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# **MATHEMATICAL MODELING AND EXPERIMENTAL VALIDATION OF HEAT EXCHANGERS OPERATING AS A BOILER USING VOLUME ELEMENTS METHOD**

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**Abstract.** *This work presents a mathematical modeling using volume elements (VEM) with experimental validation of a set of heat exchangers operating as a boiler. The heat energy used to heat water comes from the burning of solid urban waste in the incinerator. The first heat exchanger functions as an economizer, whereby the liquid water comes into indirect contact with the combustion gases of the waste and heats up to temperatures near the phase change temperature. The second heat exchanger is a phase change device, where the preheated water raises its temperature to the phase change temperature at a given pressure. The third is a superheater that allows only superheated steam to be produced. The objective is to analyze the system's performance and compare the results obtained with the experimental data. The mathematical modeling is performed considering the set in mixed regime, where the phase change step is treated in quasi-steady state, while the other phases (subcooled fluid and superheated vapor) operate in transient regime. The mathematical model is explained by a system of ordinary differential equations integrated with time, with precision and low computational time. Empirical correlations are used to quantify heat transfer coefficients for both the phase change and monophasic processes. The mathematical model was developed considering the heat transfer between the fluids circulating in the heat exchangers. Experimental validation was performed on a set of heat exchangers operating as a boiler. Experimental data were collected for different operating conditions, including mass flow rate and fluid temperature. The numerical simulation results were compared with the experimental data and showed good agreement. The VEM mathematical model presented good accuracy in predicting the system's performance. The results suggest that the VEM mathematical modeling can be a useful tool in analyzing and designing heat exchanger systems in boilers. The experimental validation confirmed that the set of heat exchangers operating as a boiler can present good thermal performance and that the VEM mathematical modeling can be an effective tool for the design and analysis of these systems.*

**Keywords:** *mathematical modeling, volume elements, heat exchangers, boiler, experimental validation, system performance*

## **1. INTRODUCTION**

In an increasingly interconnected world, our reliance on electronic devices has grown exponentially. Whether it's for the comfort they provide or the convenience of accomplishing tasks with a simple touch or voice command, society has embraced these technologies. However, this digital revolution comes at a cost: a surge in electricity consumption and the escalating levels of waste generated and discarded daily.

The transition from traditional mediums to electronic alternatives is evident in our daily lives. Tablets have replaced books, laptops have supplanted magazines and newspapers, and smartphones have become the primary means of communication through social networks. Yet, these conveniences have consequences that must be acknowledged. The escalating energy consumption and waste production are intrinsically linked to the economic activity within a society. They reflect our collective behavior and the dynamics of industrial, commercial, and service sectors. As our economies thrive and individuals embrace technological advancements, the demand for electricity surges, leading to a considerable strain on resources and the environment.

The pursuit of sustainable methods for reusing and treating Municipal Solid Waste (MSW) has emerged as a prominent global trend. To address the pressing challenges posed by increased consumption rates in the coming years, innovative technologies are being integrated into existing systems. These advancements aim to mitigate the unfavorable consequences of escalating waste generation. Research indicates a concerning projection that, by the year 2050, approximately 3.5 billion tons of solid urban waste will be produced due to population growth and accelerated urbanization. To counteract this trend, efforts are being made to implement efficient waste management strategies and environmentally conscious approaches to ensure a more promising future (Kaza et al., 2018).

An innovative alternative for the utilization of discarded materials is harnessing the potential energy inherent in them. This can be achieved through a process known as waste-to-energy (WtE), where the energy capacity stored in waste is effectively converted into electrical energy, heat, and/or fuel during the treatment process. By adopting WtE, we can significantly reduce the volume of materials that would otherwise end up in landfills or other storage facilities, while simultaneously tapping into the energy that would have otherwise gone to waste. This sustainable approach enables the transformation of accumulated energy in the waste into valuable forms, such as heat and electrical energy (Magnaleli et al., 2020).

Given the context presented above, it becomes evident that one of the most significant contemporary challenges lies in achieving enhanced energy production while upholding environmental preservation. In this context, the development of efficient equipment, such as heat exchangers, becomes crucial as they enable energy utilization with minimal waste. By employing advanced heat exchangers, industries can substantially improve their processes, leading to increased efficiency and reduced environmental impact. This, in turn, plays a vital role in curbing the emission of polluting gases, making a positive contribution towards a greener and sustainable future (Kuruneru et al., 2021).

In general, mathematical modeling finds applications at both the system and component levels (Sage, 1992). Various classifications exist in the literature for these elements, such as: i) Qualitative modeling: This approach allows for the accurate observation of trends in responses, although it may have limited precision when it comes to absolute values and local variables. (Woods and Lawrence, 1997; Vargas et al., 2001). ii) Quantitative modeling: This type of modeling offers precision not only in response trends but also in the values of local variables (Woods and Lawrence, 1997; Vargas et al., 2001). iii) High-order and low-order modeling: Shapiro (2003) proposed these categories to differentiate between complex models (high-order) and simplified models (low-order). iv) Concentrated and distributed modeling: Trivelato (2003) and Kaiser (2004) introduced this classification to distinguish between models that consider concentrated parameters and those that account for distributed parameters. By understanding and applying these diverse modeling approaches, researchers and practitioners can effectively analyze systems and components, leading to informed decisions and optimized designs for a wide range of engineering applications.

This study proposes a reduced-order mathematical model with the aim of obtaining the thermal responses of a system composed of three heat exchangers operating as a boiler, providing superheated steam. These are the economizer, evaporator, and superheater, which are physically separated. Mathworks Inc.'s software, Matlab®, was used to simulate the system's responses. Using experimental data obtained from the work of Eriksen and Schroeder in 2017, experimental validation was performed, allowing the observation of the system's responses in relation to a real physical system.

## 2. SYSTEM IN ANALYSIS

A variety of operational characteristics of the equipment being analyzed have been acquired from the manufacturer. Additionally, a schematic representation showcasing the different subsystems comprising the bioenergetic engineering system, which is installed at the Center for Research and Development in Self-Sustainable Energy at the Federal University of Paraná, is available for reference. The figure below illustrates the existing system in NPDEAS.

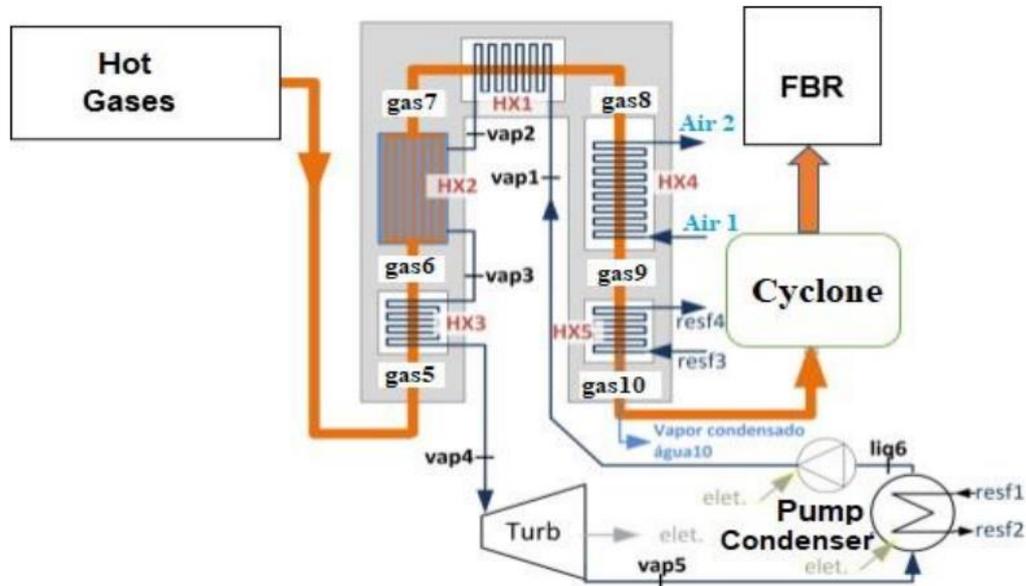


Figure 1. Illustration of hot gases, heat exchangers, Rankine cycle and photobioreactors.  
Available from: Galante, 2019.

Hot gases are generated through the processing of Municipal Solid Waste (MSW) derived from UFPR. In order to achieve complete combustion, preheating using liquefied petroleum gas (LPG) as fuel in the primary combustion chamber is necessary. While the supply of quality air and its proper distribution for combustion facilitates the burning of MSW, any material that undergoes incomplete combustion is directed to the lower part below the grid, resulting in the generation of the majority of the ashes, approximately 80%. Subsequently, the gases pass through the post-combustion chamber, where the material comes into contact with LPG again to ensure the complete oxidation process. During this step, the use of liquefied petroleum gas as fuel is crucial to maintain the temperature above 900 °C, thereby preventing the release of dioxins and the formation of nitrous oxide.

The primary function of the incinerator is to provide an appropriate disposal method for the waste by converting its chemical energy into the thermal energy of the combustion products. As a crucial component of this process, the high-temperature gases are directed to a series of three heat exchangers, namely HX3, HX2, and HX1. In this stage, water flows in the opposite direction to the gases and is initially heated in the TR HX1 before being directed to HX2, where it undergoes a phase change process. To ensure that only steam enters the turbine, the water passes through a third heat exchanger, HX3, where it is overheated.

A cyclone is installed before the FBR (Fluidized Bed Reactor) to reduce the ash content in the flue gases. In this configuration, approximately 20% of the ash mass is removed by this device. The latest addition to the plant's equipment is the FBR, which facilitates the cultivation of microalgae using flue gases rich in agro-industrial waste diluted in water as a culture medium. The FBR system depicted here consists of six units, each with a capacity of 10 m<sup>3</sup> of culture medium.

### 3. MATHEMATICAL MODELING IN A REDUCED ORDER FOR HEAT EXCHANGERS

The mathematical model presented in this article proposes a reduced-order approach for the heat exchangers (HX1, HX2, and HX3). It is based on the fundamental principles of classical thermodynamics, which encompass the conservation of mass, conservation of energy, and heat transfer concepts. The discretization process involves dividing the system domain into finite centered volumes called volume elements. Unlike methods such as finite differences, finite elements, and finite volumes, the size of each volume element does not need to be small to ensure numerical stability and result accuracy. This section describes the mathematical modeling for the three analyzed heat exchangers and outlines the associated simplifications.

#### 3.1 Mathematical Modeling of Heat Exchangers HX1, HX2 and HX3

Figures 2, 3, and 4 depict the schematic diagrams of the three heat exchangers. In Figure 2, the division of volume elements is shown, but this step will be omitted in the subsequent figures as it follows the same procedure. The mathematical model is derived by applying the principles of conservation of mass and energy to the system.

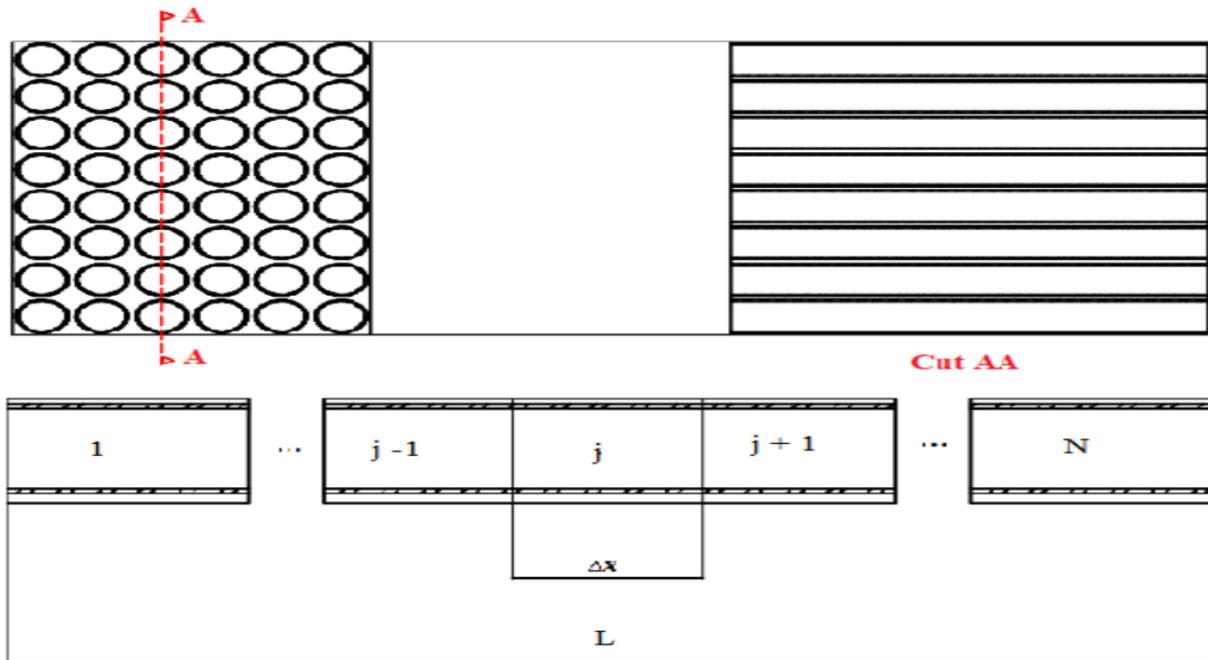


Figure 2. Schematic diagram of recuperative heat exchangers (XH1) and the division of the system into volume elements.

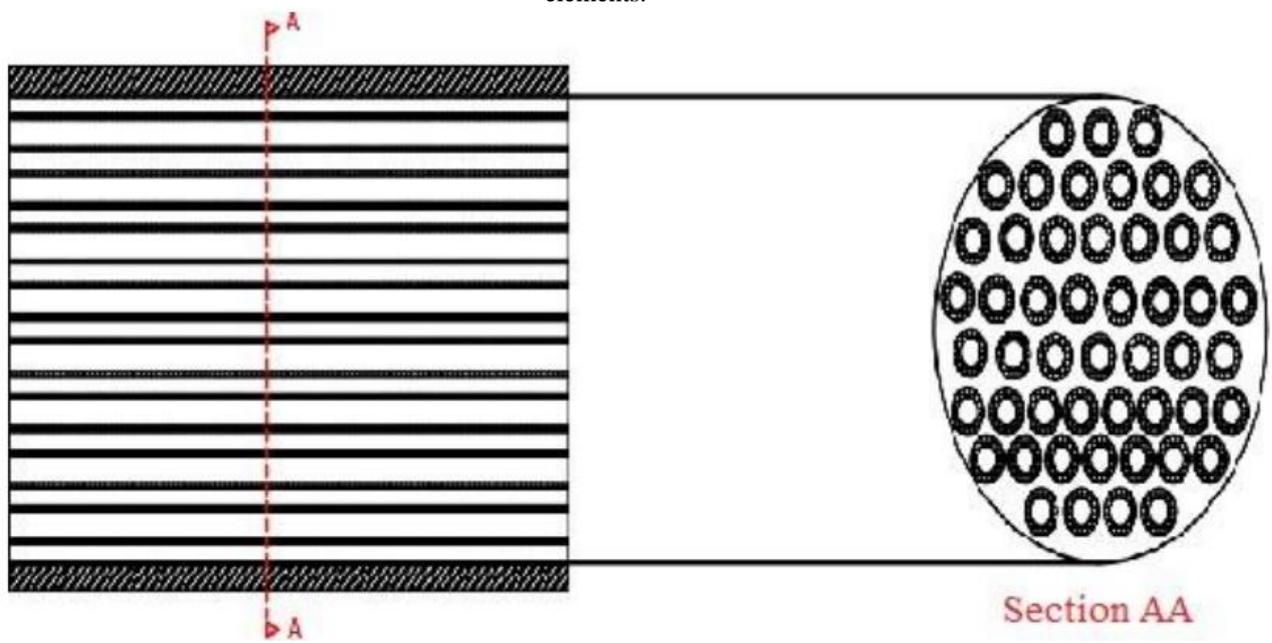


Figure 3. Schematic diagram of recuperative heat exchangers (XH2).

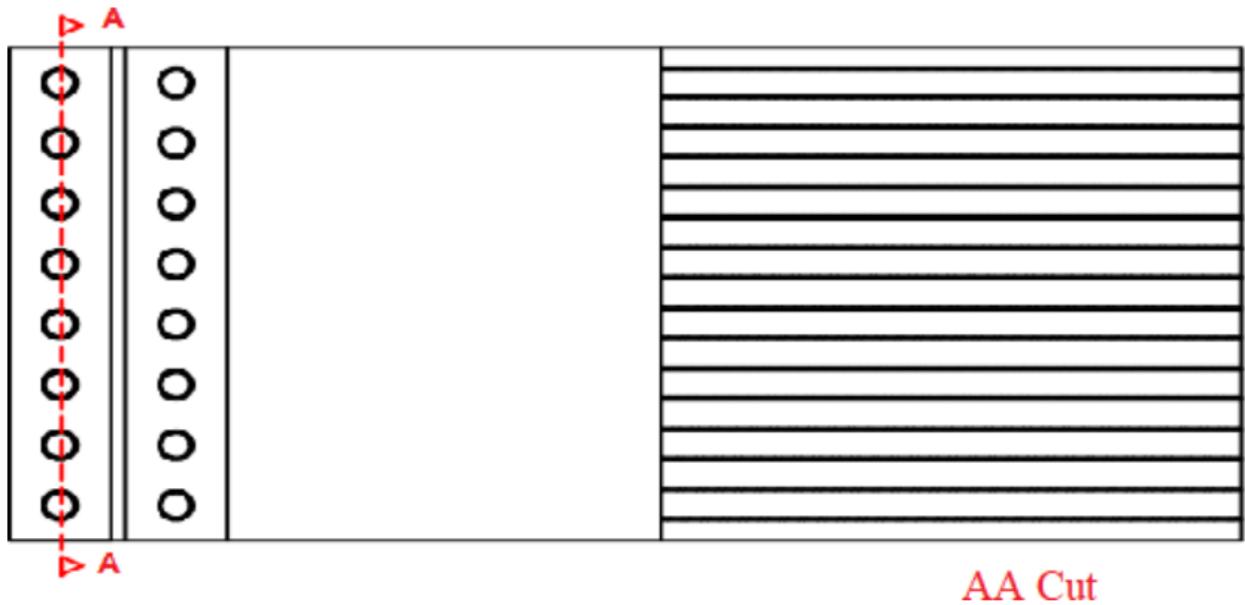


Figure 4. Schematic diagram of recuperative heat exchangers (XH3).

The equipment was initially divided into volume elements, as shown at the bottom of Figure 2. Figure 5 provides further details on the mass and energy transfers within each volume element, revealing a mixed behavior involving both fluid and solid components. To characterize the functionality and performance of each heat exchanger and determine the phase of the fluid (in this case, water) within them (whether it is subcooled, undergoing phase change, or superheated), each volume element is further divided into five subsystems. These subsystems are as follows:

1. Tube (solid part)
2. Hot gases
3. Subcooled liquid
4. Phase change
5. Superheated steam.

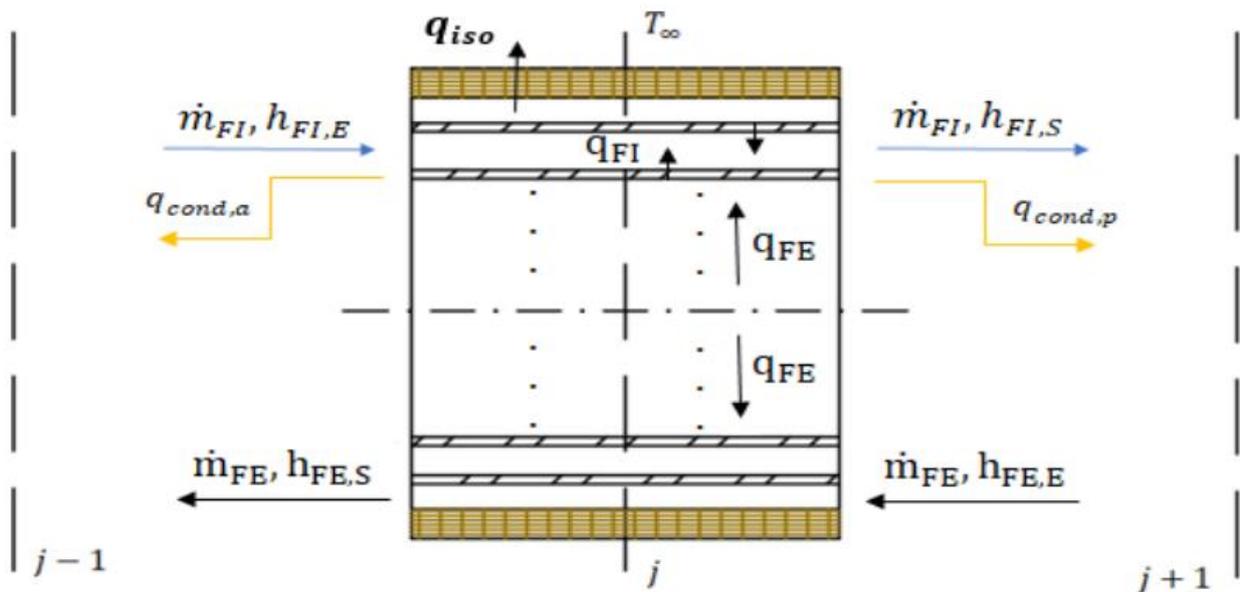


Figure 5. Detail of the volume element, where the mass and heat interactions between subsystems can be observed.

It is important to highlight that each subsystem exists within all three heat exchangers (HX1, HX2, and HX3), but their presence and characteristics may vary across the different exchangers. For instance, HX1 functions as an economizer, utilizing energy from waste combustion gases to heat water to temperatures close to saturation. As a result, subsystems 4 and 5, which pertain to phase change and superheated steam, respectively, are not applicable in HX1 since only liquid is present. The division into subsystems remains essential for all three exchangers as it enables strategic mapping of the

phase change point, where the liquid reaches saturation temperature and the vapor fraction (X) starts to vary until reaching unity. The subsequent step involves the mathematical modeling of each subsystem within the volume element, as illustrated in Figure 6. This analysis employs principles of combined mass and energy conservation, as well as the evaluation of energy interactions between subsystems. Equations are employed to quantify these interactions accurately.

Subsystem 1: Tube (solid part): The 1st Law of Thermodynamics is applied to system 1, according to the diagram above, the following equation is obtained:

$$m_{T,j} \cdot c_T \cdot \frac{dT_{T,j}}{dt} = q_{FE,j} + q_{Cond,a,j} + q_{Cond,p,j} - q_{FI,j} \quad (1)$$

Where

$$q_{FE,j} = h_{FE,j} \cdot A_{ET,j} \cdot (T_{FE,j} - T_{T,j}) \quad (2)$$

$$q_{Cond,a,j} = -k_{T,j} \cdot A_{AT,j} \cdot \frac{(T_{T,j} - T_{T,j-1})}{\Delta x} \quad (3)$$

$$q_{Cond,p,j} = -k_{T,j} \cdot A_{AT,j} \cdot \frac{(T_{T,j} - T_{T,j+1})}{\Delta x} \quad (4)$$

$$q_{FI,j} = h_{FI,j} \cdot A_{IT,j} \cdot (T_{T,j} - T_{FI,j}) \quad (5)$$

Subsystem 2: Hot gases (external flow): Applying the first law of thermodynamics to subsystem two, which consists of the hot gases that were processed in the incinerator and in the post-combustion chamber, as shown in Figure 6, and assuming variables without a subscript for the hot fluid, we have:

$$m_{FE,j} \cdot c_{v,FE} \cdot \frac{dT_{FE,j}}{dt} = \dot{m}_{FE,j} \cdot c_{p,FE} \cdot (T_{FE,j+1} - T_{FE,j}) - q_{FE,j} - q_{ISO,j} \quad (6)$$

Where

$$q_{ISO,j} = (U \cdot A)_{ISO,j} \cdot (T_{FE,j} - T_{\infty}) \quad (7)$$

$$(U \cdot A)_{ISO,j} = \left[ \frac{1}{h_{FE,j} \cdot A_{iISO}} + \frac{\ln\left(\frac{d_{eISO,j}}{d_{iISO,j}}\right)}{2 \cdot \pi \cdot k_{ISO} \cdot L} + \frac{1}{h_{\infty,j} \cdot A_{eISO}} \right] \quad (8)$$

$$A_{eISO} = \pi \cdot d_{eISO,j} \cdot \Delta x \quad (9)$$

$$A_{iISO} = \pi \cdot d_{iISO,j} \cdot \Delta x \quad (10)$$

The boundary conditions are:

$$T_0 = T_E \text{ and } \frac{\partial T}{\partial x} = 0 \quad (11)$$

Subsystem 3: Subcooled liquid: Again the 1st law of thermodynamics is applied to subsystem 3, as seen in Figure 6, providing the following equation:

$$m_{FI,j} \cdot c_{FI} \cdot \frac{dT_{FI,j}}{dt} = \dot{m}_{FI,j} \cdot c_{p,FI} \cdot (T_{FI,j-1} - T_{FE,j}) + q_{FI,j} \quad (12)$$

$$q_{FI,j} = h_{FI,j} \cdot A_{FI,j} \cdot (T_{T,j} - T_{FI,j}) \quad (13)$$

Where

$$A_{FI,j} = \pi \cdot \frac{d_{FI,j}^2}{4} \quad (14)$$

$$m_{FI,j} = \rho_{FI,j} \cdot \pi \cdot \frac{d_{IT,j}^2}{4} \cdot \Delta x \quad (15)$$

The boundary conditions are:

$$T_{r,n} = T_{r,E} \quad \text{and} \quad \frac{\partial T_{r,o}}{\partial x} = 0 \quad (16)$$

Subsystem 4: Phase change: For the mathematical modeling of the phase change process, a quasi-permanent regime will be considered, that is, the deviations that a given property undergoes in time is considered to be infinitesimal when comparing their changes in space. Thus, applying energy conservation, one has to  $\frac{dE}{dt} = 0$  during the integration interval,  $\Delta t$ , which is justified for small amounts of  $\Delta x$ .

$$0 = \dot{m} \cdot X_{j-1} \cdot h_v + \dot{m} \cdot (1 - X_{j-1}) \cdot h_l - \dot{m} \cdot X_j \cdot h_v - \dot{m} \cdot (1 - X_j) \cdot h_l + q_{FI,j} \quad (17)$$

$$q_{FI,j} = h_{FI,j} \cdot A_{FI,j} \cdot (T_{T,j} - T_{FI,sat,j}) \quad (18)$$

Subsystem 5: Superheated steam: The modeling for subsystem 5 follows the same process as for subsystem 3, however, now the physical properties needed to quantify the quantities will be done for superheated steam.

$$m_{FI,v,j} \cdot c_{v,FI} \cdot \frac{dT_{FI,v,j}}{dt} = q_{FI,v,j} + \dot{m}_{FI,v,j} \cdot c_{p,FI} \cdot (T_{FI,j-1} - T_{FE,j}) \quad (19)$$

In which

$$q_{FI,v,j} = h_{FI,v,j} \cdot A_{FI,j} \cdot (T_{FI,v,j-1} - T_{FI,v,j}) \quad (20)$$

The mathematical model was computationally implemented using MATLAB® (developed by MathWorks Inc.). The objective was to obtain an approximate solution to the system of basic equations mentioned earlier. It is essential to highlight that this set of equations exhibits a mixed characteristic. Specifically, for subsystem 4, operating under nearly steady-state conditions, the resulting equation for each EV becomes algebraic. To address this, the iterative Gauss-Seidel method was applied, facilitating the approximate solution of these equations.

Furthermore, it is important to note that the heat transfer coefficients for water and hot gases were assumed to be constant. This means that any changes in these coefficients, such as those associated with the phase change of liquid water, were not considered a priori.

The equations developed for the other subsystems can be explicitly integrated in relation to time using an adaptive time step with the fourth and fifth order Runge-Kutta method (KINCAID and CHENEY, 1991). The time step is automatically traversed according to the local truncation error, which is kept below a compatible tolerance of  $10^{-6}$ .

## 4. RESULTS AND DISCUSSION

In Section 2, we provide an overview of the operating plant at NPDEAS, where the system under study is installed. The heat exchangers, HX1, HX2, and HX3, respectively, are depicted in Figure 1, illustrating the path taken by the hot gas as it comes into indirect contact with the superheater, evaporator, and economizer. Subsequently, a comprehensive mathematical model was developed and employed to yield systematic results. This section serves a dual purpose:

- i) to furnish a detailed plant description;
- ii) to validate the model through experimental comparison and analysis.

### 4.1 Model experimental validation

Validating any mathematical model intended for practical use is an essential step to establish its credibility. To achieve this, experimental results from a heat recovery steam generator (HRSG) driven SRC power plant were employed (Eriksen and Schroeder, 2017). These experimental findings play a crucial role in corroborating the accuracy and reliability of the mathematical model, thus enhancing its applicability in real-world scenarios.

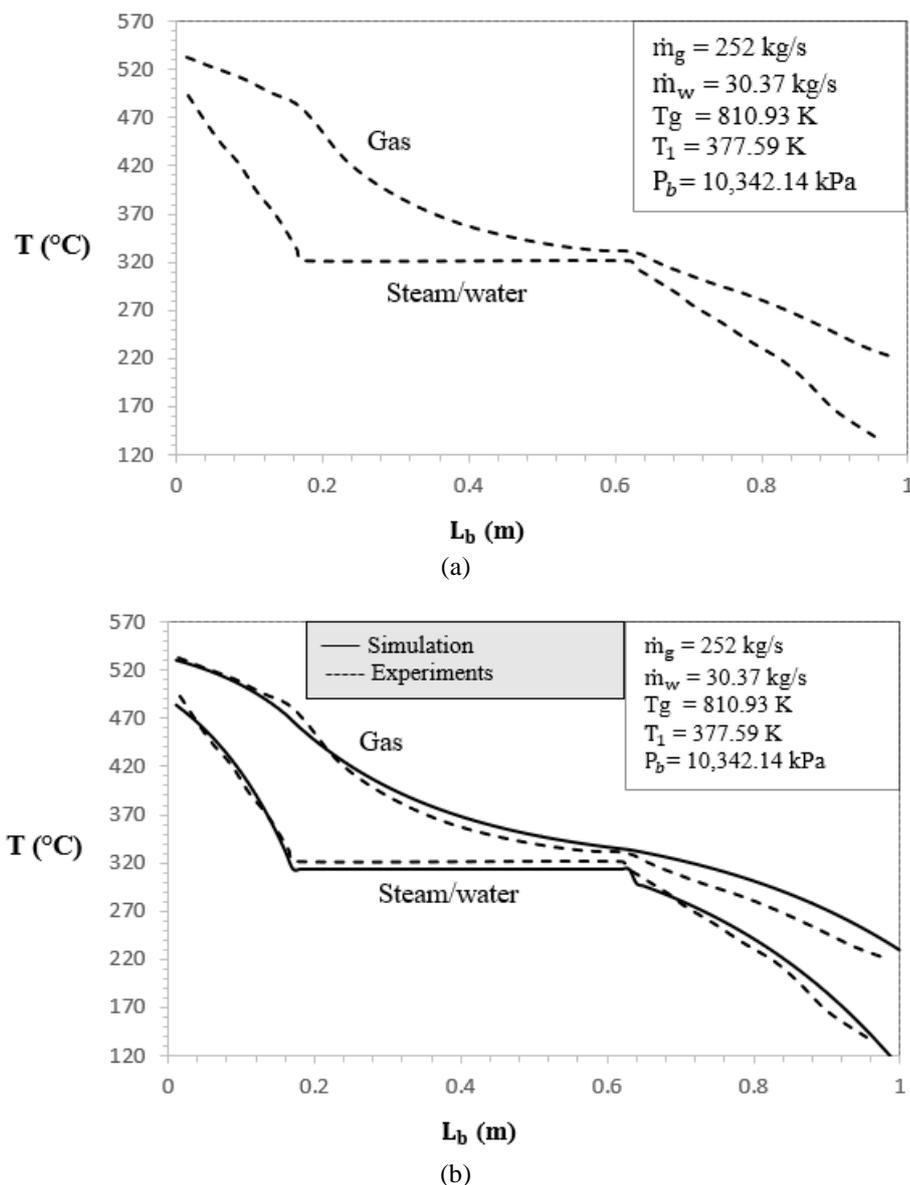


Figure 6. The model's experimental validation included: a) Using original temperature measurements from an HRSG-driven SRC plant by Eriksen and Schroeder in 2017, converted to SI units for accuracy. b) Comparing the model's dimensionless temperature results with nondimensionalized experimental data to assess its performance.

Consequently, it is consistent that the mathematical model presented in this study was successfully experimentally validated. With model validation successfully established, the study moved on to perform fundamental optimization and parametric analysis for the SRC plant. This critical step allows us to explore and optimize system performance, making informed decisions based on the validated model predictions.

## 5. CONCLUSION

In this study, it was possible to demonstrate the validity and reliability of a mathematical model of an urban solid waste treatment plant for power generation with phase change without the use of computational fluid dynamics through successful experimental validation. The results obtained from the original temperature measurements of the heat recovery steam generator (HRSG) coupled to a SRC plant, carried out by Eriksen and Schroeder in 2017, were converted to International System (SI) units to ensure the consistency and accuracy of the information. Furthermore, the comparison of the numerical predictions of the model's temperatures with the dimensionless experimental data was essential to evaluate the performance of the model. Obtaining results with good accuracy and low computational cost (experimental validation occurred with only 300 volume elements) and reliable through the validated model is a significant contribution to research in energy systems and process optimization.

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