COB-2023-1004
SPECTRAL ANALYSIS OF FATIGUE IN DRILL STRINGS UNDER TORSIONAL VIBRATIONS

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Abstract. In oil exploration and production, drilling is one of the most important processes. Undesirable vibrations, intrinsic to the drilling process, can lead to fatigue failure, resulting in loss of efficiency due to downtime and, consequently, loss of productivity and high costs. The present work proposes the analysis of fatigue in drill strings under torsional vibrations with the stick-slip phenomenon, considering the frequency domain (or spectral) approach as an alternative to the time domain approach. The stick-slip occurs when the torque imposed by the top drive is not enough to overcome the resistance offered by the enviroment and, in the process of rock-bit interaction, the bit becomes stationary for a period, causing large fluctuations in the angular speed and, in turn, in shear stress, characterizing failure by fatigue. Spectral Methods for fatigue life analysis, although recent, have shown great potential in several applications and therefore, there is much to be explored given their viability in engineering problems due to the lower computational cost and time processing, being advantageous for complex problems with large amounts of data. In this approach, concepts such as Power Spectral Density (PSD) are used to calculate the Spectral Moments, employed in the various methods of this nature for determining fatigue life. The Dirlik and Tovo-Benasciutti Methods, being widely used not only in academia, have proved to be advantageous. These methods are compared with the rainflow method, used as a reference.

Keywords: Drill Strings Dynamics, Torsional Vibration, Stick-Slip, Fatigue, Spectral Analysis

1. INTRODUCTION

The drilling process is one of the most complex and expensive activities in the oil and gas industry. During this process, undesirable vibrations are present due to several factors. Vibrations in these structures can be of three types: axial, lateral and torsional (Ritto *et al.*, 2017). Associated with these vibrations, large stress amplitudes can occur, leading to fatigue failure. Torsional vibrations, when critical, are associated with the *stick-slip* phenomenon, characterized by the bit become stationary for a period of time due to the inability to win the resistance offered by the environment, leading to large variations in angular velocity and therefore in shear stress.

The objective of this work is the analysis of fatigue in drill strings subject to torsional vibrations and the *stick-slip* phenomenon considering the classical approach (*rainflow* method, in the time domain) and the frequency domain (or *spectral*) approach as an alternative. These methods, although recent, have shown great potential in a large number of applications in engineering. The *Dirlik* (Dirlik, 1985) and *Tovo-Benasciutti* (Benasciutti and Tovo, 2006) methods were considered due to their good approximation in several works, for example, to compare the different methods or in applications to specific problems (see for example Mršnik *et al.* (2013) and Zorman *et al.* (2023)).

This manuscript is organized as follows. Section 2 presents the theoretical background, with a mathematical model to describe the dynamics of drill strings subjected to torsion and the fatigue analysis in the time and frequency domains. For this last, are present the concepts used for random processes and the *Dirlik* and *Tovo-Benasciutti* methods. Section 3 presents some results considering different values for the angular velocity imposed by the *top drive* and the *weight on bit (WOB)*, important parameters to be controlled in the drilling process. The fatigue life is calculated and the frequency domain methods are compared with the *rainflow* method. Section 4 presents the conclusions.

2. THEORETICAL BACKGROUND

2.1 Mathematical Model for Torcional Vibrations in Drill Strings

To simulate the *stick-slip* phenomenon, the drill string was modeled as a simple torsional pendulum with one degree of freedom (1-DOF) with torsional stiffness k_t, damping c_t, inertia J and angular position θ. It is considered that there is no fluid between the bit and the bottom of the well and the speed Ω imposed by the *top drive* is constant. For the bit-rock interaction, the model proposed by Ritto *et al.* (2017) was adopted. This model is illustrated in Figure 1.
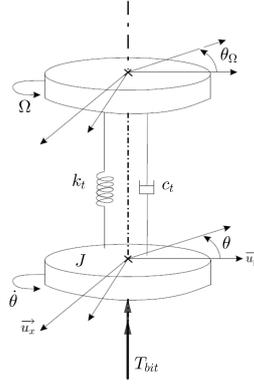

Figure 1: Torsional Pendulum Model Representation (Adapted from Navarro-Lopez and Suarez (2004)).

The governing equation, therefore, is expressed by

$$J\ddot{\theta}_{bit}(t) + c_t\dot{\theta}_{bit}(t) + k_t\theta_{bit}(t) = c_t\Omega + k_t\Omega t + T_{bit}, \tag{1}$$

where T_{bit} is the torque on bit (Ritto *et al.*, 2017) given by

$$T_{bit} = -b_0 WOB \left(tanh(b_1\dot{\theta}_{bit}) + \frac{b_2\dot{\theta}_{bit}}{1 + b_3\dot{\theta}_{bit}^2} \right), \tag{2}$$

being WOB the weight on bit and b_0, b_1, b_2 and b_3 positive constants that depend on the rock properties and characteristics of the bit. Once the angular position θ is determined, we can evaluate the shear stress

$$\tau = G\frac{(\theta_\Omega - \theta)}{L}r^{ext}, \tag{3}$$

where G is the shear modulus, θ_Ω the *top drive* angular position with L and r^{ext} being the length and external radius of the *drill pipe* respectively. From the signal generated by the stress and its amplitudes, the fatigue life is estimated.

2.2 The Time Domain Approach to a Fatigue Life Estimation

The fatigue life is estimated using the *Whöler S-N curve* (Wöhler, 1970), given by

$$\sigma = CN^k, \tag{4}$$

with C and k being material parameters, σ the stress amplitude and N the number of cycles until the crack initiation. The *rainflow* method (Matsuishi and Endo, 1968) is a counting method that transforms a historical data of loads into a series of reverse loads (Ogrinec *et al.*, 2019), used in loads with variable amplitude.

For the damage accumulation is commonly adopted the *Palmgren-Miner* linear damage accumulation rule, expressed by

$$D = \sum_{i=1}^{N} D_i = \sum_{i=1}^{N} \frac{n_i}{Ni}, \tag{5}$$

where for each class of stress i, D_i is the accumulated damage, n_i the number of cycles accounted and N_i the number of cycles until nucleation occurs.

2.3 The Frequency Domain Approach to a Fatigue Life Estimation

Fatigue life analysis in the frequency domain is based on the *Power Spectral Density* (*PSD*) which is obtained from the loading using the *Fourier Transform*. From the PSD, the spectral moments applied in the methods are calculated and then estimated fatigue life. Several methods have been proposed and, in the present work, we will use the methods of *Dirlik* and *Tovo-Benasciutti* because they are generally most used and characterized by the approximation of the stress cycles through a *Probability Density Function* (*PDF*). Next, we briefly present the properties of a random process relevant to this new methodology presented in this work, as well as the methods used in it.

2.3.1 Random Process Properties

Random loads in the frequency domain are defined from the *Power Spectral Density* (*PSD*), represented by $S_{XX}(\omega)$, where ω is the frequency in rad/s. In general, the unilateral *PSD* is used, characterized by the positive part and expressed by $G_{XX}(\omega) = 2S_{XX}(\omega)$ (Zorman *et al.* (2023)). The statistical properties of a stationary random process are expressed by the *spectral moments*, defined by

$$m_i = \int_0^{+\infty} \omega^i G_{XX}(\omega)d\omega. \tag{6}$$

From these moments, we can calculate the positive slope zero crossing v_0, the expected peak frequency v_p and the bandwidth parameter α_i, where

$$v_0 = \sqrt{\frac{m_2}{m_0}}, \qquad v_p = \sqrt{\frac{m_4}{m_2}} \qquad \text{and} \qquad \alpha_i = \frac{m_i}{\sqrt{m_0 m_{2i}}}. \tag{7}$$

These parameters are fundamental for the estimation of fatigue life in the frequency domain, whose methods discussed here will be presented below.

2.3.2 The Narrowband Method

Assuming that the stress amplitude σ can be treated as a continuous random variable, the accumulated damage for *narrowband process* is defined (Dirlik and Benasciutti, 2021) as

$$D = \frac{v_p T}{C} \int_0^{+\infty} \sigma^k \rho(\sigma)d\sigma \tag{8}$$

or, per time unit T according to Zorman *et al.* (2023)

$$\bar{D} = \frac{v_p}{C} \int_0^{+\infty} \sigma^k \rho(\sigma)d\sigma, \tag{9}$$

where $\bar{D} = D/T$, being T the observation time, v_p the expected peaks, σ the stress amplitude with C and k being fatigue parameters of the materials.

The equation (8), once the analytical expression of $\rho(\sigma)$ is known, can be obtained in the closed form (Dirlik and Benasciutti, 2021). Many concepts used to estimate fatigue in frequency domain had its origins in Rice's research. One is the probability density function (PDF) of the peak amplitude for a general broadband (Rice, 1945). For narrowband processes, peaks and valleys present symmetry in relation to the value average and, each peak coincides with a cycle (in this process also v_p coincides with v_0) and, from (8) the accumulated damage is expressed by

$$D_{NB} = \frac{v_0 T}{C}(\sqrt{2m_0})^k \Gamma\left(\frac{k+2}{2}\right), \tag{10}$$

being $\Gamma(\cdot)$ is *Euler's Gamma function* where

$$\Gamma(z) = \int_0^\infty t^{z-1}e^{-z}dt. \tag{11}$$

Note, that according to Bendat (1964), the failure will occur for $D = 1$, therefore, making $T = T_{life}$ the life is expressed by

$$T_{life} = \frac{C}{v_0(\sqrt{2m_0})^k \Gamma\left(\frac{k+2}{2}\right)}. \tag{12}$$

The methods presented below, as well as others found in the literature, were developed with the aim of extending this theory to problems characterized by *broadband* processes.

2.3.3 The Dirlik Method

The *Dirlik* Method (Dirlik, 1985) approximates the stress cycle distribution by means of the combination of an exponential distribution and two Rayleigh distributions, being an empirical method based on several simulations and, in addition to being able to be applied both to *narrowband* and *broadband* process, is widely used. Dirlik's *PDF* is given by

$$\rho_{DK} = \frac{1}{\sqrt{m_0}} \left[\frac{C_1}{Q} e^{-\frac{Z}{Q}} + \frac{C_2 Z}{R^2} e^{-\frac{2Z^2}{2R^2}} + C_3 Z e^{-\frac{Z^2}{2}} \right], \tag{13}$$

where Z is the normalized amplitude, x_m is the average frequency and C_1, C_2, C_3, Q and R are model adjustment parameters. The associated expressions are shown below.

$$Z = \frac{s}{2\sqrt{m_0}} \qquad\qquad x_m = \frac{m_1}{m_0} \sqrt{\frac{m_2}{m_4}},$$

$$C_1 = \frac{2(x_m - \alpha_2^2)}{1 + \alpha_2^2} \qquad C_2 = \frac{1 - \alpha_2 - C_1 + C_1^2}{1 - R} \qquad C_3 = 1 - C1 - C2 \tag{14}$$

$$R = \frac{\alpha_2 - Cx_m - C_1^2}{1 - \alpha_2 - C_1 + C_1^2} \qquad Q = \frac{1.25(\alpha_2 - C_3 - C_2 R)}{C_1}$$

In this way, the accumulated damage in the Dirlik model is

$$D_{DK} = \frac{v_p T}{C} (\sqrt{m_0})^k \left[C_1 Q^k \Gamma(1 + k) + (\sqrt{2})^k \Gamma\left(\frac{k+2}{2}\right) (C_2 |R|^k + C_3) \right].$$

2.3.4 The Tovo-Benasciutti Method

Tovo and Benasciutti (Benasciutti, 2004) proposed an approximation based on the linear combination of the lower and upper limits of damage intensity, which is expressed by

$$D_{TB} = [b + (1 - b)\alpha_2^{k-1}]\alpha_2 D_{NB}, \tag{15}$$

where b is a factor obtained through numerical simulations and proposed in two ways:

$$b_{TB_1} = min\left\{ \frac{\alpha_1 - \alpha_2}{1 - \alpha_1}, 1 \right\} \tag{16}$$

and

$$b_{TB_2} = \frac{(\alpha_1 - \alpha_2)[1.112(1 + \alpha_1 \alpha_2 - (\alpha_1 + \alpha_2))e^{2.11\alpha_2} + (\alpha_2 - \alpha_1)]}{(\alpha_2 - \alpha_1)^2}. \tag{17}$$

3. SIMULATIONS, RESULTS AND DISCUSSIONS

Aiming at simulating the dynamics of a drill string and the *stick-slip* phenomenon, the parameters obtained from Ritto and Ghandchi-Tehrani (2019) were used, which are presented in the Table 1. For the evaluation of the fatigue life, the material parameters C and k were obtained from Netto *et al.* (2008) and are presented in Table 2. For the simulations for the calculation of fatigue life, the steady state was considered and they were carried out with the support of the *FLife (Vibration Fatigue by Spectral Methods)* library of *open source research in structural dynamics* (Zorman *et al.*, 2023).

Table 1: Parameters used in the simulations for torcional model.

L	4733.6	Pipe Lenght, *m*
r_i	0.0595	Inside Radius of the Drill Pipe, *m*
r_e	0.07	Outside Radius of the Drill Pipe, *m*
E	220.00	Modulus of Elasticity, *GPa*
S_{ut}	690.0	Tensile Strength, *GPa*
S_y	580.0	Yield Strength, *GPa*
ν	0.29	Poisson's Ratio
ξ	0.25	Damping Rate
b_0	5671.0	Model Parameter, *m*
b_1	0.4775	Model Parameter, *s*
b_2	8.8754	Model Parameter, *s*
b_3	4.5595	Model Parameter, s^2

Table 2: Material parameters used to calculate fatigue life.

k	3.0	S-N Curve Inverse Slope
C	$4.16 \cdot 10^{12}$	S-N Curve Intercept, MPa^k

Figure 2 presents results for four different configurations, where for the speed $\Omega = 5.0 rad/s$ the weight on bit was varied using $50.0kN$, $60.0kN$, $80.0kN$ and $100.0kN$. It can be observed that the angular position θ (*a*) presents a linear behavior, observing the stationary regime and, in relation to the relative position (*b*) there is a greater difference $\theta_\Omega - \theta$ with the increase of the weight on bit after a short time interval and the angular speed converges to that imposed by the rotary table Ω as per (*c*). Since the shear stress (*d*) is a function of the variation in angular position, as seen in (3), greater amplitudes are associated with greater values of *WOB*. These configurations present scenarios without cyclic stresse after the transient, with the stress amplitude around the mean tending to zero.

Figure 3 presents the phenomenon of *stick-slip* for different values of weight on bit ($120.0kN$, $200.0kN$, $300.0kN$ and $500.0kN$) considering $\Omega = 4.0 rad/s$ being its occurrence due to high weights on bit and low rotations. According to (a) and (c), respectively, it can see the period of time when the bit doesn't present angular displacement and then, angular speed (characteristic of this phenomenon). From (b) and (c) there are amplitudes increasing with the weight on bit, increasing then the shear stress (d) and therefore, affecting in fatigue life.

Figure 4 presents the *PDF's* obtained by the methods of *Dirlik* and *Tovo-Benasciutti* corresponding to the scenarios presented in Figure 3. It is observed, with the increase of the weight on bit, the decrease of *PDF* values associated with a greater range of stress amplitude.
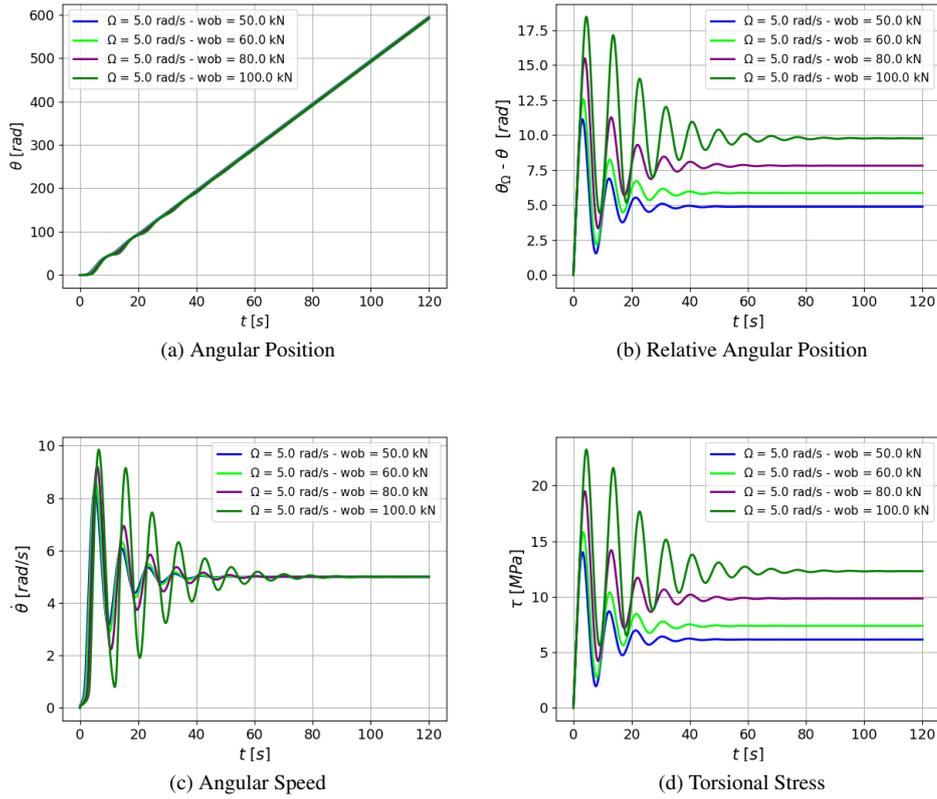
(a) Angular Position

(b) Relative Angular Position

(c) Angular Speed

(d) Torsional Stress

Figure 2: Operating conditions without *Stick-Slip*.

(a) Angular Position

(b) Relative Angular Position

(c) Angular Speed

(d) Torsional Stress
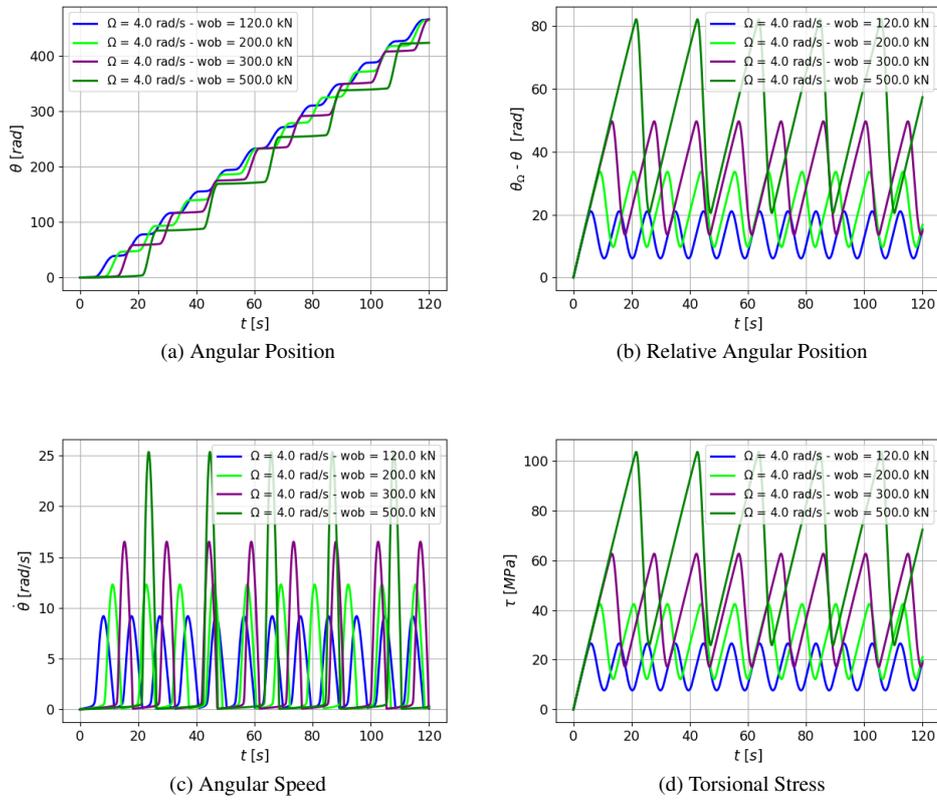
Figure 3: Operating conditions with *Stick-Slip*.

(a) $\Omega = 4.0rad/s$ and $WOB = 120.0kN$

(b) $\Omega = 4.0rad/s$ and $WOB = 200.0kN$)

(c) $\Omega = 4.0rad/s$ and $WOB = 300.0kN$

(d) $\Omega = 4.0rad/s$ and $WOB = 500.0kN$
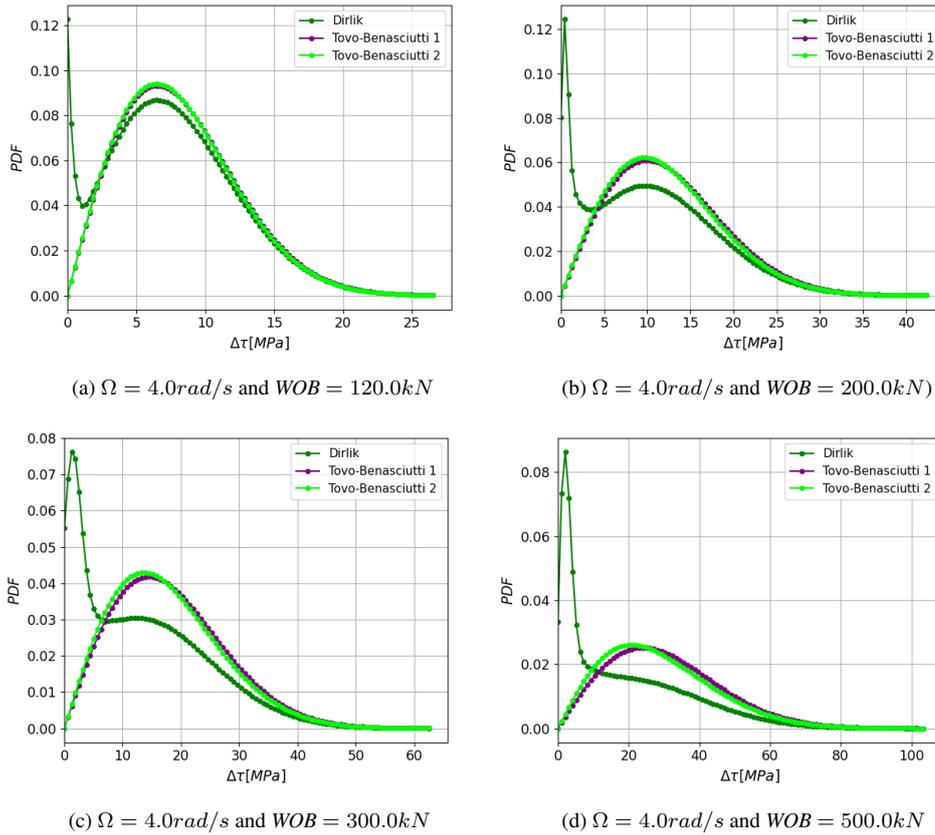
Figure 4: *PDF's* associated with *Dirlik* e *Tovo-Benasciutti* methods for different operation settings.

Figures 5 to 8 presents the fatigue life (in seconds) respectively, for the *rainflow*, *Dirlik*, *Tovo-Benasciutti 1* and *Tovo-Benasciutti 2* methods while Figures 9 to 11 presents the relative erros considering the *rainflow* method as a reference. Values of Ω from $4.0rad/s$ ($38.19rpm$) to $16.0rad/s$ ($152.78rpm$) were considered here, and for the weight on bit *WOB* an interval of $50.0kN$ to $500.0kN$. For infinite life, the limit value of $1.0 \cdot 10^{10}$ seconds was established. It is observed that, for a given value of Ω, the life decreases by increasing the weight on bit. The same occurs with the increase of Ω for a given weight on bit until a scenario in which the estimate reaches the configuration for infinite life, which can be justified by the fact that the imposed rotation wins the resistance offered by the medium. Figure 12 presents some of this results on a logarithmic scale for a better view to the methods treated here.
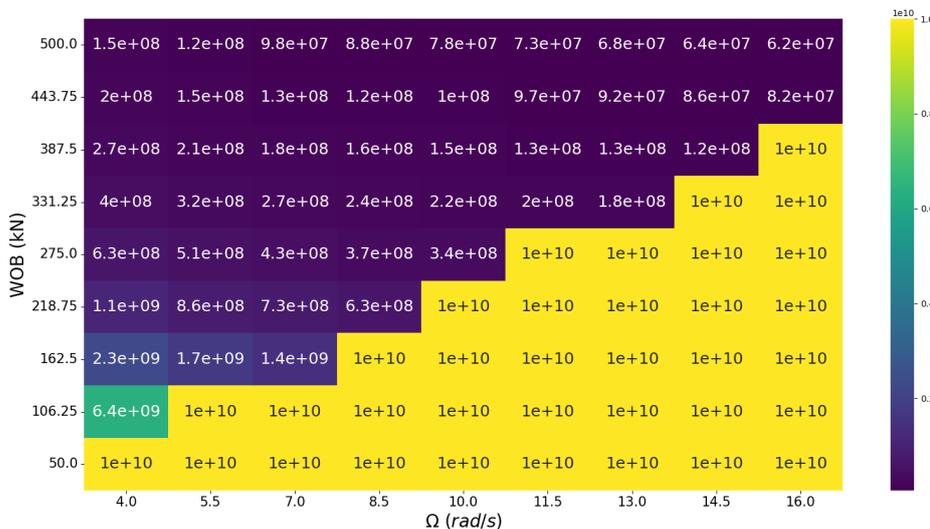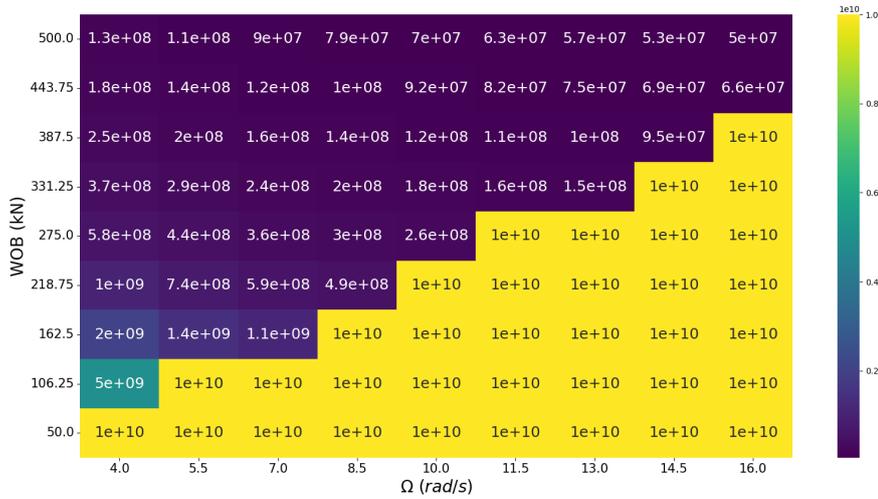

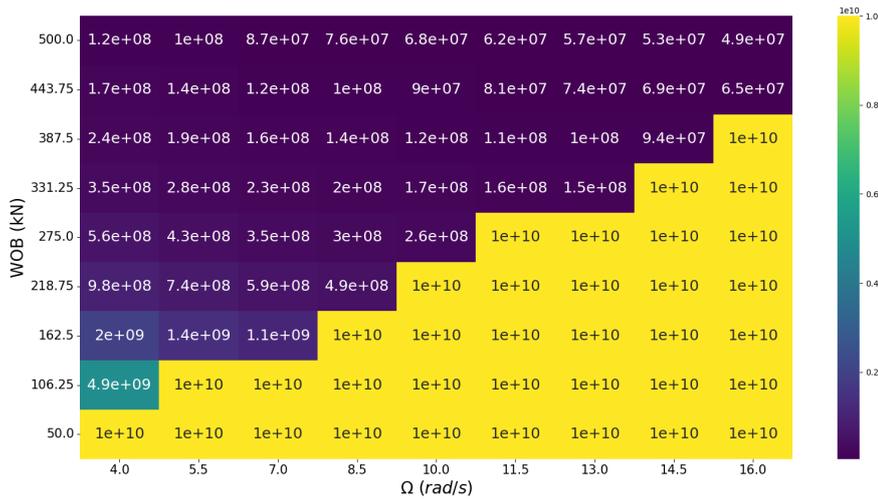Figure 5: Rainflow - Life (s)

Figure 6: Dirlik - Life (s)

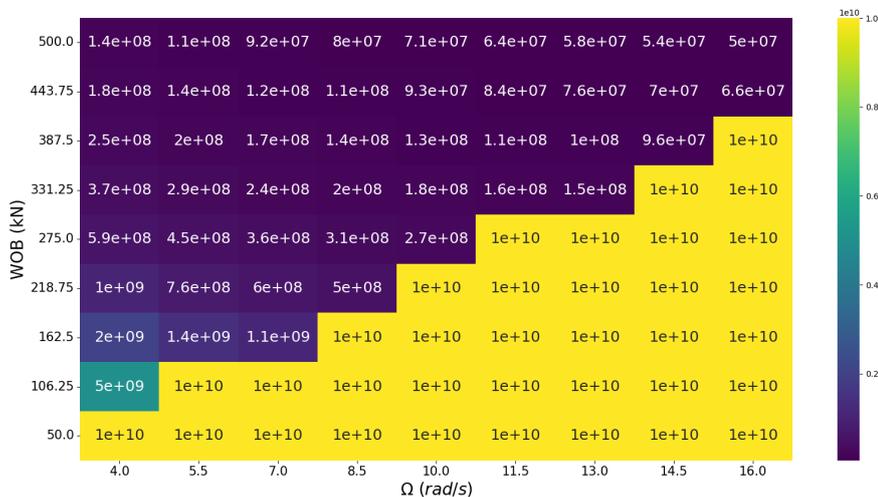Figure 7: Tovo-Benasciutti 1 - Life (s)
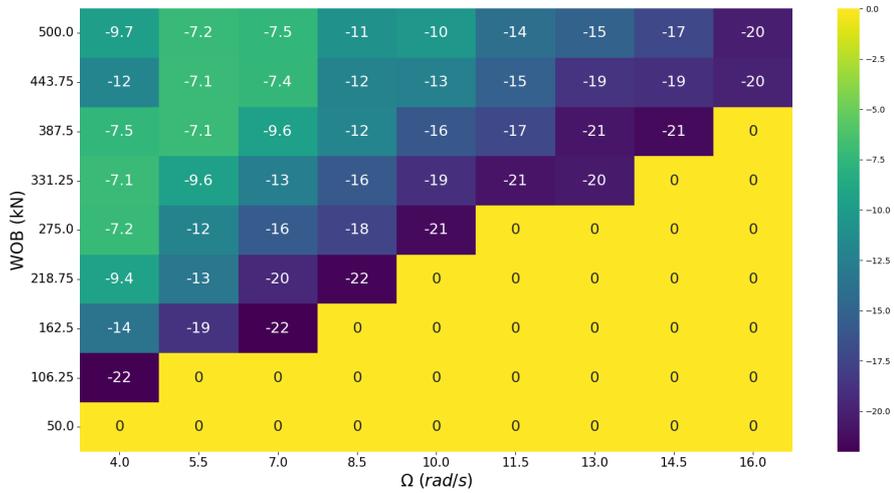

Figure 8: Tovo-Benasciutti 2 - Life (s)
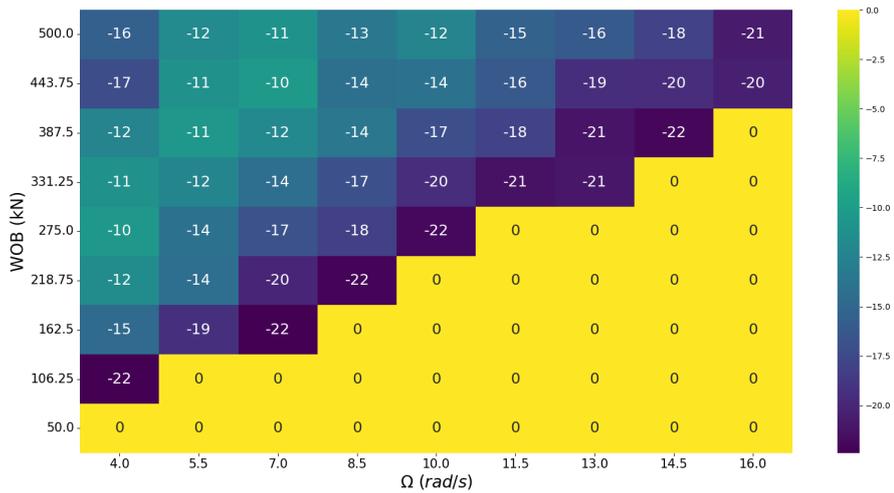
Figure 9: Dirlik - Rel.Error (%)

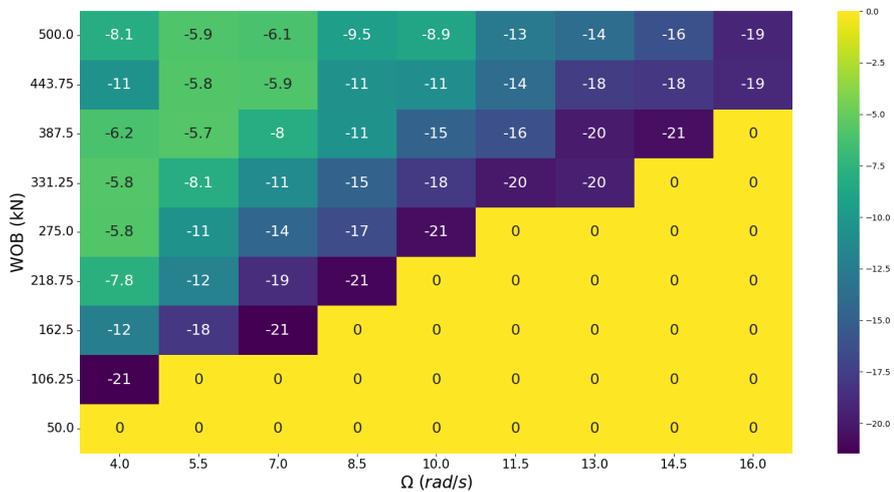Figure 10: Tovo-Benasciutti 1 - Rel.Error (%)
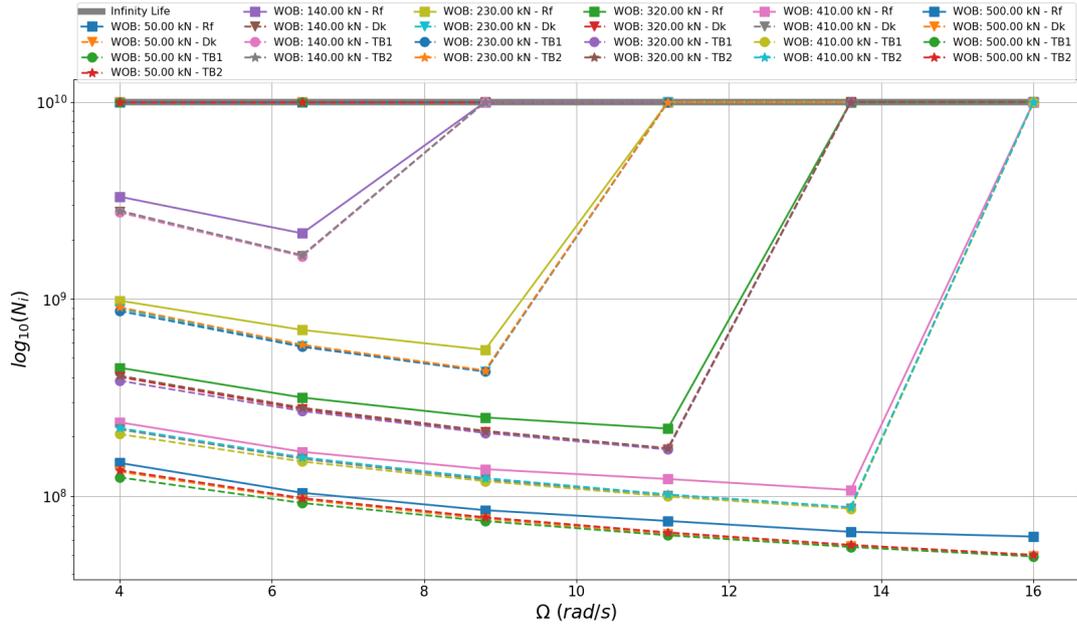

Figure 11: Tovo-Benasciutti 2 - Rel.Error (%)

Figure 12: Fatigue life as function of Ω and *WOB* for *rainflow* (Rf), *Dilik* (Dk), *Tovo-Benasciutti 1* (TB1) and *Tovo Benasciutti 2* (TB2) methods.

Although the processing time depends on the characteristics and configurations of each computer and other factors, Table 3 presents, as an example, some values for some cases regarding the methods used.

Table 3: Calculation Time Used in Methods (s) ($\Omega = 4.0 rad/s$ - $WOB = 500.0 kN$)

Observation Time (s)	Rainflow	Dirlik	Tovo-Benasciutti 1	Tovo-Benasciutti 2
1200.000	$1.0409 \cdot 10^{-3}$	$0.0000 \cdot 10^0$	$0.0000 \cdot 10^0$	$0.0000 \cdot 10^0$
12000.000	$7.3194 \cdot 10^{-3}$	$0.0000 \cdot 10^0$	$0.0000 \cdot 10^0$	$0.0000 \cdot 10^0$
120000.000	$8.4953 \cdot 10^{-2}$	$0.0000 \cdot 10^0$	$0.0000 \cdot 10^0$	$0.0000 \cdot 10^0$
1200000.000	$9.1788 \cdot 10^{-1}$	$0.0000 \cdot 10^0$	$0.0000 \cdot 10^0$	$0.0000 \cdot 10^0$
12000000.000	$2.3867 \cdot 10^1$	$3.8707 \cdot 10^{-3}$	$1.0643 \cdot 10^{-3}$	$2.6686 \cdot 10^{-3}$

4. CONCLUSIONS

The present work proposed to analyze fatigue in drill strings subjected to torsional vibrations with the *stick-slip* phenomenon. The fatigue life was computed in the time domain by the *rainflow* method and in the frequency domain using the *Dirlik* and *Tovo-Benasciutti* methods. The results for the spectral approach were conservative with the second method *Tovo-Benasciutti* presenting the best results in addition to these methods presenting a shorter simulation time. Therefore, the spectral approach to deal with fatigue life has shown, although studies and analyzes must be carried out, as a viable and efficient alternative to many engineering problems.

5. ACKNOWLEDGEMENTS

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