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An Educational Platform for Tripteron/Triflex II Robots: A Step towards the Development of New Parallel Manipulators

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Abstract. *The use of manipulator robots has become increasingly common and is closely linked to the pursuit of innovation and improvement in production processes in order to enhance competitiveness. This trend has created a demand for the development of new, robust mechanisms and solutions suitable for industrial environments. This study focuses on the structural and kinematic characteristics of three-degree-of-freedom parallel robots derived from the Tripteron robot developed by Laval University. The Tripteron system was entirely designed in this work, and the Triflex II system was adapted from an existing mechanism belonging to the laboratory, which was also attached to the novel modular actuation platform. To construct the prototypes, a modular actuation platform was developed, to which the robots' legs and platform were attached. The paper includes a review of the kinematics of robots, with a particular focus on the kinematics of the Tripteron robot, followed by an introduction of fundamental concepts of mechanism theory, with an emphasis on the analysis of redundant constraints, exemplified by the derivation of the Triflex II robot. Furthermore, an educational platform model of the Tripteron/Triflex II robots is presented, which is based on the Arduino microcontroller and stepper motors. This platform was developed to assist in the study and development of new parallel robot models, kinematics study, and embedded control. Two arduino routines are provided in this work: one that allows the robot platform to be controlled via two joysticks, and another that implements its inverse displacement equations. This study is a valuable contribution to the development of new parallel manipulators, and it has educational purposes as well. The Tripteron and Triflex II robots have simple kinematic equations and actuation decoupling, making them suitable as didactic examples for various concepts in the field of parallel robotics. The educational platform model of these robots, based on the Arduino microcontroller, provides a practical tool for students and researchers to explore and gain a deeper understanding of parallel robotics, kinematics, and embedded control.*

Keywords: *Parallel robots, Tripteron, Triflex II, mechanism design, kinematics, arduino.*

1. INTRODUCTION

The rising prominence of parallel robots in industrial, academic, and research environments is a clear indication of their increasing adoption. This trend is driven by the constant pursuit of innovations and advancements in production processes, aimed at enhancing competitiveness, Abarca and Elias (2023). To effectively address these tasks, it is imperative to develop innovative mechanisms and resilient solutions that can effectively cater to the evolving requirements and challenges at hand.

Within this context, the Raul Gunther Laboratory (LAR) at UFSC, where this study was conducted, has been actively engaged in research endeavors focused on enhancing the utilization of parallel robots in both industrial and academic domains. Notably, one of the ongoing research projects is the Triflex project, which centers on investigating the structural and kinematic characteristics of parallel robots with three degrees of freedom, exemplified by the Tripteron robot developed at Laval University in Figure 1.

From the Triflex project, a self-aligning manipulator was developed based on the Tripteron robot, called Triflex II, Simas *et al.* (2017). The Triflex II robot consists of a parallel mechanism PRRR + PRRU + PRRS, where P represents a prismatic joint, R represents a revolute joint, S represents a spherical joint, and U represents a universal joint. On the other hand, the Tripteron robot features a parallel mechanism PRRR + PRRR + PRRR. This robot is shown in Figure 1.

The fact that the Triflex II robot is self-aligning implies that there are no constraints acting on the same degree of freedom within its mechanism. Generally, the presence of redundant constraints in mechanisms and assemblies entails both advantages and disadvantages compared to mechanisms with exact constraints. The main advantage of the self-aligning feature of the Triflex II is that it provides tolerance to small dimensional manufacturing errors.

Within the scope of the Triflex research, the laboratory possesses an ABS model of the Triflex II robot. However,



Figure 1: Tripteron robot developed at Laval University.

the current drive system employed in the model does not allow the comprehensive validation of its kinematic and dynamic characteristics. Therefore, there exists a critical need to develop an enhanced platform that enables a thorough evaluation of the Triflex II robot's kinematics and dynamics, facilitating a more accurate understanding of its performance capabilities.

Furthermore, the Tripteron and Triflex II mechanisms exhibit well-defined inverse kinematic equations and actuation decoupling. Therefore, they can be utilized as didactic examples for various concepts in the field of parallel robotics. Thus, they constitute an important platform for teaching core topics in parallel robotics and for the development of new self-aligning parallel mechanisms.

In this context, the primary objective of this research is to develop and construct easily replicable, maintainable, and adaptable prototypes of the Tripteron and Triflex II robots. These prototypes serve the purpose of teaching fundamental concepts in parallel robotics while providing an accessible platform for testing and developing novel parallel mechanisms, actuation strategies, and control techniques. By achieving this objective, the aim is to contribute to the advancement of parallel robotics research and to promote the dissemination and democratization of knowledge in the field.

2. Mechanism Mobility and Self-alignment

In this section, fundamental concepts of mechanism theory are introduced, focusing on the analysis of redundant constraints. The purpose of this chapter is to demonstrate that the Tripteron robot and the Triflex II robot, which are the objects of study in this work, exhibit similar mechanisms, with the mechanism of the Triflex II being obtained from the Tripteron robot through the appropriate substitution of joints, thus making the Tripteron self-aligning.

The *mobility* (M) or *degrees of freedom* (DoF) of a mechanism can be defined as the number of independent parameters required to fully specify the configuration of the kinematic chain in space, with respect to a chosen reference link.

A rigid body moving in space can exhibit six distinct motions: three rotations and three translations, as shown in Figure 2a. In the plane, it can exhibit three distinct motions: two translations and one rotation, as shown in Figure 2b. These values represent the degrees of freedom associated with spatial and planar movements respectively.

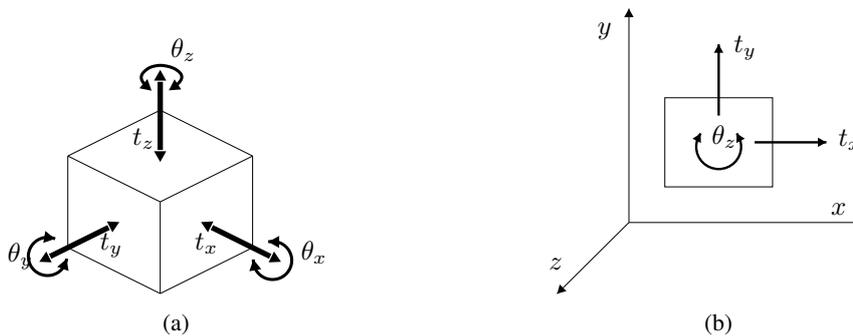


Figure 2: Allowed motions for a rigid body in the space (a) and in the plane (b).

One of the most diffused mobility criterion in the literature is the Grübler-Kutzbach criteria, Gogu (2005).

$$M = \lambda(n - j - 1) + \sum_{i=1}^j f_i \quad (1)$$

where λ represents the helicoid system dimension ($\lambda = 3$ in the planar case and $\lambda = 6$ in the spatial case), n is the number of links in the mechanism, j is the number of joints, and f_i represents the degrees of permitted motion for joint i .

The Tripteron robot exhibits three degrees of freedom, corresponding to the three translations along the x , y , and z axes, $M = 3$ mobility. Applying the Grübler-Kutzbach criterion:

$$M = \lambda(n - j - 1) + \sum_{i=1}^j f_i = 6 \cdot (11 - 12 - 1) + 12 = 0 \quad (2)$$

Applying the Grübler-Kutzbach criterion to the Tripteron yields a mobility value of $M = 0$, which is deemed absurd. This incongruity arises due to the presence of redundant constraints in the Tripteron, where multiple constraints affect the same degree of freedom. Consequently, it is inappropriate to employ the Grübler-Kutzbach criteria in such instances. Remarkably, the Tripteron robot stands as a renowned counterexample to this criterion.

The Ozol criterion Ozol (1962) is a reformulation of the Grübler-Kutzbach criterion, taking into account the redundant constraints that act on the mechanism. According to the Ozol criterion:

$$M = \lambda(n - j - 1) + \sum_{i=1}^j f_i + q \quad (3)$$

where q represents the number of constraints that act in parallel or the number of redundant constraints. Therefore, the Ozol criterion compensates for the degrees of freedom related to the redundant constraints, which were erroneously subtracted in the previous formula.

For instance, for the Tripteron mechanism which exhibits three redundant constraints:

$$M = \lambda(n - j - 1) + \sum_{i=1}^j f_i = 6 \cdot (11 - 12 - 1) + 12 + 3 = 3 \quad (4)$$

In order to determine the number of redundant constraints of a mechanism, two groups of methodologies are found in the literature: topological analysis-based methodologies and geometric analysis-based methodologies.

Methodologies rooted in topological analysis encompass RESHETOV (1979) and Blanding (1999). In contrast, geometric analysis-based approaches, particularly those founded on the helicoid formulation, such as DAVIES (1970), offer an alternative perspective.

A mechanism is said to be *self-aligning* if it can be assembled without introducing forces and deformations, even if the bodies forming the mechanism have linear and angular dimensions different from those in the design, RESHETOV (1979). Furthermore, a mechanism that possesses this property is free from redundant constraints.

RESHETOV (1979) describes a methodology for analyzing the mobility of a mechanism and determining the redundant constraints. The objective of this methodology is to eliminate redundant constraints by increasing the degrees of freedom of a set of joints without altering the kinematics of the mechanism. The method, proposed by RESHETOV (1979), is based on the analysis of fundamental circuits, a technique that had been introduced by SZYDŁOWSKI (2000) and Shamaidenko RESHETOV (1979).

The following proposition is stated by RESHETOV (1979):

Proposition 1 *For a single-loop mechanism, the presence of all the three angular mobilities is a necessary condition for the loop to close without strain, i.e. $f_{rx} \geq 1$, $f_{ry} \geq 1$ e $f_{rz} \geq 1$, where f_{rx} , f_{ry} and f_{rz} are the rotational mobilities along respectively x , y and z -axes.*

Proposition 2 *A linear mobility can be replaced by angular mobility about an axis perpendicular to the direction of linear mobility.*

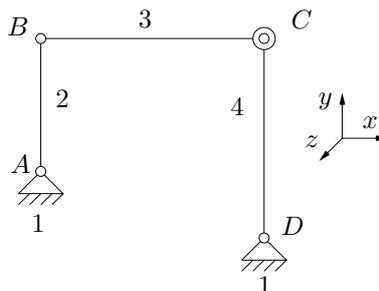


Figure 3: Spatial four-bar mechanism with 3 revolute couplings a , b , d and a spherical one c

Table 1 presents the analysis of mobility and redundant constraints of the spatial four-bar mechanism shown in Figure 3. The left-hand part of the table lists the linear mobilities f_{tx} , f_{ty} and f_{tz} , while the right-hand part reports the

Table 1: Mobility and redundant constraint analysis of mechanism in Figure 3

$F_N = 1$			
$f_{tx} = 0$	← a	—	$f_{rx} = 1$
$f_{ty} = 0$	← b	—	$f_{ry} = 1$
$F_{tz} = 0$		—	$f_{rz} = 4$
\downarrow		$C_N = 1$	\uparrow c c abcd

angular mobilities f_{rx} , f_{ry} and f_{rz} . The corresponding couplings that allow each specific freedom are listed beside each mobility.

All linear mobilities are zero in the left-hand part of the table since no direct linear mobility is allowed by the joints of the mechanism. Along the z-axis, the three revolute couplings a , b and d each allow one freedom. Additionally, the spherical coupling c enables three angular mobilities along each coordinate axis. Therefore, the right-hand side of Table 1 indicates the number of angular mobilities with their respective couplings.

However, the Reshetov methodology presents limitations when applied to spatial mechanisms and the Tripteron robot again stands as a renowned counterexample to this methodology. Furthermore, given the limitations of the Reshetov methodology, particularly in identifying redundant constraints, Carboni (2015) proposed a modification that enables its application to redundant robots. This modified methodology is briefly explained next.

1. First, all the circuits of a mechanism must be generated. In order to perform this task, any set of fundamental circuits of the mechanism can be used to generate all circuits through linear combinations. An algorithm that generates all the circuits of a mechanism is proposed in MARTINS and CARBONI (2008).
2. For each circuit:
 - Create a table with linear and angular mobilities, assigning the angular and linear mobilities provided by the circuit joints. Each circuit must be analyzed independently. For each circuit, only the distribution of mobilities is examined to verify the presence of redundant constraints. The mobilities of joints shared by two circuits are independently assigned to both circuits.
 - Check if all angular mobilities satisfy $f_{rx} \geq 1$, $f_{ry} \geq 1$, and $f_{rz} \geq 1$; otherwise, each missing angular mobility is computed as a redundant constraint.
 - Analyze the linear mobilities: missing linear mobilities are compensated, if possible, with an additional angular mobility along an axis orthogonal to the axis of the missing linear mobility.
3. The number of redundant constraints in the mechanism is determined by adding the redundant constraints of each circuit.
4. Given the value q of the redundant constraints, the mechanism mobility is calculated using the Ozol criterion:

$$M = \lambda(n - j - 1) + \sum_{i=1}^j f_i + q$$

2.1 Applying the modified Reshetov Method, MARTINS and CARBONI (2008), as a design tool

In this section it is shown how to apply modified Reshetov Method in order to obtain novel self-aligned configuration derived from an existing parallel robot. In order to illustrate the methodology the Tripteron robot is modified according to its steps obtaining the Triflex II. Although the method has been developed by Carboni (2015), the example here in presented is an original contribution of this work.

Considering the circuits $ABCDEFGH$, $ILMNEFGH$, and $ABCDILMN$, in Figure 4, these circuits respectively are represented in lines 1 – 3, 3 – 6, and 6 – 9 of in Table 2.

From the analysis of the three circuits in Table 2, $q = 3$. Thus, there exists three redundant constraints corresponding to rotations around x , y , and z axes. Specifically:

- For the first circuit, there are only revolute joints around x and z , resulting in a redundant constraint related to rotation around the y -axis.
- For the second circuit, the revolute joints are around y and z , leading to a redundant constraint in relation to rotation around the x -axis.

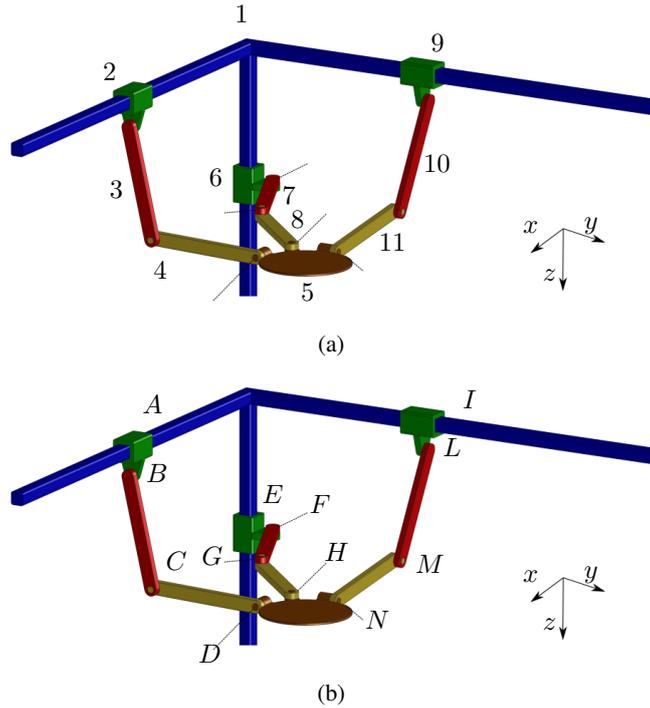


Figure 4: Tripterion robot, its links (a) and joints (b).

Table 2: Modified Reshetov Method, MARTINS and CARBONI (2008), applied to the Tripterion Robot.

Translation	Before Compensating	After Compensating	Compensating	After Compensating	Before Compensating	Rotation
f'_x	1(A)	1(A)	✓	1 (B)	2 (BD)	f''_x
f'_y	0	1(D)		0	0	$f''_y \downarrow$
f'_z	1(E)	1(E)	✓	1 (H)	2 (GH)	f''_z
f'_x	0	1(G)		0	0	$f''_x \downarrow$
f'_y	1(I)	1(I)	✓	1(N)	2 (MN)	f''_y
f'_z	0	1(M)		1 (F)	1 (F)	f''_z
f'_x	0	0		1(C)	1(C)	f''_x
f'_y	0	0		1(L)	1 (L)	f''_y
f'_z	0	0		0	0	$f''_z \downarrow$

- For the third circuit, the revolute joints are around x and y , resulting in a redundant constraint related to rotation around the z -axis.

Thus, in order to list self-aligning configurations through joint substitution, the redundant rotation constraint in each circuit must be compensated. In other words, it is necessary to determine joints that provide the missing rotation in each circuit.

In general, a mechanism can exhibit general and local (or passive) mobilities. The mobility that is relevant to all its elements is referred to as general mobility. On the other hand, the mobility that is relevant to only a restricted set of elements in the mechanism is called local mobility. Therefore, in order to avoid passive mobilities, several options for joint substitution in the Tripterion are presented below, where R represents an original revolute joint, S represents a spherical joint, and D represents a double-revolute joint.

1. By replacing RR in different arms with SS, a configuration presenting dangerous mobility would be generated; thus, this case will be disregarded.
2. By replacing RR in different arms with one S and one D, a self-aligning position could be achieved. However, in order to achieve self-alignment, the double-revolute joint must be oriented in a way that preserves the original rotational freedom it is replacing.
3. By replacing RRR in different arms with SSS, a configuration presenting dangerous mobility would be generated; thus, this case will be disregarded.

4. By replacing RRR in different arms with DDD, a self-aligning configuration can be obtained as long as the same conditions discussed in 2 are observed.

The Triflex II, illustrated in Figure 5, is obtained by considering the following joint substitutions for the second case explained above:

- Replace B with a spherical joint.
- Replace L with a double-revolute joint oriented in a way that allows rotation in y and rotation in x .



Figure 5: Triflex II mechanism.

The example demonstrates that through the modified Reshetov Method, the mechanism of Triflex II and other mechanisms can be derived from the Tripteron by appropriate joint substitutions, making the Tripteron self-aligning. The same process can be applied to other redundant parallel robots, serving as an important tool for designing new mechanisms and robots.

3. Educational platform design

The methodology applied for the project development is the Integrated Product Development Project [PRODIP] Romano (2003), developed at the Product Development Core [NEDIP] of the Federal University of Santa Catarina.

In the PRODIP methodology, the product development is envisioned in phases and sub-phases. The main phases are: planning, design, and implementation. The planning phase is divided into product planning and project planning. The design phase includes the informational, conceptual, preliminary, and detailed design stages. The implementation phase, on the other hand, encompasses production preparation, launch, and validation. Various methods are suggested for the execution of each stage.

It is worth noting, however, that this is a product methodology that does not require the execution of all stages, aiming for flexibility. It is known as a reference methodology as it seeks to systematize and provide means for innovation to occur without compromising creative freedom. In the following sections, the main steps for designing the proposed manipulator platform are presented, utilizing the PRODIP methodology.

3.1 Work Planning

The work planning focused on defining the stages of the product development process to be adopted. It was decided that the system requirements and proposed solutions would be gathered through a morphological matrix, followed by detailed design and prototyping stages.

3.2 Conceptual Design

In the conceptual design phase, the objective is to generate solutions that meet the specified requirements identified in the previous stages. This phase involves identifying the functions that the product should perform and exploring various solutions, their combinations, and initial design concepts. The goal is to conceptualize and propose design solutions that can effectively address the identified needs and constraints.

The system morphological matrix obtained in this step is presented in Table 3.

3.3 Solution Outline

After considering the possibilities, it was decided to use the Arduino Uno as the embedded system. This choice was primarily based on familiarity with the system and the availability of libraries and examples in the literature.

The Nema 17 stepper motor, A4988 stepper motor driver, and CNC shield v3 were chosen for actuation and transmission. These components, available in the laboratory, were selected regarding their commercial availability, increasing the platform design feasibility. With the use of the driver and CNC shield v3, the electronic system is compact and suitable for

Table 3: Morphologic Matrix

Embedded System	Arduino	Raspberry
Actuators	Servomotor	Nema 17, A4988 and Cnc shield
Transmission	Belt	Spindle
Structure and layout	Stainless steel profile	ABS
Energy supply	Battery	DC power supply
Sensing	Limit switches/sensors	

the requirements of the assembly, eliminating the need for custom circuit board fabrication and facilitating maintenance. This assembly is further explained in Section 4.5.2.

The Tripteron and Triflex II robots were fabricated using ABS material as a design requirement. However, the platform to which the robot is fixed was constructed using stainless steel profile.

A 12V DC power supply with a current capacity of 15A was used to power the motors. A 9V battery was employed to supply power to the Arduino. Only micro limit switches were used for sensing, with one switch placed on each arm to provide a reference system for each Nema 17 motor, enabling the implementation of kinematic equations for this first platform version.

3.4 Project Details

The overview of the designed system is presented in Figure 6. It illustrates the proposed solution for the Tripteron system in the classical configuration, where the prismatic joints are mutually orthogonal.

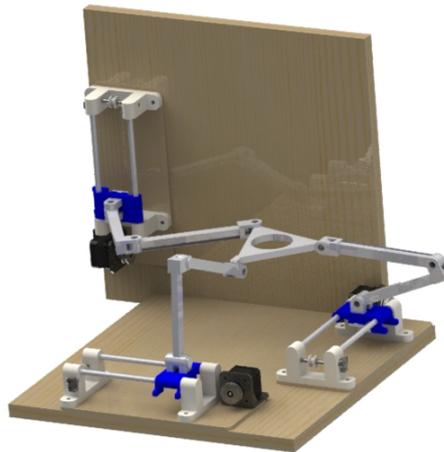


Figure 6: The overview of the structure for the Tripteron/Triflex II systems.

As the Tripteron and Triflex II robots are symmetric parallel robots in their actuation, the system was modeled with three interchangeable sets of actuation and transmission. One of these sets is shown in Figure 7.

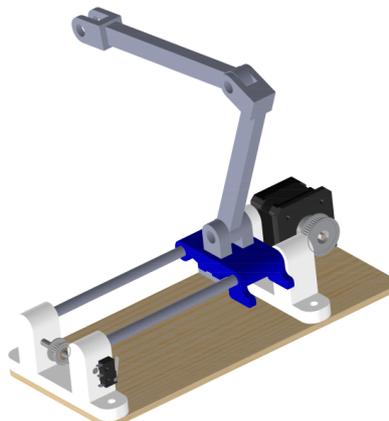


Figure 7: Base-Platform Mechanical Drive Assembly.

The actuation is performed as follows: a 6.5 mm wide timing belt moves the slider relative to the linear guide. The belt is connected to a Nema stepper motor through a 20-tooth timing pulley, using an ABS pulley. Additionally, the slider is designed with an integrated system for belt fixation and adjustment, as shown in Figure 8.



Figure 8: (a) The Slider and its integrated belt fixation and adjustment system. (b)

The electronic system consists of a Nema 17 stepper motor, A4988 stepper motor driver, CNC shield v3, Arduino Uno, limit switches, and two joysticks.

4. Implementation and Results

The proposed system was implemented according to the design specifications and requirements. During testing and operation, the system demonstrated successful performance and functionality. The joystick control allowed intuitive manipulation of the robot's movements, while the embedded implementation of the kinematic equations enabled fairly accurate and precise positioning for academic purposes. The limit switches provided reliable sensing for the reference system of each Nema 17 motor.

The implemented system showcased the desired features, such as self-alignment and manipulator kinematics, enabling effective teaching and research in the field of robotics. Further experimentation and analysis are underway to validate and evaluate the system's performance in various applications and scenarios.

Figure 9 displays the Tripteron robot prototype.

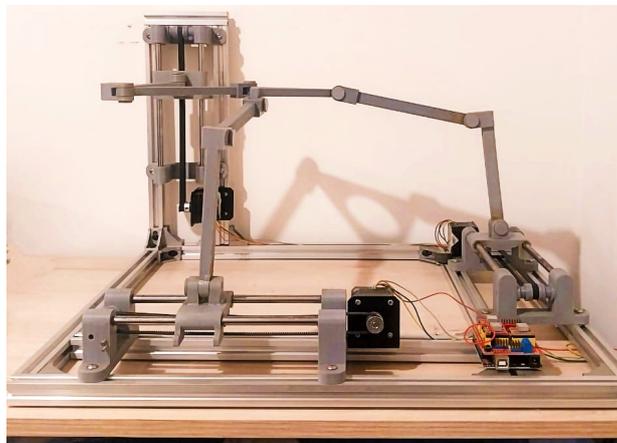


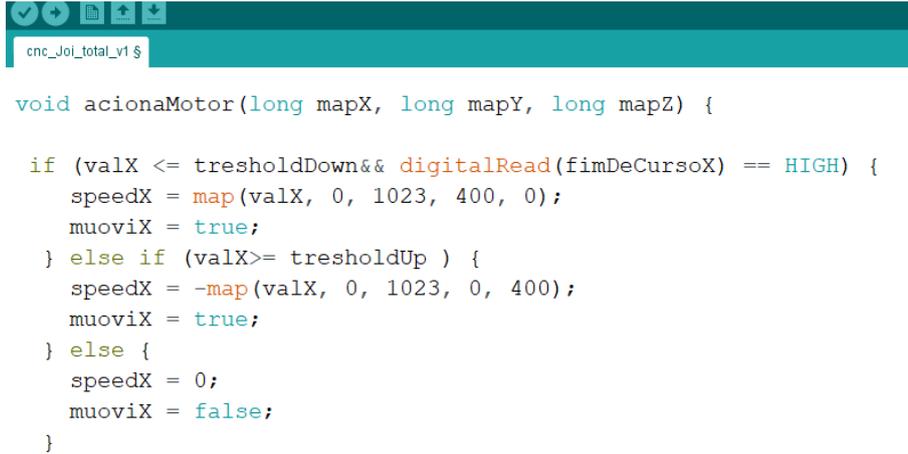
Figure 9: Tripteron robot.

4.1 Software Implementation

The movement of the platform can be controlled by the joysticks. The joysticks provide input signals that are processed by the system to determine the desired motion. In order to interpret the joystick movements as accelerations for the stepper motors and as the direction of rotation, the AccelStepper.h library are utilized. This library provides methods for controlling up to four stepper motors. Additionally, the Bounce2.h library was used to enable the digital switch of the joystick. Figure 10 presents a function, partially depicted, designed to control the motors using the joystick signals.

In order to implement the kinematic equations, the necessary mathematical expressions were derived based on the Tripteron/Triflex II configuration and the desired movement of the platform. These equations were then translated into code within the software framework.

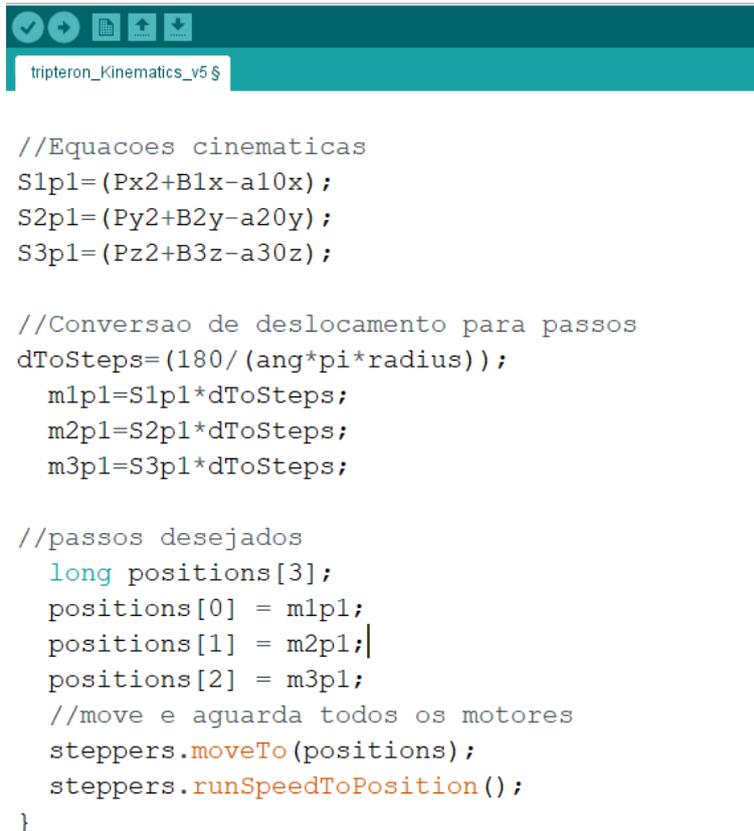
The implementation involves calculating the required displacements for each slider to achieve the desired position and orientation of the platform. This calculation takes into account factors such as the lengths of the links, the angles of rotation, and the geometrical relationships between the various components of the system.



```
cnc_joi_total_v1 $  
  
void acionaMotor(long mapX, long mapY, long mapZ) {  
  
  if (valX <= tresholdDown && digitalRead(fimDeCursoX) == HIGH) {  
    speedX = map(valX, 0, 1023, 400, 0);  
    muoviX = true;  
  } else if (valX >= tresholdUp) {  
    speedX = -map(valX, 0, 1023, 0, 400);  
    muoviX = true;  
  } else {  
    speedX = 0;  
    muoviX = false;  
  }  
}
```

Figure 10: A function to control the motors using the joysticks signals.

Once the necessary displacements are determined, they are translated into the corresponding number of steps for each stepper motor. This conversion takes into account the pulley radius and the number of steps per motor rotation. The stepper motors are then actuated using the calculated number of steps to achieve the desired movement of the platform.



```
tripteron_Kinematics_v5 $  
  
//Equacoes cinematicas  
S1p1=(Px2+B1x-a10x);  
S2p1=(Py2+B2y-a20y);  
S3p1=(Pz2+B3z-a30z);  
  
//Conversao de deslocamento para passos  
dToSteps=(180/(ang*pi*radius));  
  m1p1=S1p1*dToSteps;  
  m2p1=S2p1*dToSteps;  
  m3p1=S3p1*dToSteps;  
  
//passos desejados  
  long positions[3];  
  positions[0] = m1p1;  
  positions[1] = m2p1;  
  positions[2] = m3p1;  
  //move e aguarda todos os motores  
  steppers.moveTo(positions);  
  steppers.runSpeedToPosition();  
}
```

Figure 11: Arduino implementation of the inverse kinematic equations.

5. Conclusions

In this research, a platform for educational and research purposes in the field of robotics is developed and implemented. The platform demonstrated its self-aligning capability, kinematic movements, and intuitive control through joystick inputs and inverse kinematic equations. Its modular and reconfigurable nature allowed for flexibility in adapting to various research experiments and investigations.

However, the project's main contribution is providing valuable insights into the design and development process of self-aligned parallel robots, offering a methodology that can be applied to future robot configurations and advancements,

the Tripteron/Triflex II system, for educational and research purposes in the field of robotics. The project not only resulted in the implementation of a functional platform but also provided a concise methodology for the development of novel self-aligned parallel robot configurations. It provides valuable insights into the design and development process of self-aligned parallel robots, offering a methodology that can be applied to future robot configurations and advancements.

6. Further Work

While the developed platform and methodology have laid a solid foundation, there are exciting opportunities for further work and research. Specifically, future efforts should focus on enhancing the platform's capabilities and exploring advanced control strategies.

One avenue for improvement is the integration of additional sensing capabilities, such as vision systems or force/torque sensors. This will enable the platform to perceive its environment in real-time, allowing for more advanced control techniques and applications.

Moreover, further research is needed to explore and develop advanced control strategies for the platform. This could include trajectory planning, motion coordination, or adaptive control algorithms, allowing for more precise and dynamic control of the platform's movements.

By expanding the platform's capabilities and exploring advanced control strategies, researchers and students will have the opportunity to delve into complex robotic tasks and push the boundaries of self-aligned parallel robot configurations.

In conclusion, this project has not only resulted in the development of a functional platform but has also provided a concise methodology for the development of novel self-aligned parallel robot configurations. The platform serves as a valuable educational and research tool, while the methodology offers insights and guidance for future advancements in the field of robotics.

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