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A STRATEGY TO FORM SUPER ATTENUATION BANDS IN BEAMS USING ARRAYS OF RESONATORS

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Abstract. *The design of a meta-structure using mechanical vibration absorbers (resonators) to improve the vibration reduction of the system and its surroundings has been an active research topic lately. Typically, the interaction between the host structure and the vibration absorbers simultaneously creates a scattering mechanism and adds an anti-resonance to the frequency response of the structure. This latter mechanism makes these devices attractive as they can be designed to operate in a wide range of frequencies. However, since the width of the attenuation band is usually small, a large vibration absorber mass is required. To overcome this issue, one design strategy that has attracted attention is the combination of both effects to form a wide attenuation band, called super attenuation band. Recent works in finite mono-coupled structures with an array of equally spaced vibration absorbers have shown that when correctly tuned, the two effects are combined and form a super attenuation band. However, when this structural design is applied to a periodic beam, the effectiveness of creating such a super attenuation band has not been achieved due to the multi-coupling mechanism of the host structure. This paper investigates a strategy to form a super attenuation band in a meta-structure comprising a beam with an array of vibration absorbers attached at arbitrary positions of the host structure. The main objective is to determine the physical factors that govern the attenuation band and how to obtain it. Using the dynamic stiffness and receptance methods, the transversal displacement transmissibility for a beam with a single resonator is determined and the effect of the attachment position is investigated numerically. Analytical expressions are derived to support the numerical investigation of the effect of changing the vibration absorbers natural frequency. This analysis is extended for a beam structure with an array of resonators attached at arbitrary positions. The results show that the smooth super attenuation band can only occur when each optimally tuned vibration absorber is attached to a position where the beam exhibits a nodal point.*

Keywords: *Super attenuation band, meta-structure, resonators*

1. INTRODUCTION

The use of vibration absorbers to mitigate or reduce the amplitude level of vibration has been a hot research topic in the area of structure dynamics (Kela and Vähäoja, 2009). Typically, due to the interaction between the vibration absorber and the host structure, these devices create a scattering mechanism and add an anti-resonances component to the structure dynamics (Gonçalves et al., 2021), acting as a filter and reducing the vibration amplitude by several orders of magnitude. The main advantage is that the frequency at which this anti-resonance occurs is controlled by the vibration absorber and can be adjusted to occur at any particular frequency. However, its bandwidth is narrow, and the performance of the device depends upon the absorber mass. Different approaches have been investigated to overcome this limitation, e.g., using arrays of vibration absorbers with the same (Silva et al., 2020) or different (Brennan, 1997; Hu et al., 2021) tuned frequency or using a combined force and moment vibration absorber (Toit et al., 2021).

An alternative is to form a smooth super attenuation band. This phenomenon has been investigated in mono-coupled structures such as rod and lumped systems comprising symmetric cells (Cleante et al., 2022) and asymmetric cells (Cleante et al., 2023). It consists of combining the scattering mechanism with the local resonance attenuation band to form a single, yet wide attenuation band. This phenomenon has been also exploited in infinite metamaterial beams. Xiao et al. (2013) showed that using an array of vibration absorbers, due to the multi-coupled characteristic of the structure, only a quasi-super attenuation band is formed.

In this context, this paper investigates the use of an array of resonators to create a super attenuation band in beam structures. The dynamic stiffness method is employed to determine the transverse displacement transmissibility, and numerical investigations are conducted to explore the formation of a super attenuation band in beams with a single vibration absorber as well as with an array of resonators. To achieve this, the optimum tuning frequency of the system is determined and hence the required natural frequency of the vibration absorber is also determined. This paper is organised as follows: following the introduction, the methodology to investigate the problem is described. A numerical investigation into the formation of the super attenuation band using a single vibration absorber located at different positions on the beam is presented. Later, the analysis is extended to explore the effect of use of an array of vibration absorbers is carried out before closing with conclusions.

2. PROBLEM STATEMENT

In this section a model to describe the dynamics of a beam structure with an array of resonators, as shown in Fig. 1, will be derived. The interest is to evaluate the transverse displacement transmissibility from the left to the right of the beam. The system consists of a uniform homogeneous Euler-Bernoulli beam of length l with forces F, moments M, transverse displacement W and angular displacement θ, at each end of the beam, where the subindex L and R denote left and right, respectively. The resonator consists of a damped single degree-of-freedom vibration absorber of mass m_a with a spring of stiffness s_a and a material loss factor of η_a.

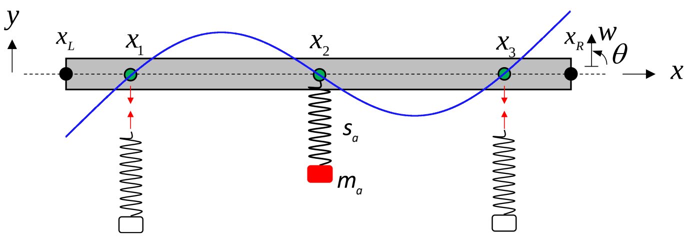

Figure 1. Schematic of one-dimensional beam with resonators attached to it.

This beam is divided in four elements of length l_e where the inner node coordinates denoted as x_1, x_2 and x_3 corresponds to the position where the displacement of the transverse mode shape of a free-free beam at the natural frequency $l/\lambda \approx 1.25$ is motionless (the reader is suggested to revisit references to find more details about the natural frequencies and mode shapes of a beam, e.g., (Blevins, 1995)). The system is modelled in the frequency domain using the dynamic stiffness method. For each beam element, the vector of normalised forces and moments, $\mathbf{f} = \{\hat{F}_L \;\; \hat{M}_L \;\; \hat{F}_R \;\; \hat{M}_R\}^{\mathrm{T}}$, where $\hat{F} = F/EI\beta^3$ and $\hat{M} = M/EI\beta^2$, is related to the vector of displacements and rotations $\mathbf{w} = \{W_L \;\; \theta_L/\beta \;\; W_R \;\; \theta_R/\beta\}^{\mathrm{T}}$ by the dynamic stiffness matrix $\mathbf{D}_e$, such that $\mathbf{f} = \mathbf{D}_e\mathbf{w}$, where (Gardonio and Brennan, 2004)

$$\mathbf{D}_e = \begin{bmatrix} -K_{11} & -P & K_{12} & V \\ -P & Q_{11} & -V & Q_{12} \\ K_{12} & -V & K_{11} & P \\ V & Q_{12} & P & Q_{11} \end{bmatrix}, \tag{1}$$

in which

$K_{11} = [\cos(\beta^* l_e/2)\sinh(\beta^* l_e/2) + \sin(\beta^* l_e/2)\cosh(\beta^* l_e/2)]/R,$

$K_{12} = [\sin(\beta^* l_e/2) + \sinh(\beta^* l_e/2)]/R,$

$P = \sin(\beta^* l_e/2)\sinh(\beta^* l_e/2)/R,$

$V = [\cos(\beta^* l_e/2) - \cosh(\beta^* l_e/2)]/R,$

$Q_{11} = [\cos(\beta^* l_e/2)\sinh(\beta^* l_e/2) - \sin(\beta^* l_e/2)\cosh(\beta^* l_e/2)]/R,$

$Q_{12} = \left[\sin\left(\beta^* l_e/2\right) - \sinh\left(\beta^* l_e/2\right)\right]/R,$

$R = \cos\left(\beta^* l_e/2\right)\cosh\left(\beta^* l_e/2\right) - 1,$

where $\beta^* = (1 - \frac{1}{4}\mathrm{j}\eta)\beta$ is the complex wavenumber and $\beta = (\omega^2 \rho A/EI)^{1/4}$ is the flexural wavenumber of the beam, ω is the angular frequency, ρ, E, A and I are, the density, the Young's modulus, the cross-sectional area and the second moment of area of the beam and η is the material loss factor of the beam.

The complete structure can be modelled using an assembling procedure similar to the finite element method, where for each inner node a vibration absorber can be attached. The global dynamic stiffness matrix $\mathbf{D}$ is given by

$$\mathbf{D} = \sum_{e=1}^{N} \mathbf{B}_e^T \mathbf{D}_e \mathbf{B}_e + \sum_{i=1}^{n} \mathbf{C}_i^T D_{\text{att}} \mathbf{C}_i , \tag{2}$$

where $\mathbf{B}_e$ and $\mathbf{C}_i$ are the Boolean matrices, N is the number of beam elements and n is the number of attached resonators. D_{att} is the non-dimensional dynamic stiffness of the force vibration absorber, which is given by (Salleh and Brennan, 2007)

$$D_{\text{att}} = \frac{\mu\beta l\left(1+j\eta_a\right)}{1+j\eta_a-\left(\beta l/\beta_a l\right)^4}, \tag{3}$$

where $\mu = m_a/\rho Al$ is the ratio between the mass of the vibration absorber and the mass of the beam, and β_a is the flexural wavenumber of the beam at the natural frequency of the vibration absorber, i.e., when $\omega = \omega_a = \sqrt{k_a/m_a}$. The displacement vector at each node, i.e., at each cell connection, is given by

$$\mathbf{w} = \mathbf{Rf} , \tag{4}$$

where $\mathbf{R} = \mathbf{D}^{-1}$, is the inverse of the dynamic matrix. The transverse displacement transmissibility of the structure from the left to the right can be determined by

$$T_W = \frac{W_R}{W_L}, \tag{5}$$

where W_L is the transverse point receptance, the receptance where the excitation force is applied, and W_R is the transverse transfer receptance, the receptance at the opposite end where the force is applied.

3. SUPER ATTENUATION BAND FORMATION

In this section, the formation of a super attenuation band using a single resonator attached to a beam span will be investigated. To achieve this, the vibration absorber mass ratio is assumed to be $\mu = 2$, which is a relatively large value for practical engineering applications but, as we are interested to understand the physics behind the use of vibration absorber to form the super attenuation band, this size of mass is helpful to visualise the dynamic effects on the structure.

3.1 Attached to the centre of the beam

Consider first a vibration absorber attached to the centre of the beam at the position $x_2 = 0.5$ as shown in Fig. 1. In order to investigate the dynamic effect of attaching a vibration absorber, the transverse displacement transmissibility given by Eq. (5) is calculated numerically as a function of l/λ, where λ is a flexural wavelength of the beam and l/λ_a, where λ_a is the flexural wavelength at the natural frequency of the vibration absorber, for a mass ratio of $\mu = 2$; l/λ is set to vary from 0.2 to 2 and l/λ_a is set to vary from 0.75 to 1.25, both with increments of 0.01. The material loss factors η and η_a are set to 0. The top-view of the modulus of the transverse displacement transmissibility is plotted in Fig. 2. The frequencies limiting the attenuation bands in 0 dB as a function of the natural frequency of the vibration absorber are determined numerically and are plotted in Fig. 2 as black solid lines. Also plotted as a blue dashed line is the natural frequency of the vibration absorber and as a thin blue solid line is the tuned frequency, l/λ_t, which is the flexural wavelength at the frequency where the interaction between the host structure and the vibration absorber creates an anti-resonance in the system.

It is seen in Fig. 2 that due to the dispersive characteristic of the beam and the dynamic interaction with the vibration absorber, the resulting tuned frequency of the system occurs at a frequency higher than the natural frequency of the vibration absorber. Moreover, it can be noticed that with the evolution of the vibration absorber natural frequency there is a tuning condition at which the super attenuation band is potentially formed, which occurs at $l/\lambda_t \approx 1.25$. The dynamic response of the system at this tuned condition is better examined in Fig. 3(ii), which shows the transverse displacement transmissibility with its corresponding point and transfer receptances. For the purpose of comparison, two additional cases, where the system is not correctly tuned, with 90% and 120% of the correctly tuned condition, are also plotted in Figs. 3(i) and 3(iii), respectively. The frequency response plots for the bare beam (without vibration absorber) are also plotted as in grey lines.

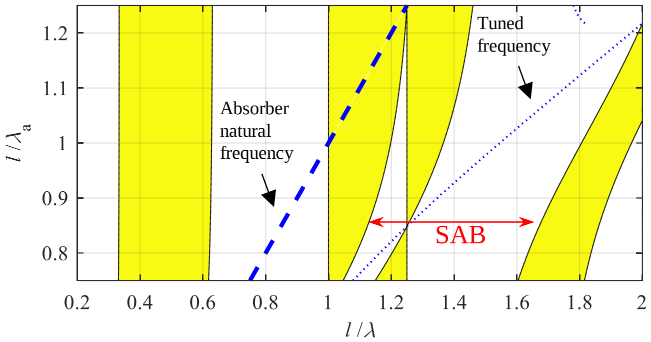

Figure 2. Top-view of the displacement transmissibility as a function of the excitation frequency, l/λ, and of the vibration absorber natural frequency, l/λ_a, assuming $\mu = 2$ and $\eta = \eta_a = 0$. The yellow shaded area is for $|T| > 0$ dB. The blue dashed line is the vibration absorber natural frequency and the blue dotted line is the resulting tuning frequency.

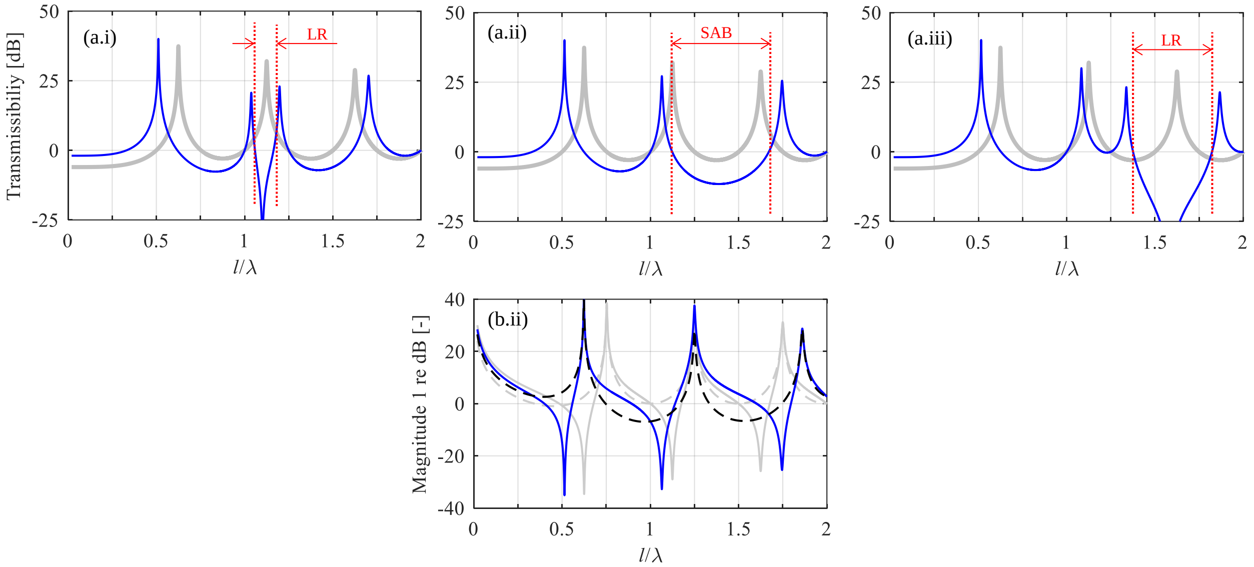

Figure 3. (a) Transverse displacement transmissibility; and (b) point (blue solid line) and transfer (black dashed line) receptances for a beam with a vibration absorber attached to its centre when tuned to (.i) $l/\lambda_a = 0.76$; (.ii) correctly tuned to form the super attenuation band, $l/\lambda_a = 0.8484$; and (.iii) $l/\lambda_a = 1$, assuming $\mu = 2$ and $\eta = 0.01$ and $\eta_a = 0.008$. The grey lines are for the frequency response plots of the bare beam.

Examining Figs. 3(a.1) and 3(a.iii), which are the cases where the system is not correctly tuned, it is clear that there is a range of frequency that exhibits a sharp dip in attenuation. This occurs due to the local resonance effect added by the vibration absorber. Of particular interest is the case shown in Fig. 3(a.ii). When the natural frequency of the vibration absorber is correctly tuned, such that the system's tuned frequency occurs at $l/\lambda_t \approx 1.25$, a smooth super attenuation band is formed, resulting in a considerable improvement in its attenuation properties such as the width of the band and the magnitude of attenuation within the band, when compared to the transverse displacement transmissibility of the bare beam. At this condition there is a superposition of the vibration mode added by the vibration absorber with the anti-symmetric mode of the beam. As a result, within the frequency range considered, as shown in Fig. 3(b.ii), only three resonance frequencies are observed as it is for the case of the bare beam.

The mechanism to form the super attenuation band can also be explained as follows. Consider the diagram box shown in Fig. 4. It represents the beam structure shown in Fig. 1 as a system with one input (where the excitation force is applied) and 4 outputs. Suppose the vibration absorber is only attached to the output represented by x_2. Thus, by force equilibrium $F_A = -F_2$. Using the appropriate boundary conditions for each of node in this system, the point receptance, which is the receptance at the position where the force is applied, is given by

$$\frac{X_F}{F_L} = R_{LL} - \frac{R_{L2}R_{2L}}{R_{22}+R_{att}}, \qquad (6)$$

where R_{LL}, R_{L2}, R_{2L} and R_{22} are the receptance elements of the global recepatance matrix of the bare beam given in Eq. (2). As shown in Fig. 3, the condition to the super attenuation be formed is when the tuned frequency, $l/\lambda_t = 1.25$, corresponds to a resonance frequency. In order for this condition to occur, the denominator of Eq. (5) must be equal to zero, resulting in

$$R_{\text{att}} = -R_{22}\,. \tag{7}$$

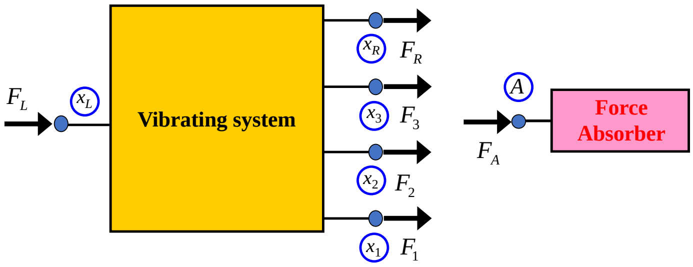

Figure 4. Block diagram representing the beam structure in Fig. 1.

Examining Eq. (7), it becomes clear that a matching of receptances is required between the vibration absorber with the dynamic of the bare beam at the frequency of $l/\lambda = 1.25$. The vibration absorber and the beam must have the same magnitude but be opposite in phase. Substituting the inverse of Eq. (3) into (7) gives

$$\frac{l}{\lambda_a} \approx \frac{1.25}{\sqrt[4]{1-2.5\pi\mu R_{22}}}\,, \tag{8}$$

which it is a relation to determine the required vibration absorber natural frequency to correctly tune the system so that the super attenuation band is formed. For the condition discussed in Fig. 3, $\mu = 2$, l/λ_a is found to be, approximately, 0.8484. Note that, because the system is linear, if the vibration absorber is attached to the nodes x_1 or x_3, the relation to determine the natural frequency of the corresponding vibration absorbers are simply

$$\frac{l}{\lambda_a} \approx \frac{1.25}{\sqrt[4]{1-2.5\pi\mu R_{11}}} \text{ or } \frac{l}{\lambda_a} \approx \frac{1.25}{\sqrt[4]{1-2.5\pi\mu R_{33}}}\,, \tag{9a,b}$$

where R_{11} and R_{33} are, respectively, the point receptances at the positions x_1 and x_3 of the bare beam.

3.2 Vibration absorber attached at different positions of the beam

To assess the possibility of forming a super attenuation band when the vibration absorber is placed at a position different from the beam centre, a numerical investigation on the dynamic response of the system is conducted.

Consider again the beam shown in Fig. 1. In addition to the beam centre, there are two other positions, x_1 and x_3, where the deflection shape of the bare beam is motionless at the frequency of $l/\lambda = 1.25$ and the vibration absorber can be attached to potentially form the super attenuation band. These positions correspond approximately, to $x_1 = 0.132$ and $x_2 = 0.868$ from the left-end of the beam, respectively. Assuming the absorber mass ratio is $\mu = 2$, from Eqs. (8a) and (8b) the required vibration absorber natural frequency for both attachment positions are found to be the same and, approximately, $l/\lambda_a = 0.7728$. The transverse displacement transmissibilities and their corresponding point and transfer receptances for the device attached at x_1 or x_3 are plotted, respectively, in Figs. 4(i) and 4(ii) assuming $\eta = 0.01$ and $\eta_a = 0.008$ to avoid sharp resonant peaks. For the purpose of comparison, the transverse displacement transmissibility of the bare beam is also plotted as grey solid lines.

It is clear from Fig. 5 that the smooth super attenuation band is formed when the vibration absorber is attached at a position different from the beam centre. However, it is noticed that the difference in position result in different dynamic behaviour. When the vibration absorber is positioned close to the vibration source, as shown in Fig 5(a.1), a super attenuation band with wide bandwidth and with a large attenuation is achieved. As a side effect, there is a slightly increase in the transmissibility at frequencies lower than the super attenuation band. When the vibration absorber is positioned far from the vibration source, as shown in Fig. 5(a.ii), a super attenuation band is still formed but the improvement in the attenuation band properties is not so significant as when the vibration absorber is positioned close to the vibration source.

However, for frequencies lower than the super attenuation band at some frequencies a reduction in the transmissibility is clearly observed.

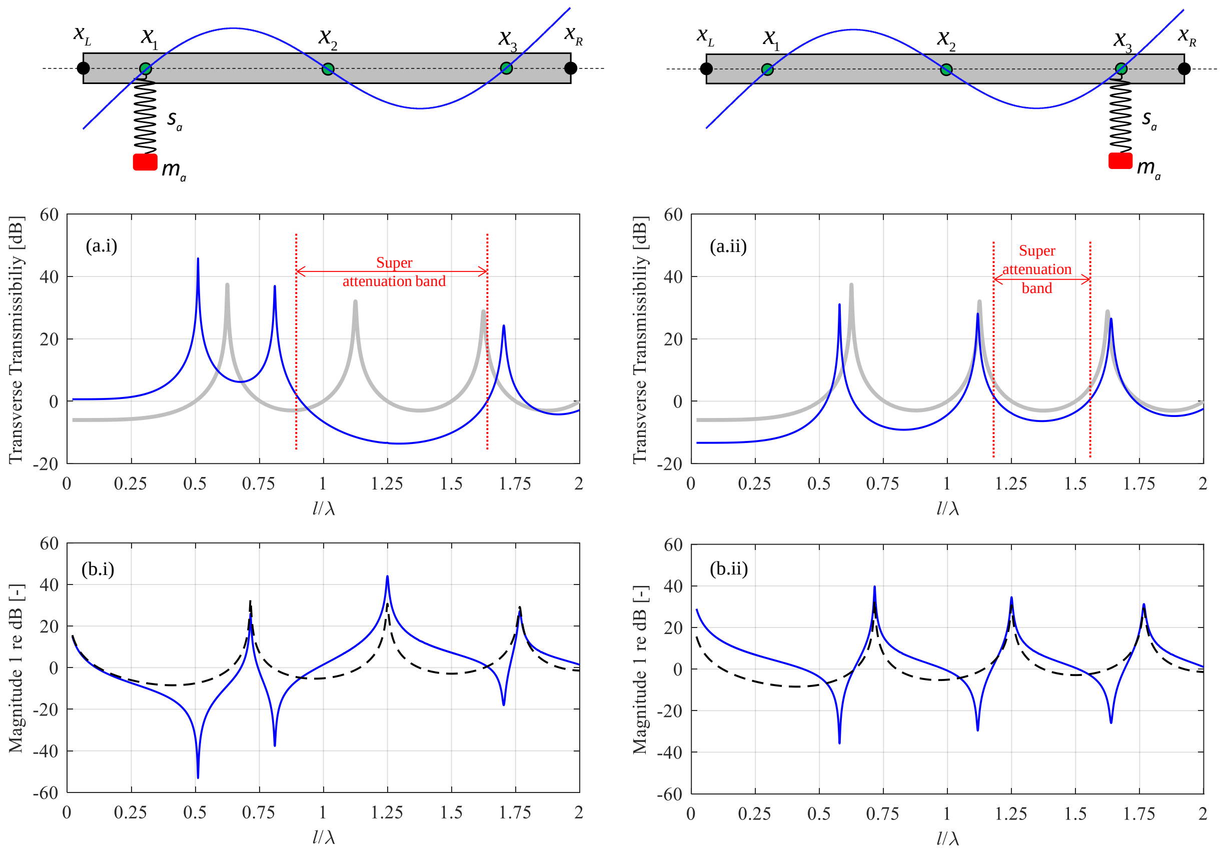

Figure 5. (a) Transverse displacement transmissibility (the grey line is for the bare beam); and (b) point (blue solid line) and transfer (black dashed line) receptances for a beam with a vibration absorber attached at (.i) x_1 or (.ii) x_3 when correctly tuned to form the super attenuation band, assuming $\mu = 2$ and $\eta = 0.01$ and $\eta_a = 0.008$.

It is worth noting in Figs. 5(b.1) and 5(b.ii) that for both cases, as for the case of the vibration absorber attached to the centre of the beam, the tuned frequency of the system coincides with the free-free natural frequency of the beam, overlapping the vibration mode added by the vibration absorber with the anti-symmetric mode of the system.

This analysis demonstrates that when the vibration absorber is attached to a position at which the bare beam is motionless and is precisely tuned to the natural frequency of the beam corresponding to the anti-symmetric mode, the super attenuation band is formed whether the device is attached to the beam centre or at position different from the beam centre. This suggests that within the same beam span, a structure comprising multiple resonators can be used to create a super attenuation band.

4. SUPER ATTENUATION BAND FORMATION IN A BEAM WITH AN ARRAY OF RESONATORS

It was seen that the super attenuation band can be formed within the same beam span by attaching the vibration absorber to a position where deflection shape of the bare beam is motionless. In this section, the super attenuation band will be investigated considering a single beam span with an array of two vibration absorber attached at different positions.

Considerer again the beam shown in Fig. 1. One vibration absorber is attached at position x_1, and the second at x_3, located at distances of $x_1 = 0.132$ and $x_2 = 0.868$ from the left-end of the beam, respectively. It is intrinsic to assume that the optimal natural frequency for the vibration absorbers should be the same as for the cases where they are evaluated individually, as seen in Eqs. 7 and 8. However, when a second device is attached to the structure, the matching of impedance is no longer solely related to the bare beam but encompasses the dynamic of the beam plus the vibration absorber. To correctly determine this new optimum condition for both vibration absorber, the natural frequency of each device obtained from Eq. 8 is used as an initial condition, and later the natural frequency of each device is refined manually until the local resonance (the sharp dip) vanishes from transverse displacement transmissibility, which were found to be $l/\lambda_{a1} = 0.7728$ and $l/\lambda_{a3} = 0.7651$.

To examine the dynamic of the system at this condition, the transverse displacement transmissibility and the point and transfer receptances of the structure are plotted in Fig. 6 for $\mu = 2$ and $\eta = 0.01$ and $\eta_a = 0.008$. Also plotted, as a grey solid lines and grey dashed lines, are the transmissibilities for the cases where the vibration absorber is only attached at x_1 and x_3, respectively.

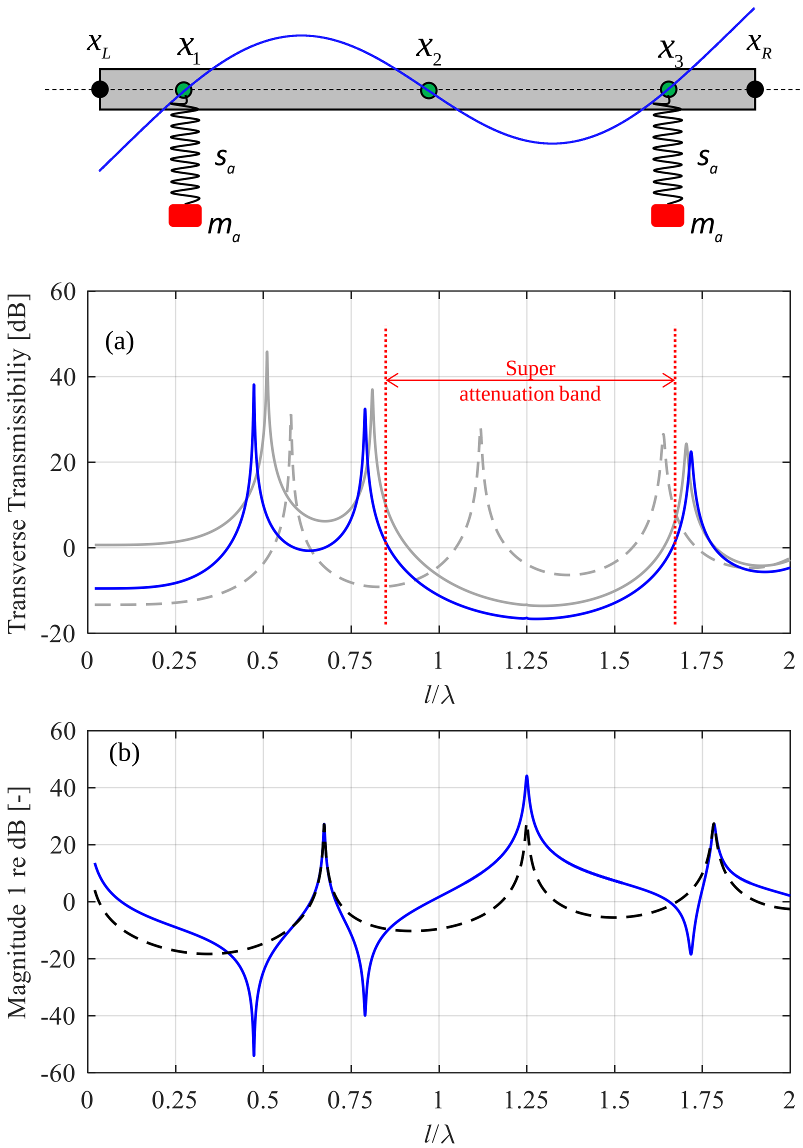

Figure 6. Frequency plots of a beam with a vibration absorber attached to positions x_1 and x_3 correctly tuned to form the super attenuation band: (a) transverse displacement transmissibility; and (b) point (blue solid line) and transfer (black dashed line) receptances. The grey solid and dashed lines in the transmissibility are, respectively, for the beam with the vibration absorber attached only at x_1 and only at x_3.

It is clear from Fig. 6(a) that a super attenuation band is formed in a beam with an array of two vibration absorbers. When compared with the super attenuation band created by attaching a single vibration absorber near the vibration source (solid grey line), the properties of attenuation, such as the width of the band and the amplitude of attenuation, shows a small improvement. However, as a vibration absorber is also attached farther from the vibration source, the attenuation for frequencies below the super attenuation band was also improved. Note, from the point and transfer receptances shown in Fig. 6(b), that once again the formation of the super attenuation band implies a superposition of modes, combining the vibration modes added by the two vibration absorbers with those of the beam. As a result, within the considered frequency range, only three resonance peaks are observed, similarly to the cases of a bare beam and a beam with only one vibration absorber attached.

This clearly demonstrates that using an array of resonators it is possible to create a super attenuation band when they are correctly tuned, but it also shows the advantages of distributing the resonators along the beam as it is capable to improve the attenuation properties of the structure at lower frequencies.

5. CONCLUSION

This paper investigated the formation of a super attenuation band in a beam using an array of resonators (vibration absorbers) attached at different positions. The interest was to improve the attenuation properties of a beam structure. To achieve this, a large absorber mass was considered with the purpose of better visualise the physics behind the use of vibration absorbers. The transverse displacement transmissibility was analysed using the dynamic stiffness matrix method for two cases: a single vibration absorber attached at different positions along the beam, and an array of two vibration absorbers attached to the beam. It was demonstrated that a super attenuation band can be formed when a single vibration absorber is attached to a position where the transverse displacement of the bare beam is motionless, and the tuned frequency of the system coincides with the natural frequency of anti-symmetric mode of the free-free beam. In this condition, a matching of receptances between the vibration absorber and the bare beam is required. Furthermore, when an array of two vibration absorbers is attached to the beam, a super attenuation band can also be achieved by precisely tuning each vibration absorber to a specific frequency. The analysis on the transverse transmissibility revealed that spreading the resonators along the beam improves the attenuation properties of the super attenuation band. Additionally, it enhances the attenuation properties at frequencies below the super attenuation band, making this beam meta-structure more suitable for a wide range of engineering applications.

6. ACKNOWLEDGEMENTS

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