

COBEM2023-1248 Creation of a algorithm using the Blade Element Momentum (BEM) methodology for the aerodynamic analysis of horizontal axis wind turbines

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Abstract. *This paper presents a new methodology for conducting two-dimensional analyses of horizontal-axis wind turbines. Initially, the classical theory called Blade Element Momentum (BEM) was applied in the development of a Python algorithm. In this methodology, there is no need for other tools; the objective is to perform the analysis using only the algorithm. The logic of the code involves inputting fluid properties such as viscosity and density, as well as the geometric properties of the turbine, including radius, chord, and angular relationships. The software XFOIL is used to perform the aerodynamic calculations, with the Tip Loss Correction Factor, also applied to achieve more accurate analyses of the induced drag caused by blade tips. Furthermore, unlike other works, this methodology applies Montgomerie extrapolation, ensuring that the analysis is conducted to obtain a wider range of lift and drag coefficients. This allows for the determination of Thrust, Torque, and Total Power values. The code was validated with the literature, resulting in satisfactory values of the power coefficient.*

Keywords: *Wind Energy, Horizontal Axis Wind Turbine - HAWT, Blade Element Momentum - BEM, Tip Loss Correction Factor, Montgomerie Extrapolation.*

1. INTRODUCTION

The study of mathematical models applied to wind turbine projects in recent years is generally based on BEM, and its justification is its simplifications, which consider that the blade can be analyzed as a number of independent stream tubes, the spanwise flow is negligible, the flow is considered axisymmetric, and usually, the cascade effect is not taken into account. The velocity is determined through the application of momentum conservation, which originates from the aerodynamic polar curve. The results of the BEM method are highly dependent on the accuracy of the lift and drag coefficients, C_l , C_d . Typically, during the operational period of a wind turbine, due to variations in wind speed, the angle of attack, α , can reach high values and thus enter the post-stall condition. Consequently, according to the study by Vaz *et al.* (2011), it is necessary to analyze all possible situations for the efficiency of the HAWT design. For the development of an analysis methodology for HAWT, a high level of knowledge in aerodynamics is essential. It is also important to investigate current analysis methods to examine their input and output parameters, and subsequently establish criteria to focus on the studies that could best contribute to the construction of this work. In this regard, the summaries of the references used to form a critical framework on the topic addressed in this research will be presented.

In Coutinho (2017), it is proposed to estimate the performance of a horizontal axis wind turbine using a classical BEM theory coupled with data obtained from computational fluid dynamics (CFD) simulations, and to compare the results with experimental data found in the literature. Thus, to estimate the performance of each turbine configuration, CFD simulations were conducted to obtain the aerodynamic coefficients of specific airfoils. Subsequently, an algorithm was programmed to perform the turbine performance calculations. As the study focused heavily on airfoil analyses, the results indicate that the turbine utilizing the LS0417 airfoil exhibits the best performance among the cases studied.

In Oliveira (2019), the objective is to aerodynamically design a 300W wind turbine surrounded by a power intensifier and evaluate the gains this new approach can bring compared to the design of the same wind turbine without the presence of this equipment. The methodology adopted consisted of implementing the BEM in a programming language within the Visual Basic for Applications (VBA) environment of the Excel software. This was applied to perform the calculations of

the rotor blades, and for the diffuser sizing, the study by Ohya (2008) was used as a reference. The results demonstrated that it was possible to confirm that the electrical power converted by a wind turbine with a diffuser is higher compared to the same wind turbine without a diffuser. The study showed an approximately five fold gain in power, dP , conversion for the former case compared to the latter. Therefore, the use of a low-power wind turbine surrounded by a diffuser presents considerable advantages in terms of converted power, dP , despite the increased drag resulting from the presence of the power intensifier.

In Giljarhus (2020), the application of BEM in Python was performed. The author used experimental data as a basis for validation. However, it is important to mention that the article does not employ tools to calculate the aerodynamic coefficients, necessitating the adoption of these values as input parameters, thereby limiting the usability of the code. Similarly, the output parameters of the code are thrust, dT , torque, dQ , and total Power, P , however, the output parameters of power coefficients, C_p , and torque, C_q , are not included in this study.

In Jinane Radi (2022), a mathematical model BEM concept was developed with the aim of optimizing the chord and twist distribution along the span of a $20kW$ HAWT blade using the BEM method. Design parameters of the blade, such as the optimal lift and drag coefficients, C_l , C_d , chord c , twist angle, axial induction factor, a , and tangential induction factor, a' , for a horizontal axis wind turbine were estimated using a MATLAB code. The methodology involved the implementation of BEM in MATLAB, focusing on the modeling and design of a HAWT. The blade geometry parameters, including the Prandtl tip loss correction, F , were implemented with the assistance of MATLAB. The results of the analysis were found to be satisfactory, as there was a decrease in chord length and twist angle along the blade span. However, it was observed that the maximum values achievable for the axial and tangential induction factors, a , a' , were 0.3325 and 0.175, respectively. The conclusion is that this approach can be effectively implemented, but further refinement and validation are necessary to ensure coherence for HAWT analysis under different blade design characteristics.

The objective of this study is to develop a new design methodology for HAWT based on the Blade Element Momentum (BEM) Theory, utilizing the XFOIL software as an integral part of the code. Unlike previous works, which rely on other software or literature research to obtain aerodynamic coefficients, this algorithm uses the airfoil itself as an input parameter instead of aerodynamic polar curve, as commonly done in the work by Giljarhus (2020). The Montgomerie Extrapolation was implemented to obtain the Lift and Drag coefficients, C_l , C_d , in a range of -180° to 180° angle of attack, α . In order to increase accuracy, a correction method was also incorporated, considering the tip loss factor, resulting in more realistic and precise values for HAWT. The input parameters include fluid properties such as viscosity μ , and density ρ , as well as geometric properties of the turbine such as chord, c , radius r , and mounting angle. As output parameters, the algorithm provides axial and tangential induction factors, a , a' , thrust, dT , torque, dQ , power, dP as well as the corresponding torque and power coefficients, C_q , C_p . Thus, the proposed algorithm offers an integrated and self-contained approach for calculating the aerodynamic coefficients and performance of Horizontal-Axis Wind Turbines.

2. METHODOLOGY

This section presents the methodology and tools required for developing the code in this study, which was based on BEM and implemented along with the XFOIL software to perform aerodynamic calculations. The Montgomerie extrapolation technique was also employed to obtain lift and drag, coefficients, C_l , C_d for a range of 360° angle of attack, α . In addition, tip loss correction factor F , and the code was validated against existing literature. This approach aimed to establish a novel design methodology for the analysis of two-dimensional horizontal axis wind turbines.

2.1 Blade Element Momentum

As the classical theory of wind turbine rotor aerodynamics, the BEM method (also known as Strip theory or Glauert/Wilson method) combines the Momentum theory and Blade Element theory, the blade is divided into several sections and each section sweeps an annular area when the rotor rotates. These annuli are separated and no interaction between each other. In other words, the stream tube is decomposed along different radius positions and each annulus has its own momentum balance. By dividing the wind turbine blades into annular blade elements and applying one-dimensional linear momentum conservation to the annular elements, the forces and power are calculated and integrated based on the sectional airfoil lift and drag coefficients the blade is divided into N sections (Tang *et al.*, 2015). In each section, there are forces and velocities acting in different ways. To initiate the explanation of the BEM theory, it is crucial to examine the Fig. 1, which delineates the key angles governing the performance of wind turbine blades. Therefore, the angles under consideration are:

- β is blade pitch angle, angle between the plane of rotation and the chord line
- α is angle of attack, Angle between the chord line and the relative wind W .
- ϕ is flow angle, $\phi = \alpha + \beta$

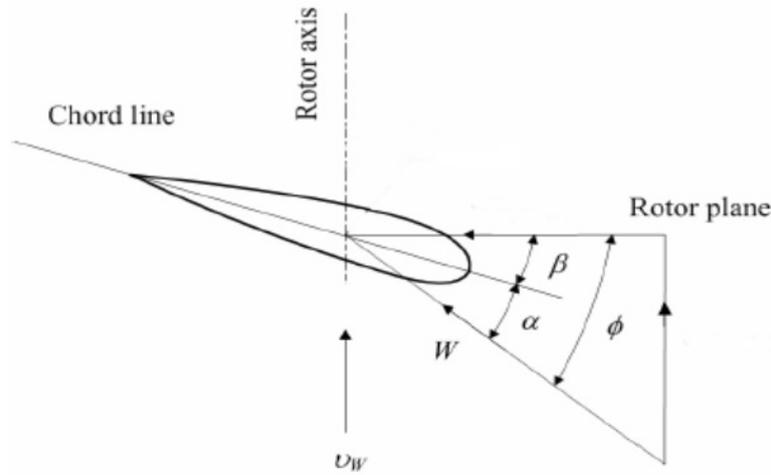


Figure 1. Definition of the main angles of a wind turbine blade. Adapted of Vaschetti *et al.* (2013)

As stated by the BEM Theory, the origins of the forces acting on the blade are aerodynamic in nature, given that the blade is manufactured using airfoil profiles. According to Bernoulli's Law, the fluid passing over the airfoil will experience different flow conditions on the upper surface (extrados) and lower surface (intrados), resulting in a pressure difference and subsequently generating lift and drag forces. Therefore, it is possible to decompose these forces with respect to the normal and Tangential axes, leading to the following equations Eq. (1) and Eq. (2)

$$C_n = C_l \cdot \cos \phi + C_d \cdot \sin \phi \quad (1)$$

$$C_t = C_l \cdot \sin \phi - C_d \cdot \cos \phi \quad (2)$$

Where ϕ is the flow angle, C_l is the lift coefficient, and C_d is the drag coefficient. With the results from the flow velocity and the angular velocity, we can begin formulating the velocity triangle, where its resultant is the relative velocity, defined by the equation:

$$W = \sqrt{\frac{(\omega \cdot r \cdot (1 - a))^2}{(U \cdot (1 + a'))^2}} \quad (3)$$

Where, ω is the angular velocity, r is the local radius, U is the flow velocity, a is the axial induction factors, where the axial induction factor is a measure of the influence of drag and lift forces on the airflow around a surface. It represents the axial velocity induced by vortices on the lifting surface. and a' is the tangential induction factor, on the other hand, the tangential induction factor measures the tangential velocity induced by the vortices. These factors are used to model the effects of air flow over the airfoil surface, and defined by the equation "Eq. (4)" and "Eq. (5)", respectively:

$$a = \frac{1}{\frac{4 \cdot (\sin \phi)^2}{\sigma \cdot C_n} + 1} \quad (4)$$

$$a' = \frac{1}{\frac{4 \cdot \sin \phi \cdot \cos \phi}{\sigma \cdot C_t} - 1} \quad (5)$$

C_n is the normal coefficient, and C_t is the tangential coefficient, which have already been mentioned in Eq. (1) and Eq. (2), and the σ is Rotor Solidity, represented by equation Eq. (6).

$$\sigma = \frac{B \cdot c}{2 \cdot \pi \cdot r} \quad (6)$$

Thus, the BEM method enables the analysis of the rotor through the velocity triangle, where the axial and tangential induction factors, a , a' are already known. This allows for the formulation of equations to determine the forces acting in these directions, which are represented by equation Eq. (7) and Eq. (8)

$$F_n = \frac{1}{2} \cdot B \cdot C_n \cdot \rho \cdot W^2 \cdot c \cdot dr \quad (7)$$

$$Ft = \frac{1}{2}.B.Ct.\rho.W^2.c.dr \quad (8)$$

Where B is number of blades, ρ is air density, W the resultant velocity, c the blade local chord, dr division of radius into n parts from the center of the rotor.

Similarly, according to the BEM, it is worth noting that we can also formulate the axial and tangential induction factors, a , a' , to determine the thrust dT , and torque forces dQ , which are the main objectives of the theory. These equations are demonstrated in Eq. (9) and Eq. (10), respectively:

$$dT = \frac{1}{2} \cdot \frac{\rho.B.U^2.(1-a)^2.c.Cn.dr}{(\sin \phi)^2} \quad (9)$$

$$dQ = \frac{1}{2} \cdot \frac{\rho.B.U.(1-a).\omega.r.(1+a').c.Ct.r.dr}{\sin \phi \cdot \cos \phi} \quad (10)$$

Therefore, power will be defined as the multiplication of torque dQ and ω the angular velocit, Eq. (11).

$$dP = dQ.\omega \quad (11)$$

Moreover, it is possible to obtain the coefficients of power and torque by integrating these components, thus obtaining the total power P , and torque Q . Their coefficients are represented by equations Eq. (12) and Eq. (13), respectively.

$$Cp = \frac{P}{\frac{1}{2}.\rho.U^3.\pi.R^2} \quad (12)$$

$$Cq = \frac{Q}{\frac{1}{2}.\rho.U^2.\pi.R^3} \quad (13)$$

Where, P represents the total power, Q represents the total torque, and R represents the full rotor radius.

Due to the low pressure on the upper surface of the airfoil, the flow at the blade tip tends to flow upwards, resulting in a decrease in lift. Several models have been used to account for this loss at the tip, but the most commonly used model is based on the Prandtl Method, which involves introducing a correction factor into the original BEM theory. The Prandtl tip loss factor serves as a correction for the assumption of an infinite number of blades in BEM, as assuming the rotor consists of an infinite number of blades leads to an infinite number of tip vortices, resulting in a cylindrical wake structure. Thus, the curvature of the undisturbed flow near the wake tip would be allowed, leading to a complete uniform flow. In the case of potential two-dimensional flow, the flow field around a circle can be described by the superposition of a uniform elemental flow with a doublet and vortex flow (Ramdin, 2017). Therefore, the tip loss correction factor is defined by Eq. (14).

$$F = \left(\frac{2}{\pi}\right) \cdot \arccos \left[\left(\exp \left(-\frac{(B/2) \cdot [1 - (r/R)]}{(r/R) \cdot \sin \phi} \right) \right) \right] \quad (14)$$

In this way, it is necessary to apply the correction factor F to the axial and tangential induction factors factors of the BEM, a , a' , which is represented by Eq. (15) and Eq. (16).

$$a = \frac{1}{\frac{4.F.(\sin \phi)^2}{\sigma.Cn} + 1} \quad (15)$$

$$a' = \frac{1}{\frac{4.F.\sin \phi \cdot \cos \phi}{\sigma.Ct} - 1} \quad (16)$$

2.2 Extrapolation Montgomerie Method

Initially, to calculate the performance of the turbine blade using the BEM method, it is necessary to know the lift and drag coefficient values, as normally these values are calculated based on software such as XFOIL. It is common that these data are usually obtained from a limited range of angles of attack, α . It is necessary to extrapolate the initial data to obtain the lift and drag coefficients, Cl , Cd , respectively, for the entire interval range that a horizontal axis wind turbine can work, which are used to extrapolate the Viterna and Montgomerie methods, both methods are the relatively common ones adopted to complement the BEM theory, and the conclusion of the study by Mahmuddin *et al.* (2017) was that the Viterna method was formulated based on the potential flow theory and is simpler than the Montgomerie method, however, even though it is more complicated than the Viterna method, the Montgomerie method has been shown to have greater accuracy than the Viterna method. Thus, justifying the adoption of Montgomerie in the present work.

The implementation of extrapolation is fundamental for the excellence of the project, since in wind turbine technology, the stall of the blades can occur due to strong winds, or weather conditions that were not foreseen for a certain region. In this way, understanding the behavior of the aerodynamic coefficients at different values of, α , angle of attack is fundamental, since through this information it is possible to limit the excessive power, P . The types of wind turbines are related to power regulation and transmission, in power regulation, there are two main types: "stall" and "pitch." In the "stall" method, the airflow is temporarily interrupted by adjusting the turbine blade geometry, thereby reducing power production, in the "pitch" method, the pitch angle of the blades is changed to limit the rotational speed and protect the turbine, these strategies are used to control power production and ensure the safe operation of wind turbines, being chosen according to site conditions and project requirements.(Nath *et al.*, 2018).

The Montgomerie method is formulated based on the assumption that there exists some potential flow-like behavior in a real airfoil around a 0° angle of attack, α . At higher angles of attack, the airfoil performance behaves like a basic thin plate. For intermediate angles, a transformation function, f is used to simulate the behavior. The overall performance is an interpolation between the behavior of the thin plate and the potential flow curve (Mahmuddin *et al.*, 2017). In this way, it is possible to define the Montgomerie method using the following Eq. (17).

$$Cl = f.t + (1 - f).s \quad (17)$$

Where, f is determined from the original Lift Coefficient Cl vs angle of attack α , and f is defined by Eq. (18), t is the straight line function being a tangent when the angle of attack is equal to zero, resulting in a tangential behavior around $Cl(0)$ and $Cd(0)$, and s is an interpolation between the thin plate behavior.

$$f = \frac{1}{1 + k.\Delta\alpha^4} \quad (18)$$

Where k is defined by equation Eq. (19).

$$k = \left(\frac{1}{f_2} - 1 \right) \cdot \frac{1}{(\alpha_2 - \alpha_m)^4} \quad (19)$$

The point α_m it can be computed using the following Eq. (20):

$$\alpha_m = \frac{\alpha_1 - G.\alpha_2}{1 - G} \quad (20)$$

Where G is represented in 'Eq. (21)'. ..

$$G = \sqrt[4]{\frac{\frac{1}{f_1} - 1}{\frac{1}{f_2} - 1}} \quad (21)$$

Where f_1 and f_2 are values that need to be obtained from the original Lift Coefficient Cl vs angle of attack, α curve, the way to extract these points is connecting the already obtained points, which are the pre-stall aerodynamic polar curve.

2.3 Algorithm Design Methodology

In the developed algorithm, which can be observed in Fig. 2, the first step is to define an aerodynamic profile. The algorithm reads and plots the airfoil profile, and with the implementation of XFOIL, it performs calculations for pre-stall aerodynamic polar data. These calculations are done within a range of -20° to 20° α , angle of attack, which is more than sufficient for performing extrapolation using the montgomerie method, resulting in an angle of attack range of -180° to 180° . With the complete aerodynamic behavior of the airfoil determined, the fluid properties and rotor geometric properties are then inputted to initiate the equations defined in "Eq. (1)" of this study. The algorithm also includes the implementation of correction methods to enhance accuracy.

Therefore, using equations Eq. (9), it is possible to determine the thrust, while equations Eq. (12) and Eq. (13) define the values of the power and torque coefficients. These parameters are crucial as they serve as the primary means to compare with other design methodologies such as Computational Fluid Dynamics – CFD and experimental approaches. Furthermore, this code has been validated against the existing literature to ensure the proper programming of the equations and to validate the algorithm for use in the analysis of two-dimensional preliminary designs of horizontal-axis wind turbines.

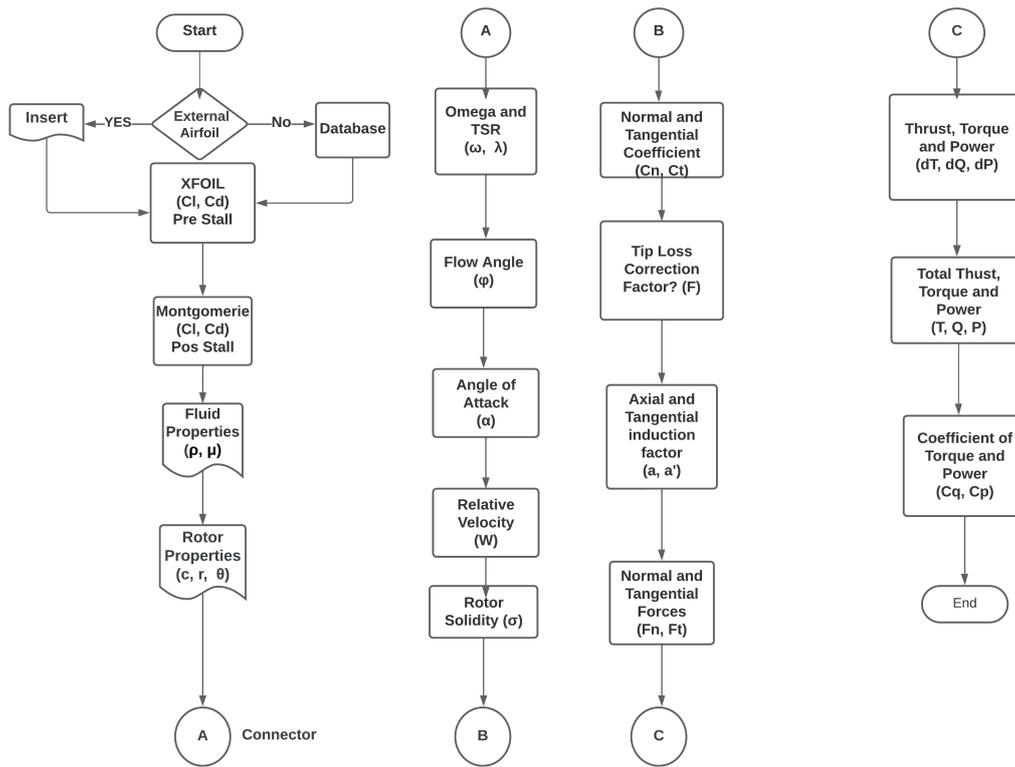


Figure 2. Flowchart illustrating the essential steps of the algorithm, from data input to the calculation of thrust, torque, and their respective coefficients.

2.4 Input parameters

Using Wenzel (2007) as a reference, the initial input parameters were the fluid properties. The air density, denoted by ρ , was set to 1.275 kg/m^3 , and the dynamic viscosity, denoted by μ , was set to $17.2 \times 10^{-6} \text{ m}^2/\text{s}$. The rotational speed (RPM) was set to 635.56, and the number of blades, B was set to 3. The NACA23018 airfoil was used as the input parameter in this study. Furthermore, it is worth noting that in Wenzel (2007) work, the blade was divided into 50 sections, each section having its corresponding radius, chord, and pitch angle. While all 50 stations were used in the calculations, Tab. 1 only represents 11 stations. The rationale behind this is to better understand and visualize the behavior of each radius, r , chord, c , and pitch angle, β , by selecting a subset of stations, we can focus on specific data points. By applying these input parameters, the aim was to obtain results similar to those in the reference work. With these data, it was possible to validate the new methodology and calculate the errors.

Table 1. Initial geometric parameters used in the algorithm

Sections	Radius, m	Chord, m	Pitch, rad
1	0.150	0.157	0.350
5	0.199	0.142	0.257
10	0.261	0.122	0.175
15	0.322	0.105	0.117
20	0.384	0.092	0.075
25	0.445	0.081	0.043
30	0.507	0.073	0.018
35	0.568	0.066	-0.002
40	0.629	0.060	-0.018
45	0.691	0.055	-0.032
50	0.752	0.050	-0.043

3. RESULTS AND DISCUSSION

To verify the quality of the methodology and, consequently, the reliability of the numerical results, a comparison was performed with the work by Wenzel (2007), which focused on an important parameter: a , the axial, and a' , tangential induction factors. This comparison was conducted because these factors have the greatest influence on the normal and tangential forces, as illustrated in Fig. 3 and Fig. 4. The values obtained were found to be very similar to the reference literature. The axial induction factor, a , exhibited a minimum error of 0.033% and a maximum error of 17.389% in one section, however, up to section 25, the percentage difference remained below 10%, while the tangential induction factor, a' , had a minimum error of 6.71% and a maximum error of 19.21%. In regard to the tangential induction factor, a' , the obtained values closely approached the reference values. This is particularly relevant since these results proved to be satisfactory for the constants reliant on the tangential induction factor, a' . Furthermore, the thrust, dT , values were also compared, as shown and compared along the secondary axis in Fig. 3. The minimum error for thrust was 3.999%, with a maximum error of 24.014%. nevertheless, only in 7 out of the 50 sections analyzed, the percentage differences were higher, exceeding 20

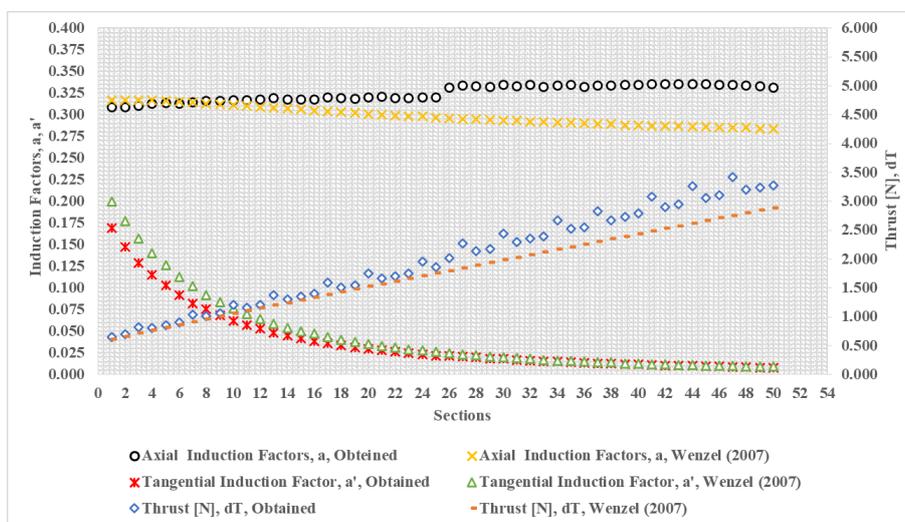


Figure 3. Representation of the output parameters without the Prandtl Method F

Additionally, the numerical results for torque, dQ , and power, dP were compared, as these parameters are of importance in the preliminary analysis of a horizontal-axis wind turbine, as they are analyzed to assess the project's feasibility. Thus, it was possible to observe from Fig. 4 that the data is consistent with Wenzel (2007) literature, as their behavior in relation to the data was as expected, with errors consistently falling within a moderate range. The torque, dQ values had a minimum error of 0.042% and a maximum error of 7.558%. Likewise, for power, dP , values, which are dependent on torque, the minimum error was 0.0482% and the maximum error was 7.520%. Therefore, based on these values and Fig. 4, considering that this larger error above 7% occurred in only 5 sections. It can be concluded that the methodology has proven to be highly efficient for the most important parameters of the turbine.

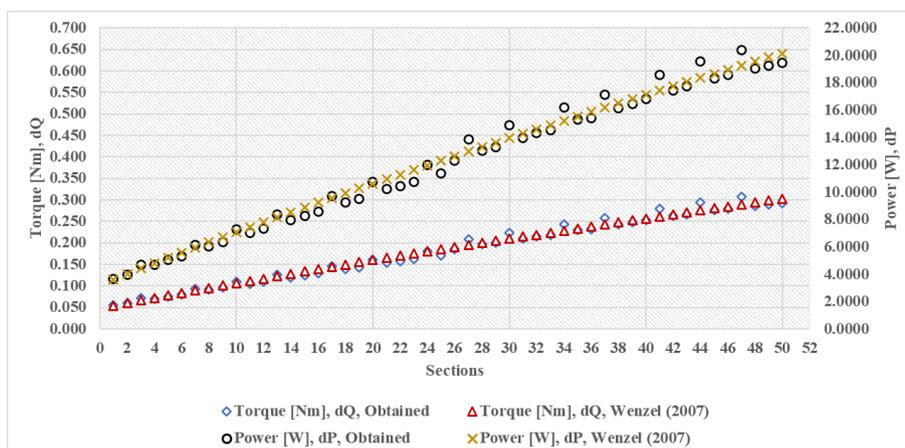


Figure 4. Representation of the output parameters, without the Prandtl Method F

In this methodology validation, the values were compared both with and without the correction method. Thus, in a secondary analysis, the axial and tangential induction values were examined in relation to the blade Prandtl tip loss factor, F , to validate the correction methods and understand how this method affects the forces acting on the rotor blade. According to Fig. 5, it can be concluded that these values are closely approaching the results of Wenzel (2007). The axial induction factor a exhibited a minimum error of 0.23% and a maximum error of 10.81%, while the tangential induction factor a' had a minimum error of 1.74% and a maximum error of 18.64%. Furthermore, the thrust, dT , relationships were also compared along a secondary axis, with the minimum error being 6.09% and the maximum error being 18.09%.

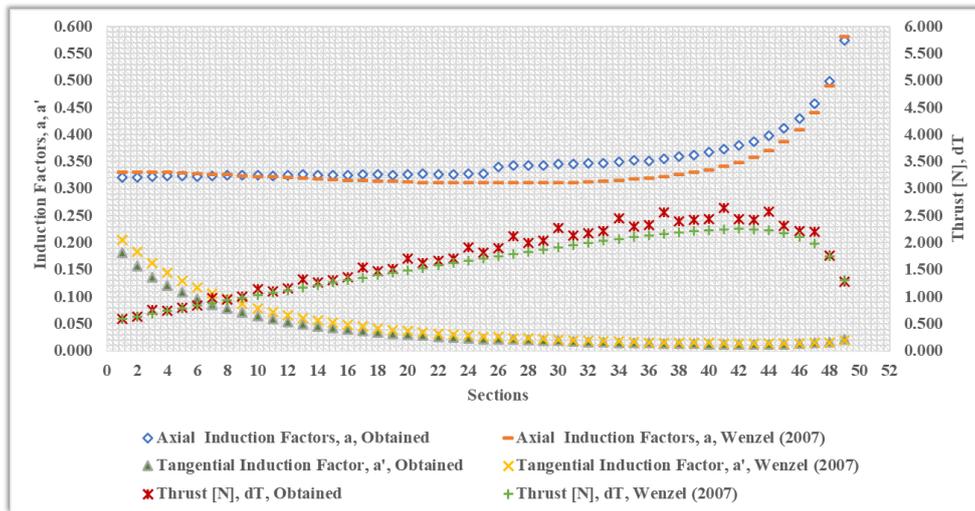


Figure 5. Representation of the output parameters with the Prandtl Method F

The analysis of torque and power results plays a crucial role in enhancing the performance of renewable energy generation systems, especially in horizontal-axis wind turbines. Torque, dQ , is a fundamental measure that represents the rotational force resulting from the interaction between the wind and the turbine blades, and it is essential for assessing the turbine's ability to convert the kinetic energy of the wind into mechanical energy. In turn, power, dP , is a measure of the rate at which this mechanical energy is converted into electricity. Obtaining accurate values for torque and power is crucial to ensure the efficiency and proper performance of the design. In this regard, the use of the Prandtl tip loss factor, F , is of utmost importance as it allows for the correction of effects that can negatively impact the accuracy of the results. The errors found in torque, dQ , ranged from a minimum of 0.26% to a maximum of 32.58%, while errors in power ranged from a minimum of 0.127% to a maximum of 32.503%. As Fig. 6 demonstrates through the behavior of the data, it is important to conduct careful analyses and correct any discrepancies to obtain values that are closer to reality, thereby optimizing the performance of horizontal-axis wind turbines.

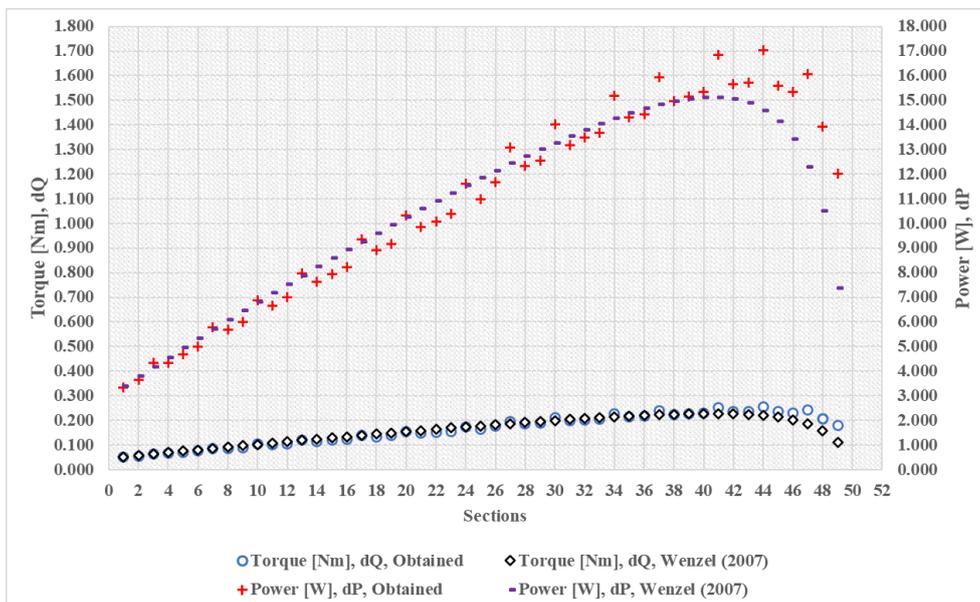


Figure 6. Representation of the output parameters, with the Prandtl Method F

The methodology employed for calculating the forces and performance of each section of the wind turbine blade enabled the integration of these parts, thereby providing the overall values for rotor performance. The results obtained, as presented in Tab. 2, demonstrated excellent proximity to the reference values. The error found for thrust, T , was 13.350%. It is important to highlight that the total rotor torque, Q , was the value that closely approached the reference, with an error of only 1.073%. Consequently, the obtained power, P , values were highly satisfactory, resulting in an error of 1.097%, and 2.537% for the Torque coefficient, C_q . These results confirm that the employed methodology is highly reliable and produces results extremely close to the reference values, thus validating the accuracy and robustness of the adopted approach.

Table 2. The output parameters, without the Prandtl Method F

Output Parameters	Obtained	Wenzel (2007)	Error (%)
Total Thrust. N (T)	100.489	88.654	13.350
Total Torque. Nm (Q)	9.131	9.230	1.073
Total Power. W (P)	606.761	613.490	1.097
Torque Coefficient (C_q)	0.107	0.110	2.537
Power Coefficient (C_p)	0.536	0.560	4.333

Furthermore, through the correction method validated by reference Wenzel (2007), it is possible to obtain more accurate results regarding the actual efficiency of a wind turbine. This is because the Prandtl tip loss factor serves as a correction for the assumption of an infinite number of blades in BEM theory. By comparing Tab. 2 and Tab. 3, it can be observed that variations occurred in the values of the torque and power coefficients, C_t , C_p , respectively. Upon analysis, it was found that C_t decreased by 0.107 to 0.094, with an error of 6,000% compared to the literature, and C_p decreased by 0.536 to 0.471, with an error of 1,875%. These variations occurred due to the application of the Prandtl F Method. These results demonstrate the effectiveness of the correction method, as the obtained values are in excellent agreement with the reference, allowing for a more accurate assessment of the performance of the horizontal-axis wind turbine.

Table 3. The output parameters, with the Prandtl Method $F_{protect}$

Output Parameters	Obtained	Wenzel (2007)	Error (%)
Total Thrust, N (T)	84.624	77.731	8.868
Total Torque, Nm (Q)	8.023	7.840	2.335
Total Power, W (P)	533.142	520.710	2.387
Torque Coefficient (C_q)	0.094	0.100	6.000
Power Coefficient (C_p)	0.471	0.480	1.875

4. CONCLUSIONS

This article presented an innovative methodology for the two-dimensional analysis of horizontal-axis wind turbines (HAWTs). The procedure takes into account fluid properties as well as the geometric relationships of the turbine rotor to obtain turbine performance parameters as results. An important contribution of this methodology is the implementation of the XFOIL software within the algorithm itself, along with the Montgomerie extrapolation technique for analyzing aerodynamic polar curve. Furthermore, the incorporation of the Prandtl tip loss factor allows for a better approximation of the actual performance values of the wind turbine. The methodology was validated by comparing the obtained results with the reference Wenzel (2007). The results were very satisfactory, as all the parameters in relation to the reference, which can be observed in Tab. 2 and Tab. 3, came very close to the expected values. The use of this methodology is particularly recommended for preliminary analyses as it helps to avoid errors resulting from a lack of knowledge about the expected turbine performance.

In conclusion, it was found that the methodology presents several advantages by implementing the BEM theory, XFOIL, The Prandtl tip loss factor F , and the Montgomerie Extrapolation in the code. These approaches allowed for obtaining the output parameters, including the axial and tangential induction factors a , a' , respectively, thrust, dT , torque, dQ , and power, dP . As a result, the analysis process became more streamlined and efficient. The incorporation of the BEM theory provided accurate modeling of the aerodynamic behavior of the blades, while the use of Xfoil allowed for detailed evaluation of lift and drag characteristics. Additionally, the tip loss factor considered the influence of the blade tips, resulting in more precise estimates of the output parameters. The Montgomerie Extrapolation, in turn, offered a reliable technique to extrapolate the results obtained under limited test conditions to a wider operating range. By

combining these approaches, our methodology provided a comprehensive and robust approach for wind turbine analysis, simplifying the process of obtaining the parameters of interest. Furthermore, future studies can compare the obtained results with other approaches, such as computational fluid dynamics (CFD) simulations and experimental data, to assess the consistency and accuracy of the methodology in different contexts. (Olczak *et al.*, 2016)

5. ACKNOWLEDGEMENTS

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7. RESPONSIBILITY NOTICE

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