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COMPARATIVE NUMERICAL STUDY BETWEEN TWO FLEXIBLE ROTOR BALANCING METHODOLOGIES WITHOUT TEST MASS: NEURAL NETWORKS AND AUGMENTED KALMAN FILTER

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Abstract: *Among the most often failures in rotating machines, unbalance stands out, which occurs due to the irregular distribution of inertia along the axis of the rotor, that is, when the center of mass of the axis does not coincide with its geometric center. To mitigate this inconvenience, balancing techniques based on vibratory responses have been developed, either in the time or frequency domain. Among the main contributions in this field, we highlight the Modal Balancing, Influence Coefficient Method, and Method of four rounds without phase. Despite their wide application in industry, these techniques have limitations regarding the requirement of test masses. In this context, the main objective of this work is to perform a comparative study between rotor balancing techniques without the use of test masses. The first is based on the application of Artificial Neural Networks (ANN), which seeks to estimate the correction masses and the corresponding angular positions using the vibration responses of the rotor operating under normal conditions. It is worth mentioning that the ANN training process is performed using the vibration responses of the unbalanced rotor as inputs and the correction masses and the corresponding angular positions, as outputs, obtained through the application of a regular balancing technique (Influence Coefficient, for example). The second methodology is based on the implementation of the Augmented Kalman Filter (AKF) in conjunction with the optimization of the uncertainty matrices. Once the unbalance forces have been estimated, the associated correction masses are applied. The results showed that both methodologies, ANN and AKF, were able to perform the balancing satisfactorily, however, the ANN showed more promise. In addition, the ANN proved to be more interesting in terms of its computational costs, since it does not need the mathematical model of the rotor to be applied.*

Keywords: *Balancing based on model, Balancing based on signals, Artificial Neural Networks, Augmented Kalman Filter*

1. INTRODUCTION

By definition, the balancing of rotating machines consists of the use of masses, addition or removal, along the axis, to balance the unbalance forces present during the operation of such equipment. The main objective of this procedure is to make its geometric center and center of mass coincide, in this way, the vibration amplitudes will be reduced, consequently, reducing the efforts applied to the bearings and other structures (Ye *et al.* (2018)). Balancing techniques are divided into two major groups, signal-based methodologies (Influence Coefficient, Four Rounds Without Phase, Seven Rounds Without Phase, among others) and model-based methodologies (Modal Balancing and Unified Approach, for example). Due to their simplicity, signal-based methodologies are widely used in the industrial context, especially when it comes to small machines, in which the number of stops for application of the test masses is not a problem, and when there is little non-linearity in the relationship between the unbalance forces and the vibration responses. However, when it is intended to balance large machines, in which each machine stop represents a significant loss of resources, or when the dynamic behavior of the machine is nonlinear, these techniques do not present satisfactory efficacy, as discussed by Zhang *et al.* (2018). That said, an alternative solution to reduce the need for test masses is the use of model-based techniques, which can be found some applications in rotating machines in the works of, Cavallini Jr (2007), Cavallini Jr (2010), Rende (2015a), Rende (2015b) e Cavallini Jr (2016). However, they present as main disadvantage the need for a representative

model, otherwise the balance will also be impaired. In this sense, several methodologies were developed in order to improve both balancing modalities.

Villafañe Saldarriaga (2003) in his study, he proposed the use of two optimization techniques, Genetic Algorithm and Artificial Neural Networks. The methods basically consist of obtaining the response to the unbalance of the rotor, which is simulated by a model in Finite Elements. In both cases, the unbalance masses and their angular positions are the design variables of the optimizations. Next, it is enough to solve an inverse problem in which the obtained masses are installed in the balancing planes at 180° of the response obtained by the optimizations, in order to minimize the vibration amplitudes, the authors evaluated the methodology and discussed its limitations.

Villafañe Saldarriaga *et al.* (2009) developed an alternative methodology for balancing rotors in conditions in which vibration responses and unbalance forces no longer present a linear relationship, it is worth mentioning that in these cases conventional balancing methods do not present good accuracy. The technique consists of the determination of a transfer function obtained by the Artificial Neural Network, in which it relates the vibration responses with their unbalance forces. The new approach has been validated numerically and experimentally, proving its efficacy.

Villafañe Saldarriaga *et al.* (2011) aiming at the context in which the vibration response and the unbalance force are related in a non-linear way, the authors proposed an alternative methodology to correlate these two parameters, the method consists of simulating the rotor by means of the FEM in unbalance conditions based on the real parameters of the machine, such as stiffness and damping, then a pseudo-random optimization (GA) technique is used, having as objective function the vibration responses and the unbalance conditions (mass and angular position) as design variables, finally, the methodology was evaluated numerically and experimentally.

Carvalho *et al.* (2018) studied and developed a new methodology of revised influence coefficient, which consists of the pre-processing of the uncertainties that affect the measurements in the rotating machines, the data were evaluated from the perspective of fuzzy logic, so that the vibration responses recorded in a long period were considered by means of a fuzzy transformation, since the unbalance condition was determined by means of defuzification, which are finally introduced into the algorithm of the influence coefficient, in order to obtain the masses and angular positions of correction, the results showed the effectiveness of the method, when compared with the conventional one.

In this context, the present contribution aims to mitigate the need for the use of test masses, and a representative model in the rotor balancing process. For this, two balancing methodologies without masses of tests are evaluated. One through the application of Artificial Neural Networks (ANN), which has the advantage of being able to incorporate the nonlinearities present in flexible rotors, in addition to not using mathematical models. The other is the use of FKA which, because it uses the combination of real data and a mathematical model, has the ability to estimate the forces of unbalance in any degree of freedom, and unlike neural networks, does not need to investigate the relationships between the forces of unbalance and the vibration responses or the construction of a database.

Therefore, this work is organized as follows: the section 2 presents the FE modeling used in the definition of AKF and, as it is a numerical study, it will also be used in the generation of samples for ANN. The section 3 deals with the fundamental concepts of an ANN and the balancing methodology employed, the section 4 presents the formulation of FKA and its application in the identification of forces in rotating machines, the section 5 presents the main results and makes a comparison between both methodologies. Finally, the conclusions are summarized in the section 6.

2. ROTOR FINITE ELEMENT MODEL

Due to the need of the mathematical model in the implementation of the Augmented Kalman Filter, and to obtain a parameter of comparison for the techniques of balancing via Artificial Neural Networks, a bench was modeled by means of finite elements, for this, the procedure described by (Lalanne and Ferraris, 1998) was adopted, which consists of the calculation of the kinetic and potential energies of all the components of the rotor, as well as the virtual work of external forces. Expressions of kinetic energy are required to characterize disks, axis, and unbalance mass, while strain energy is also required to characterize the axis. Then the elementary equations are combined to get the general equation of the rotating machine, so that we can analyze the behavior of the rotor at each node. The general equation describing the dynamic behavior of the rotor is obtained from the sum of the elementary matrices of disk, shaft, bearings, and unbalance, as expressed by the Equation 1,

$$[M]\{\ddot{q}(t)\} + [C + \Omega G]\{\dot{q}(t)\} + [K_1 + \dot{\Omega}K_2]\{q(t)\} = F_u(t), \quad (1)$$

where $[M]$ is the mass matrix, $[C]$ is the damping matrix, $[G]$ is the matrix associated with gyroscopic effects, $[K_1]$ is the stiffness matrix, $[K_2]$ is the stiffness matrix due to transient motion, and $[q(t)]$ is the displacement vector. In addition, $F_u(t)$ represents the result of the weight, unbalance, and reaction forces present in bearings. As previously mentioned, in the implementation of the AKF, it needs a mathematical model of the system, so, as it is a theoretical evaluation, two numerical models will be used, one representing a real system, and the other representing the model used in the construction in the AKF. Figure 1 presents the finite element model, modeled using the ROSS library, it was discretized into 38 elements, 1000 mm long ($E=205$ GPa, $\rho=7850$ kg/m³, $\nu=0.29$), in the model was incorporated two disks D_1 (node #12) and D_2 (node #23).

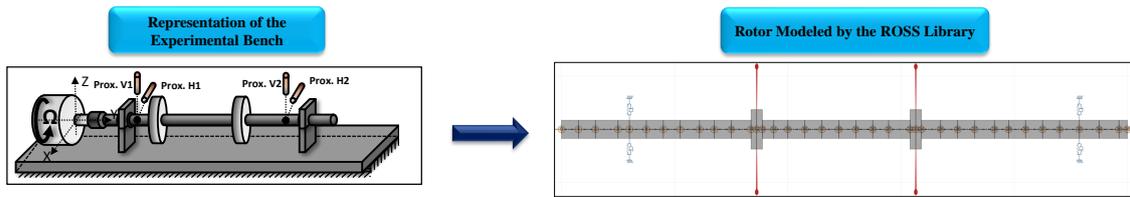


Figure 1. Finite Element Representation of the Model Under Study.

3. ARTIFICIAL NEURAL NETWORKS

In the present contribution, the neural network seeks to investigate the relationship between vibration responses and unbalance forces. Importantly, a neural network basically consists of three types of layer, namely: input layer, hidden layers, and output layer. The number of neurons and number of layers depend almost exclusively on the type of problem to be solved. However, optimization techniques and design patterns can serve as tools to assist in the construction of the network pattern in question (Vyas and Satishkumar, 2001). In general, the neurons present in the hidden layers will process and thus learn a certain pattern, they receive the inputs x_n which are multiplied by the weights w_k and the product is added to the bias b_k , this result becomes the argument of the activation function $f(\cdot)$, which results in the output vector y_k , as represented in Figure 2.

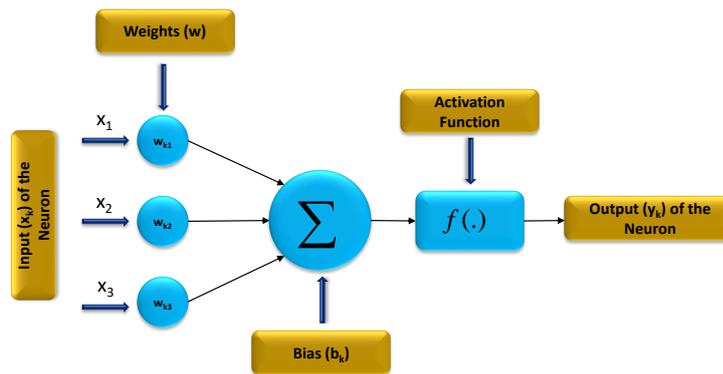


Figure 2. Schematic Representation of a Neuron.

In the field of balancing rotating machines, methodologies can be divided into two broad categories, model-based methods, and signal-based methods, i.e., vibration responses. Model-based methods have the advantage of not requiring test masses to estimate the relationship between vibration responses and the forces present in machines, nor of holding a database, as in the case of neural networks. However, these methods depend heavily on the representativeness of the real system, that is, the more representative the model, the more accurate its estimates of the forces will be. Signal-based methods, on the other hand, investigate this relationship more directly, usually assuming that the vibration response and the unbalance forces have a linear relationship, which is not always true. In this context, neural networks are presented as powerful tools in this investigation, since they can deal with the nonlinearities present in the mechanical system.

Multilayer Perceptrons (MLPs) is a class of feedforward artificial neural networks, in which systems of interconnected perceptrons, the basic unit in Artificial Neural Networks, apply nonlinear transformations over the linear combination of their inputs (Figure 2). This approach aims to obtain a nonlinear mapping between input and output vectors, and it is the superposition of many simple nonlinear transformations that allows MLPs to approach extremely complex behaviors. Figure 3 provides a schematic representation of a conventional feedforward MLP applied to rotor balancing.

In addition to the great versatility of neural networks, one of their limitations is the need for a large number of test samples, so that their estimates present high accuracy. And this can make it difficult to apply, since in real systems, usually, the sample number is quite limited. To deal with this inconvenience, the methodology developed by Pereira Neto *et al.* (2023) was applied, in which virtual samples can be obtained from a few experimental samples.

In essence, the method consists of creating virtual samples from real samples, varying the angular positions of the masses and phases of the unbalancing conditions. It is worth mentioning that in this method the IC is used only to determine the positions and masses of corrections, another method could be used for this purpose. Figure 4 presents the general flowchart for the technique employed.

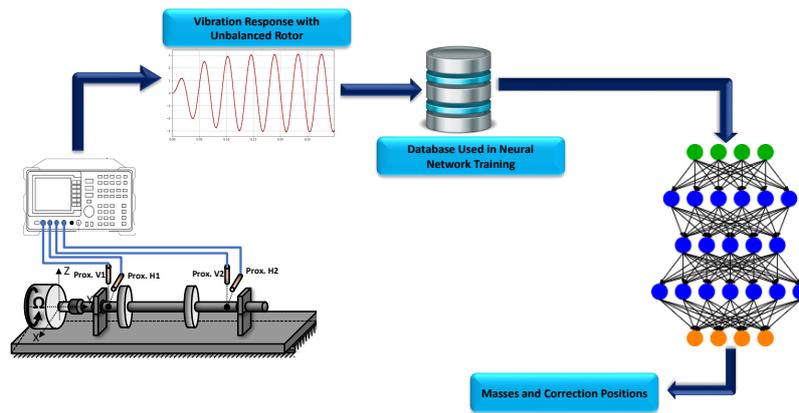


Figure 3. Representation of the balancing method using conventional ANN.

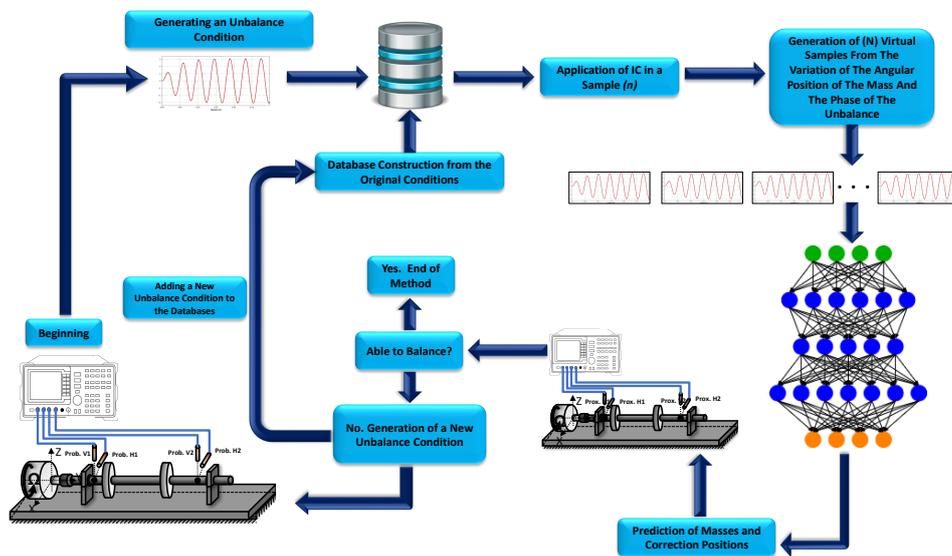


Figure 4. Representation of the balancing method using ANN with virtual sample generation proposed by Pereira Neto *et al.* (2023).

4. KALMAN FILTER

Among the main limitations of model-based balancing techniques is that they rely heavily on the accuracy of the modeling of the real system, which may limit them to simple systems, or the need for a prior study of the dynamic behavior of the system. In both situations, these methodologies face difficulties in implementations, especially at the industrial level. That said, one way to reduce this dependence is to combine the estimates obtained by the mathematical model and the measurements made in the real system. Thus, the Augmentation Kalman Filter presents itself as a viable tool for the incorporation of rational data into the model without the need to know the dynamic behavior of the rotor. By definition, AKF is a state estimator, so in the rotor-dynamics conjecture, the Equation 1 should be rewritten according to the following procedures. Figure 5 presents the general idea of how the Kalman Filter works. In essence, the technique consists of the application of the conventional Kalman Filter in the Augmented State Space, in which the forces cease to be control variables and become state variables. Next, the procedure for the construction of the Augmented State Space model applied to rotating machines will be demonstrated, and the definitions and equation of the conventional Kalman Filter will be presented.

For the modeling of mechanical systems, the Augmented State Space is obtained from the discrete time equation added to an unknown uncertainty vector w_k as being a stochastic process variable $\{w_k \in \mathbb{R}^{n_s}\}_{k=0}^{\infty}$, where n_s is related to the number of states, furthermore, A and B refer to the state and input matrices, respectively, as presented by the Equation 2.

$$x_{(k+1)} = Ax_{(k)} + BFu_{(k)} + w_{(k)} \quad (2)$$

As previously mentioned, the forces present in the system must be incorporated into the problem as state variables, for this, the Equation 2 is complemented with an expression that directly relates the force vectors at times k and $(k + 1)$, as

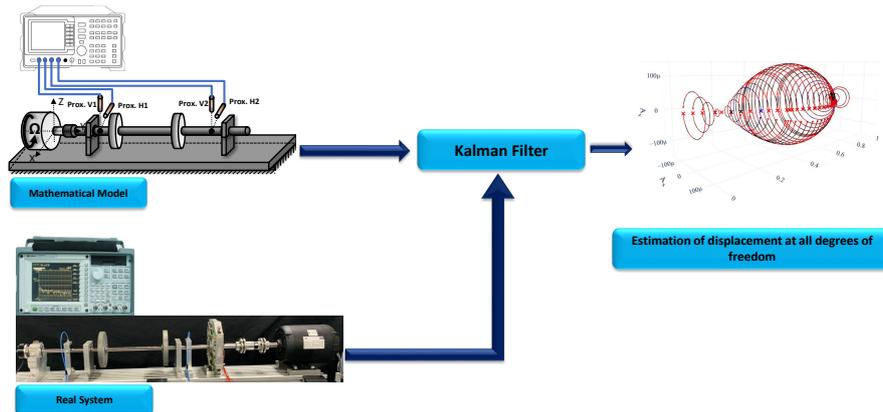


Figure 5. Schematic representation of the application of the Augmented Kalmana Filter.

expressed by the Equation 3.

$$F_{u(k+1)} = F_{u(k)} + \eta(k) \quad (3)$$

The Equation 3 allows the estimation of the time history of the force through an appropriate choice of the covariance matrix of the process $\eta(k)$. Therefore, to obtain the Augmented State Space, simply combine the equations 2 and 3 and redefine the state vector X^a , as follows.

$$X_{(k)}^a = \begin{bmatrix} x_{(k)} \\ F_{u(k)} \end{bmatrix} \quad (4)$$

So that one can define AKF by the Equation 5.

$$x_{(k+1)}^a = A^a X_{(k)}^a + \xi(k) \quad (5)$$

Where the Augmented State Matrix $A_a \in \mathbb{R}^{(n_s+n_p) \times (n_s+n_p)}$ can be defined as,

$$A^a = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \quad (6)$$

and the noise vector $\xi(k) \in \mathbb{R}^{(n_s+n_p)}$, assumed to be Gaussian averaging zero, which combines the uncertainties arising from the modeling $w(k) \in \mathbb{R}^{n_s}$, as well as the uncertainties associated with the forces $\eta(k) \in \mathbb{R}^{n_p}$, as expressed by the Equation 7.

$$\xi(k) = \begin{bmatrix} w(k) \\ \eta(k) \end{bmatrix} \quad (7)$$

Similar to the conventional KF definition, the discrete-time observation equation with an increased uncertainty vector $v_{(k)}^a$, can be defined as expressed by the 8 equation, calls for the replacement of $X_{(k)}$, by $X_{(k)}^a$, and H by H^a , thus defining the Augmented State observation equation,

$$y_{(k)} = H^a X_{(k)}^a + v_{(k)} \quad (8)$$

where, $y_{(k)} \in \mathbb{R}^{n_d}$, represents the experimental output vector, the matrix $H^a \in \mathbb{R}^{n_d \times (n_s+n_p)}$, is assembled from the output and feed matrices C and D, respectively, Equation 9.

$$H_a = [C \quad D] \quad (9)$$

Briefly, the Augmented State Space equations can be formulated as expressed by the following equations.

$$x_{(k+1)}^a = A^a X_{(k)}^a + \xi(k) \quad (10)$$

$$y_{(k)} = H^a X_{(k)}^a + v_{(k)} \quad (11)$$

4.1 Conventional Kalman Filter Formulation

One can define KF as a linear recursive estimator, developed so that it is optimal for a minimal variance. In order to have a proper understanding of Kalman's algorithm, it is necessary to elucidate some definitions. Initially, $\hat{X}_{(k|s)}^a$ can be defined as the estimate of $\hat{X}_{(k)}^a$ at a given instant $\{y_{(n)}\}_{n=0}^s$. It should be noted that $\hat{X}_{(0|-1)}^a$ is related to the initial estimate of $\hat{X}_{(k)}^a$ at a time $k = 0$. In addition, the error propagation matrix can be defined based on the Expectation of the difference in the actual and estimated data, as follows:

$$P_{(k|s)} = \mathbb{E}[(x_{(k)}^a - \hat{x}_{(k)}^a)(x_{(k)}^a - \hat{x}_{(k)}^a)^T] \quad (12)$$

Furthermore, it can be assumed that the covariance matrix of the initial error $P_{(0|-1)}$, as well as the initial estimate of the $\hat{X}_{(0|-1)}^a$ state are known. As such, note that the initial state $\hat{X}_{(0)}^a$ is considered a random variable and does not need to be known in advance. The uncertainty processes $\{w_{(k)} \in \mathbb{R}^{n_s}\}_{k=0}^{\infty}$, $\{v_{(k)} \in \mathbb{R}^{n_s}\}_{k=0}^{\infty}$ and $\{\eta_{(k)} \in \mathbb{R}^{n_s}\}_{k=0}^{\infty}$ are considered stationary, mutually uncorrelated stochastic processes with zero mean. Their covariances are considered known and are represented by matrices $Q \in \mathbb{R}^{n_d \times (n_s + n_p)}$, $R \in \mathbb{R}^{n_d \times (n_s + n_p)}$ and $S \in \mathbb{R}^{n_d \times (n_s + n_p)}$, respectively, and are defined by applying the Expectation function to the following uncertainties.

$$\mathbb{E}\{w_{(k)}w_{(s)}^T\} = Q, \quad \mathbb{E}\{v_{(k)}v_{(s)}^T\} = R, \quad \mathbb{E}\{\eta_{(k)}\eta_{(s)}^T\} = S \quad (13)$$

It is noteworthy that the matrices Q and R are associated with the uncertainties of the mathematical model estimates, and of the experimental data, respectively. They are responsible for guiding the Kalman gain, that is, if at the instant k , the algorithm will favor the estimates of lower uncertainty. When it comes to AKF, a new matrix is defined arising from the uncertainties associated with the forces of the system; With these forces cannot be measured directly, the uncertainties are associated with the model, being S a regulatory matrix. Normally, this matrix is defined through a regulation process, called L curve, however, this method will not be used in this contribution. Figure 6 shows the flowchart the Augmented Kalman Filter.

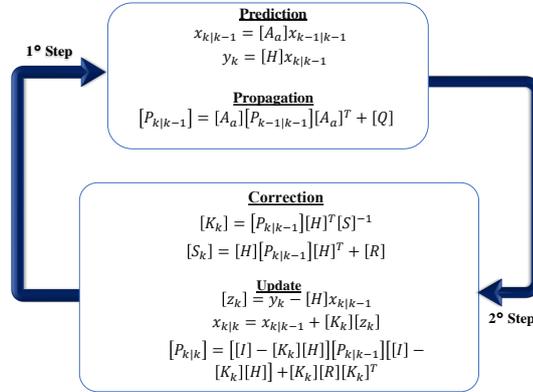


Figure 6. Schematic Representation of the Augmented Kalman Filter.

The accuracy of the AKF estimate is directly related to the choice of these matrices, therefore, an optimization process was carried out, in the matrices Q , R and S are the design variables, and the objective functions are defined as expressed by the mono-objective functions:

$$R_1^2(y_1, \hat{y}_1) = 1 - \frac{\sum_{i=1}^n (y_1(i) - \hat{y}_1(i))^2}{\sum_{i=1}^n (y_1(i) - \bar{y}_1(i))^2} \quad (14)$$

$$R_2^2(y_2, \hat{y}_2) = 1 - \frac{\sum_{i=1}^n (y_2(i) - \hat{y}_2(i))^2}{\sum_{i=1}^n (y_2(i) - \bar{y}_2(i))^2} \quad (15)$$

$$R_3^2(y_3, \hat{y}_3) = 1 - \frac{\sum_{i=1}^n (y_3(i) - \hat{y}_3(i))^2}{\sum_{i=1}^n (y_3(i) - \bar{y}_3(i))^2} \quad (16)$$

where R_1 represents the difference between the actual displacements, y_{1_i} , located in the (nodes #8 and #28), and the displacements estimated by the AKF, \hat{y}_{1_i} , obtained at the same positions. Since the forces cannot be measured directly, we used the AKF force estimates and the Equation 1 to compose the second objective function R_2 , where y_{2_i} represents the displacements obtained by the mathematical model caused by the force F_u , and \hat{y}_{2_i} represents the displacements estimated by the AKF, also in the positions of the sensors. R_3 is also defined by the difference between the displacements obtained by the Equation 1, caused by the forces F_u , differing only that in this case, all degrees of displacement freedom are evaluated, rather than being evaluated only on the sensors. Finally, the *Pareto* curve was constructed based on the values found during the multi-objective optimization by the compromise function, Lobato *et al.* (2016).

5. RESULTS AND DISCUSSION

In this section, the main results of estimating the unbalancing forces with both methodologies will be presented. As this is a numerical analysis, a white signal-noise rate (SNR), with 1%, was added to the signal generated by the finite element model. In this case, the efficacy of the methods, ANN and AKF, in this condition was evaluated. For the analysis in question, it was decided to generate 10 samples with different unbalance conditions, 5 of which were intended for ANN training, and 5 for its validation, as presented in the section 3, it should be noted that these samples were generated under the same conditions in which the AKF was applied. After the training of the network, the estimation of the unbalance forces is carried out, in this case the masses and correction positions are determined. Figure 7 presents a comparison of the masses of angular positions of the unequilibrium conditions, real and estimated by ANN (nodes 12# and 23#).

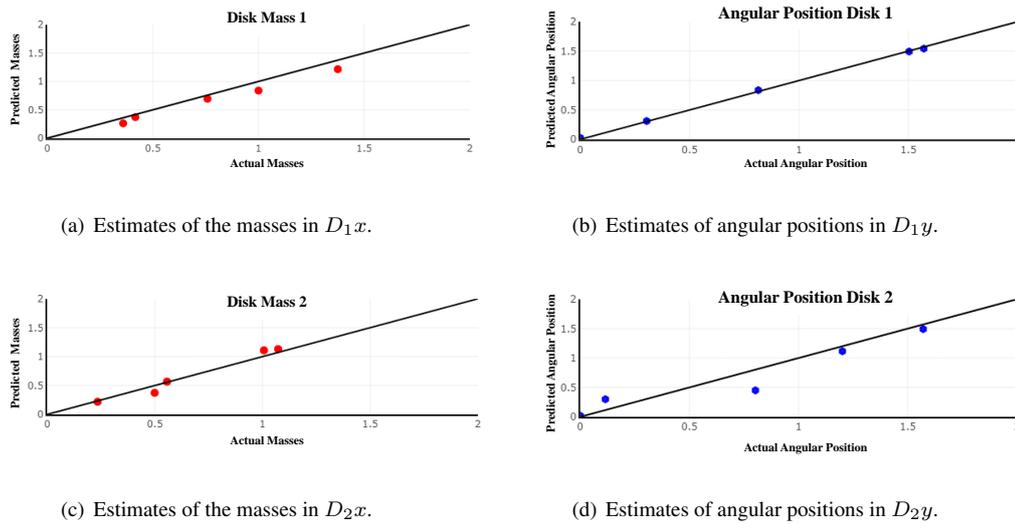


Figure 7. Comparison between the data with white noise of intensity of 1% and the signals predicted by ANN under five different conditions.

In the case of the application of AKF, there is no need to build a database, the estimate of the force can be performed directly, just by collecting the signals in the sensors. Figure 7 shows the comparison between the displacements with 1% of SNR and the signals predicted by the AKF at the sensor positions (nodes 8# and 28#). After the implementation of AKF, the unbalance forces on the disks were estimated, D_1 and D_2 , with this, one can determine the masses and positions of corrections. Figure 9 presents a comparison between the actual forces and the estimates by the AKF.

It can be seen that both methodologies were able to estimate the unbalance forces present on the Disks (nodes 12# and 23#) with good accuracy, however, due to the intrinsic characteristics of the methods, they present advantages and disadvantages in relation to each other. For example, AKF does not need the construction of databases, once the optimization process of the uncertainty matrices Q , R and S has been carried out, it is enough to apply the Kalman algorithm to obtain the forces and displacements in any degree of freedom, however, this is a costly process and its accuracy is affected by the location of the sensors, mathematical model used and by the dynamic behavior of the rotating machine (Shrivastava and Mohanty, 2018). In the case of ANN, if on the one hand, it does not depend on the use of any model of the rotating machine (which significantly decreases the processing time), on the other hand, it requires the construction of a database, which usually makes its use inefficient in some real applications. In our case, due to the methodology employed, the number of samples used was reduced, 5 for training, but still, it represents a disadvantage when compared to the AKF, in this regard.

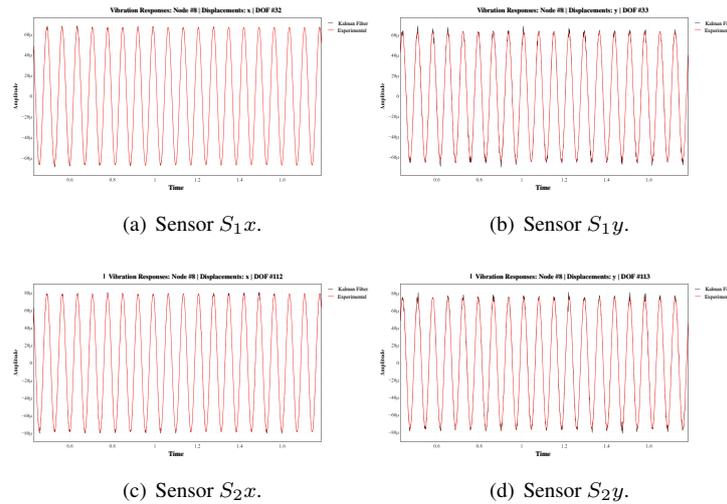


Figure 8. Comparison between the data with white signal-noise rate of 1% and the signals predicted by the AKF in the positions of the sensors.

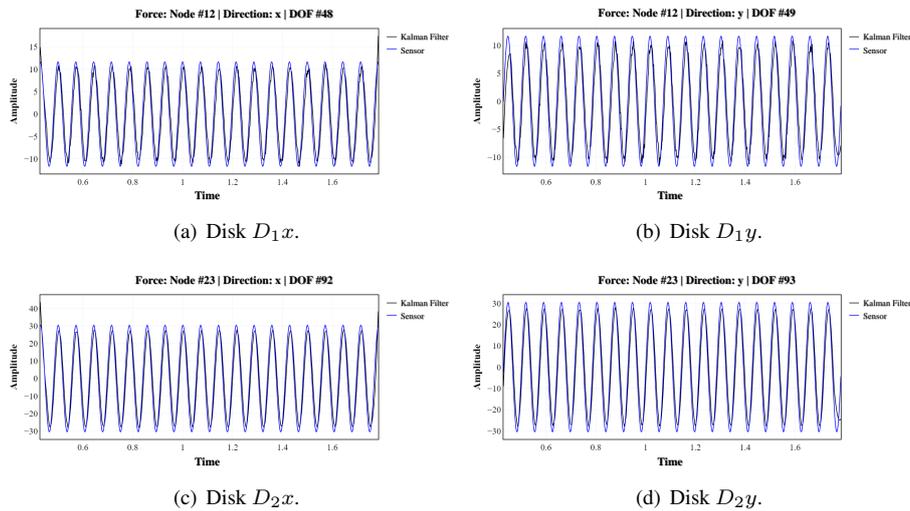


Figure 9. Comparison between the data with white signal-noise rate of 1% and the signals predicted by the AKF in the positions of the Disks, before and after the balancing.

6. CONCLUSION

In the present work, a numerical analysis between two balancing methodologies without test masses was performed, being AKF, a model-based method and ANN, a signal-based methodology. For the study in question, they were evaluated under the same conditions (with white signal-noise rate of 1 %). By analyzing the results, it is noted that ANN requires a much shorter computational time, on average 5 % of that used by AKF. Since ANN does not require a representative rotor model, this method can be applied to any rotating machine, provided that the Network understands the relationship between the unbalance forces and the rotor vibration responses. However, this methodology requires a small database. The AKF can be used online, because it estimates the unbalance forces based only on the displacements measured in the sensors, and the mathematical model, that is, unlike the ANN, there is no need to understand the relationship between the unbalance forces and the vibration responses, nor the need to apply test masses. Therefore, both methodologies are presented as alternatives to the balancing of rotating machines without mass of tests, however, due to the simplicity of operation, the low computational effort and the absence of mathematical models of the rotating machine, the ANN presents itself as the best alternative, especially when the location of the unbalance planes is not previously known. Last but not least, the authors intend to carry out new experimental investigations, in order to validate both methodologies and evaluate the convergence in relation to the numerical results.

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