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COB-2023-1252 EQUIVALENCE AMONG BRINKMAN, SINGLE AND DOUBLE CONTINUUM MODELS IN THE DESCRIPTION OF SINGLE PHASE FLOW IN 2D VUGGY POROUS MEDIA

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Abstract. *This article compares two porous media fluid flow description methods applied to heterogeneous reservoirs composed of architectural elements of high permeability and porosity, called fractures and karsts. Both are characteristic of Pre-Salt carbonate rocks, the main reservoirs in Brazil. These elements' existence brings complexity into characterization and, hence, increases the uncertainty in field production prediction curves. The Reservoir Engineering numerical simulators usually used to generate these curves model the flow behavior inside the cavities in an approximate manner. Moreover, due to the kilometric scale of oil reservoirs, these models use cells that comprehend the three media (porous matrix, karst and fractures). This study evaluates the equivalence of black oil and Brinkman models, which is still not widely used. The Brinkman model describes the flow through the porous matrix and through the cavities and fractures with a single equation without the need for boundary conditions along the interface between the two domains. With this objective, a two-dimensional single-phase Brinkman simulator was designed based on the finite elements method. The pressure propagation and flow velocity values obtained while simulating fluid flow inside selected layers from Lajedo Arapuá's carbonate formation were compared to those obtained with a black oil simulator. Studies were performed using dual-continuum models seeking, through variation of the fracture effective permeability and the shape factor, convergence to the single-continuum reference model that was selected after Brinkman's results comparative analysis. The results show a slight variation between the two methods when the karst system is composed of sparse and disconnected vugs. In contrast, conduit-shaped karsts with complex configurations increase the variation in pressure wave propagation and flow velocity values between models, especially in scenarios where matrix permeability values were closer to karst permeability values in the black oil model. The dual-continuum analysis showed that it is possible to obtain similar results to those obtained by a heterogeneous black oil model with karst system characterization through homogeneous and even ten times coarser models. It was also possible to conclude that effective fracture permeability was sufficient as a fitting parameter to achieve equivalent results to those from the single-continuum model, using threshold criteria. The abstract should describe the objectives, context, and significance of the research, methods, results, and main conclusions of the paper in about 200 words. It should not include formulas or references to a bibliography. It must be written in only one paragraph.*

Keywords: *Brinkman Equation, Finite Element Method, Reservoir Simulation, Dual-Continuum Model, Carbonate Reservoirs.*

1. INTRODUCTION

At the end of 2021, Pre-Salt carbonate reservoirs represented approximately 74% of Brazil's total equivalent oil production. The success in Pre-Salt exploration led Brazil to 8th among world's top-producing countries. The high productivity observed is a consequence of large original volumes and high porosity, permeability, and thickness values. Besides these characteristics, carbonate rocks usually present heterogeneities like fractures and karsts that contribute to high production rates but, at the same time, worsen the oil sweep effect and recovery factor. A specific type of karstic feature called vug corresponds to empty spaces inside the porous matrix, where free flow of fluids occurs. These elements add complexity to the description of flow inside the reservoir and thus bring more uncertainty to the field production prediction curves, which are important to generate the field development plan.

Reservoir numerical simulators commonly used by the industry in actual fields have considerable difficulty in describing flow inside reservoirs composed by porous matrix and free-flow regions, which is the case for many carbonate reservoirs. In these cases, using the Darcy equation, an equivalent permeability value should be used to represent both the flow through the porous matrix and the vugs. Other approaches, such as dual-continuum models, were developed to describe the porous matrix and the free-flow region separately, and then couple the solutions through transfer terms.

The parameters of single and dual-continuum models should be defined to optimize the accuracy of flow description through a heterogeneous porous medium. Ideally, the description of the flow through vugs, fractures and porous matrix should rely on more accurate models, like the Brinkman model. This study aims to test the equivalence of Brinkman, single and dual-continuum models in describing heterogeneous porous media flow. To do so, a steady and unsteady Brinkman model was created using the finite element method. The predictions obtained using Brinkman model were compared to those obtained with a single-continuum heterogeneous black oil model. The black oil models that showed a good approximation to the Brinkman model had their results selected as objective to a dual-continuum matching analysis where fracture permeability and fracture spacing were used as fitting parameters, concluding whether dual-continuum models could represent the selected flow conditions.

2. SINGLE AND DUAL-CONTINUUM MODELS

Numerical reservoir simulation is a method that calculates the time evolution of pressure and fluid saturations fields. The pressure field is obtained by solving the pressure diffusivity Eq. (1). In the equation, k stands for permeability, p for pressure, c_t for total compressibility ($c_t = c_f + c_r$, where c_f and c_r are the fluid and the rock compressibility, respectively), μ for fluid viscosity, ϕ for porosity and t for time.

$$\nabla \cdot (k\nabla p) = c_t \mu \phi \frac{\partial p}{\partial t} \tag{1}$$

The dual-continuum models were developed to describe flow inside a reservoir containing heterogeneities such as fractures and karsts (Barenblatt et al., 1960). This type of model solves the porous media flow equations for a matrix grid (index m) and a fractured grid (index f), presenting specific properties with the addition of a fracture aperture parameter in the fracture grid case. In double porosity and double permeability models, the flows through the two domains are coupled by a transfer term added to both diffusivity equations.

$$\begin{cases} \nabla \cdot (k_m \nabla p_m) - \alpha\left(p_m - p_f\right) = c_{tm}\mu\phi_m \frac{\partial p_m}{\partial t} \\ \nabla \cdot \left(k_f \nabla p_f\right) + \alpha\left(p_m - p_f\right) = c_{tf}\mu\phi_f \frac{\partial p_f}{\partial t} \end{cases} \tag{2}$$

The transfer factor α is equal to the matrix permeability times a shape factor, which many authors have studied. The chosen equation for the shape factor in this study is the one introduced by Kazemi et al. (1976) and presented below, where L is the fracture network aperture in each direction.

$$\sigma = 4\left[\frac{1}{L_x^2 + L_y^2 + L_z^2}\right] \tag{3}$$

3. BRINKMAN MODEL

Henry Brinkman (1949a, 1949b) proposed an equation to represent the flow of a fluid on a dense swarm of particles that successfully represents the flow inside a porous matrix that contains vugs and fractures. The proposed equation recovers Darcy's equation in the porous matrix, and the Stokes equation in the free-flow regions. Besides delivering more accurate results compared to equivalent properties models (Oda, 1986; Sitharam et al., 2001) there is no need for boundary conditions between both regions as in Darcy-Stokes model (Yao et al., 2010; Arbogast and Lehr, 2006; Peng et al., 2007).

Brinkman equation (4) disregards gravitational effects and considers Newtonian incompressible fluid in a creeping flow regime. Since this study considers single-phase flow in very low total compressibility values, the equation applies. The term μ' is Brinkman's effective viscosity, which equals the fluid viscosity μ in high porosity regions (Auriault, 2009; Gulbransen et al., 2009).

$$\nabla p = -\frac{\mu}{k}\boldsymbol{u} + \mu' \nabla^2 \boldsymbol{u} \tag{4}$$

Brinkman's equation and the continuity equation below form the differential equations system of the Brinkman model.

$$\frac{\partial}{\partial t}(\rho\phi) + \nabla \cdot (\rho \boldsymbol{u}) = 0 \tag{5}$$

The finite element method was used to solve the SDE (System of Differential Equations). Triangular elements from the Lagrangean family were used with three and six degrees of freedom for pressure and velocity fields, respectively. The solution method was implemented in Python, using the DOLFIN library from the FEniCS project (Langtangen and Logg, 2016). The variational formulation of both equations, where q_i and $\boldsymbol{v_i}$ are the weight functions for pressure and velocity, respectively, are shown below. In both equations, Γ represents the boundaries of the porous media and $\boldsymbol{n}$ is the normal vector to this boundary.

$$R_{mi} = \frac{\phi \rho c_t}{\Delta t} \int_\Omega (p^n - p^{n-1}) q_i d\Omega + \int_\Gamma (\boldsymbol{n} \cdot \boldsymbol{u}) q_i d\Gamma = 0 \tag{6}$$

$$R_{ci} = \frac{\mu}{\mathrm{k}} \int_\Omega (\boldsymbol{u} \cdot \boldsymbol{v_i}) d\Omega + \int_\Gamma \boldsymbol{n} \cdot (p \cdot v_i) d\Gamma - \int_\Omega p(\nabla \cdot \boldsymbol{v_i}) d\Omega + \int_\Omega \mu[\nabla \boldsymbol{u} : (\nabla \boldsymbol{v_i})^T] d\Omega = 0 \tag{7}$$

The model and implementation were validated by comparing to the analytical solution of an unsteady one-direction flow in a homogeneous porous media (Balhoff, 2016) and the velocity profile near the boundary between empty channel (Navier-Stokes equation) and porous media (Darcy equation) to the Beavers and Joseph (1967) experiment and achieving the same velocity values as Yao et al. (2010).

4. SIMULATION SCENARIOS

A prebuilt dynamic model was selected to generate triangular finite elements and regular quadrilateral cartesian grids at which the Brinkman and the black oil model were solved to describe a 2D single phase flow. The Lajedo Arapuá is a Jandaíra's carbonate formation outcrop in the northeast of Brazil and has been used as subject to several studies, including developing a structural model containing its karst and fracture systems (Quadros, 2018) and a black oil simulation model (Machado, 2020). Layers of the dynamic model were selected to represent different configurations of karst systems: segregated vugs, continuous karsts, and a mix between the first two with higher complexity and closed regions. These layers were then used in a 2D reservoir simulation, disregarding the layer thickness (Figure 1). The geometry scale of the problem was reduced by a factor of one hundred to enable solution convergence and short simulation times while still permitting the desired comparison between the models. The size of the finite elements was defined by controlling edge size at external and internal boundaries and maximum edge size inside the whole grid using an algorithm created by Dali (2019).

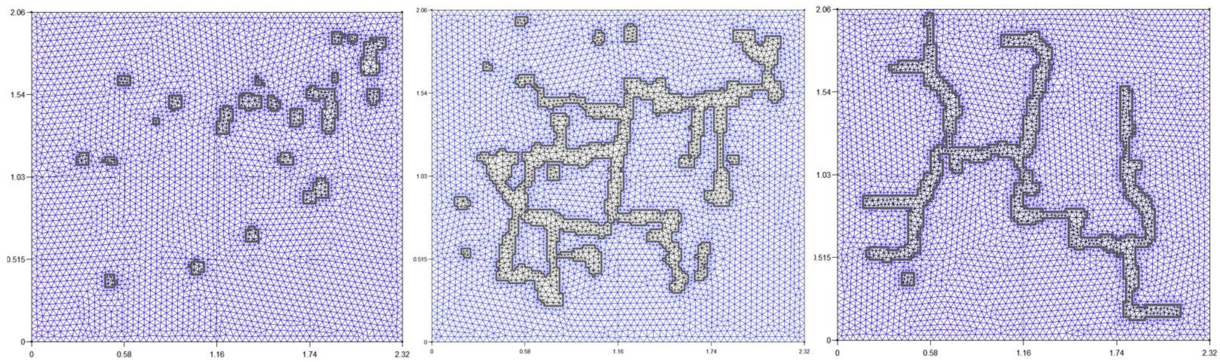

Figure 1. Three finite element grids used in the study.

The two-dimensional flow was simulated by imposing a 300 kgf/cm² (29.4 MPa) pressure differential between the left and right boundaries of each domain. There is no flow at the top and bottom boundaries. The initial conditions are zero velocity and uniform pressure equal to 488 kgf/cm² (47.9 MPa) through the hole domain.

The original single-continuum model's porosity, permeability, rock compressibility, and fluid parameters were respected in the Brinkman model (Table 1). Then, different values of matrix permeability were used in both models to evaluate a range of vug (karst) and matrix permeability ratios $\left(\frac{k_v}{k_m}\right)$, from 50 to 50000. The vug permeability was kept at the value of 50 Darcy.

Table 1. Rock, porous matrix, karst system, and fluid property original values.

Property	**Porous Matrix**	**Karst**
Porosity, %	40	100
Permeability, mD	10	50000
Rock compressibility at initial pressure, 1/(kgf/cm²)	3e-5	3e-4

Fluid compressibility at initial pressure, 1/(kgf/cm²)	3e-4	3e-4
Fluid viscosity, mPa.s	0.6	0.6

5. SINGLE-CONTINUUM DISCUSSION AND RESULTS

The flow achieves a steady state after five minutes in the simulations with original 10 mD porous matrix permeability. In the first hundred seconds, the results of the Brinkman and the black oil simulation show a range of 9 to 35% of variation between them, reaching a range of 6 to 12% at a steady state after 250 seconds in the simulation.

The visual comparison between the resulting velocity magnitude maps of the Brinkman and black oil simulations with original permeability values showed excellent correspondence at steady state (Figure 2). As expected, the vug regions represented preferential paths with high-velocity magnitudes.

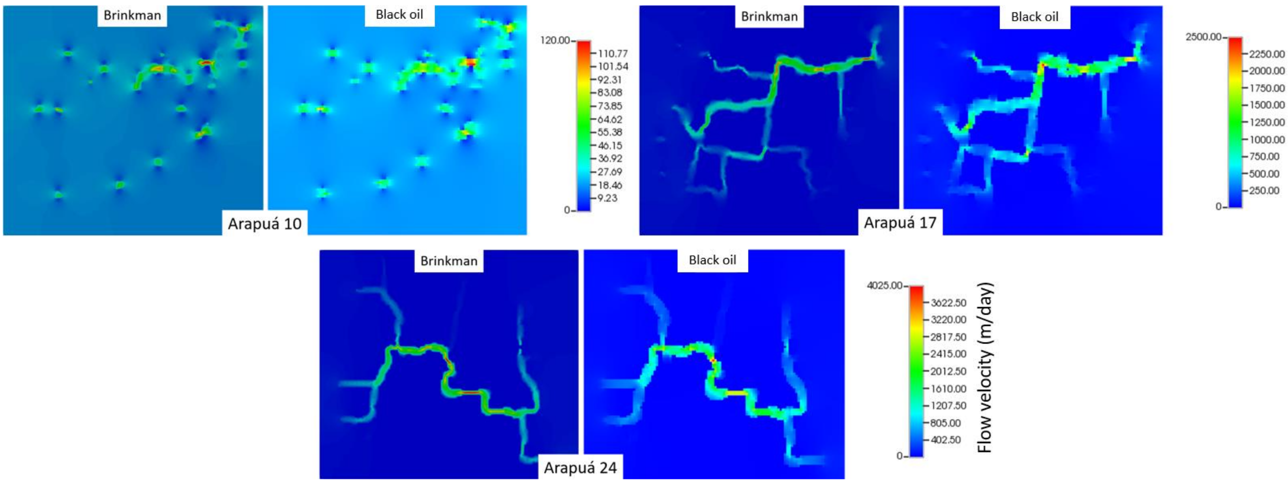

Figure 2. Velocity maps comparison for the three generated grids, $k_v = 50000$ mD, $k_m = 10$ mD.

Figure 3 shows the velocity magnitude maps of the Brinkman simulations for the different domains and different matrix permeability values, from 1 to 1000 mD. In the case of segregated vugs, the velocity scale growth is proportional to the enhancement of matrix permeability, showing that in this scenario, the fluid flow is dominate by the matrix. This is observed since the maps maintain almost exact colors in all the maps and a scale that is shifted by one order of magnitude between each case. In the other two scenarios, high velocities are developed inside the karst system. In the porous matrix, high velocities are found at the entrance and exit of the grid. Regions of the porous matrix surrounded by continuous karsts show very low-velocity magnitudes.

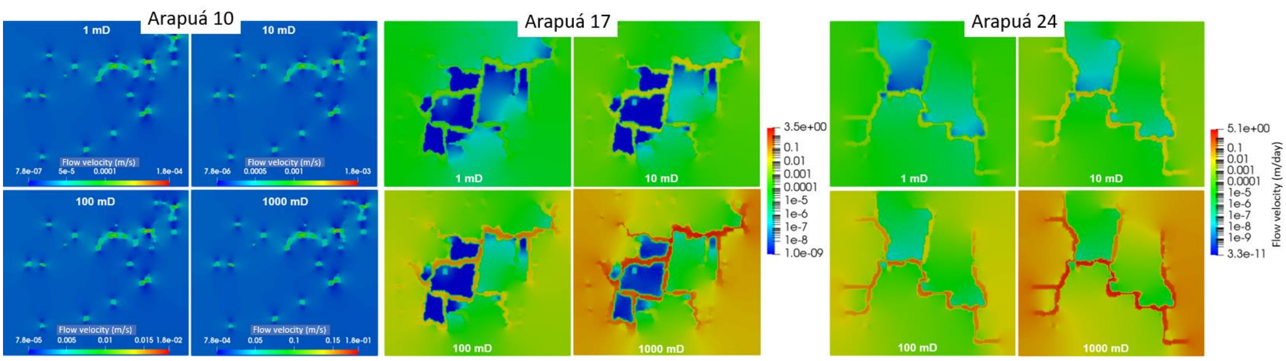

Figure 3. Velocity maps for each matrix permeability value.

The pressure maps also reveal similar behavior between the two models (Figure 4). The pressure distribution in each matrix permeability scenario in the Brinkman simulation (Figure 5) shows that a homogenization of the pressure waves occurs, with the progressive increase of matrix influence in the fluid flow following the increase of matrix permeability values. This does not happen in the black oil simulation where there is more resistance to the flow at the karst system due to the use of the Darcy equation in the hole domain.

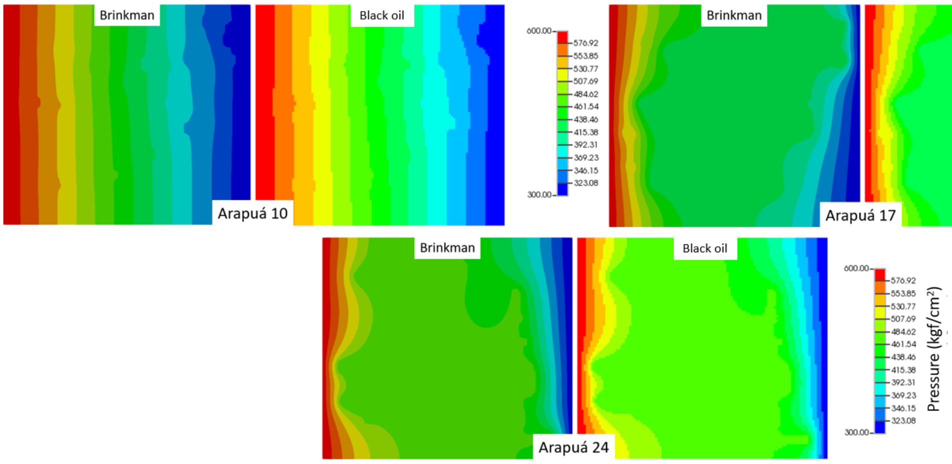

Figure 4. Pressure maps comparison between black oil and Brinkman simulations.

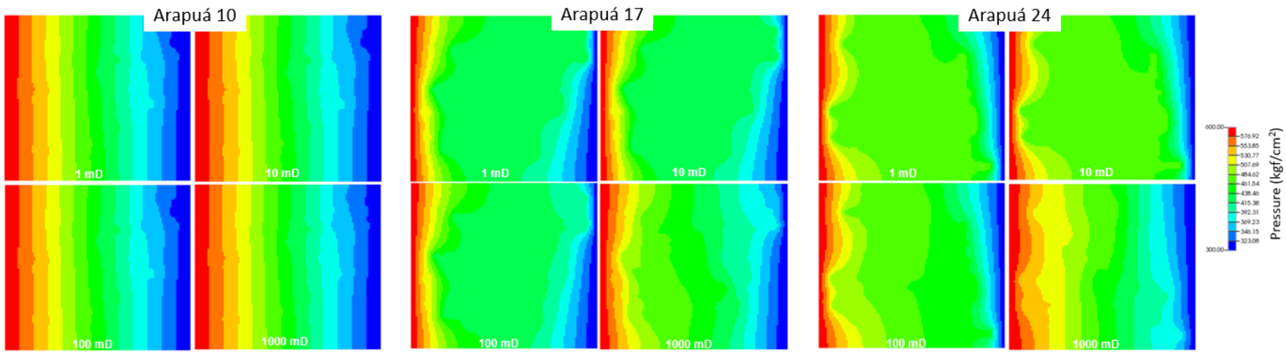

Figure 5. Pressure maps for each matrix permeability value in Brinkman simulations.

A first quantitative analysis was performed by comparing the velocity profiles at the domain exit at steady state predicted by both models. In the case of segregated vugs, the results in all matrix permeability scenarios were similar (Figure 6). In the other two cases, as the relation $\left(\frac{k_v}{k_m}\right)$ got smaller, the difference between the velocity profiles increased (Figures 7 and 8).

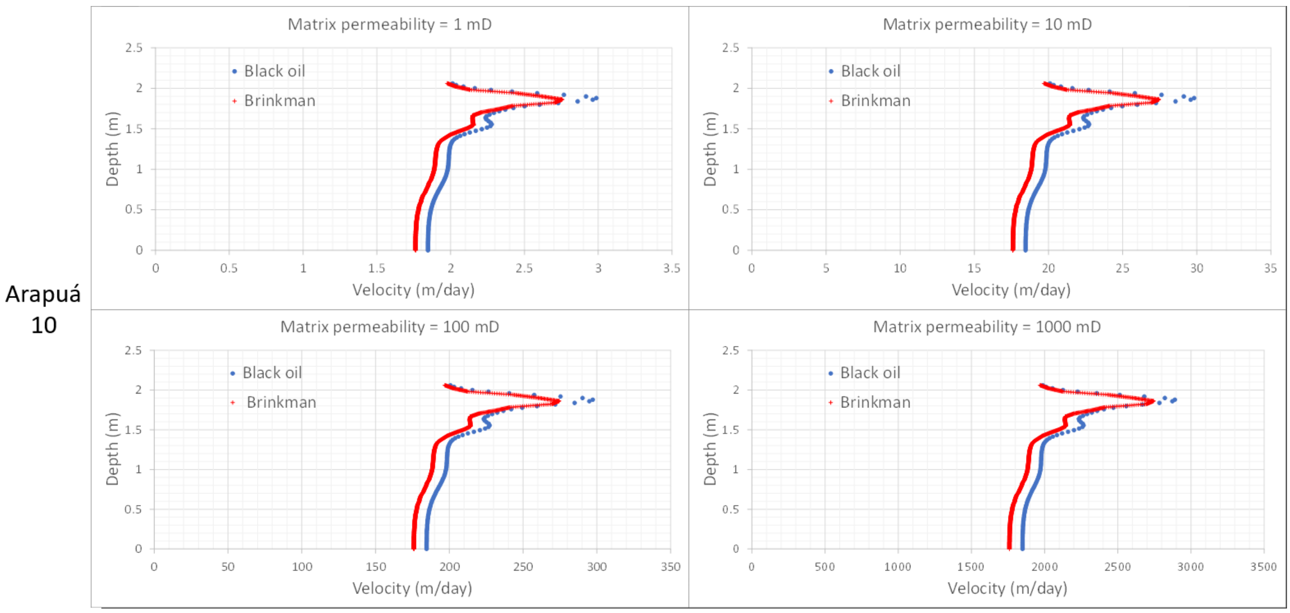

Figure 6. Velocity profile comparison at four different matrix permeability values using the 10th layer of the Lajedo Arapuá model.

Arapuá 17

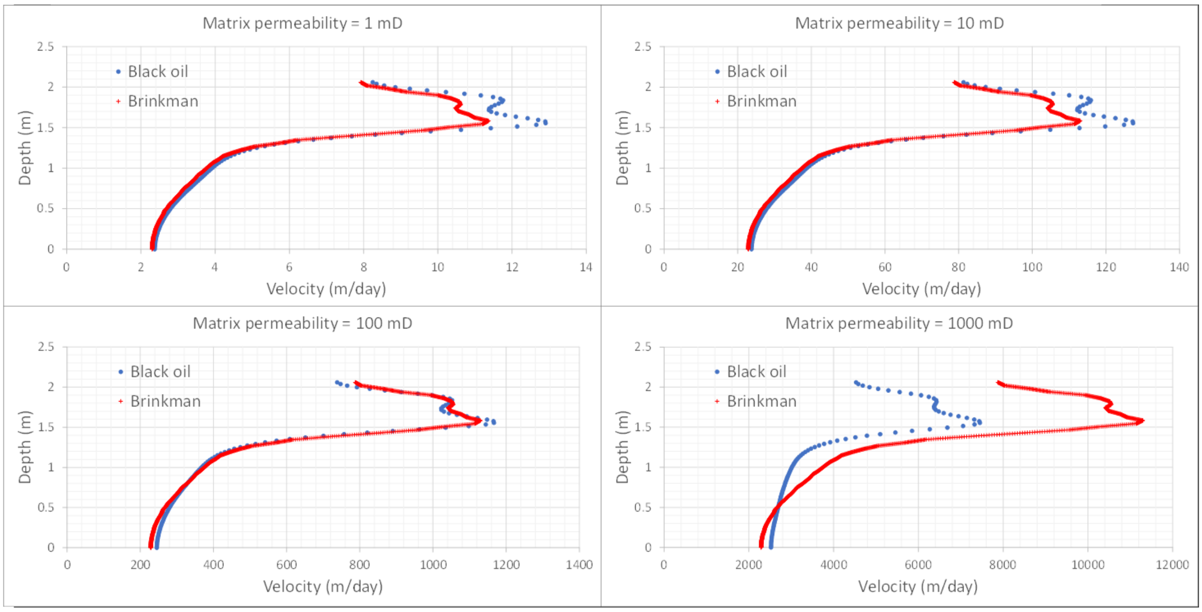

Figure 7. Velocity profile comparison at four different matrix permeability values using the 17th layer of the Lajedo Arapuá model.

Arapuá 24

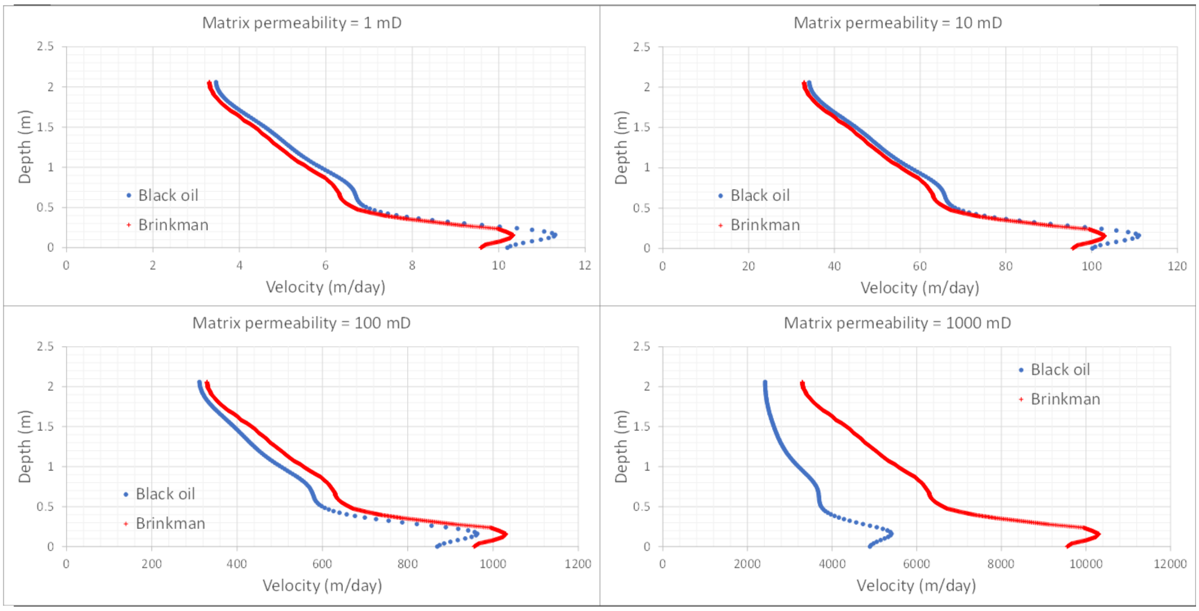

Figure 8. Velocity profile comparison at four different matrix permeability values using the 14th layer of the Lajedo Arapuá model.

Table 2 shows the root mean square deviation (RMSD) and the scatter index (SI) values of each comparison used to quantify the observed variation. Approximately 5% of SI is because in the black oil simulator, the velocity is calculated using the rate value obtained through well model coupling. This difference was considered small for study's goals and was not treated.

Table 2. RMSD and SI between black oil and Brinkman results for flow velocity profile of each simulation scenario.

Layer	Matrix permeability, mD	RMSD	SI
Arapuá 10	1	0.10	5.16 %
	10	1.05	5.36 %
	100	10.38	5.29 %
	1000	92.32	4.70 %

Arapuá 17	1	0.61	11.06 %
	10	5.57	10.20 %
	100	20.30	3.72 %
	1000	2210.68	40.51 %
Arapuá 24	1	0.39	6.68 %
	10	2.96	5.03 %
	100	54.78	9.32 %
	1000	2753.28	46.85 %

To better understand the velocity variations between the two models, the pressure variation along a horizontal line connecting the inlet and outlet boundaries was examined at different scenarios. In the case with segregated vugs, the pressure values showed the same descending linear behavior, with almost constant regions where the section crossed the vugs (Figure 9). In the other two cases (Figures 10 and 11), the central region of the grid presented constant pressure, and as the ratio $\left(\frac{k_v}{k_m}\right)$ got smaller, the Brinkman model showed no change in the pressure values, confirming what was observed in the pressure maps, while the black oil model progressively approximated a linear variation. The more abrupt pressure differential in the Brinkman model near the external boundaries explains the variation observed in the velocity profiles in the cases with more complex karst systems.

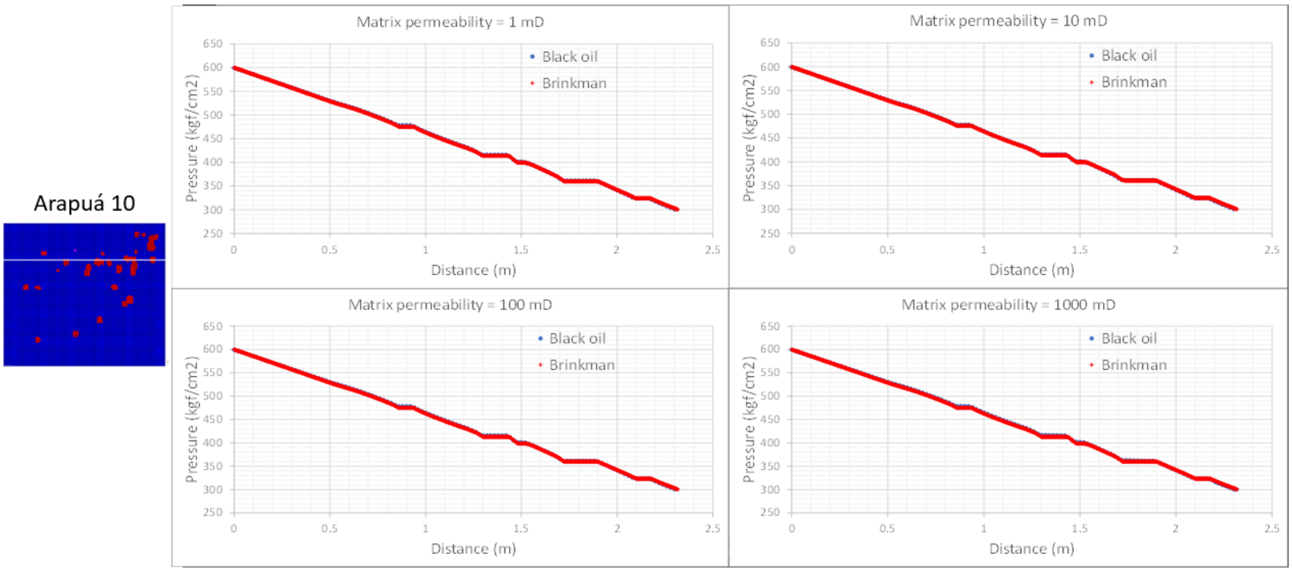

Figure 9. Pressure in linear section at four different matrix permeability values using the 10th layer of the Lajedo Arapuá model.

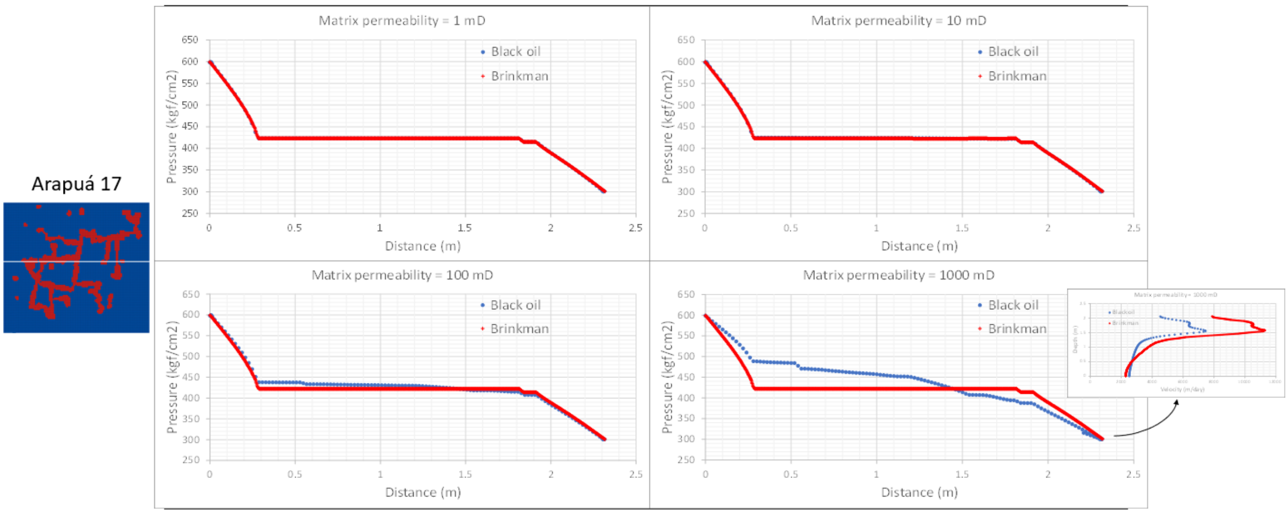

Figure 10. Pressure in linear section at four different matrix permeability values using the 17th layer of the Lajedo Arapuá model.

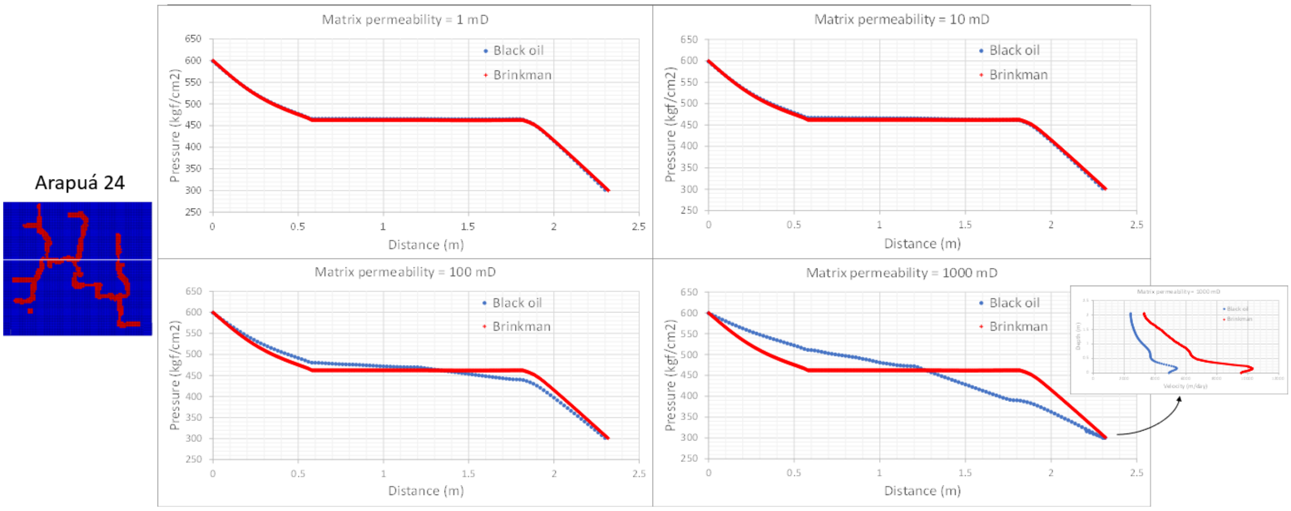

Figure 11. Pressure in linear section at four different matrix permeability values using the 14th layer of the Lajedo Arapuá model.

The production rates of the two models also showed the same high variation tendency at smaller $\left(\frac{k_v}{k_m}\right)$ scenarios and very close results for all matrix permeability values in the segregated vug case, showing a slight influence of the vugs in the fluid flow (Table 3).

Table 3. Rate values of each simulated scenario and the variation between them.

Layer	Matrix permeability, mD	$Q_{Brinkman}$ (m³/d)	$Q_{black\ oil}$ (m³/d)	Variation
Arapuá 10	1	4.05	4.25	4.87 %
	10	40.43	42.48	5.07 %
	100	404.22	424.50	5.02 %
	1000	4042.18	4226.53	4.56 %
Arapuá 17	1	11.36	12.17	7.14 %
	10	113.97	120.42	6.59 %
	100	1129.22	1126.47	-0.24 %
	1000	11291.89	8203.42	-27.35 %
Arapuá 24	1	12.08	12.81	5.98 %
	10	120.50	125.82	4.41 %
	100	1204.74	1101.17	-8.60 %
	1000	12047.17	6958.49	-42.24 %

6. DUAL-CONTINUUM DISCUSSION AND RESULTS

To conclude if a homogeneous dual-continuum model can describe fluid flow inside a karstified porous media, a matching analysis was performed using the 10 mD of matrix permeability single-continuum results as reference values since they represent a good approximation of the Brinkman model results and matrix permeability value close to the actual average value in the formation. The fracture permeability and shape factor of the dual-continuum model were treated as fitting parameters. The ranges were 1 to 50000 mD for fracture permeability and 30000 to 3000000 m^{-2} for shape factor. A regular grid with the same number of cells was used (11948 cells), in addition to two upscaled scenarios with 5 and 10-times coarser grids. These coarser grids aimed to verify whether a more straightforward model could achieve results with the same quality and lower computational cost. The analysis tried to match the flow rates at the domain's left and right boundaries, minimizing an objective function using an experimental design. Thirteen matching studies were conducted with specific configurations to enable results comparison:

1. Homogeneous variation of both parameters inside the grid.
2. Independent variation of the parameters inside and outside the mapped karst system.
3. Same as study 1 in a 5 times coarser grid.
4. Same as study 1 in a 10 times coarser grid.
5. Variation of the parameters only inside the mapped karst system.

6. Same as study 1, altering only fracture permeability (shape factor at mean value).
7. Same as study 2, altering only fracture permeability (shape factor at mean value).
8. Same as study 3, altering only fracture permeability (shape factor at mean value).
9. Same as study 4, altering only fracture permeability (shape factor at mean value).
10. Same as study 1, altering only fracture permeability (shape factor at study 1 result value).
11. Same as study 2, altering only fracture permeability (shape factor at study 2 result value).
12. Same as study 3, altering only fracture permeability (shape factor at study 3 result value).
13. Same as study 4, altering only fracture permeability (shape factor at study 4 result value).

Observing the global variation calculated by the simulation software, it becomes clear that the results improve considerably in the possibility of mapping the karstic system. This does not happen in the case of segregated vugs, where the karst system has less influence on the porous media fluid flow. It is also noticeable that for the layers selected, it is possible to work with coarser grids without prejudice to the results. This is not true only for the case with segregated vugs with a grid ten times coarser, where the cell size competes with the vug size.

Study 5 showed that it is possible to match the reference data using a double continuum model only inside the mapped karstic system region with the same quality as expanding the double continuum model to the hole grid and changing fracture permeability and shape factor independently inside and outside the karstic system. Studies 6 to 13 showed that a match with the same quality could be obtained by the double continuum models changing only the permeability factor, keeping the shape factor at a mean value inside the specified range or at the best match value of the first four studies.

7. CONCLUSIONS

This study aimed to test the equivalence between a more accurate but more resource-intensive porous media flow model, as the Brinkman model and the black oil model widely used in the petroleum industry, and the capability of a double continuum model to reach the same results with or without the mapping of the karstic system present in the reservoir.

The simulations performed using the developed Brinkman model and the commercial black oil model describing porous media flow inside layers of the Lajedo Arapuá's outcrop model showed equivalence when segregated vugs constituted karstic system. Whereas the karstic system presented more complexity, with connected conduit shape vugs, the analysis showed that there is a range of $\left(\frac{k_v}{k_m}\right)$ ratio, which goes from 500 to infinity according to the selected scenarios, where a black oil model can describe fluid flow inside a karstified reservoir.

The scenario with $\left(\frac{k_v}{k_m}\right) = 5000$ was selected as reference data to test if a double continuum model could achieve the same results as a single-continuum one with karstic system modeling. The simulations showed that homogeneous double continuum models can describe fluid flow inside karstified reservoirs where vugular porosity is scattered. Suppose field data indicates a high probability of a karstic system with higher connectivity and more complex configuration. In that case, one should ideally seek a discrete representation of the karstic system to achieve more accurate results, according to the results achieved in the matching analysis of these cases.

The double continuum models showed good results in coarsening tests, except for the 10 times coarser grid of the layer with segregated vugs. This can represent simulation time saving without loss of quality. The study also concluded that fracture permeability dominates the simulation results compared to the shape factor, enabling less complex matching analysis with less fitting parameters.

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