeper understanding of the global sensitivity of rotor system parameters, which is applied to improve the design of these components in various applications. In summary, the contributions of the present paper are the sensitivity analysis in a frequency range and a comparison between two rotordynamics models, one non-rotating and the other rotating; this comparison shows the gyroscopic effect on the sensitivity indices. Therefore, automatically the axis coupling of stiffness and damping occurs.

In the following, Section 2 explains the rotordynamic modeling and Section 3 analyzes the numerical results. Finally, the concluding remarks are made in the Section 4.

2. ROTORDYNAMICS

Studies on rotordynamics are currently being widely conducted, a study of mathematical theory is seen in Tiwari (2017); also demonstrated in the reduction model field by Lacerda (2023) and helicopters case by Sumida *et al.* (2019). The dynamics of rotor systems are inherently complex, and the rotor itself can be divided into its constituent parts, including the disks, bearings, and shaft. The equation of motion commonly employed in rotordynamics is expressed as follows:

$$M\ddot{q}(t) + (C + \Omega G)\dot{q}(t) + Kq(t) = f(t), \quad (1)$$

where $q(t) \in R^n$ is the generalized displacement vector and $f(t) \in R^n$ is the forcing vector. M , C , and K are the symmetric mass, damping, and stiffness $n \times n$ matrices, respectively, which include the disk and bearing components, and G is the gyroscopic anti-symmetric $n \times n$ matrix.

Figure 1 shows the rotor model constructed using the Finite Element Method (FEM) simulated in the ROSS. The rotor was discretized into 13 Timoshenko beam elements, as presented in Lalanne and Ferraris (1998). It should be noted that only the lateral vibration of the rotor is taken into account, while axial and torsional vibrations are disregarded. Table 1 provides an overview of all the rotor parameters and their respective characteristics.

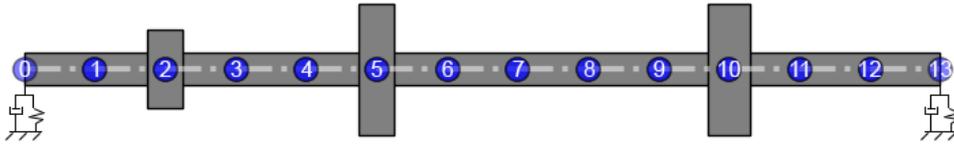


Figure 1: The rotor model for simulations.

Parameters	Values
density (ρ)	7850 kg/m ³
Young's modulus (E)	211 × 10 ⁶ Pa
Poisson's ratio (ν)	0.3
length(L)	1.3 m
diameter (D)	0.05 m
disc 1 and 2 widths	0.05 m
disc 3 width	0.06 m
disc 1 diameter	0.12 m
disc 2 and 3 diameters	0.20 m
discs unbalance	5 × 10 ⁻⁴ kg m
bearings x -stiffness	5 × 10 ⁷ N/m
bearings y -stiffness	7 × 10 ⁷ N/m
bearings x -damping	5 × 10 ² Ns/m
bearings y -damping	7 × 10 ² Ns/m
# of finite elements	13
finite element length	0.1 m

Table 1: Details of the rotor analyzed; see Fig.1.

2.1 Uncertainty Quantification and Propagation

Uncertainty can be incorporated into the model's parameters or introduced into the system matrices (Soize, 2000; Ritto *et al.*, 2011; Cavalini Jr. *et al.*, 2015; Batou and Nabarrete, 2018; Oliveira *et al.*, 2023). In this study, the main focus lies on parameter uncertainty in rotodynamics, its propagation, and the resulting impacts on system behavior. The applications of uncertainties are specifically selected for both bearing stiffness and damping. Figure 2 illustrates the implementation of a truncated normal distribution for the stiffness along the X-axis (a); the stiffness along the Y-axis (b); the damping along the X-axis (c); the damping along the Y-axis (d). For all parameters, including the stiffness and damping of both bearings, the mean values are presented in Table 1, with the standard deviation being one-tenth of the respective mean value for bearing's stiffness and one-fifth of the respective mean value for bearing's damping.

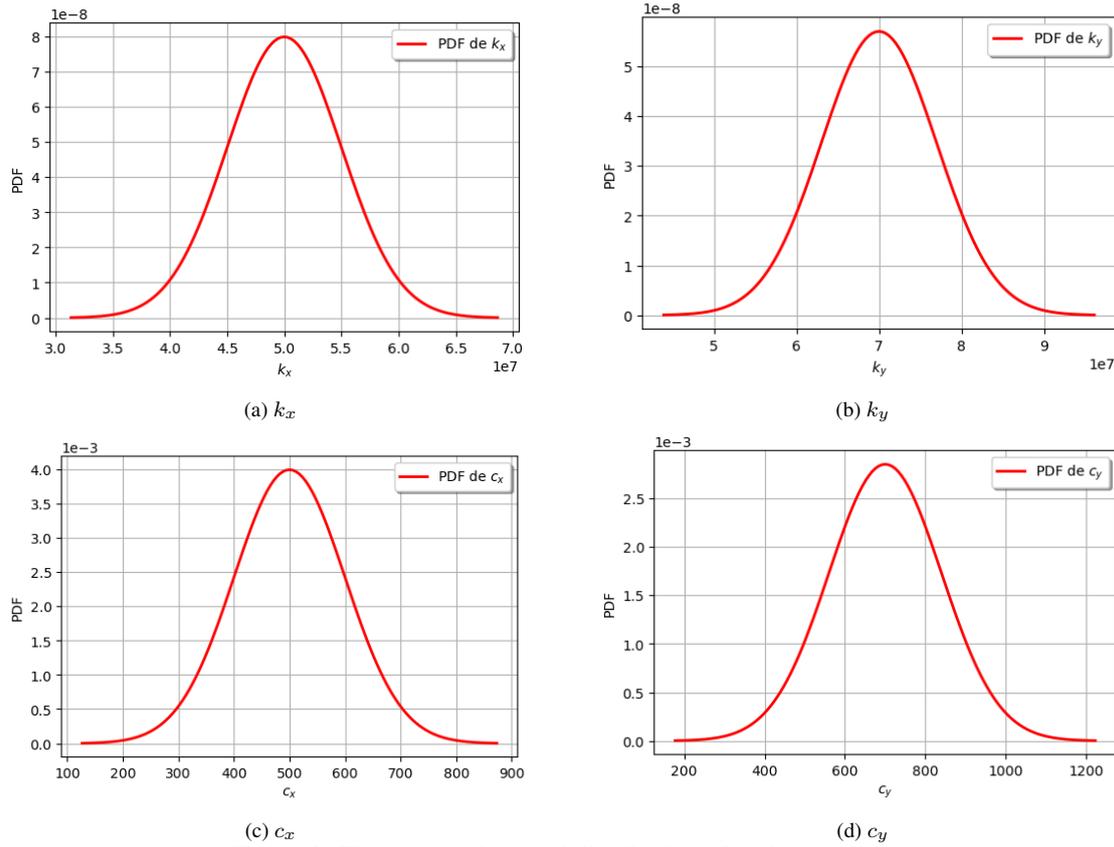


Figure 2: The truncated normal distribution of each parameter.

2.2 Sensitivity analysis

Sensitivity analysis is a method employed to assess the influence of uncertainties in each parameter. The mathematical procedure involves varying a single parameter while keeping the others constant, and subsequently altering each of the remaining parameters individually (Saltelli *et al.*, 2008). This analysis enables the examination of the interplay between uncertainties associated with multiple parameters. In Garoli *et al.* (2021), a sensitivity analysis was conducted on magnetic bearings. By utilizing a functional decomposition of the function's variance, sensitivity analysis aims to explore (Sobol, 2001):

$$V(Y) = \sum_i^d V_i(Y) + \sum_{i < j}^d V_{ij}(Y) + \dots + V_{1,2,\dots,d}(Y) \quad (2)$$

the conditional variances are expressed as:

$$V_i(Y) = V[E(Y|x_i)] \quad V_{ij}(Y) = V[E(Y|x_i x_j)] - V_i(Y) - V_j(Y) \quad (3)$$

Then, SOBOL indices are calculated by:

$$S_i = \frac{V_i(Y)}{V[Y]} \quad S_{ij} = \frac{V_{ij}(Y)}{V[Y]} \quad (4)$$

S_i corresponds to the first-order term which appraises the contribution of the i -th parameter, while S_{ij} corresponds to the second-order term which informs about the contribution of interactions between the i -th and the j -th parameters. These equations can be generalized to compute higher order terms; however, they are expensive to compute and their interpretation is complex. This is why only first order indices are provided.

Total order indices represent the global contribution of the parameters to the variance of the Quantities of Interest(QoI) and are defined as:

$$S_{T_i} = S_i + \sum_j S_{ij} + \sum_{j,k} S_{ijk} + \dots = 1 - \frac{V[E(Y|x_{\sim i})]}{V[Y]} \quad (5)$$

First order indices sum to at most 1, while total order indices sum to at least 1. If there are no interactions, then first and total order indices are equal, and both first and total order indices sum to 1.

3. RESULTS OF ROTORDYNAMICS SIMULATIONS

To simulate the rotor model (Figure 1), the open-source software ROSS was utilized. This software enables various types of rotor simulations and analyses. This section of the paper presents the simulation results. The Equation 6 shows the system transfer matrix, which its inverse results in all FRF's of the model.

$$H = -\omega^2 M + \omega j(C + \Omega G) + K \quad (6)$$

3.1 Simulations with non-rotating rotor

First case, where $\Omega = 0$ in Equation 6, when the rotor is not rotating. Figure 3 displays a Stochastic Frequency Response Function (FRF) with 99% probabilistic envelope, where the stochastic envelope represents the effect of uncertainties applied to the rotor's parameters. By considering the bearing stiffness and damping as probability distributions, it becomes evident how these uncertainties impact the FRF peaks, for both the response amplitudes and the natural frequencies of the system. The FRF considers an excitation and measurement along the y-axis at nodes 2 and 12, respectively. In this simulations, the stochastic response shows three peaks.

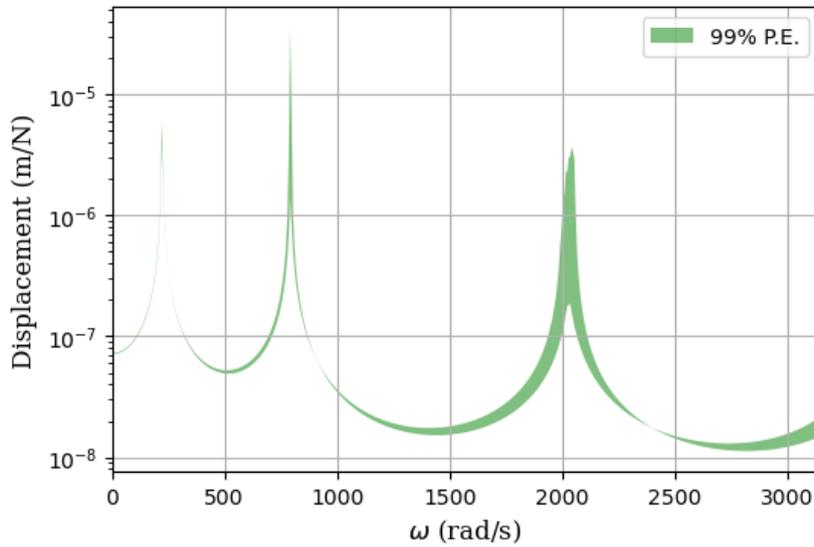


Figure 3: Stochastic FRF due to uncertainties on bearings, with non-rotating rotor. Excitation and measurement at nodes 2 and 12, both in y-axis.

Figure 4 illustrates the total SOBOL indices for each parameter across all analyzed frequencies, applying a 3000 size sample, providing valuable insights into the system's sensitivity to parameter variations. An interesting observation is the notably high value associated with the stiffness of the y-axis, indicating its significant impact on system dynamics; while in x-axis has none impact in this case of observation being in y-axis.

Figure 5 offers a detailed comparison between the first and total SOBOL indices for the k_y parameter across all frequencies. This variance analysis underlines that when k_y varies, the concurrent changes in other parameters result in a collectively higher effect, emphasizing the complex interplay among these parameters.

For an enhanced visual representation and an in-depth understanding of the system's behavior, Figure 6 combines both the stochastic FRF and the total SOBOL' indices. This composite figure clearly highlights that variations in the total SOBOL' indices for k_y are most prominent at the peaks of the stochastic FRF.

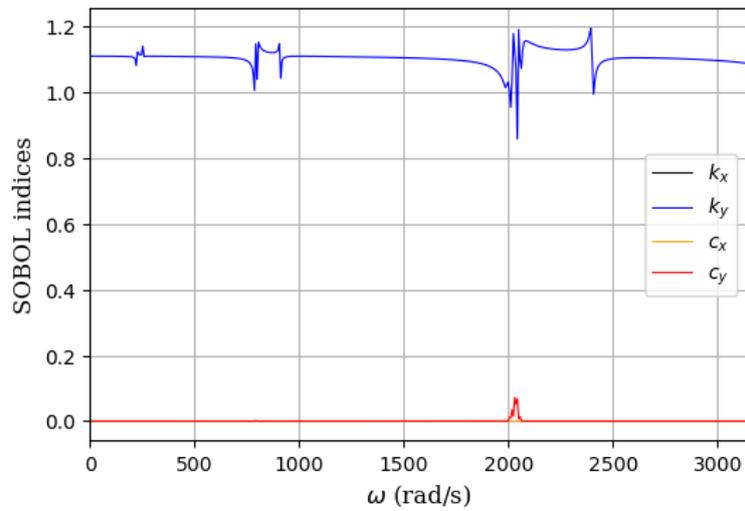


Figure 4: Total order SOBOL indices.

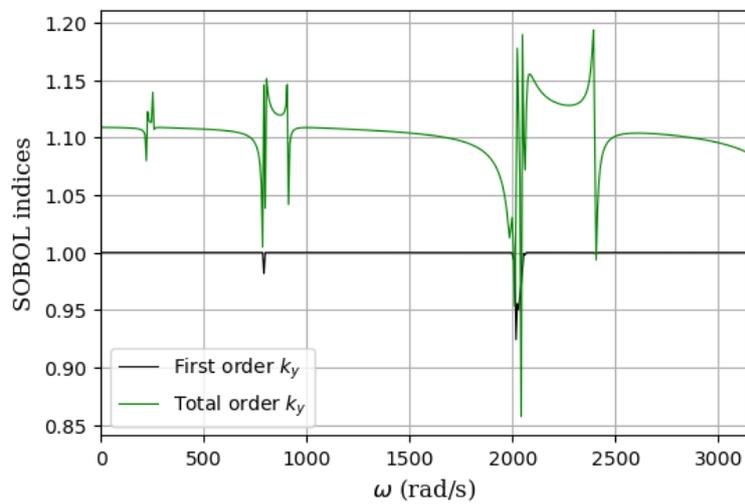


Figure 5: Comparison between first and total order on the k_{xx} .

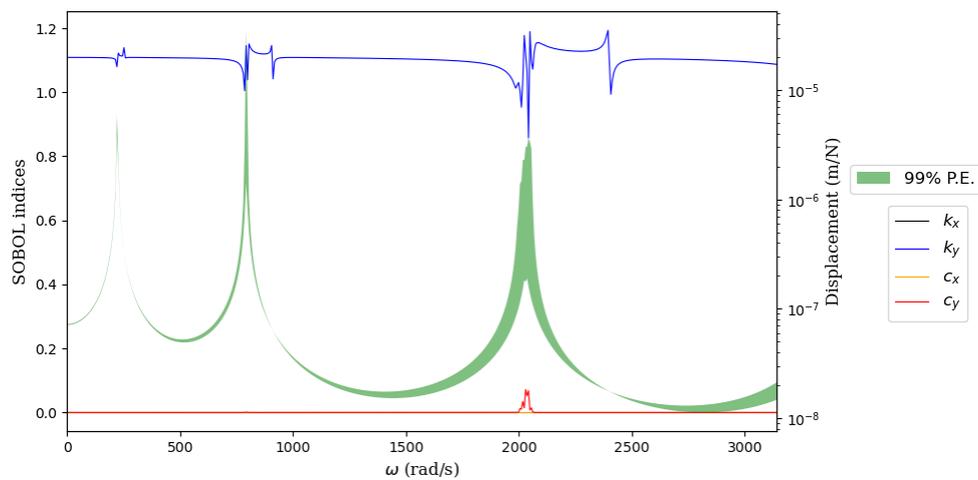


Figure 6: SOBOL indices and Stochastic FRF.

3.2 Simulations with rotating rotor

In the second scenario, described by Equation 6 with $\Omega = \omega$, we consider a rotating rotor. As depicted in Figure 7, the FRF provides valuable insights into the system's behavior when uncertainties are applied to the rotor's parameters. In

this dynamic case, the stochastic envelope clearly illustrates the effects of parameter uncertainties. These uncertainties are introduced in both bearing stiffness and damping, and their influence on the FRF peaks becomes evident as the rotor rotates. The FRF is excited and measured along the y-axis, precisely at nodes 2 and 12, revealing the system’s dynamic response in this configuration. When comparing Figure 7 to the stochastic FRF obtained in the previous scenario with a static rotor, Figure 3, an interesting observation emerges; the response exhibits more peaks, indicating an increased number of natural frequencies within the same frequency range. This dynamic behavior underscores the intricate relationship between rotor rotation and the system’s frequency characteristics.

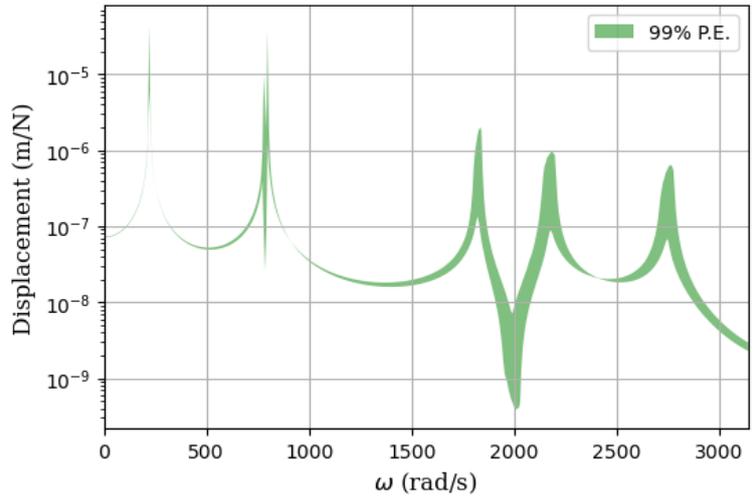


Figure 7: Stochastic response due to uncertainties on bearings, with gyroscopic effect. Excitation and measurement at nodes 2 and 12, both in y-axis.

Figure 8 illustrates the total SOBOL indices for each parameter across all analyzed frequencies, applying a 3000 size sample. With simulations considering the rotor rotating is exhibit an alternation of the parameter with the higher value, between k_y and k_x . The values were set as decoupled; this effect is associated with the gyroscopic coupling. This coupling is given only when the rotor is rotating, and the phenomena consists in that is not possible decoupling the vibrations modes of the system. Also, is possible to notice that while the frequency increases, the gyroscopic coupling effect increasing as well. In this simulations, the gyroscopic coupling is perceptibly seen after the 1500 rad/s .

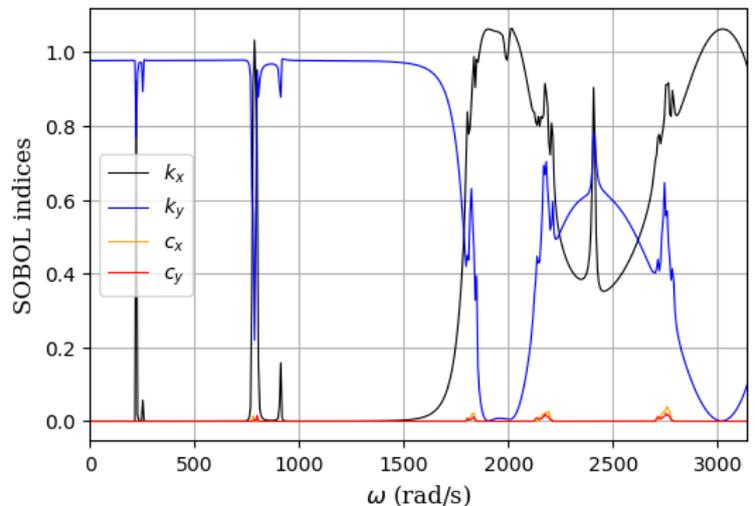


Figure 8: Total order SOBOL indices for the rotating rotor.

Figure 9 provides a comparison between the first and total SOBOL indices for the k_y parameter across all frequencies. This variation confirms that when k_y varies, the other parameters simultaneously exhibit a greater combined impact compared to when k_y varies alone. While, Figure 10 presents both the stochastic FRF and the total SOBOL’ indices. This Figure underscores the pivotal role of stochastic FRF in the analysis. Notably, it reveals that significant variations in the total SOBOL’ indices of k_x and k_y coincide precisely with the peaks of the stochastic FRF. It’s crucial to emphasize that the rotor’s bearings remain decoupled concerning stiffness and damping factors. The noteworthy increase in the indices

for k_x can be attributed solely to gyroscopic coupling effects. This phenomenon arises due to the specific orientation of the FRF excitation and measurement, which occurs along the y-axis. This insight is of paramount importance in comprehending the intricate interactions within the system. It elucidates the unique role that gyroscopic coupling plays in influencing the system's behavior, particularly in relation to the stiffness parameter k_y .

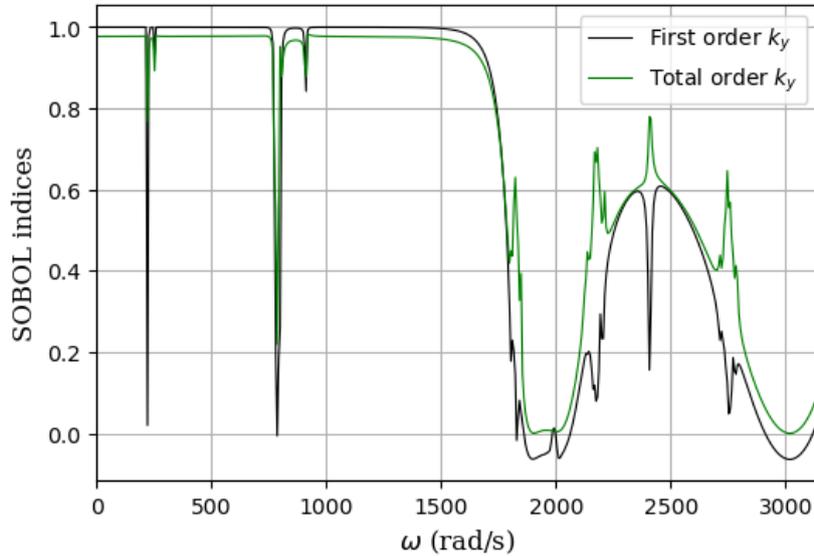


Figure 9: Comparison between first and total order on the k_{xx} .

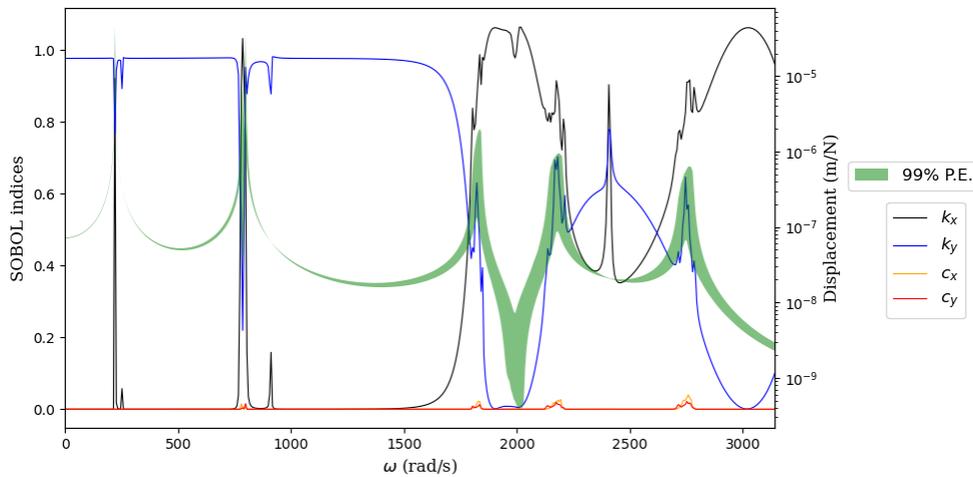


Figure 10: SOBOL indices and Stochastic FRF.

4. CONCLUSIONS

This paper explores the application of Global Sensitivity Analysis to assess parametric uncertainty in rotodynamics. By utilizing open-software designed for rotodynamics, uncertainty propagation, and Monte Carlo simulations, a stochastic Frequency Response Function (FRF) was obtained for two cases; a non-rotating rotor and a rotating rotor. The SOBOL indices were calculated to perform sensitivity analysis for all frequencies of the range, and a comparison was made between the first and total indices for the y-axis bearing stiffness in each case.

The stiffness had a more pronounced influence than damping, even when considering a lower standard deviation; this is evident from the results of SOBOL' indices. Additionally, the findings reveal a significant variation in the parameter indices value along the stiffness axis at the peaks of the FRF, which correspond to the rotor's natural frequencies. Therefore, this variation can be attributed to the presence of gyroscopic coupling. This effect increase as the frequency increases; this consequence is clearly shown in the simulation results and can be measured by the sensitivity analysis.

5. REFERENCES

- Batou, A. and Nabarrete, A., 2018. “Nonparametric probabilistic approach of uncertainties with correlated mass and stiffness random matrices”. *Mechanical Systems and Signal Processing*, Vol. 111, pp. 102–112. doi: 10.1016/j.ymssp.2018.03.049.
- Cavalini Jr., A.A., Lara-Molina, F.A., Sales, T.d.P., Koroishi, E.H. and Steffen Jr., V., 2015. “Uncertainty analysis of a flexible rotor supported by fluid film bearings”. *Latin American Journal of Solids and Structures*, Vol. 12, No. 8, p. 1487–1504. ISSN 1679-7825. doi:10.1590/1679-78251582.
- Denimal, E. and Sinou, J.J., 2021. “Advanced kriging-based surrogate modelling and sensitivity analysis for rotordynamics with uncertainties”. *European Journal of Mechanics - A/Solids*, Vol. 90, p. 104331. ISSN 0997-7538. doi: <https://doi.org/10.1016/j.euromechsol.2021.104331>.
- Dutfoy, A., Dutka-Malen, I., Pasanisi, A., Lebrun, R., Mangeant, F., Sen Gupta, J., Pendola, M. and Yalamas, T., 2009. “Openturns, an open source initiative to treat uncertainties, risks’n statistics in a structured industrial approach”.
- Fu, C., Sinou, J.J., Zhu, W., Lu, K. and Yang, Y., 2023. “A state-of-the-art review on uncertainty analysis of rotor systems”. *Mechanical Systems and Signal Processing*, Vol. 183.
- Garoli, G.Y., Pilotto, R., Nordmann, R. and de Castro, H.F., 2021. “Identification of active magnetic bearing parameters in a rotor machine using bayesian inference with generalized polynomial chaos expansion”. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Vol. 43, No. 12. doi:10.1007/s40430-021-03287-9.
- Lacerda, G.N., 2023. *Uncertainty propagation analysis in reduced-order rotor systems using random matrices (in Portuguese)*. Master’s thesis, Graduate Program in Mechanical Engineering, Federal University of Rio de Janeiro, Rio de Janeiro, Brasil.
- Lalanne, M. and Ferraris, G., 1998. *Rotordynamics prediction in engineering*. Number v. 2 in Rotordynamics prediction in engineering. John Wiley. ISBN 9780471972884.
- Lara-Molina, F., Cavalini, A., Koroishi, E. and Steffen, V., 2019. “Sensitivity analysis of flexible rotor subjected to interval uncertainties”. *Latin American Journal of Solids and Structures*, Vol. 16, No. 4. doi:10.1590/1679-78255476.
- Oliveira, R.M., Maldonado, D.J.G., Batou, A. and Ritto, T.G., 2023. “Application of random matrix theory combined with the singular value decomposition to journal bearings uncertainty analysis”. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Vol. 45, No. 213. doi:10.1007/s40430-023-04141-w.
- Ritto, T., Lopez, R., Sampaio, R. and Souza De Cursi, J., 2011. “Robust optimization of a flexible rotor-bearing system using the campbell diagram”. *Engineering Optimization*, Vol. 43, No. 1, pp. 77–96. doi:10.1080/03052151003759125.
- Rubinstein, R. and Kroese, D., 2011. *Simulation and the Monte Carlo Method*. Wiley Series in Probability and Statistics. Wiley. ISBN 9781118210529.
- Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D., Saisana, M. and Tarantola, S., 2008. *Global sensitivity analysis: The primer*. doi:10.1002/9780470725184.
- Sawicki, J. and Genta, G., 2001. “Modal uncoupling of damped gyroscopic systems”. *Journal of Sound and Vibration*, Vol. 244, No. 3, pp. 431–451. ISSN 0022-460X. doi:<https://doi.org/10.1006/jsvi.2000.3473>.
- Sobol, I., 2001. “Global sensitivity indices for nonlinear mathematical models and their monte carlo estimates”. *Mathematics and Computers in Simulation*, Vol. 55, No. 1-3, pp. 271–280. doi:10.1016/S0378-4754(00)00270-6.
- Soize, C., 2000. “Nonparametric model of random uncertainties for reduced matrix models in structural dynamics”. *Probabilistic Engineering Mechanics*, Vol. 15, No. 3, pp. 277–294. doi:10.1016/S0266-8920(99)00028-4.
- Sumida, I.Y., Velho, H.F.C. and Ritto, T., 2019. “Helicopter main rotor dynamics by neuro-fuzzy classification with bred vector”. In *CONFERENCE OF COMPUTATIONAL INTERDISCIPLINARY SCIENCE - CCIS 2019*. Atlanta, Georgia, United States.
- Timbó, R., Martins, R., Bachmann, G., Rangel, F., Mota, J., Valério, J. and Ritto, T.G., 2020. “Ross - rotordynamic open source software”. *Journal of Open Source Software*, Vol. 5, No. 48, p. 2120.
- Tiwari, R., 2017. *Rotor Systems: Analysis and Identification*. CRC Press.

6. RESPONSIBILITY NOTICE

The authors are solely responsible for the printed material included in this paper.