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A STOCHASTIC MODELING OF AEROVISCOELASTIC SYSTEMS SUBJECTED TO UNCERTAINTIES FOR SUBSONIC STABILITY ANALYSIS.

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Abstract. *To reduce the environmental impact in the world, governments and international organizations have proposed several strategies to produce more efficient products and solutions. The aeronautical and aerospace industries have proposed to reduce their CO_2 emissions to 0 by 2050. To achieve this goal, manufacturers are using solutions such as new blade engines, Sustainable Aviation Fuel (SAF), lighter materials, and new geometric wing shapes that are more efficients. However, to avoid the instability effects due to the structural and aerodynamic modifications of the resulting aeroelastic systems, such as the flutter phenomenon, one strategy may be the use of vibration control techniques. In this context, passive control using viscoelastic material is an interesting strategy to be used in such situations due to its low cost and application facilities. Moreover, due to the inherent variabilities of the structural and aerodynamic parameters appearing in these systems, it is also necessary to propose an efficient stochastic modeling methodology of them for dealing with more realistic applications of industrial interest. Thus, this work aims to model a stochastic aeroviscoelastic system of a plate-like wing in a subsonic regime to increase the stability of this type of structure by combining the stochastic finite element modeling and the Doublet Lattice Method (DLM). It will also quantify the effect of increasing mass and stiffness purely related to the addition of layers, and then the system including the damping of the viscoelastic material. It is shown the envelopes of solutions in terms of the flutter speed boundary and the degree of influence of structural and aerodynamic parameters on the critical flutter speeds of a panel system treated with a passive constraining layer.The results have shown an increase of 31 % flutter speed with a 38 % increase in mass. Although the ratio (speed gain/mass gain) is below one in this case, with partial treatment it can even reach 2, proving that it can be advantageous to treat aeronautical panels with layers of viscoelastic material.*

Keywords: *Viscoelastic Material, Stochastic Finite Element Method, Vibration Control, Aeroelasticity.*

1. INTRODUCTION

To reduce the environmental on the world, governments and international organizations have proposed several strategies to produce more efficient products and/or solutions. For example, the aeronautical and aerospace industries have proposed to reduce their CO_2 emissions to 0 by 2050, based on the year 2005 (ICAO, 2021). To achieve this goal, they have increased the use of lighter materials combined with new geometries to the structure and aerodynamic surfaces, such as wings. However, the resulting system is more susceptible to dangerous aeroelastic instabilities, such as the flutter phenomenon. Within this field, several researchers (WRIGHT; KIDNER, 2004; GRIPP; RADE, 2018) have suggested interesting strategies to control the flutter phenomenon on these new light flexible structures to achieve greater reliability and safety of such systems in service. Among the number of control strategies available in the open literature, the use of viscoelastic materials has great advantages such as the application facilities, fewer maintenance costs, and robustness (CUNHA-FILHO et al., 2016). In this case, several works have focused on studying vibration control using viscoelastic material, such as those by Mozaffari-Jovin, Firouz-Abadi, and Roshanian (2015) and Fazelzadeh, Ghavanloo, and Pouresmaeeli (2015).

However, most of the works dealing with aeroviscoelastic systems have considered the effects of the supersonic stability analysis by using the Linear piston Theory (CUNHA-FILHO, 2019). Moreover, none of the proposed aeroelastic models have considered the effects of inherent parametric uncertainties for more realistic applications, which has motivated the present study. This lack of studies can be explained in part by the difficulty in modeling the frequency and temperature-dependent viscoelastic material properties of subsonic aeroviscoelastic systems to predict the flutter boundary.

Hence, this work proposes the use of the so-called Stochastic Finite Element Method (SFEM) combined with the DLM to model the three-layer sandwich panel system subjected to the subsonic regime and parametric uncertainties, simultaneously. In the quest for the viscoelastic model, it is retained herein the Fractional Derivative Model (FDM), and the envelopes of the critical flutter speeds are predicted by using an improved version of the well-known pk method.

2. METODOLOGY

This section will present finite element modeling, the Karhunen-Loeve model for the stochastic structural system, and the aerodynamic model used to couple the structural system with the DLM.

There are some limitations in the final plate case study, such as having a high aspect ratio (length/thickness) to avoid shear locking, and also limitations in the DLM, as they are derived from an aerodynamic potential model, where viscosity effects are not considered.

2.1 Background on Modeling of Stochastic Sandwich Viscoelastic Plates

In this work, the First Order Shear Deformation Theory (FSDT) presented by Reddy (1985) will be used, where the displacements of the elastic layers (base and constraint, as in Fig.1) are defined according to the Kirchhof-Love plate theory (CUNHA-LIMA, 2019), and the displacements of the viscoelastic core are a combination of the displacements of the elastic layers through the assumption that such layers are perfectly glued and have 8 degrees of freedom.

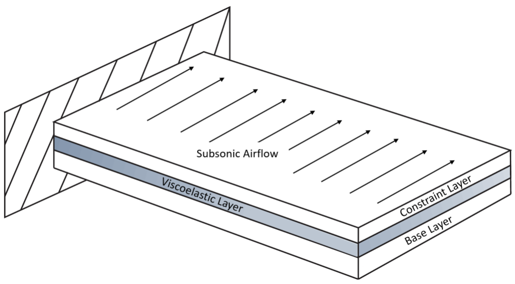


Figure 1: Aeroviscoelastic System.

Therefore, the finite element has the ordering degrees of freedom, namely: longitudinal displacements **u** and ***v*** in the directions ***x*** and ***y***; transverse displacement ***w***; and the rotations θ_x and θ_y. As there is no sliding between the layers, we have the following kinematic relationships described by Eq. 1. where (k) represents the k_{th} layer. The strain field ϵ, considering small displacements, is given by Eq. 2. For elastic layers ($k = 1, 3$), we had Eq. 3 and for viscoelastic layer ($k = 2$) the Eq. 4.

$$\{U^{(k)}\} = \begin{Bmatrix} u^{(k)}(x,y,z,t) \\ v^{(k)}(x,y,z,t) \\ w^{(k)}(x,y,z,t) \end{Bmatrix} = \begin{Bmatrix} {u_0}^{(k)}(x,y,z,t) - (z - z^{(k)})\frac{\partial w(x,y,t)}{\partial x} \\ {v_0}^{(k)}(x,y,z,t) - (z - z^{(k)})\frac{\partial w(x,y,t)}{\partial y} \\ w(x,y,z,t) \end{Bmatrix} \tag{1}$$

$$\epsilon^{(k)} = \mathbf{D}U^{(k)} \tag{2}$$

$$\epsilon^{(1,3)} = [\epsilon_x \quad \epsilon_y \quad \tau_{xy}] \tag{3}$$

$$\epsilon^{(2)} = [\epsilon_x \quad \epsilon_y \quad \tau_{xy} \quad \tau_{xz} \quad \tau_{yz}] \tag{4}$$

Once we find the nodal displacement field $\boldsymbol{U}$, we have that Hooke's law, strain energy relations, and kinetic energy give us the stiffness (Eq. 5 and Eq. 6) and mass matrices (Eq. 7). Where E and ρ represent the elastic modulus and density, respectively, and **D** and **N** are the matrixes of differential operators and shape functions, respectevily. Observating that the viscoelastic layer has a dependece of frequency and temperature.

$$\boldsymbol{K}^{(1,3)} = \int_V \boldsymbol{D}^{(1,3)^T} E^{(1,3)}\, \boldsymbol{D}^{(1,3)}\, dV \tag{5}$$

$$\boldsymbol{K}^{(2)}(\omega, T) = \int_V \boldsymbol{D}^{(2)^T} E^{(2)}(\omega, T)\, \boldsymbol{D}^{(2)}\, dV \tag{6}$$

$$\boldsymbol{M}^{(k)} = \int_V \boldsymbol{N}^{(k)^T} \rho^{(k)} \boldsymbol{N}^{(k)}\, dV \tag{7}$$

2.2 Stochastic Finite Element Method (SFEM)

The stochasticity theory used is developed by Karhunen-Loeve and the implementation in the MEF is done according to Ghanem, R. and Spanos, P.D. (1991). These variables are explained in Cunha-Filho (2018) by a process dimension stochastic. Thus, assume that thicknesses are random processes that can be expressed as the sum of a deterministic mean part with a zero mean random part (Eq. 8, Eq. 9 and Eq. 10), such as:

$$h_1 = \overline{h}_1(x,y) + \alpha_1(x,y,\theta) \tag{8}$$

$$h_2 = \overline{h}_2(x,y) + \alpha_1(x,y,\theta) \tag{9}$$

$$h_3 = \overline{h}_3(x,y) + \alpha_1(x,y,\theta) \tag{10}$$

Where, $\overline{h}_j(x,y)$ is the average thickness of layer j. Another hypothesis made is that this mean does not depend on the position, that is, $\overline{h}_j(x,y) = \overline{h}_j$. $\alpha_j(x,y)$ stochastic process of layer j, with zero mean and covariance to the four-dimensional exponential function $C_j(x_1,y_1,x_2,y_2) = exp(-\frac{|x_1-x_2|}{L_{x,j}} - \frac{|y_1-y_2|}{L_{y,j}}).L_{x,j}$ the correlation length in the x direction of layer j and $L_{y,j}$ the correlation length in the y direction of layer j.

Using the KL expansion of the stochastic processes j up to an order N, one can rewrite for each layer j, as Eq. 11:

$$h_j \approx \overline{h}_j + \sum_{r=1}^{N} \sqrt{\lambda_{j,r}} f_{j,r}(x.y)\xi_{j,r}(\theta) \tag{11}$$

The truncation of the infinite series from the KL decomposition to an N order of Eq. 11 creates a small error. To decrease the error, it is better to get a high enough order of the KL decomposition, and thus guarantee the good convergence of the solution. However, the higher the order, the longer the computation time associated with stochastic FRF calculations. For a two-dimensional stochastic process, the KL discretization error is even greater and that it is useless to increase the KL order. It is better to increase the correlation lengths then.

The stochastic processes can be defined over the real elements (e.g. the entire plate), or over the reference element. The simple stochastic plate study showed that there is not much difference between the two ways of doing it. However, since in the case where the processes are defined on the reference element, the stochastic matrices do not depend on the element, the implementation becomes easier and the calculation time is faster. Therefore, in this work, this strategy is adopted.

Correlation lengths in x and y could depend on the layer. However, a correlation length equal to the length of the eigenfunction definition interval is generally used (here it is equal to 2 in both directions). Therefore, in this work, the same correlation lengths are taken for the three layers. So it won't depend on the element either.

With these choices, the index j of Eq. 12 can disappear in the eigenvalues and the eigenfunctions. However, it remains in the random variables, as they depend on the uncertainty (their standard deviation) imposed in the analysis for each layer. There is then:

$$h_j = \overline{h}_j + \sum_{r=1}^{N} \sqrt{\lambda_r} f_r(x.y)\xi_{j,r}(\theta) \tag{12}$$

2.3 Viscoelastic Material

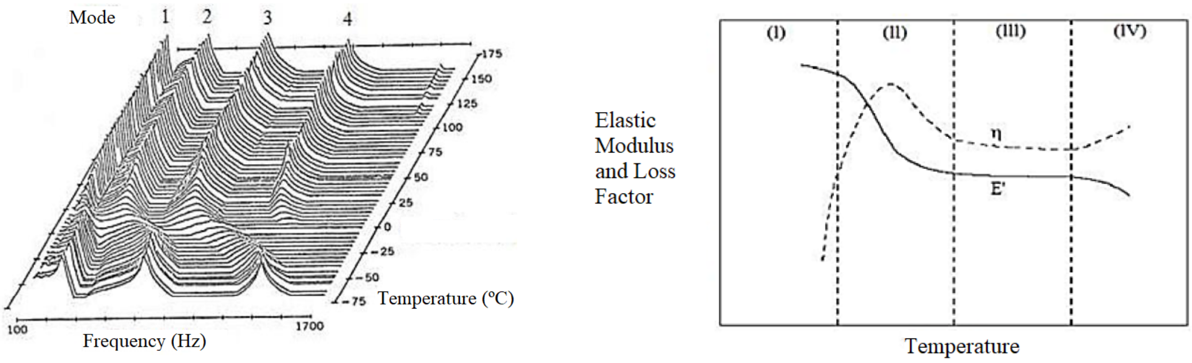
Viscoelastic materials are high-frequency and temperature-dependent as demonstrated by Nashif (1985). Fig. 2a illustrates the experimental FRFs of a free-standing beam treated with viscoelastic material.

It is noted that temperature exerts a greater influence than frequency on the treated system. For this reason, the viscoelastic material can present 4 different states: glassy state (I), transition state (II), rubbery state (III), and fluid state (IV). Figure 2b shows the four states in a graph of the storage module (E) and the loss factor (η) as a function of temperature.

For a fixed temperature close to the glass transition temperature (within transition region II), measurements of the viscoelastic characteristics give the storage modulus, E' , and the loss factor, η. The viscoelastic characteristics at different temperatures can be associated with each other through the Principle of Frequency-Temperature Superposition (PSFT). Symbolically represented by:

$$G(\omega_r, T_0) = E(\alpha_t\omega, T_0) \tag{13}$$

$$\eta(\omega_r, T_0) = \eta(\alpha_t\omega, T_0) \tag{14}$$



(a) FRFs of a beam with viscoelastic material. (b) Viscoelastic behavior of E' and η vs temperature

Figure 2: PSFT - Nashif (1985).

where $\omega_r = \alpha_t(T)\omega$ is the reduced frequency, ω corresponds to the excitation frequency, $\alpha_t(T)\omega$ is the displacement factor that depends on the temperature of the viscoelastic material, and T_0 is the reference temperature. From Eq. 13 and 14 it is possible to obtain the curves of the viscoelastic material, known as monograms (Figure 2b).

Finally we get the equation motion on frequency domain, as expressed on Eq. 15. Viscoelastic characteristics at different temperatures can be associated with each other through the Frequency-Temperature Superposition Principle (PSFT) as explained. To represent the complex modulus G as a function of reduced frequency and temperature for the 3M ISD112 viscoelastic material to be used in this work, obtained from Borges (2019), we get the Eq. 16.

$$[\boldsymbol{K}_e(\theta) + G(\omega, T, \theta)\overline{\boldsymbol{K}}_v(\theta)]\, \boldsymbol{U}(\omega, T, \theta) = \boldsymbol{F}(\omega) \tag{15}$$

$$G(\omega, T, \theta) = 0.4307 + \frac{1200}{\left[1 + 3.24\left(\frac{i\omega\alpha(T(\theta))}{1543000}\right)^{-0.18} + \left(\frac{i\omega\alpha(T(\theta))}{1543000}\right)^{-0.6847}\right]} \tag{16}$$

2.4 Doublet Lattice Method (DLM)

The DLM method initially proposed by Albano and Rodden (1969), considers the elementary solution of non-stationary flow via a pressure dipole, which has intensity varying harmonically over time. To this end, a panel discretization is used and a line of dipoles is positioned at 1/4 of the chord of each panel (x direction). Therefore, a control point, half span, and 3/4 of the chord are taken on each panel to analyze the induced speed. Fig. 3 shows a generic representation of a wing discretization and a panel detail. The application of the null normal flow condition applied at the control point allows us

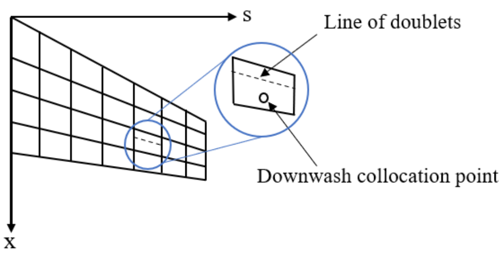


Figure 3: Discretization in panels in the DLM method - Adapted from Borges (2019).

to obtain the dimensionless pressure field, which represents the dimensionless induced speed. Once we have the speed, we can calculate the pressure distribution on the panel ΔC_p as Eq. 17 through the Aerodynamic Influence Coefficient Matrix [AIC] and the upwash $\{W\}$.

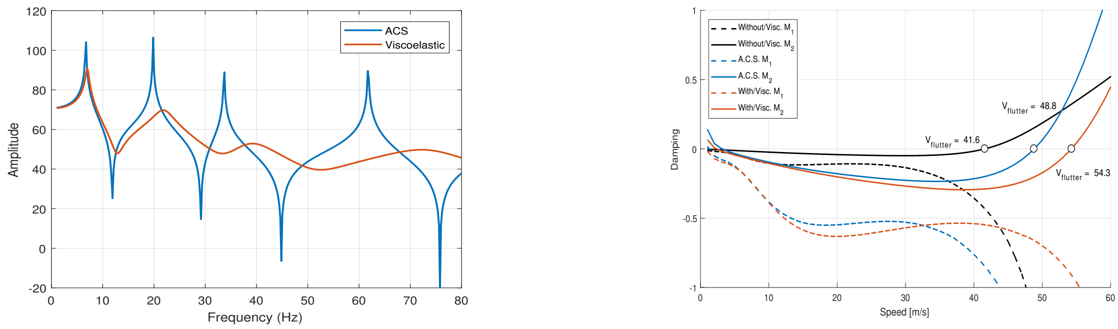
$$\{\Delta C_p\} = \{\boldsymbol{AIC}\}\{W\} \tag{17}$$

After using the standard FE element modeling procedures based on the node connectivity and coupling with the DLM model, the global equations of motion of the stochastic viscoelastic system are given as Eq. 18.

$$\boldsymbol{M}(\theta)\, \ddot{\boldsymbol{u}}(t, \theta) + [\boldsymbol{K}_e(\theta) + G(\omega, T, \theta)\boldsymbol{K}_v(\theta)]\, \boldsymbol{u}(t, \theta) = \boldsymbol{F}_{aero}(t, \theta) \tag{18}$$

3. RESULTS

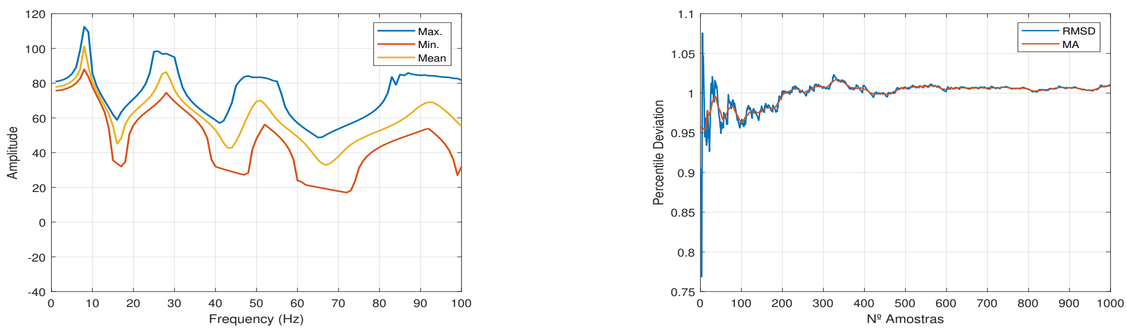
By solving Eq. 15 using pk method, it is possible to generate the V-g diagrams, as Fig. 4b where is possible to see the 3 gains separated: a plate without viscoelastic, the associated conservation system (A.C.S.), which consider only the increase of mass and stiffness due to treatment, and the plate with the full viscoelastic system. Also, the amplitudes of the frequency response functions (FRFs) of a sandwich plate treated with a passive constraint layer for two situations, namely: the dissipative system and the A.C.S., where the modulus of the viscoelastic material at low frequency (independent of frequency and temperature) is assumed. Also, we obtained the FRF's of the structural stochastic system as in the Fig. 5a and the number of samples determined through the Fig. 5b through root mean square deviation (RMSD), and the mean average (MA) with a frequency of 25. From the analysis of Fig. 4b, it can be seen that it has aeroelastic terms, there is a reasonable gain in the $flutter$ speed of the system with viscoelastic treatment, showing its efficiency to act in the subsonic regime. Despite the viscoelastic plate having a mass of 838.5 g against 607.5 g of the plain plate, representing a mass increase of 38 %. In this case, the structural performance (speed gain/mass gain) of the viscoelastic treatment is 0.8.



(a) Deterministic FRF's.

(b) V-g diagram of plates.

Figure 4: Structural responses of the aeroviscoelastic system.



(a) Stochastics FRF's.

(b) Convergence Analysis of Stochastic System.

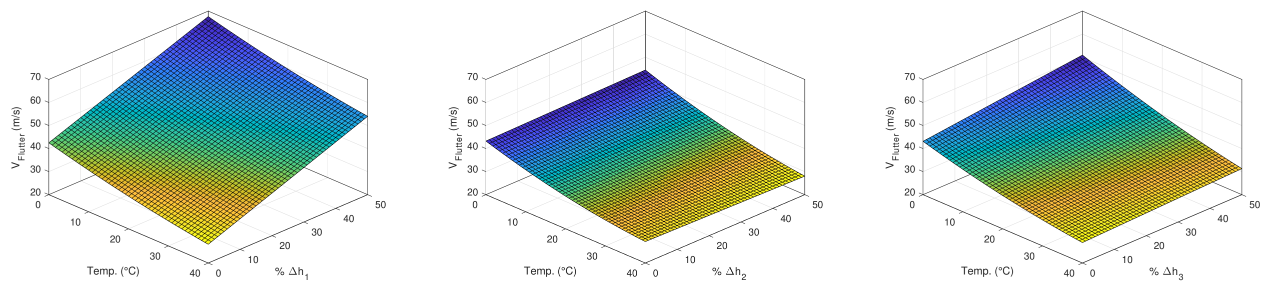
Figure 5: Stochastics Analysis.

In order to observe the behavior of the $flutter$ speed as a function of the base layer thickness variation, a percentage thickness variation Δh_1 from 0 to 50 % of the initial thickness was adopted. As the multilayer board also depends on the operating temperature (due to the viscoelastic), it was also sought to observe this influence, where both are shown on the surface of Fig. 6a. The variation of $V_{flutter}$ with increasing thickness is greater than with temperature. Because the speed gain with increasing thickness is approximately 70 % while at the temperature it is around 30 %. Unlike the base layer, the viscoelastic layer, by itself, has little influence on the stability of the aeroviscoelastic system, as shown in Fig. 6b. Similar to the base layer, the constraint varies proportionally with increasing thickness, however, due to its low thickness characteristics, its behavior concerning temperature is similar to the viscoelastic layer, as shown in Fig. 6c.

4. CONCLUSION

It shows the envelopes of solutions in terms of the flutter speed boundary on Fig.4b, where we can see that the ACS can offer a gain of 17%, and when we have the complete system (viscoelastic), this gain becomes 31%. This difference applies because viscoelastic acts more considering frequencies and temperatures, which cause its G modulus to increase significantly, above the rigidity modulus of the purely elastic plate.

The percentage of flutter increase desired by the designer can be achieved by observing Fig.6 for a temperature range,



(a) Base Layer (b) Viscoelastic Layer (c) Constraint Layer

Figure 6: Parametric Analysis.

thus acting as an initial starting point for further optimization with partial treatment. The increase in mass in the system is also relevant. Despite the flutter/mass ratio being better in the treated system. Mass reduction techniques with partial treatment of the plate are being studied to guarantee a better structural efficiency ratio.

Although the ratio (speed gain/mass gain) is below one in full treatment, with partial treatment it can even reach 2, proving that it can be advantageous to treat aeronautical panels with layers of viscoelastic material.

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6. REFERENCES

Albano, E. and Rodden, W. P., 1969, "A Doublet-Lattice Method for Calculating Lift Distributions on Oscillating Surfaces in Subsonic Flows", AIAA J., Vol. 7, p. 279-285.

Borges, D. M., 2019. *Desenvolvimento De Uma Metodologia De Análise Da Estabilidade De Sistemas Aeroviscoelásticos Empregando O Método Doublet Lattice*. Master's thesis, Graduate Program in Mechanical Engineering, Federal University of Uberlândia, Uberlândia, Brasil. p. 34.

Cunha-Filho, A.G., De Lima, A.M.G., Donadon, M.V., Leão, L.S., 2016, "Flutter suppression of plates using passive constrained viscoelastic layers", Mechanical Systems and Signal Processing, v. 79, p. 99-111.

Cunha-Filho, A.G., Briend, Y.P.J.,De Lima, A.M.G., Donadon, M.V., 2018, "An efficient iterative model reduction method for aeroviscoelastic panel flutter analysis in the supersonic regime", Mechanical Systems and Signal Processing, v. 104, p. 575-588.

Cunha-Filho, A.G., 2019, "Transient approach to the effects of viscoelastic damping on the aeroelastic stability of aeronautical structures.", Doctoral Thesis, Federal University of Uberlândia, Brazil.

Ghanem, R. and Spanos, P.D., 1991, "Stochastic Finite Elements: A Spectral Approach", Dover Publications.

Gripp, J.A.B., Rade, D.A., 2018, "Vibration and noise control using shunted piezoelectric transducers: A review", Mechanical Systems and Signal Processing, v. 112, p. 359-383.

ICAO, 2021, "Latest News and Highlights",
https://www.icao.int/Newsroom/Pages/ICAO-welcomes-new-netzero-2050-air-industry-commitment.aspx.

Mozaffari-Jovin, S., Firouz-Abadi, R.D., Roshanian, J., 2015, "Flutter of wings involving a locally distributed flexible control surface", Journal of Sound and Vibration, v. 357, p. 377-408.

Nashif, A.D., Jones, D.I.G. and Henderson, J.P., 1985. "Vibration Damping". Wiley, New York.

Pouresmaeeli, S., Ghavanloo, E., Fazelzadeh, S.A., 2015, "Vibration analysis of viscoelastic orthotropic nanoplates resting on viscoelastic medium", Composite Structures, v. 96, p. 405-410.

Reddy, J.N., 1985, "Review of the Literature on Finite-Element Modeling of Laminated Composite Plates," Journal of Shock and Vibration Digest, Vol. 17. No. 4.

Wright, R.I.;Kidner, M.R.F.,2004, "Vibration Absorbers: A Review of Applications in Interior Noise Control of Propeller Aircraft",Journal of Vibration and Control, SAGE Publications Inc., v. 10, p. 1221-1237.

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