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Improved stochastic modeling of hygrothermal influences on the buckling response of laminated composite plates

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Abstract. *The effects of moisture and temperature on the buckling response of laminated composite plates are numerically investigated in this paper. Due to the complexity of physical mechanisms which determine the environmental and operational conditions in many cases of interest, temperature and moisture variations around a structure can be hardly controlled, and can be more appropriately characterized as random quantities. Hence, in the context of Uncertainty Quantification (UQ), this paper aims at investigating the influence of space-dependent stochastic hygrothermal conditions on the buckling resistance of composite laminate plates. Differently from the majority of previous investigations, uncertainty quantification takes into account the combined influences of the environmental effects on the properties of the material and the hygrothermally-induced stresses. For this purpose, a model based on the Rayleigh-Ritz approach under the hypotheses of the Classical Lamination Theory is proposed to perform buckling analysis considering hygrothermal and mechanical influences, using the micromechanical approach to account for the degradation of material properties as a consequence of space-dependent random variations in environmental factors. The Karhunen-Loève expansion is used to discretize thermal and moisture fluctuations as stationary two-dimensional random fields. Then, sampling-based statistics are presented for the critical buckling load adopting different values of standard deviations and correlation lengths associated to the random fields. From the simulations scenarios analyzed, it is observed an increase in dispersion of buckling resistance as both correlation length and standard deviation are increased. Also, the convenience of accounting for random environmental uncertainties in the analysis and design of reliable and robust composite structures is highlighted.*

Keywords: *hygrothermal-mechanical buckling, uncertainty quantification, random field, micromechanical modeling, laminated composite plates.*

1. INTRODUCTION

The use of composite materials has been continuously increased in a variety of engineering areas in the last decades, in particular the aerospace industry. Benefits including high strength-to-weight ratios, attractive behavior regarding impact and corrosion resistance, larger design space, and lower thermal conductivity, among others, can explain their increasingly demand in the quest for weight savings combined with structural performance. In aerospace applications, severe environmental conditions are common and, consequently, composite materials are generally exposed to temperature and moisture variations, which may have significant effects on their structural responses (Gibson, 2012).

Physical mechanisms of heat conduction and moisture diffusion associated to either the surrounding conditions or normal system operation can degrade the composite material properties. Matrix elastic and hygrothermal properties are more affected by temperature and/or moisture changes than those of the embedded fibers. According to Gibson (2012), increase in temperature causes softening of the polymer matrix and, if the glass transition region (transition from glassy to rubbery behavior) is achieved, the structural requirements can be seriously compromised. Such undesirable behavior can be aggravated when the glass transition is brought to lower temperatures as a result of moisture increase.

In addition, the temperature and/or moisture increase leads to expansion/swelling of the polymer matrix, whereas contraction is engendered by a decrease in these quantities. As fibers are practically insensitive to hygrothermal changes, residual stresses develop, leading to modifications of stress and strain distributions within the composite.

The influence of hygrothermal stresses on the structural behavior of conventional unidirectional fiber laminates has been investigated by many researchers. In (Whitney and Ashton, 1971), the analyses of bending, vibration, and buckling of symmetric and unsymmetric laminates, including the effect of expansional strains induced by temperature and swelling agent, are conducted under the hypotheses of the Classical Lamination Theory (CLT) and a generalization of the Duhamel-Neumann form of Hooke's law. Different moisture concentrations and temperatures are considered in the work of Patel *et al.* (2002), where a higher-order finite element model is employed to evaluate the structural behaviour of deflection, buckling and natural frequency of thick composite laminates, accounting for degradation in lamina material properties at elevated temperature and moisture. Also regarding natural frequencies and buckling characteristics of composite laminated plates under hygrothermal effects, Amoushahi and Goodarzian (2018) compared delaminated plates with non-delaminated ones for different boundary conditions under increased temperature and moisture concentration, showing that reduction rate of buckling and natural frequency in the presence of delamination was intensified by environmental changes.

An extensive literature regarding the analysis of laminated composites and sandwich structures subjected to hygrothermal conditions is presented in a benchmark review conducted by Garg and Chalak (2019). Important outcomes of many researches are highlighted and categorized according to the type of displacement field, structural analysis, thermal/hygro profile, and method used during hygro-thermal analysis, also providing directions for future studies in themes to be further explored.

In many cases of practical interest, the use of purely deterministic approaches in analyses involving material properties and induced stresses under hygrothermal conditions, as in the case of the previous cited studies, may become inadequate. In fact, complex combinations of physical mechanisms of heat and moisture transfer between the material and the surrounding media can hardly be controlled, which makes more appropriate to consider the temperature and moisture variations at different points of a structure as space-distributed random quantities. Such stochastic characterization is even more justified when it comes to composite materials, as significant variability in material properties can arise from environmental influences, in addition to uncertanties engendered by their complex manufacturing process, which propagate to the overall stiffness and strength of the material. This highlightes the importance of incorporating uncertainties in the performance assessment of composite structures (Zhou *et al.*, 2017).

Random variations of lamina thickness and material properties of straight-fiber laminate plates subjected to uniform temperature/moisture changes were considered in the investigation of hygrothermal buckling loads conducted by Singh and Verma (2009). Using the first order perturbation technique (FOPT) in the probabilistic analysis approach combined with the finite element method, variabilities in thickness demonstrated to have greater impact on buckling response scattering under hygrothermal effects than individual random changes in material property.

Kumar *et al.* (2015) also analyzed the hygrothermal buckling response of laminated composite plates with randomness in the material properties, considering both temperature independent and dependent thermoelastic properties. The influence on hygrothermal buckling load of boundary conditions, thickness and aspect ratios, fiber volume fraction and changes in temperature and moisture concentration is investigated in various numerical simulations.

The adequate stochastic characterization of some random quantities can require the space-dependent (1-, 2-, or 3-dimensional) variability representation, as previously mentioned regarding variations in temperature and moisture conditions throughout a structure. The concept of random fields generally involves discretization techniques, the most popular of which is the Karhunen-Loève Expansion (KLE) (Ghanem and Spanos, 1991).

Investigations considering spatially varying system properties of composite laminates discretized by KLE have been conducted in some studies. In (Naskar *et al.*, 2018), the effect of spatial randomness of micro and macro-mechanical material properties based on KLE is quantified to characterize the probabilistic descriptions of stochastic dynamics and stability of composite laminates. Adopting an uncertainty propagation strategy based on Monte Carlo simulation, where a surrogate model was developed to decrease the computational intensiveness, both types of cascading effect in stochasticity (micro and macro-scale approach) were compared considering the influence of different boundary conditions and correlation lengths of the random field.

Recently, Parviz *et al.* (2022) investigated spatially varying stochastic mechanical properties of laminated composite plates as dependent of temperature. Then, using KLE to discretize the stochastic thermal field, stochastic thermal buckling analysis was performed for different plate aspect ratios and coefficients of variations (COV) of temperature gradient, showing significant influence of uncertainty in temperature distribution on the stability of composite plates.

In the context delineated above, the present paper aims to evaluate the effect of stochastic space-dependent temperature and moisture variation on the buckling response of laminated composite plates. The main contribution is the consideration of spatial randomness of hygrothermal conditions influencing both the material properties and induced stresses, which, to the best of authors' knowledge, have not been considered so far. Thermal and moisture fluctuations throughout a composite plate are independently discretized by KLE as stationary two-dimensional random fields. Consequent randomness in environmental-dependent material properties, modeled using the micromechanical approach, and in hygrothermally-induced stresses are included in the plate modeling based on Rayleigh-Ritz method under the hyphotheses of the classical lamination theory (CLT). The uncertainty quantification of the buckling load considers different standard deviations and correlation lengths assigned to the random fields, and stochastic numerical analyses are performed by Monte Carlo Simulation (MCS).

2. FORMULATION

2.1 Temperature and moisture random fields discretized by KLE

In a rectangular plate with dimensions $a \times b$, illustrated in Figure 1, the stochastic temperature distribution is represented as a two-dimensional random field, described using KLE in terms of the normalized coordinates $\zeta = 2x/a$ and $\eta = 2y/b$, as follows (Ghanem and Spanos, 1991):

$$T(\zeta,\eta,\Theta) = \bar{T} + \Delta T(\zeta,\eta,\Theta) = \bar{T} + \sum_{n=1}^{M_T} \sqrt{\lambda_{Tn}}\xi_{Tn}(\Theta) f_{Tn}(\zeta,\eta) \ , \tag{1}$$

where $\bar{T}$ is the mean temperature, Θ belongs to the sample space of an appropriate random space, and the variation ΔT denotes zero-mean fluctuations of the random field about the mean value. A truncated series is kept in KLE using M_T terms, a set of zero-mean independent Gaussian random variables ξ_{Tn}, and the eigenvalues λ_{Tn} and eigenfunctions f_{Tn}, which are solutions of the Fredholm integral equation associated with a determined covariance function.

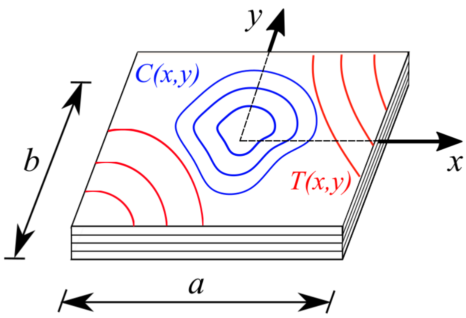


Figure 1: Rectangular laminated composite plate under non-uniform temperature and moisture distributions.

Only certain types of covariance function have analytical solutions to the integral eigenvalue problem, such as the exponential one, which is adopted in this study and has been extensively used in the literature. Hence, the applicable integral eigenvalue problem is expressed as follows:

$$\int_{-1}^{1}\int_{-1}^{1} e^{-|\zeta_1-\zeta_2|/l_{T\zeta}-|\eta_1-\eta_2|/l_{T\eta}} f_{Tn}(\zeta_2,\eta_2) d\zeta_2 d\eta_2 = \lambda_{Tn} f_{Tn}(\zeta_1,\eta_1) \ , \quad n = 1,...,M_T \ , \tag{2}$$

where $l_{T\zeta}$ and $l_{T\eta}$ are the correlation lengths in the ζ and η directions, respectively.

The solution of Eq. 2 above is found in details in (Ghanem and Spanos, 1991).

Similarly, the two-dimensional random field representing the stochastic moisture distribution can be discretized by KLE as follows:

$$C(\zeta,\eta,\Theta) = \bar{C} + \Delta C(\zeta,\eta,\Theta) = \bar{C} + \sum_{n=1}^{M_C} \sqrt{\lambda_{Cn}}\xi_{Cn}(\Theta) f_{Cn}(\zeta,\eta) \ , \tag{3}$$

where $\bar{C}$ is the mean moisture content in the matrix material and ΔC represents the zero-mean variation of the moisture random field. The truncated KLE comprehends M_C terms, a set of zero-mean independent Gaussian random variables ξ_{Cn}, and the eigenvalues λ_{Cn} and eigenfunctions f_{Cn} solutions of a integral equation similar to Eq. 2, as indicated below, where $l_{C\zeta}$ and $l_{C\eta}$ are the correlation lengths in the ζ and η directions, respectively:

$$\int_{-1}^{1}\int_{-1}^{1} e^{-|\zeta_1-\zeta_2|/l_{C\zeta}-|\eta_1-\eta_2|/l_{C\eta}} f_{Cn}(\zeta_2,\eta_2) d\zeta_2 d\eta_2 = \lambda_{Cn} f_{Cn}(\zeta_1,\eta_1) \ , \quad n = 1,...,M_C \ . \tag{4}$$

2.2 Micromechanical modeling of hygrothermal and material properties

The elastic and hygrothermal properties of a single lamina are predicted using the micromechanical approach, where dependence of temperature and moisture content is taken into consideration. Changes in hygrothermal conditions predominantly affect the polymeric matrix, for which the property degradation are estimated using the following empirical matrix mechanical and hygrothermal property retention ratios, respectively, duly adapted to account for randomness in temperature and moisture content (Chamis and Sinclair, 1981; Chamis, 1987):

$$F_m(\zeta,\eta,\Theta) = \frac{P}{P_o} = \left[\frac{T_{gw}(\zeta,\eta,\Theta) - T(\zeta,\eta,\Theta)}{T_{go} - T_o}\right]^{0.5} \ , \tag{5}$$

$$F_h(\zeta,\eta,\Theta) = \frac{1}{F_m(\zeta,\eta,\Theta)} \ , \tag{6}$$

where P refers to the matrix strength or stiffness after hygrothermal degradation and P_o is the reference matrix strength or stiffness before degradation. Considering all temperatures in $^\circ F$, T_o denotes the test temperature at which P_o is measured, and T_{go} and T_{gw} are, respectively, the glass transition temperatures for reference dry and wet, corresponding to property P, conditions. According to Chamis (1987), T_{gw} can be obtained as follows with the inclusion of space-dependent randomness in moisture content:

$$T_{gw}(\zeta,\eta,\Theta) = [0.005C^2(\zeta,\eta,\Theta) - 0.10C(\zeta,\eta,\Theta) + 1.0]\, T_{go} \ . \tag{7}$$

As a consequence, based on (Gibson, 2012), the spatially stochastic tensile moduli along and perpendicular to the fiber direction, shear modulus, longitudinal and transverse coefficients of thermal and hygroscopic expansion for a single lamina can be written, respectively, as:

$$E_1(\zeta,\eta,\Theta) = E_{f1}V_f + F_m(\zeta,\eta,\Theta)E_mV_m \ , \tag{8}$$

$$E_2(\zeta,\eta,\Theta) = F_m(\zeta,\eta,\Theta)E_m\left[(1-\sqrt{V_f}) + \frac{\sqrt{V_f}}{1-\sqrt{V_f}(1-F_m(\zeta,\eta,\Theta)E_m/E_{f2})}\right], \tag{9}$$

$$G_{12}(\zeta,\eta,\Theta) = F_m(\zeta,\eta,\Theta)G_m\left[(1-\sqrt{V_f}) + \frac{\sqrt{V_f}}{1-\sqrt{V_f}(1-F_m(\zeta,\eta,\Theta)G_m/G_{f12})}\right], \tag{10}$$

$$\alpha_1(\zeta,\eta,\Theta) = \frac{E_f\alpha_fV_f + F_m(\zeta,\eta,\Theta)E_mF_h(\zeta,\eta,\Theta)\alpha_mV_m}{E_fV_f + F_m(\zeta,\eta,\Theta)E_mV_m} \ , \tag{11}$$

$$\beta_1(\zeta,\eta,\Theta) = \frac{E_f\beta_fV_f + F_m(\zeta,\eta,\Theta)E_mF_h(\zeta,\eta,\Theta)\beta_mV_m}{E_fV_f + F_m(\zeta,\eta,\Theta)E_mV_m} \ , \tag{12}$$

$$\alpha_2(\zeta,\eta,\Theta) = (1+\nu_m)F_h(\zeta,\eta,\Theta)\alpha_mV_m + (1+\nu_f)\alpha_fV_f - \alpha_1\nu_{12} \ , \tag{13}$$

$$\beta_2(\zeta,\eta,\Theta) = (1+\nu_m)F_h(\zeta,\eta,\Theta)\beta_mV_m + (1+\nu_f)\beta_fV_f - \beta_1\nu_{12} \ , \tag{14}$$

where the subscripts f and m refer to fiber and matrix, respectively, and 1 and 2 denote the quantities along the longitudinal and transverse directions of the lamina, respectively.

The volume fractions of fiber (V_f), matrix (V_m), and voids (V_v) can be related as follows:

$$V_f + V_m + V_v = 1 \ . \tag{15}$$

According to Chamis (1987), the matrix Poisson's ratio is considered not hygrothermally degraded and can be obtained, using the rule of mixture, as follows:

$$\nu_{12} = \nu_{f12}V_f + \nu_mV_m \ . \tag{16}$$

2.3 Stochastic structural plate model

Under stochastic hygrothermal environment, the constitutive stress–strain relations for a k-th unidirectional lamina of a rectangular composite laminate plate, as portrayed in Fig.1, can be written as (Agarwal *et al.*, 2006):

$$\boldsymbol{\sigma_k} = \begin{Bmatrix} \sigma_x(x,y,\Theta) \\ \sigma_y(x,y,\Theta) \\ \tau_{xy}(x,y,\Theta) \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11}(x,y,\Theta) & \bar{Q}_{12}(x,y,\Theta) & \bar{Q}_{16}(x,y,\Theta) \\ \bar{Q}_{12}(x,y,\Theta) & \bar{Q}_{22}(x,y,\Theta) & \bar{Q}_{26}(x,y,\Theta) \\ \bar{Q}_{16}(x,y,\Theta) & \bar{Q}_{26}(x,y,\Theta) & \bar{Q}_{66}(x,y,\Theta) \end{bmatrix}_k (\boldsymbol{\epsilon^0} + z\boldsymbol{\kappa} - \boldsymbol{\epsilon^H}), \tag{17}$$

where $\bar{Q}_{ij}$ are transformed reduced stiffness coefficients, z is the distance from neutral surface in the direction of plate thickness, and the vectors of mid-plane strains, curvatures and hygrothermal strains are denoted by $\boldsymbol{\epsilon^0}$, $\boldsymbol{\kappa}$, and $\boldsymbol{\epsilon^H}$, respectively. Such strain vectors are expressed as follows, considering T_o and C_o as the temperature and moisture related to measured properties P_o, respectively, and w_0 as the transverse displacement field to be described later on:

$$\boldsymbol{\epsilon^0} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix};\ \boldsymbol{\kappa} = \begin{Bmatrix} -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix};\ \boldsymbol{\epsilon^H} = \begin{Bmatrix} \varepsilon_x^H \\ \varepsilon_y^H \\ \gamma_{xy}^H \end{Bmatrix} = \begin{Bmatrix} \alpha_x(x,y,\Theta)[T(x,y,\Theta)-T_o] + \beta_x(x,y,\Theta)[C(x,y,\Theta)-C_o] \\ \alpha_y(x,y,\Theta)[T(x,y,\Theta)-T_o] + \beta_y(x,y,\Theta)[C(x,y,\Theta)-C_o] \\ \alpha_{xy}(x,y,\Theta)[T(x,y,\Theta)-T_o] + \beta_{xy}(x,y,\Theta)[C(x,y,\Theta)-C_o] \end{Bmatrix}.$$

The resultant forces (N_x,N_y,N_{xy}) and moments (M_x,M_y,M_{xy}) in the composite laminate plate are then given as:

$$\begin{Bmatrix} N_x(x,y,\Theta) \\ N_y(x,y,\Theta) \\ N_{xy}(x,y,\Theta) \\ ----- \\ M_x(x,y,\Theta) \\ M_y(x,y,\Theta) \\ M_{xy}(x,y,\Theta) \end{Bmatrix} = \left[\begin{array}{c|c} [A(x,y,\Theta)] & [B(x,y,\Theta)] \\ ------ & ------ \\ [B(x,y,\Theta)] & [D(x,y,\Theta)] \end{array} \right] \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ --- \\ -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} - \begin{Bmatrix} N_x^H(x,y,\Theta) \\ N_y^H(x,y,\Theta) \\ N_{xy}^H(x,y,\Theta) \\ ----- \\ M_x^H(x,y,\Theta) \\ M_y^H(x,y,\Theta) \\ M_{xy}^H(x,y,\Theta) \end{Bmatrix}, \tag{18}$$

where A_{ij}, B_{ij}, and D_{ij} are coefficients of the extensional, bending-extensional coupling, and bending stiffness matrices, respectively. Hygrothermal forces, denoted by N_x^H, N_y^H, and N_{xy}^H, and hygrothermal moments, represented by M_x^H, M_y^H, and M_{xy}^H, are expressed as:

$$\left(\begin{Bmatrix} N_x^H(x,y,\Theta) \\ N_y^H(x,y,\Theta) \\ N_{xy}^H(x,y,\Theta) \end{Bmatrix}, \begin{Bmatrix} M_x^H(x,y,\Theta) \\ M_y^H(x,y,\Theta) \\ M_{xy}^H(x,y,\Theta) \end{Bmatrix} \right) = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \left(\begin{bmatrix} \bar{Q}_{11}(x,y,\Theta) & \bar{Q}_{12}(x,y,\Theta) & \bar{Q}_{16}(x,y,\Theta) \\ \bar{Q}_{12}(x,y,\Theta) & \bar{Q}_{22}(x,y,\Theta) & \bar{Q}_{26}(x,y,\Theta) \\ \bar{Q}_{16}(x,y,\Theta) & \bar{Q}_{26}(x,y,\Theta) & \bar{Q}_{66}(x,y,\Theta) \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x^H \\ \varepsilon_y^H \\ \gamma_{xy}^H \end{Bmatrix} \right) (1,z) dz. \tag{19}$$

Based on the assumptions of the classical lamination theory (CLT) and neglecting any stretching in the plate middle plane (i.e., $\varepsilon_x^0 = \varepsilon_y^0 = \gamma_{xy}^0 = 0$), only symmetric laminates are considered, which leads to the absence of hygrothermal moments and bending-extensional coupling. Therefore, the strain energy U of the laminate, considering stochastic hygrothermal effects, is given as follows (Tauchert and Huang, 1987):

$$U(\Theta) = \frac{1}{2} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \boldsymbol{\kappa}^T(x,y) \left[D(x,y,\Theta)\right] \boldsymbol{\kappa}(x,y)\, dydx\, . \tag{20}$$

In a stochastic version of the Rayleigh-Ritz (R-R) method, the transverse displacement field $w_0(x,y)$ of the plate can be conveniently approximated in the form of a double series as follows, using the Legendre polynomials in terms of the normalized coordinates $\zeta = 2x/a$ and $\eta = 2y/b$ (Wu *et al.*, 2012):

$$w_0(\zeta,\eta) = \boldsymbol{F}(\zeta,\eta)\boldsymbol{q} = (1-\zeta^2)^c(1-\eta^2)^c \sum_{i=0}^{I} \sum_{j=0}^{J} X_i(\zeta) Y_j(\eta) q_{ij}\, , \tag{21}$$

where the exponent c determines the boundary conditions, taking values 0, 1, and 2 for free, simply supported and clamped edges, respectively. The generalized coordinates are represented by the vector $\boldsymbol{q} = \{q_{ij}\}$ and the functions $X_i(\zeta)$ and $Y_j(\eta)$ are defined as:

$$X_i(\zeta) = \frac{1}{2^i} \sum_{k=0}^{i} \binom{i}{k}^2 (\zeta-1)^{i-k}(\zeta+1)^k \quad ; \quad Y_j(\eta) = \frac{1}{2^j} \sum_{k=0}^{j} \binom{j}{k}^2 (\eta-1)^{j-k}(\eta+1)^k\, .$$

Regarding the potential energy associated to in-plane forces due to the transverse deflection, two parts are identified: V_H, related to hygrothermal forces; and V_M, for an external uniaxial compressive load N_x^M applied in the plate. Such potential energies are expressed as below (Timoshenko and Gere, 1989):

$$V_H(\Theta) = -\frac{1}{2} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \left[N_x^H(x,y,\Theta) \left(\frac{\partial w_0}{\partial x}\right)^2 + 2N_{xy}^H(x,y,\Theta) \left(\frac{\partial w_0}{\partial x}\right) \left(\frac{\partial w_0}{\partial y}\right) + N_y^H(x,y,\Theta) \left(\frac{\partial w_0}{\partial y}\right)^2 \right] dydx\, , \tag{22}$$

$$V_M = -\frac{1}{2}\int_{-a/2}^{a/2}\int_{-b/2}^{b/2} N_x^M \left(\frac{\partial w_0}{\partial x}\right)^2 dydx \,. \tag{23}$$

Making all necessary transformations to normalized coordinates ζ and η, the principle of minimum total potential energy is applied considering U, V_H and V_M, with the displacement field w_0 defined by Eq. 21. Then, the following eigenvalue problem for stochastic buckling analysis is obtained:

$$\left[\boldsymbol{K}(\Theta) - N_x^M\,\boldsymbol{K_G} - \boldsymbol{K_H}(\Theta)\right]\boldsymbol{q} = \boldsymbol{0}\,, \tag{24}$$

where N_x^M is the eigenvalue representing the buckling loads, and the structural, geometric, and hygrothermal stiffness matrices, denoted by $\boldsymbol{K}$, $\boldsymbol{K_G}$, and $\boldsymbol{K_H}$, respectively, are given as:

$$\boldsymbol{K}(\Theta) = \int_{-a/2}^{a/2}\int_{-b/2}^{b/2} \begin{Bmatrix} -\frac{\partial^2 \boldsymbol{F}}{\partial x^2} \\ -\frac{\partial^2 \boldsymbol{F}}{\partial y^2} \\ -2\frac{\partial^2 \boldsymbol{F}}{\partial x \partial y}\end{Bmatrix}^T \left[D(x,y,\Theta)\right] \begin{Bmatrix} -\frac{\partial^2 \boldsymbol{F}}{\partial x^2} \\ -\frac{\partial^2 \boldsymbol{F}}{\partial y^2} \\ -2\frac{\partial^2 \boldsymbol{F}}{\partial x \partial y}\end{Bmatrix} dydx\,;\ \ \boldsymbol{K_G} = \int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\left(\frac{\partial \boldsymbol{F}}{\partial x}\right)^T \frac{\partial \boldsymbol{F}}{\partial x}\,dydx\,;$$

$$\boldsymbol{K_H}(\Theta) = \int_{-a/2}^{a/2}\int_{-b/2}^{b/2} \begin{Bmatrix} \frac{\partial \boldsymbol{F}}{\partial x} \\ \frac{\partial \boldsymbol{F}}{\partial y}\end{Bmatrix}^T \begin{bmatrix} N_x^H(x,y,\Theta) & N_{xy}^H(x,y,\Theta) \\ N_{xy}^H(x,y,\Theta) & N_y^H(x,y,\Theta)\end{bmatrix} \begin{Bmatrix} \frac{\partial \boldsymbol{F}}{\partial x} \\ \frac{\partial \boldsymbol{F}}{\partial y}\end{Bmatrix} dydx\,.$$

3. NUMERICAL APPLICATIONS

3.1 Model description

In this work, a square unidirectional composite laminate plate of sides 160mm, simply supported along all its edges, is assumed in the numerical analyses. Two different lamination sequences composed of 0.20mm thickness laminæ are considered: $[-45/45/90/0]_S$, named L1; and $[-45/45/-45/45]_S$, named L2.

The fiber and matrix chosen for the analyses are given in Table 1, together with the respective values of elastic and hygrothermal properties. Such values are considered to be measured at temperature T_o =75°F and moisture content C_o =0%, which are chosen as the initial environmental condition for computing degradation of lamina properties according to Eqs. 8-16. The temperature and moisture content distribution are assumed to be described by two-dimensional random fields discretized by KLE, as per Eqs. 1 and 3, respectively, with mean values of $\bar{T}$ =120°F and $\bar{C}$ =0.1%. Also, neglecting void contents in the composite, the fiber volume fraction adopted is 0.60.

Table 1: Elastic and hygrothermal properties of fiber and matrix (Gibson, 2012).

Material	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν	α (10^{-6}/°F)	β	T_{go} (°F)
Fiber (AS)	214	14	14	0.2	-0.55	0	-
Matrix (HM)	5.2	5.2	1.93	0.35	40	0.33	420

3.2 Numerical Validation

A validation of the proposed R-R-based model is deterministically carried out using the mean values of random variables, which, in the case of the present study, are zero for all of them. The number of terms used in the R-R method is determined evaluating the convergence of the lowest (critical) buckling load obtained from the resolution of Eq.24, where it was found $I = J = 8$ sufficient to yield accurate results, being kept for all further analyses.

For the two different lamination sequences, L1 and L2, critical buckling loads from R-R are compared with the counterparts obtained from finite element (FE) models built and analyzed using the commercial software NASTRAN. Satisfactory accuracy is presented in Table 2.

Table 2: Comparison of critical buckling loads from Rayleigh-Ritz (R-R) model and finite element (FE) model.

Lamination sequence	R-R (N/mm)	FE (N/mm)	Deviations
$L1$: $[-45/45/90/0]_S$	10.2635	10.2537	0.0956%
$L2$: $[-45/45/-45/45]_S$	11.3772	11.3626	0.1285%

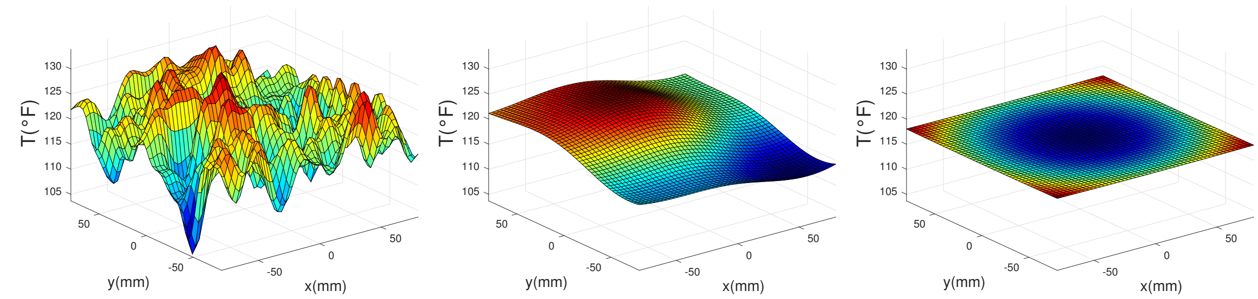
3.3 Stochastic Analysis

The stochastic spatial effect of temperature and moisture variation on the critical hygrothermal-mechanical buckling load is numerically investigated for the composite laminate plates described in section 3.1 . Uncertainty propagation is performed considering different values of standard deviations and correlation lengths, associated with both random fields of temperature and moisture discretized by KLE.

Three different values of coefficient of variation (COV), which is the ratio of the standard deviation to the mean, are adopted: $0.01, 0.05$, and 0.10. It should be emphasized that, as the mean values of all random variables are zero, the mean temperature ($\bar{T}$) and moisture content ($\bar{C}$) are used to compute standard deviation values assigned to the Gaussian random variables of temperature and moisture random fields, respectively.

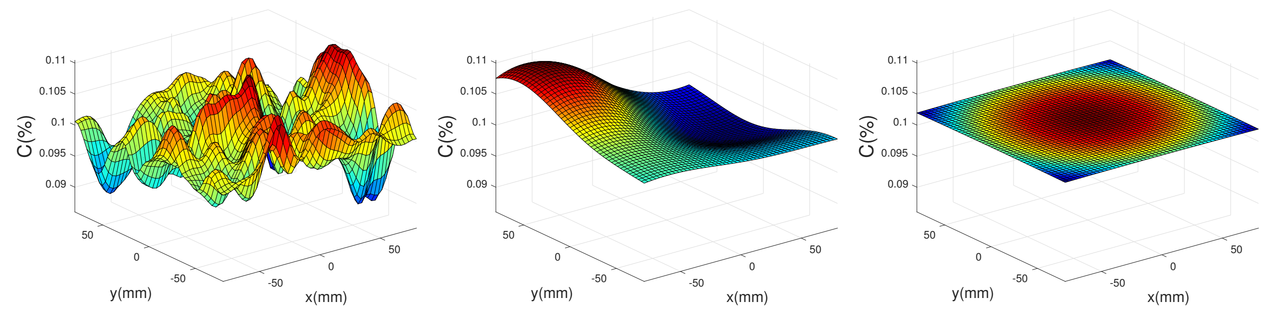
Regarding the correlation lengths (CLs), the values 0.10, 1.0 and 10.0 are considered and, in each analysis, assigned simultaneously to all correlation lengths (i.e, $l_{T\zeta} = l_{T\eta} = l_{C\zeta} = l_{C\eta}$). As the value of the correlation length influences the discretization of the random field using KLE, the appropriate choice of number of terms in the truncated series expansion is required. Sorting the eigenvalues from Eqs.2 and 4 in decreasing order, it is used the criterion of retaining only the eigensolutions satisfying $\lambda_n/\lambda_1 > 0.10$, which selects the most influential eigenvalues. Then, for correlation lengths of 0.10, 1.0, and 10.0, the number of random variables used for discretizing each random field are 200, 6, and 1, respectively.

The MCS method is combined with the Latin Hypercube Sampling (LHS) to generate 10,000 sample sets of random variables for each combination of values of COV and correlation length. For exemplification, Figures 2 and 3 illustrate the spatial distributions of temperature and moisture fields, respectively, related to the three correlation lengths analyzed and a fixed COV of 0.05. It can be noticed that, for both random fields, a high spatial frequency of variation is obtained for the smaller correlation length and, as the correlation value is increased, the spatial profile presents a smoother variation to the limit of a practically constant value when a correlation length of 10.0 is adopted.



(a) $l_{T\zeta} = l_{T\eta} = 0.10$ (b) $l_{T\zeta} = l_{T\eta} = 1.0$ (c) $l_{T\zeta} = l_{T\eta} = 10.0$

Figure 2: Samples of temperature distribution on laminate plate for the three values of correlation length considering the COV value of 0.05.



(a) $l_{C\zeta} = l_{C\eta} = 0.10$ (b) $l_{C\zeta} = l_{C\eta} = 1.0$ (c) $l_{C\zeta} = l_{C\eta} = 10.0$

Figure 3: Samples of moisture distribution on laminate plate for the three values of correlation length considering the COV value of 0.05.

The critical buckling load from Eq.24 is computed for each set of random variables generated. In Figure 4, the COV of buckling response related to each combination of standard deviation and correlation length assigned to both random fields is presented for the laminate with lamination sequence L1. It is observed greater buckling dispersion when standard deviation of random variables increases, as expected, and/or when greater values of correlation length are adopted, as larger correlation length tends to magnify the variation in scenarios of higher fluctuation since random field approximates to a single random variable behavior. Furthermore, a linear relationship between COV values adopted for random variables and those obtained for buckling response is verified.

Also in Figure 4, probability density estimates for the sample data of each analysis are depicted, where distributions similar to Gaussian are obtained for critical buckling response of all analyses. Similar results are obtained for laminates L2 in Figure 5, with the differences of slightly lower values of critical buckling dispersion and distinct mean values obtained for probability density estimates, which is in accordance with Table 2 for deterministic analysis considering mean values of random variables.

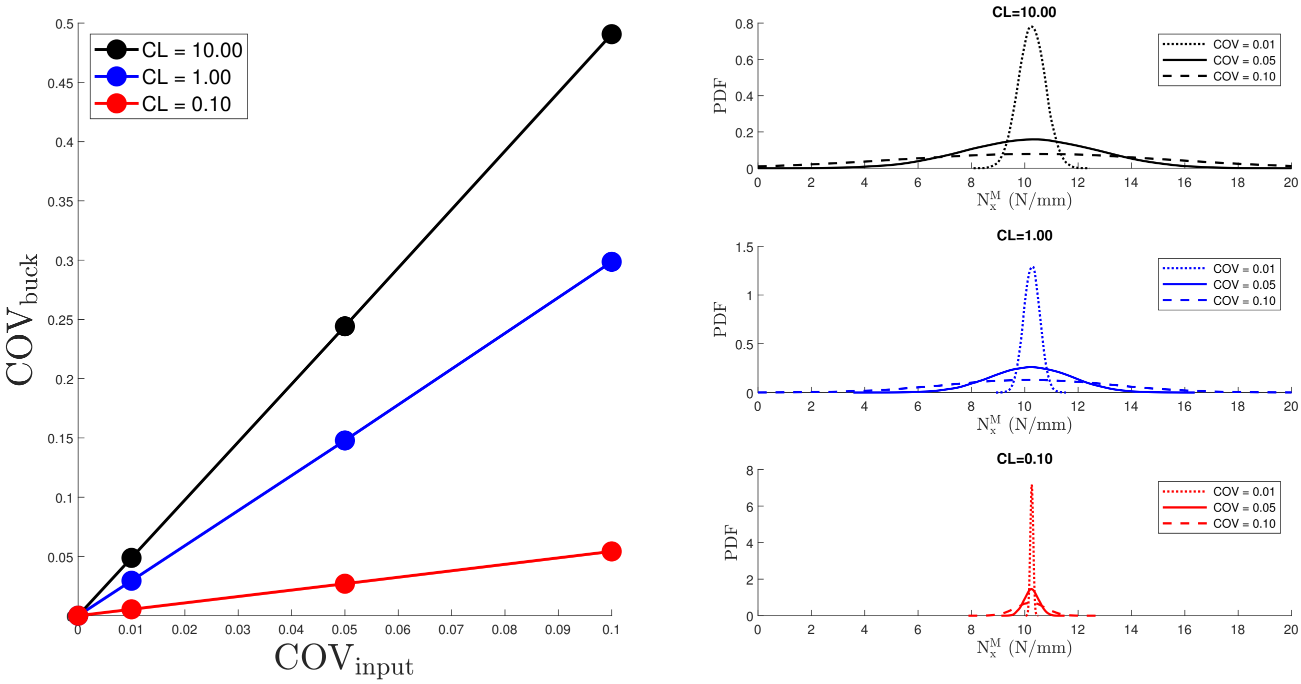


Figure 4: Results of critical buckling load dispersion for laminate L1 regarding all stochastic analyses performed.

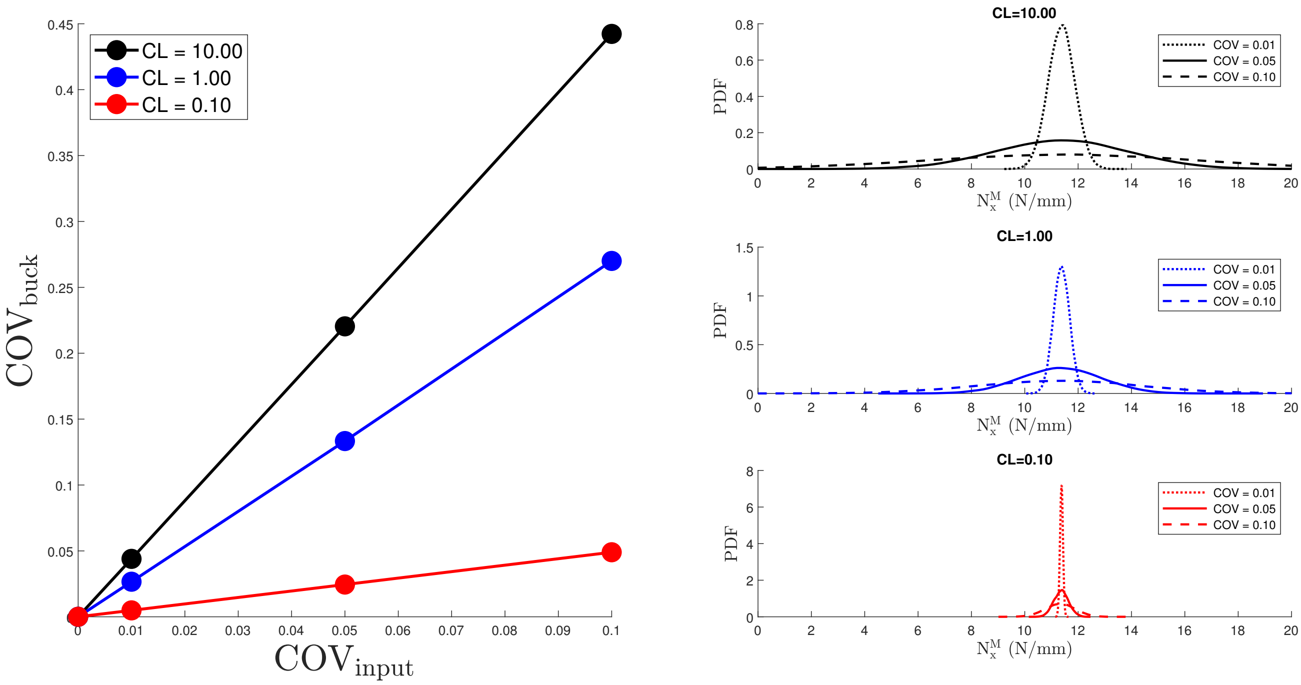


Figure 5: Results of critical buckling load dispersion for laminate L2 regarding all stochastic analyses performed.

3.4 Sensitivity Analysis

Aiming at quantifying the relative contribution of each random variable on the variation of hygrothermal-mechanical buckling load, a sensitivity analysis is performed based on sigma-normalized derivatives, which exhibit the following property (Smith, 2013):

$$\sum_{i=1}^{N_V} \alpha_i^2 = \sum_{i=1}^{N_V} \left(\frac{\dfrac{\partial N_x^M(\boldsymbol{\mu}_{\boldsymbol{\xi_i}})}{\partial \xi_i} \sigma_{\xi_i}}{\sqrt{\sum_{i=1}^{N_V} \left(\dfrac{\partial N_x^M(\boldsymbol{\mu}_{\boldsymbol{\xi_i}})}{\partial \xi_i} \sigma_{\xi_i} \right)^2}} \right)^2 = 1 \,, \tag{25}$$

where α_i and σ_{ξ_i} are the sensitivity coefficient and standard deviation related to random variable i, respectively. The vector of mean values of all N_V random variables is represented by $\boldsymbol{\mu}_{\boldsymbol{\xi_i}}$.

Tables 3 and 4 present the squared sensitivity coefficients α_i^2 associated to random variables of temperature and moisture random fields, respectively, for all considered analyses. It is important to mention that, in this work, those coefficients do not depend on the COV value as can be inferred from Eq. 25. For both lamination sequences, one can notice a strong contribution of the first random variable from temperature random field on the variation of critical buckling load, while contributions of random variables used to discretize moisture random field presented much smaller values.

Table 3: Squared sensitivity coefficients related to temperature random field.

		$\alpha_i^2(\%)$						
Lam. Seq.	CL	ξ_{T1}	ξ_{T2}	ξ_{T3}	ξ_{T4}	ξ_{T5}	ξ_{T6}	ξ_{T7-200}^{*}
	0.10	60.24	$<10^{-2}$	$<10^{-2}$	0.13	13.17	13.16	12.57
L1	1.00	98.49	$<10^{-2}$	$<10^{-2}$	0.36	0.36	0.07	-
	10.00	99.27	-	-	-	-	-	-
	0.10	60.62	$<10^{-2}$	$<10^{-2}$	0.20	12.90	12.93	12.61
L2	1.00	98.49	$<10^{-2}$	$<10^{-2}$	0.34	0.34	0.11	-
	10.00	99.27	-	-	-	-	-	-

$*$ Refers to the sum of α_i^2 related to random variables from ξ_{T7} to ξ_{T200}.

Table 4: Squared sensitivity coefficients related to moisture random field.

		$\alpha_i^2(\%)$						
Lam. Seq.	CL	ξ_{C1}	ξ_{C2}	ξ_{C3}	ξ_{C4}	ξ_{C5}	ξ_{C6}	ξ_{C7-200}^{*}
	0.10	0.44	$<10^{-2}$	$<10^{-2}$	$<10^{-2}$	0.10	0.10	0.09
L1	1.00	0.72	$<10^{-2}$	$<10^{-2}$	$<10^{-2}$	$<10^{-2}$	$<10^{-2}$	-
	10.00	0.73	-	-	-	-	-	-
	0.10	0.44	$<10^{-2}$	$<10^{-2}$	$<10^{-2}$	0.10	0.10	0.09
L2	1.00	0.72	$<10^{-2}$	$<10^{-2}$	$<10^{-2}$	$<10^{-2}$	$<10^{-2}$	-
	10.00	0.73	-	-	-	-	-	-

$*$ Refers to the sum of α_i^2 related to random variables from ξ_{C7} to ξ_{C200}.

4. CONCLUSIONS

The influence of stochastic space-dependent temperature and moisture variation on the hygrothermal-mechanical buckling response of laminate composite plates was numerically assessed in the present work. Based on the Rayleigh-Ritz approach combined with the Classical Lamination Theory, the composite plate was modeled representing each environmental fluctuation as a two-dimensional random field discretized by the Karhunen-Loève expansion. Stochastic modeling has been performed using Monte Carlo Simulation, combined with Latin Hypercube Sampling, and taking into account the combined effect of environmental variation on the material properties and hygrothermally-induced stresses.

Analyzing different values of standard deviation and correlation length associated to both temperature and moisture random fields, the stochastic critical buckling load has increasing variation as the dispersion of random field and/or correlation length are increased, regardless the lamination sequence considered. In addition, for the values adopted in the present work, the temperature variation has stronger influence on response variation than the moisture content fluctuation, in particular the first random variable used in the discretization of temperature random field.

The significant buckling response dispersion presented in the results enables to justify the importance of incorporating environmental uncertainties in the analysis and design of composite structures. Future investigations involving different types of covariance functions and non-Gaussian random fields are worth being explored.

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