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EXPERIMENTAL INVESTIGATION OF THE RAYLEIGH DAMPING APPROXIMATION IN A VIBRATING CATENARY PIPE

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Abstract. *This article deals with the Rayleigh Method to approximate viscous damping as a combination of inertial and stiffness effects in structures, common in finite element analysis (FEA) programs, such as those that numerically study the dynamics of risers, for example. In slender structures like these, often installed on a free-hanging catenary, it is known that vibration modes progressively succeed each other until immediate proximity between the natural frequencies at not-so-high order. Therefore, the objective of this work is to experimentally verify whether the Rayleigh method proportional to the linear combination of mass and stiffness in a pipeline launched in a free-hanging catenary can adequately represent the viscous damping in a wide range of natural frequencies typically excited by offshore currents. For this purpose, a 9.5 meter long pipe model launched at a height of 6.5 meters in the air and with an angle at the top end of approximately 7 degrees was characterized in terms of its natural frequencies and respective damping coefficients. For measurements, two complementary techniques were adopted: the test with a frequency exciter to obtain the natural frequencies and damping coefficients via the Frequency Response Function (FRF) and the simultaneous analysis with an analytical approach for the characterization of the respective natural modes of vibration. Subsequent comparison of the experimental results with the theory shows that the Rayleigh method is a reasonable approximation for the viscous damping of the studied catenary model, although exhibiting some limitations and presenting dependency on the approach of the method it is applied.*

Keywords: *Rayleigh Damping, Catenary Pipe, Modal Analysis, Frequency Exciter, Comparisons*

1. INTRODUCTION

According to Rabelo (2009), Brazil has one of the most relevant companies in the world considering the exploration and production of petroleum, which combines technology and the evaluation of hydrocarbons. In this scenario, as claimed by Pereira (2015), some structures are vital to enable the engineering contained in the offshore process, namely risers, which are responsible for the transportation of oil and gas between the seabed and the platform, usually floating. Therefore, when risers are launched, they assume a particular geometry due to their own weight and static characteristics, which then sets out a catenary of approximate configuration.

Due to this peculiar geometry and the inserted environment that includes hydrodynamic loads by the maritime current, the intern flow behavior, and the relative movement of the platform, some phenomena occur as a result of the high ratio between length and diameter, which defines their slenderness. By these considerations, the evaluation and modal characterization of these structures are indispensable for operational, environmental, and economic issues.

With all these arguments mentioned, it is possible to understand the excitation effects on the static equilibrium and also the effect on the dynamic amplitude response of the structure, which is invoked by the following vibrations. One of these events can be identified as a result of the Vortex-Induced Vibration (VIV) phenomenon.

Thus, it is undoubtedly necessary to identify the issue of the development and dimensioning of offshore production lines, for which interpretation of the details of the structure is essential. Structural damping, in this context, is a parameter of great influence for the consideration of certain phenomena such as the VIV, in addition to intervening in the robustness of the projects, which, according to Pesce (1997), helps in the characterization of the loads upon the occurrence of different operation cases.

In view of this, the following document explores the application of the Rayleigh viscous damping method to, analytically and experimentally, understand its performance and validity given by certain boundary conditions, geometry, and material. Therefore, the comparison of information can be interpreted in such a way that it allows for an understanding of the method with its limitations and considerations for improving predictions within the projects.

1.1 Rayleigh Damping Method

According to Clough and Penzien (2003), the easiest form to define a damping matrix is to proportionately combine the mass and stiffness matrices due to the orthogonality of the undamped modes. Therefore, assuming that viscous damping can be expressed by the combination of mass and stiffness, the Eq. (1) is valid, where a_0 and a_1 are proportionality constants that have units of $[s^{-1}]$ and [s], respectively. Also, c means the constant of viscous damping, m is the equivalent mass, and k is the corresponding stiffness.

$$c = a_0 \cdot m + a_1 \cdot k \tag{1}$$

As also reported in Clough and Penzien (2003), damping can be expressed as a function of natural frequencies according to coefficients in Eq. (2); where ξ_m and ξ_n are the damping coefficients of frequency in natural modes of vibration m and n.

$$\xi_m = \frac{1}{2}\left[\frac{a_0}{\omega_m} + a_1\omega_m\right], \qquad \xi_n = \frac{1}{2}\left[\frac{a_0}{\omega_n} + a_1\omega_n\right] \tag{2}$$

Then, isolating the factors that give the solution for these equations, we have the following solutions for a_0 and a_1.

$$a_0 = \frac{2\omega_m\omega_n}{\omega_n^2 + \omega_m^2}\left[\omega_n\xi_m - \omega_m\xi_n\right], \qquad a_1 = \frac{2\omega_m\omega_n}{\omega_n^2 + \omega_m^2}\left[-\frac{\xi_m}{\omega_n} + \frac{\xi_n}{\omega_m}\right] \tag{3}$$

Under these conditions, Figure 1 shows exactly how the method works, which also distinguishes the contributions in terms of mass and stiffness, as well as the combined curve.

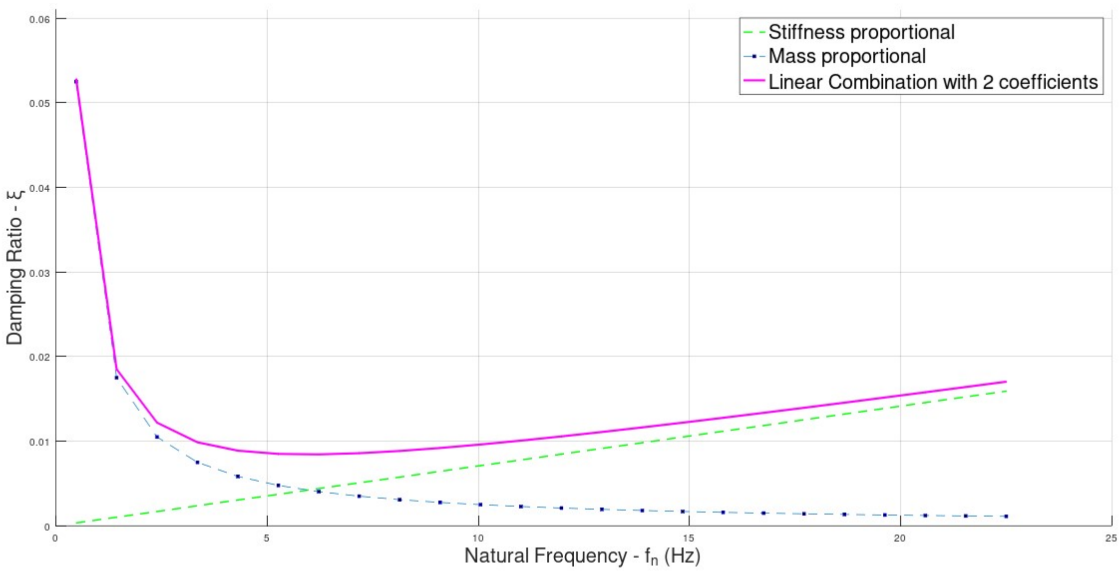

Figure 1. Rayleigh Method relation between natural frequency and damping ratio.

Although Eq. (2) is usually applied, it is known that information on the variation of damping and frequency in pipeline projects is rarely available. Then, Clough and Penzien (2003) assume that only one damping value is extended for the entire structure as $\xi_m = \xi_n = \xi$, so the coefficient equations can be adapted as follows.

$$a_0 = \frac{2\xi}{\omega_m + \omega_n} \cdot \omega_m \cdot \omega_n, \qquad a_1 = \frac{2\xi}{\omega_m + \omega_n} \tag{4}$$

In contrast to the case mentioned above, Clough and Penzien (2003) mentioned that if a lot of information about the structure is known, the method can be expressed by an extended approach that includes an infinite number of matrices and has a generalized damping value associated with any mode n, that case can be shown by Eq. (5), where the coefficients a_b and c_b need to be calibrated.

$$c = m \cdot \sum_b \left[a_b \cdot (m^{-1} \cdot k)^b\right] \equiv \sum_b c_b \tag{5}$$

According to this, the index b can be used in the range $-\infty < b < \infty$, depending on the number of terms available to characterize the structural behavior. Given that the damping coefficient can be defined using four terms, the solution equations are, therefore, rewritten as follows.

$$\xi_m = \frac{1}{2} \cdot \left[\frac{a_{-1}}{\omega_m^2} + \frac{a_0}{\omega_m} + a_1 \cdot \omega_m + a_2 \cdot \omega_m^3\right], \qquad \xi_n = \frac{1}{2} \cdot \left[\frac{a_{-1}}{\omega_n^2} + \frac{a_0}{\omega_n} + a_1 \cdot \omega_n + a_2 \cdot \omega_n^3\right] \tag{6}$$

$$\xi_o = \frac{1}{2} \cdot \left[\frac{a_{-1}}{\omega_o^2} + \frac{a_0}{\omega_o} + a_1 \cdot \omega_o + a_2 \cdot \omega_o^3\right], \qquad \xi_p = \frac{1}{2} \cdot \left[\frac{a_{-1}}{\omega_p^2} + \frac{a_0}{\omega_p} + a_1 \cdot \omega_p + a_2 \cdot \omega_p^3\right] \tag{7}$$

The following sections show the application of these theoretical subsidies in light of the modal characterization of a catenary-like vibrating pipe.

2. Experimental Setup

This section describes all the experimental issues, including the details of the pipe model, the excitation and acquisition apparatus, and the experimental and analytical methods adopted.

2.1 The Pipe Model

Before the construction of the experimental model, an analytical study was conducted on this particular geometry. To simulate a probable scenario, as stated in Pesce (1997), parametric equations were applied to describe the problem as follows.

$$x_c(s) = \frac{T_{0c}}{q} \cdot \sin^{-1}\left(\frac{qs_c}{T_0}\right), \qquad y_c(s) = \frac{T_{0c}}{q} \cdot \left(\left(1 + \left(\frac{qs_c}{T_{0c}}\right)^2\right)^{1/2} - 1\right) \tag{8}$$

Furthermore, Pesce (1997) also expresses the hanging length as in Eq. (9).

$$L_c = T_{0c} \cdot tan(\theta_c) \tag{9}$$

Taking into account these equations based on Pesce (1997), it was possible to elaborate Table 1, which includes geometric results and also the main structural data of the experimental pipe model.

Table 1. Main parameters of the experimental pipe model.

Parameter	**Value**	**Unit**
Mass	2.020	kg
Suspended length	9.50	m
Total length	11.80	m
External diameter	0.025	m
Intern diameter	0.021	m
Transversal section area	$3.14 \cdot 10^{-4}$	m^2
Inertial moment	$4.13 \cdot 10^{-9}$	m^4
Modulus of elasticity	$2.41 \cdot 10^3$	MPa

These aspects were pondered and led to the development of two graphics that describe the catenary approximation, as shown in Figure 2.

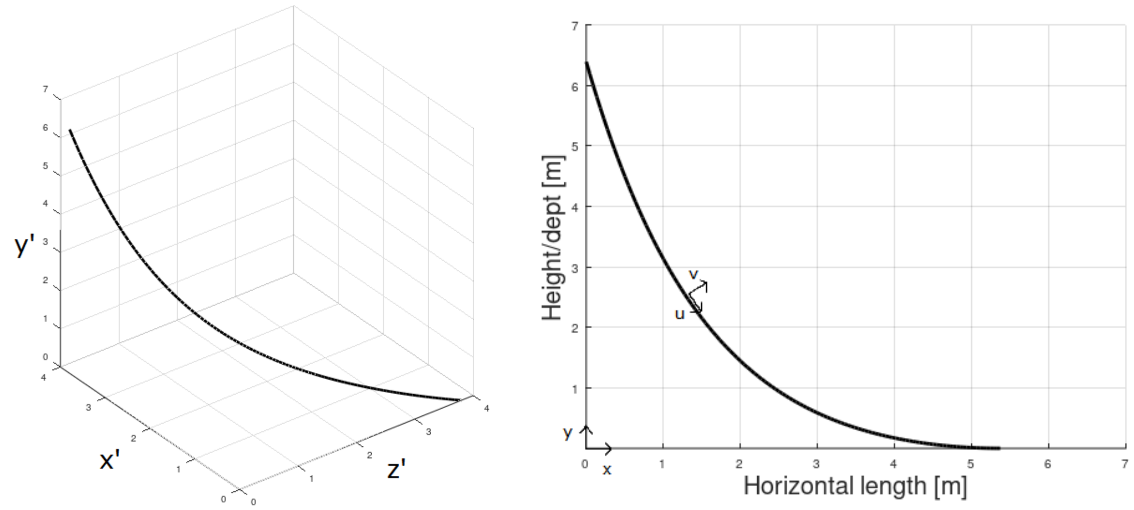

Figure 2. Analytical approximation of the catenary geometry, where: x', y', and z' are the main directions of the coordinate system.

It is important to note that the experimental pipe model used to simulate the catenary approximation was made up of polyvinyl chloride (PVC). This structure was attached to a wooden support on the wall and also restrained by a clamp on the ground. These apparatuses were chosen to limit an eventual displacement and also set the boundary conditions.

Therefore, after all the dimensions and the analytical view defined to actually build the pipe model, it was feasible to elaborate it as shown in Figure 3.

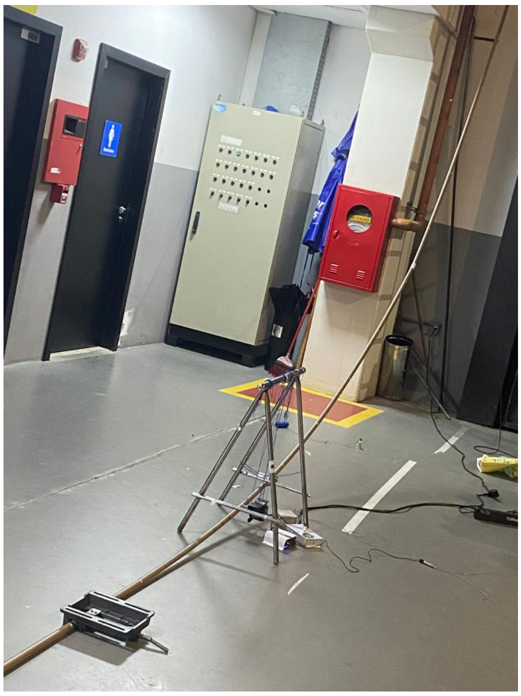

Figure 3. The experimental pipe model launched in a catenary-like geometry.

2.2 The Excitation and Acquisition Equipment

The Modal Experiment, which provides the modal analysis, required the equipment named LMS Pimento, a frequency exciter (shaker) with a force transducer and a uniaxial accelerometer. LMS Pimento software has an acquisition system, with 24 canals, which can transform the information given by an electric signal into treated information such as structural and modal analysis. More details in LMS Pimento (2003).

The shaker then uses the vibration to induce the proper behavior that is expected by the test. For this project, it was used to excite the experimental pipe model in the previously determined frequency range.

Using this excitation and acquisition equipment, shown in Figure 4, it was feasible to identify the structural behavior of vibration, particularly regarding the determination of natural frequencies, damping coefficients, and modal shapes.

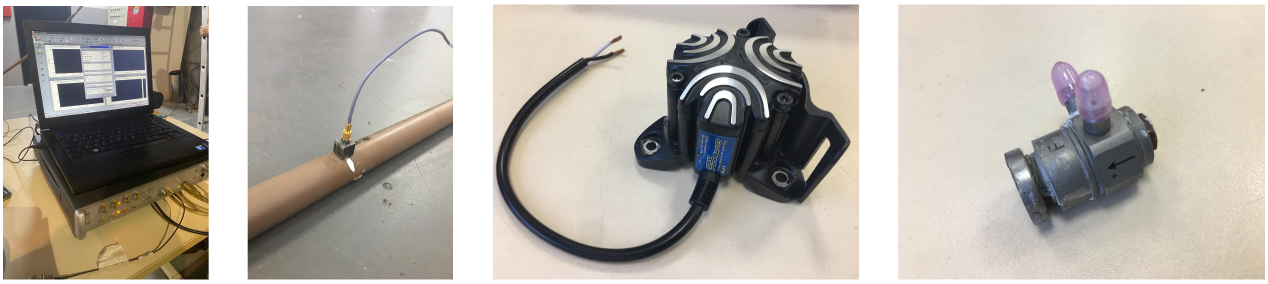

Figure 4. From left to right: the LMS Pimento, the uniaxial accelerometer, the frequency exciter, and the force transducer.

2.3 Modal Analysis

The modal analysis was performed experimentally using the output data offered by the LMS Pimento acquisition software, which generates directly the values of the natural frequencies, damping coefficients, and also the modal shapes of the vibration using geometric information. Furthermore, analyses based on the Wentzel–Kramers–Brillouin (WKB) method were also conducted to help the modal identification process through the experiments.

2.3.1 Details on the Analytical Approach for Modal Identification

The WKB method is used to analytically define the natural frequencies and modes of a catenary-like riser when the hypothesis of an inextensible curved cable can be adopted for the development of its dynamic equations. In this case, structural stress is known to be mainly influenced by geometric stiffness, unless a significantly large marine current acts on the structure.

Thus, according to Pesce *et al.* (1999), it is possible to calculate the dimensionless stress of the tube using the Eq. (10), where $\alpha = s/L$ represents a term as a function of the Lagrangian coordinate s, which characterizes the curvilinear coordinate of the system, and L is the length of the suspended cable.

$$F_c(\alpha) = \sqrt{1 + tan^2(\theta_c(\alpha))} \tag{10}$$

Moreover, in the particular and simplified case of a pure catenary, that is, without the presence of current, the structural stress is given by the Eq. (11), where $\zeta = tan(\theta(\alpha))$.

$$F_c(\zeta) = \sqrt{1 + \zeta^2}, \tag{11}$$

In this sense, to effectively calculate the natural frequency, Ω_n, Eqs. (12) and (13) are used, where n is the modal order to be defined, m is the structural mass, m_a the additional mass influenced by the presence of fluid displaced, μ is a parameter that corresponds to the $\tan(\theta_L)$ and T_0 the static traction of the suspended structure.

$$\Lambda_n \equiv n \cdot \pi \cdot \left(\int_0^{\mu} \frac{d\zeta}{\sqrt{F(\zeta)}} \right)^{-1} \tag{12}$$

$$\Omega_n = \Lambda_n \cdot tan(\theta_L) \cdot \sqrt{\frac{T_0}{(m + m_a) \cdot L}} \tag{13}$$

2.3.2 Details on the Experimental Modal Analysis

To actually start the experiment, all the equipment mentioned above had to be placed. Therefore, the experimental pipe model was subdivided into 21 equidistant points to establish the position of the accelerometer for each acquisition.

Then, modal identification was performed placing the shaker at a specific point on the model and the accelerometer at different points along the length. The shaker was allowed to excite the structure, and thus the LMS Pimento could receive the information as a function of time and, by the Fast Fourier Transform (FFT), transform it into frequency functions.

It is important to note that this procedure was carried out both for in-plane and out-of-plane vibrations of the catenary-like pipe. Furthermore, all sampling precautions have been taken to avoid the problem of aliasing and promote correct intended modal identification. It is also mentioned that, due to the limitation of the shaker to excite at frequencies below $5Hz$, it was necessary to perform standard decay tests to identify the first three natural frequencies for each direction, whether in the plane of the catenary or outside it. Furthermore, for this type of measurement, three reference points were established for measurement based on the accelerometer's response amplitude, and three repetitions were defined for each of these points, which will be analyzed further based on an uncertain method.

Then, combining the frequency values identified with standard procedures of modal analysis via LMS Pimento and via regular decay tests with those predicted by the WKB method, it was possible to complete the modal identification, including the characterization of the vibration modes as illustrated in Figure 5.

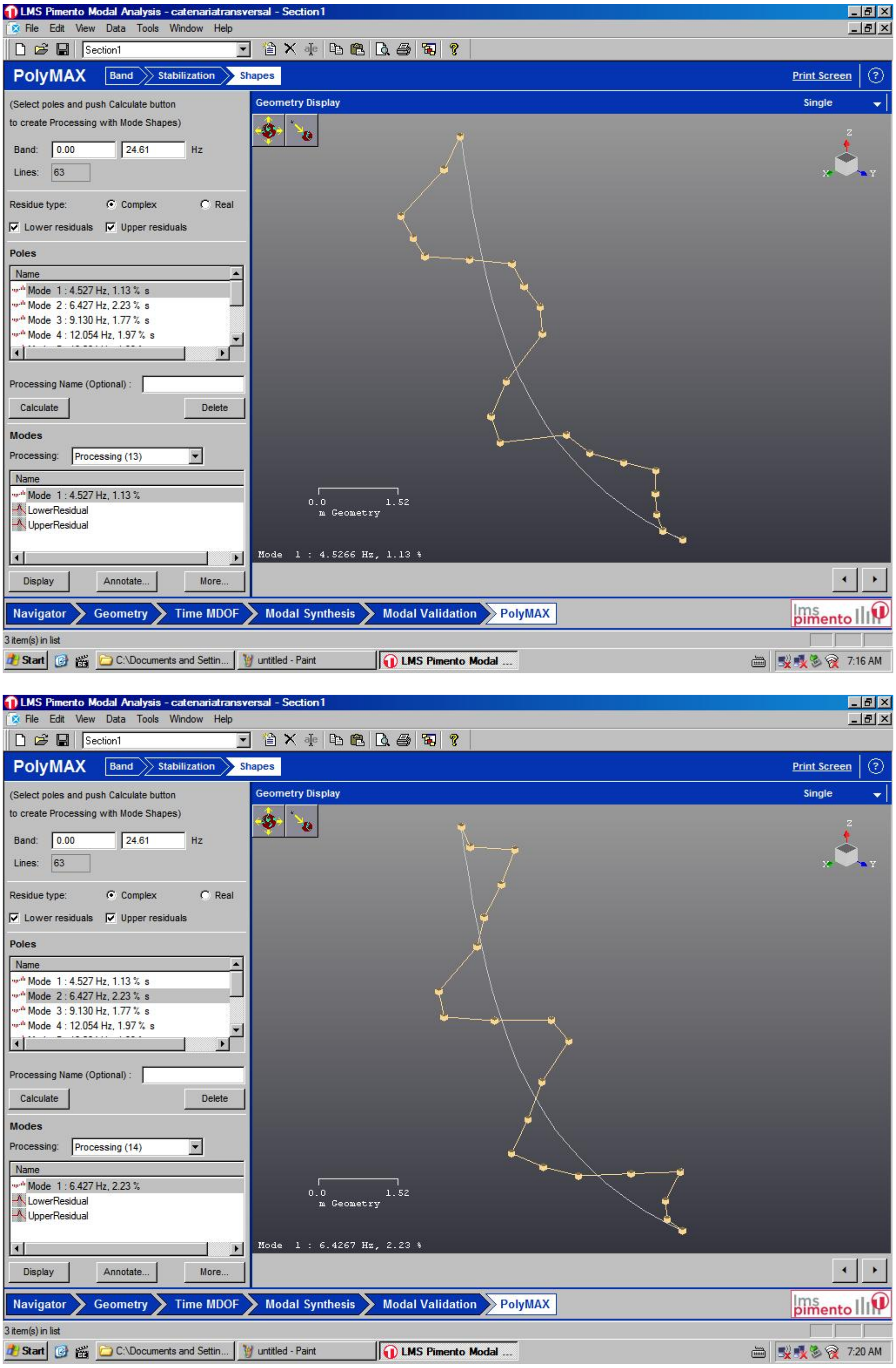

Figure 5. Examples of modal identification: fourth and fifth modes of catenary-like pipe model vibration out-of-plane.

Once the frequencies and natural modes of vibration have been identified, the respective modal damping coefficients were determined. After all the data was analyzed, the natural frequencies and respective damping coefficients resulting from the FRFs were determined and summarized in Table 2.

Table 2. Natural frequencies and damping coefficients for the tenth in-plane and out-of-plane modes of vibration.

Modal Order n	**Out-of-plane**		**In-plane**	
	Natural Frequency, f_n [Hz]	Damping Coefficient, ξ_n [%]	Natural Frequency, f_n [Hz]	Damping Coefficient, ξ_n [%]
1	0.533	4.79	1.233	4.38
2	1.433	3.33	2.570	2.62
3	2.700	2.76	3.800	1.92
4	4.500	1.13	4.470	1.19
5	6.427	2.23	7.121	1.44
6	9.130	1.77	9.935	1.83
7	12.054	1.97	13.270	2.56
8	18.284	1.39	16.829	1.76
9	20.922	0.91	19.741	1.19
10	22.028	0.69	22.459	1.70

2.3.3 Uncertainties of Experimental Measurements

Considering all the experimental results, an uncertainty analysis was carried out to assess the trust rating in the measured values, so, concepts of average and standard deviation from a triangular distribution were used. According to this approach, Table 3 presents the damping results now considering the respective average, standard deviation, and average of standard deviation values. It is worth mentioning that this analysis was carried out only to identify natural frequencies and damping of the first three natural modes, in-plane and out-of-plane of the catenary-like model, due to the need for experimental capture and subsequent data processing in an analytical way, unlike what occurred with the highest order modes.

Knowing that the coefficient of variation is defined by the ratio between standard deviation and mean of the data, from Table 3 it is possible to infer that the deviation linked to the test was negligible since presents coefficients of variation less than 1.0% for all the measurements collected. This information represents the validation of the homogeneity of the data and, therefore, expresses its reliability in relation to the experiment made. Furthermore, it is evident that the standard deviation is always two orders of magnitude lower than the averages, making the values obtained considerably reasonable for method validation. Thus, for higher-order modes of higher order than the fourth natural mode of vibration, it is possible to infer that the LMS Pimento software is able to capture information more efficiently and reliably, so that is not necessary uncertainty analysis for those modes.

Table 3. Natural frequencies and damping coefficients for the tenth in-plane and out-of-plane vibration modes, where MP means the "measurement point" along the catenary-like model.

Modal Order n	**Out-of-plane Damping %**				**In-plane Damping %**			
	MP	Average, ξ_a	Standard Deviation, $\xi_d \cdot 10^{-2}$	Average of Standard Deviation, $\xi_{da} \cdot 10^{-2}$	MP	Average, ξ_a	Standard Deviation, $\xi_d \cdot 10^{-2}$	Average of Standard Deviation, $\xi_{da} \cdot 10^{-2}$
1	8	3.969	1.1307	0.6528	8	4.102	1.1523	0.6653
	13	4.647	4.6881	0.0270	13	4.654	4.7101	2.7194
	20	6.679	3.3946	0.0119	20	7.014	1.1958	0.6904
2	8	3.200	0.1350	0.0007	8	2.667	0.1377	0.0765
	13	3.178	0.0090	0.0005	13	2.513	0.1268	0.0732
	20	3.602	0.9002	0.0005	20	2.678	0.0123	0.0071
3	8	1.656	0.0455	0.0026	8	2.063	0.2463	0.1421
	13	4.239	0.0003	0.1822	13	1.721	0.0039	0.0023
	20	2.372	1.8908	0.0109	20	1.983	0.1423	0.0823

3. COMPARISONS AND DISCUSSION

Based on the completed identification, comparisons were then made between the experimental results of Table 2 with the possible formulations of the Rayleigh damping method.

Therefore, we first consider the simple method using the range from the first to the twentieth natural mode and calibrate the Eq. (4) with a damping coefficient related only to the fundamental frequency, that is, 4.79%.

Figure 6 indicates that up to the sixth mode (and also for the ninth mode), the damping coefficients will be underestimated, while this value for the highest frequencies will be overestimated, leading to a non-conservative consideration of phenomena such as Vortex-Induced Vibrations (VIV). This can be a problem because, according to the literature, the higher the damping coefficients, the smaller the amplitudes of the VIV responses and therefore less damage to fatigue.

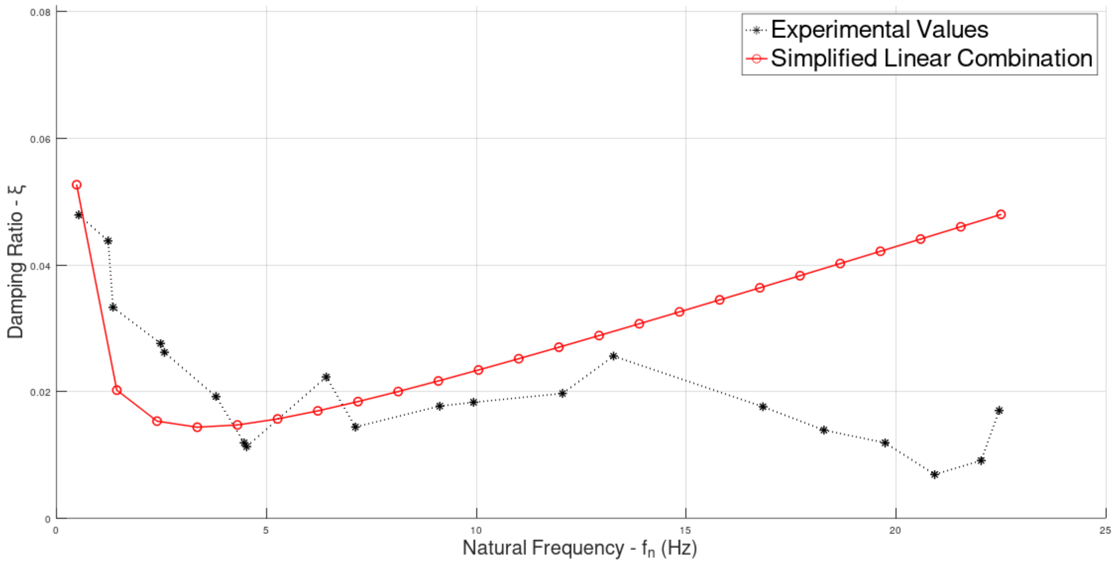

Figure 6. Rayleigh Simplified Method calibrated with only one damping information compared to the experimental relation between natural frequencies and damping coefficients.

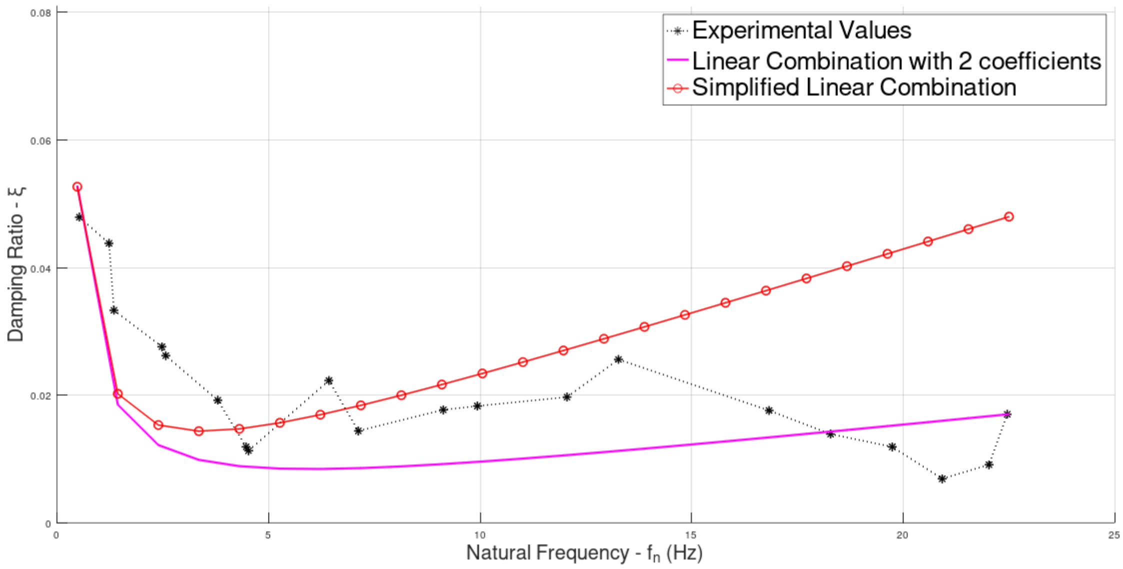

Figure 7. Comparison between the Method with two pair of information and the Simplified one.

Then, if the reference value of the damping coefficient for calibration purposes increases, the theoretical curve will shift and become more inclined. Therefore, from a design perspective, this method will become very delicate to use from a design point of view because, depending on the estimate of values, a project may be very conservative when the curve is below the real values, so it will consider higher VIV amplitudes than the need for more robust projects with more money involved.

On the other hand, if a second approach with two pairs of information in the same frequency range is adopted according to Eq. (3), the graph is as shown in Figure 7. It is now perceptible that the magenta line not only expresses a conservative point of view in terms of possible higher VIV amplitudes, but also represents a better approximation for the experimental damping coefficient as a whole.

In addition, if the extended method is applied, using the first, seventh, fifteenth, and twentieth pair of information, two new graphs can be obtained in Figure 8. By this graph, it is observed that depending on the pair of frequency and damping values that were set, the curves present different shapes, which is expected considering Eqs. (5), (6) and (7).

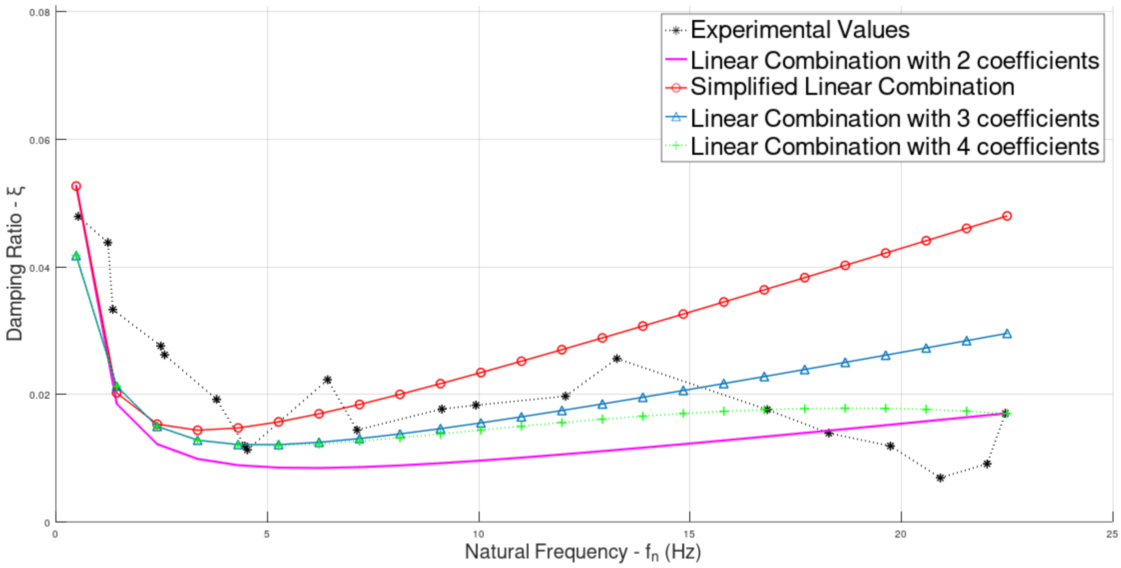

Figure 8. Comparison between all aspects of the Rayleigh Damping Method.

According to Figure 8, the approximation of the experimental values was better with four pairs of information than with others. It means that the more terms applied to the extended method, the better and more reliable the theoretical damping coefficients. In quantitative terms, the results of the Rayleigh Damping Method with two, three, and four terms can be compared by means of the mean deviation related to the experimental values.

Table 4 shows that the Rayleigh Damping Method with four terms calibrated by four pairs of information leads to a better theoretical approximation for the damping coefficient in a whole sense, at least considering the dynamic behavior of the pipe model tested here.

Table 4. Quantified comparison between different applications of the Rayleigh Damping Method.

Type of Method	**Mean deviation of points [%]**
Simplified	105.08
Classic	43.81
With 3 terms	54.94
With 4 terms	35.31

4. CONCLUSION

This work involves experimental and analytical research that interprets Rayleigh's viscous damping method, considering different analytical expansions to describe the structural modal damping of a pipe model in a catenary-like configuration. Thus, through the procedures applied and the information collected, the results pointed to the need for care in relation to the way this method is applied. In summary, the final evaluation indicates that the four-parameter method, which parameterizes the curve from four pairs of information (4 natural frequencies f_n and respective damping coefficients ξ_n), is more consistent than the others, having an overall mean deviation of approximately 35% from the experimental data. On the other hand, the least accurate method is one that requires only one damping information, having an overall mean deviation of 105%. Therefore, at least on the basis of the results with the pipe model studied here, it became clear that using a greater amount of information to calibrate the Rayleigh method that describes the viscous damping is more adequate. Based on this conclusion, for real projects based on simulations and finite element analysis, great care must be taken with the version of the Rayleigh method adopted, since it can negatively influence, for example, the prediction of the VIV amplitudes and consequently the study of fatigue due to this phenomenon.

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