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VIRTUAL SENSING OF ROTATING MACHINES USING AUGMENTED KALMAN FILTER AND ROSS MODELS

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Abstract. *Rotating machines play a fundamental role in the industry due to their adaptability and usability. However, they are susceptible to noise and vibrations, damaging equipment and production processes. Continuously monitoring and modeling rotors is crucial to ensure optimal performance. Despite this, physical methods for measuring vibration responses can be limited by the high cost of sensors, restricted access to the monitoring area, and geometric complexity, making it expensive or impossible to measure vibration responses of certain degrees of freedom (DOFs) in rotating machines. As a solution, virtual sensing (VS) techniques combine information from accurate models with monitoring data to produce more precise predictions. In this work, the Augmented Kalman Filter (AKF) method is used to estimate the vibrational response of a rotor, including unmeasurable DOFs. The AKF algorithm combines information from a finite element model with numerically vibration data to produce more accurate estimates. A Differential Evolution-based optimization process is incorporated to facilitate the execution of the AKF. The methodology can estimate vibrational behavior with a maximum error of ≤11µm, given a prediction delay, and quickly achieves stability with an error of ≤2µm at a low computational cost, which is important for the field of study.*

Keywords: *Virtual Sensing, Rotating Machines, Augmented Kalman Filter*

1. INTRODUCTION

Excessive vibrations negatively affect the efficient and safe operation of rotating machinery, such as pumps, motors, and compressors. However, during the normal operation of these types of equipment, they are subject to various forces and loads that can result in undesired vibrations. These vibrations can be simultaneously caused by multiple factors, such as mass imbalance, misalignment, mechanical clearances, component wear, and aerodynamic issues (Miettinen, 2020; Du and Sun, 2021; Vettori *et al*., 2023).

In particular, excessive vibrations of a rotor can lead to premature wear of its critical components, causing mechanical failures and reducing the machine's lifespan. Furthermore, unwanted oscillations can negatively affect the product or service quality. For example, in a production line, rotor vibrations can result in poorly manufactured or inferior finished products. Consequently, measuring and monitoring the dynamic behavior of rotating machinery for safety, economic, and production purposes (Cavallini Jr, 2013; Ganguly and Roy, 2022; Vettori *et al*., 2023).

Monitoring devices equipped with physical sensors and data acquisition drivers are typically used to monitor mechanical structures and vibration measurements of rotating machinery. These devices are positioned at strategic points of the machine to obtain precise information about the vibrations. Subsequently, vibration sensors capture the mechanical oscillations and convert them into electrical signals, which are later processed and analyzed to provide valuable information about the machine's condition (Cavallini Jr, 2013; Kalista, Liska and Jakl, 2021; Rezende *et al.*, 2023a).

In general, modern monitoring devices are designed with advanced and cost-effective technologies, such as accelerometers, which measure the acceleration of vibrations in different directions. They may also include features like thermocouples to measure temperature and proximity detectors to monitor the rotational motion of the machine (Cavallini Jr, 2013; Tiainen *et al.*, 2019).

On the other hand, data acquisition drivers are responsible for collecting the signals from the sensors and converting them into digital data that can be processed and analyzed using specialized software. These drivers are commonly connected to the physical sensors via cables, ensuring direct data transmission for analysis (Cavallini Jr, 2013; Mora *et al.*, 2023; Rezende *et al.*, 2023a).

In parallel, traditional techniques for analyzing damage in rotating machinery utilize information from key critical points of the structure. A clear example of this approach is fatigue life analysis, which uses historical records of stresses at specific locations in the system when subjected to certain operating conditions (Rezende *et al.*, 2023b).

However, in certain usage scenarios, installing measurement instruments in specific locations of a real structure may be impractical or impossible. This limitation can be attributed to various factors, such as the high cost of sensors, technical constraints related to their instrumentation, or even the hostile conditions of the surrounding environment. As a result, obtaining information about an entire system in all its degrees of freedom (DOFs) can be challenging, making Virtual Sensing (VS) techniques an attractive alternative for accurately estimating the dynamic responses of a system in non-measurable locations (Mora *et al.*, 2023; Rezende *et al.*, 2023a).

To achieve this, VS techniques enable the acquisition of measurements from the structure not directly from physical sensors but rather through the inference of data from other sensors, which can also be combined with information from approximate models to improve the estimates made (Neisi *et al.*, 2022; Mora *et al.*, 2023).

Thus, using VS in a mechanical structure can be carried out through two main approaches: data-driven techniques or model-based techniques (Mora *et al.*, 2023).

In the data-driven approach, the inference of additional information about the system is facilitated using monitoring data. In other words, data from physical sensors are used to build statistical models or machine learning (ML) algorithms, aiming to identify relevant patterns and correlations. Based on these patterns and correlations identified in the data, it is possible to generate reliable estimates for the unknown variables. As such, the data-driven approach of VS relies on the sensitivity of the data to extract valuable information and provide insights into the system in question (Sun *et al.*, 2017; Mora *et al.*, 2023; Rezende *et al.*, 2023a).

In this sense, Artificial Neural Networks (ANNs) are examples of machine learning algorithms that have been widely used in the condition monitoring of rotating machinery, mainly employed for fault identification and characterization in critical rotor components (Li, Wang and Li, 2022; Rezende *et al.*, 2023a).

Khan, Hwang, and Kim (2021), for instance, incorporate in their work a formulation of the two-dimensional Convolutional Neural Network (2D-CNN) with data augmentation techniques, which are utilized to build a machine learning model capable of predicting up to 42 damage classes in a rotor. The artificially produced data through data augmentation represents the behavior obtained from the machine via virtual sensors, and by utilizing this data in the training of the 2D-CNN, the authors were able to improve the accuracy by up to 58% during the tests, compared to the direct classification of data without using virtual sensors.

On the other hand, Bobylev *et al.* (2021) compared various ML technologies to identify the stiffness coefficients of bearing supports in a horizontal rotating machine. For this purpose, the ML models were trained with frequency-domain data (numerically generated by a simulation model). Then the machine's operating scenarios were extrapolated virtually and tested based on real measurement data.

Other works that explore the application of virtual sensing in mechanical structures through data-driven techniques include Sun *et al.*, 2017; Azzam *et al.*, 2021; Šabanovič *et al.*, 2021; Li, Wang and Li, 2022; Rezende *et al.*, 2023a; and Rezende *et al.*, 2023b.

On the other hand, model-based VS techniques intuitively require creating a numerical model capable of adequately representing the dynamic behavior of the monitored system. This model is generally built using physical properties and laws and combined with data obtained through physical sensors to estimate the unknown variables of interest (Neisi *et al.*, 2022; Mora *et al.*, 2023).

In this context, it is considered that the vibratory response of a linear structure is formed by modal contributions in which only a few natural modes are active. Thus, it becomes possible to infer the behavior of specific points in the system using virtual sensors, which are based on knowledge of the modal behavior of the system or other relevant information to estimate responses at locations where no physical sensors are installed.

In rotordynamics, it is possible to indirectly estimate the forces applied to a rotor through the structure's responses and the information obtained from its mathematical model (Cavallini Jr, 2013). However, ensuring this approach's feasibility requires using a numerical model representative of the system, which can be challenging in many real-world problems. As an alternative to this requirement, a considered approach is using stochastic algorithms, which can estimate rotor states with sufficient precision (Neisi *et al.*, 2022; Mora *et al.*, 2023).

In this sense, the Kalman Filter (KF) algorithm, proposed by R. Kalman in 1960, is a viable method for stochastic estimation. Its main application consists of estimating states, that is, the mean and covariance of a dynamic system, using information from models and real monitoring data.

This algorithm is particularly useful in scenarios with uncertainties and noise in the input data. It combines information from the system model with real observations, resulting in more accurate estimates of the system's current state (Mora *et al.*, 2023).

Due to its effectiveness, the KF is widely used in various areas such as signal processing, navigation, automatic control, object tracking, and rotordynamics. Over the years, the algorithm has been the subject of intense research, resulting in various variants and extensions aimed at improving its performance and adaptability to the specific needs of each application (Cumbo *et al.*, 2021; Neisi *et al.*, 2022).

However, it is important to note that in the field of rotordynamics, existing studies on KF (and its variants) are predominantly focused on estimating unbalance forces, aiming to minimize the effects of this fault through balancing processes. Therefore, there is a significant gap in the literature regarding studies on virtual sensing of rotors, which encompass the estimation of vibration amplitudes of the system throughout its entire extent (including physically unmeasurable regions).

Furthermore, the monitoring of rotating machinery in practical situations is often restricted to the bearing regions, which are the fixed areas of monitoring and typically distant from the critical locations of the machine. This restriction compromises the accuracy of the estimates of rotor integrity conditions and consequently hinders the proper optimization of maintenance resources.

In the present work, a study on the Augmented Kalman Filter (AKF) algorithm is made regarding its usage in virtual sensing of rotating machines. For this, a numerical case study is considered using a characteristic rotor model. Next, the differential evolution (DE) method and the compromise optimization technique are used to facilitate the KF model build and improve the estimates made. The results achieved were promising regarding utilizing KF in rotor VS, thus being able to be applied in real cases.

2. AUGMENTED KALMAN FILTER

As previously discussed, the Kalman Filter algorithm is a highly efficient stochastic technique for estimating the states of dynamic systems. In other words, by utilizing a numerical model and real measurement data, the KF can generate accurate estimates of the behavior of the analyzed structure. This approach harmoniously combines the predictive ability of the model with updated measurement information, resulting in more reliable estimates of the system states, including those that are not physically measurable (Cumbo *et al.*, 2021; Neisi *et al.*, 2022).

However, it is important to note that the KF relies on constructing a representative numerical model of the system. This model is used in formulating the state plan, which means its accuracy and fidelity are crucial to the algorithm's effectiveness. Therefore, special care must be taken in developing the model to ensure its suitability and accuracy for the analyzed system (Cumbo *et al.*, 2021; Neisi *et al.*, 2022).

In rotordynamics, the Finite Element Method (FEM) is widely used to construct numerical models of rotors, employing a second-order linear differential equation to describe the response states of a dynamically vibrating system (Cavalini Jr, 2013). Thus, by utilizing Lagrange's energy formulations (kinetic and potential), it is possible to express the characteristic linear equation of a rotor in FEM as follows:

$$M\ddot{q}(t) + (C + \Omega G)\dot{q}(t) + \left(K_m + \dot{\Omega}K_{st}\right)q(t) = S_f F(t) \tag{1}$$

where:

- ✓ M – is the rotor mass matrix;
- ✓ C – is the damping matrix;
- ✓ G – is the matrix associated with the gyroscopic effects;
- ✓ K_m – is the stiffness matrix;
- ✓ K_{st} – is the rotor stiffening matrix under the transient regime;
- ✓ S_f – is the force distribution matrix $F(t)$ in predetermined DOFs;
- ✓ Ω – is the angular speed of the rotating machine in *rad/s*;
- ✓ q – is the states vector at the instant of time *t*.

The matrices mentioned in equation 1 provide a global representation of the physical characteristics of the structure in question. Therefore, these matrices are obtained by combining the local properties of each finite element used in the model, contributing to the global description of the modeled structure.

Furthermore, in the adopted formulation, the dynamic behavior of the rotor is described by a second-order linear equation. This equation completely describes the system's dynamics and requires two state variables.

This is because a second-order equation requires two independent initial conditions to be fully solved. Therefore, in the context of rotordynamics, the state vector q used comprises the displacements and velocities imposed on each degree of freedom of the finite element model.

Thus, it is possible to obtain a reformulation of the characteristic equation of the rotor (Equation 1) so that the FEM model is rewritten in the state-space form, as follows:

$$\begin{bmatrix} \dot{q}(t) \\ \ddot{q}(t) \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}\left[K_m + \dot{\Omega}K_{st}\right] & -M^{-1}[C + \Omega G] \end{bmatrix} \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -M^{-1}S_f \end{bmatrix} \begin{bmatrix} 0 \\ F(t) \end{bmatrix} \tag{2}$$

Subsequently, by simplifying equation 2, we obtain:

$$\dot{x} = Ax + Bu \tag{3}$$

where $x = \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix}$ represents the state vector of the system, $A = \begin{bmatrix} 0 & I \\ -M^{-1}[K_m + \dot{\Omega}K_{st}] & -M^{-1}[C + \Omega G] \end{bmatrix}$ represents the state transition matrix, and $B = \begin{bmatrix} 0 \\ -M^{-1}S_f \end{bmatrix}$ represents the input matrix of the system. Meanwhile, the measured output y of the monitored system is associated with the state vector as follows:

$$y = Cx + Du \tag{4}$$

where C is the sensor selection matrix and D is the feedforward matrix, which in this case is given as null.

Notably, equations 3 and 4 address the observation of states in the continuous domain. However, the KF algorithm utilizes measurement data to adjust the estimates made by a numerical model. The resolution capability of the acquisition system impacts the formulation of the KF, making it necessary to discretize the equations that compose the state plan. This can be achieved by adopting the following relationships for the state transition matrices A and input B:

$$A_D = e^{A \cdot \Delta t} \tag{5}$$

$$B_D = A^{-1} \cdot (A_D - I) \cdot B \tag{6}$$

And, therefore, the representation of a rotating system in the state plane is given according to equation 7.

$$\begin{cases} x_k = A_D x_{k-1} + B_D u_{k-1} + w_{k-1} \\ y_k = C_D x_k + v_k \end{cases} \tag{7}$$

where w and v represent the uncertainties associated with numerical model estimates (error) and experimental measurement data (noise), respectively. For simplicity, these uncertainties are assumed to be of the Gaussian type. In addition, it is considered that the forecasts made, and the measurements do not have a direct correlation with each other, allowing the simplification of the covariance matrices of the process error Q and noise of the experimental data R to diagonal matrices (equations 8 and 9). Thus, each diagonal value of these matrices corresponds to the uncertainty associated with each state q_i of the model and each input z_i of the sensor, respectively.

$$Q = diag(w_{q_1,k}, w_{q_2,k}, \dots, w_{q_n,k}) \tag{8}$$

$$R = diag(v_{z_1,k}, v_{z_2,k}, \dots, v_{z_n,k}) \tag{9}$$

As it is a stochastic algorithm, the determination of the Q and R error covariance matrices have a strong influence on the KF sensitivity (Mora *et al.*, 2023). However, there is no predefined criterion for constructing these matrices, and different approaches can be used to determine the values of w and v. That said, the most common tuning technique, the L-curve, is often used. However, for the present work, a new approach was adopted (Section 3), involving the differential evolution method (Rezende *et al.*, 2023b) and the multi-objective compromise optimization technique (Lobato, 2008).

On the other hand, in rotordynamics, it is highly challenging, if not impossible, to characterize all excitation forces that affect a rotating machine during its operation. As a result, the force components u_{k-1} cannot easily be considered in equation 7. This requires reformulating the described state plane, which incorporates a variant of KF called the Augmented Kalman Filter (AKF).

In AKF, it is initially considered that the force values u_{k-1} are unknown. Then, these values are incorporated into the state vector according to the following approach:

$$x_{a_k} = \begin{bmatrix} x_k \\ u_{k-1} \end{bmatrix} = \begin{bmatrix} q_k \\ \dot{q}_k \\ F_k \end{bmatrix} \tag{10}$$

and, consequently, the following modifications can be established for equations 7, 8 and 9.

$$\begin{cases} x_{a_k} = A_A x_{a_{k-1}} + \zeta_{k-1} \\ y_{a_k} = C_A x_{a_k} + \eta_k \end{cases} \tag{11}$$

where:

$$A_A = \begin{bmatrix} A_D & B_D \\ 0 & I \end{bmatrix} \quad \text{e} \quad C_A = \begin{bmatrix} S_d - S_a M^{-1} K & S_v - S_a M^{-1} C & S_a M^{-1} S_f \end{bmatrix} \tag{12}$$

with S_d being the displacement observation matrix, S_v the velocity observation matrix (in this case $\cong 0$) and S_a the acceleration observation matrix ($\cong 0$).

In parallel, the Q and R error covariance matrices are also modified according to equations 13 and 14.

$$Q_A = \begin{bmatrix} Q & 0 \\ 0 & Q_u \end{bmatrix} \tag{13}$$

$$R_A = \begin{bmatrix} R & 0 \\ 0 & 0 \end{bmatrix} \tag{14}$$

being Q_u a regularization matrix of the errors associated with the forces imposed on the rotor. It is necessary to emphasize that the uncertainties involved in the prediction of the forces are considerably larger compared to the uncertainties of w and v. As a direct result of this discrepancy, the magnitude of the values of Q_u is also larger, often exceeding the 10^3 mark.

In summary, the FK algorithm is a Bayesian estimation method that allows for incrementally predicting the mean state and covariance of a dynamic structure over time (Figure 1). First, estimates are made for time k (Step #1), then these estimates are updated for time $k+1$ (Step #2). In this way, the FK algorithm enables a continuous process of adjusting the predictions, considering the new information available at each instant.

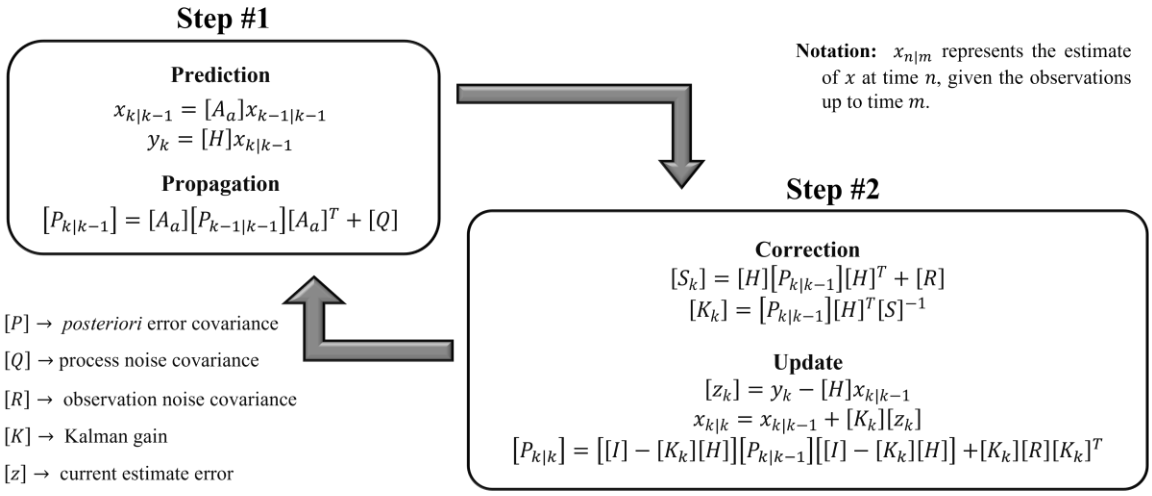


Figure 1. Augmented Kalman Filter algorithm flow.

In Step #2, the objective is to improve the predictions made by the numerical model, which were generated in Step #1. This is done by weighting the prediction error, represented by z_k, using a gain matrix called the Kalman gain, represented by K_k. Therefore, the Kalman gain is a measure of the reliability of the model's predictions.

Throughout the process iterations, it is observed that the Kalman gain tends to stabilize, indicating that the model is approaching more accurate estimates. On the other hand, the prediction error is calculated by comparing the model estimates with the actual values y_k obtained through measurement. This comparison allows evaluating the model's performance and identifying areas where the predictions are less accurate. Thus, improving the predictions in Step #2 involves using the Kalman gain to weigh the prediction error and adjust the model estimates, aiming to reduce the discrepancy between the predictions and the actual values.

Finally, during the processing of AKF, three components are estimated: the nodal displacements $q(t)$ of the rotor, the corresponding velocities $\dot{q}(t)$, and the imposed forces $F(t)$. The determination of vibration amplitudes $q(t)$ in rotating machines is complex but offers significant benefits. In this context, the use of AKF is highly relevant for rotor studies.

3. DETERMINATION OF UNCERTAINTY MATRIXES Q_A E R_A

During the implementation of the AKF algorithm, it is common to use the sensitivity of the acquisition system as a measure of the uncertainty of the measurement noise R_A. However, determining the Q_A process noise uncertainty can be more challenging as it is not always possible to directly observe the process being estimated. Despite this difficulty, in certain cases, even a relatively simple or imprecise process model can produce acceptable results if an appropriate amount of uncertainty is introduced into the process through proper selection of Q_A and R_A (Cumbo *et al.*, 2021; Neisi *et al.*, 2022).

However, due to the lack of available knowledge about error covariance matrices in practical situations, different techniques have been developed to determine them in recent years. Many of these techniques are performed offline and involve the use of other complementary state observers. On the other hand, in the present work, a new approach is formulated for the determination of the AKF error covariance matrices P, Q_A and R_A, based on optimization by differential evolution and the compromise technique (Lobato, 2008).

Differential evolution optimization is a technique that uses an iterative process to search for optimal solutions in complex search spaces, and the compromise technique is employed to balance the trade-offs between multiple mono-objective functions. Figure 2 shows the processing approach used in this work to determine the AKF uncertainty matrices.

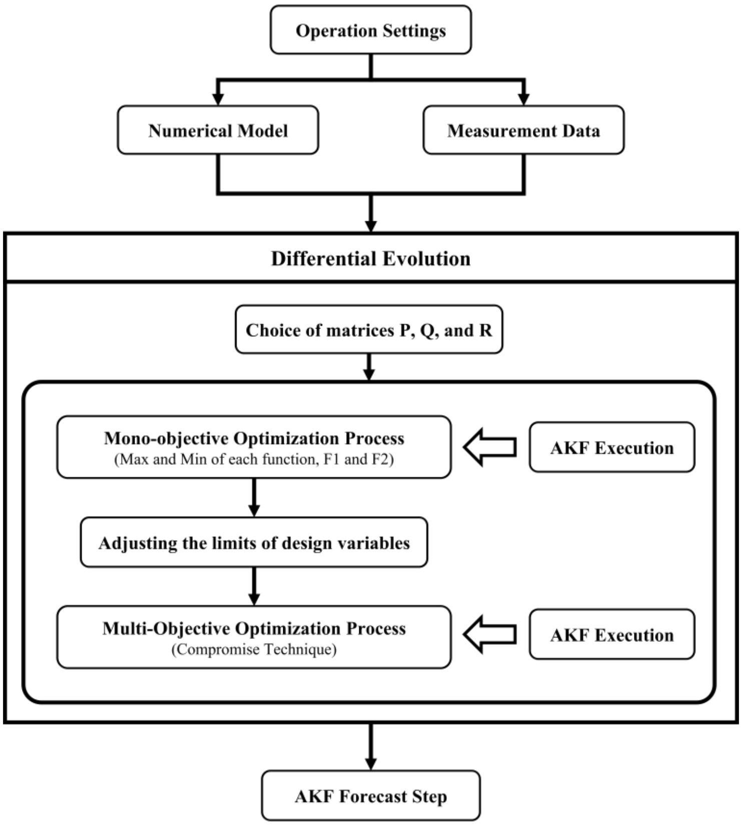


Figure 2. Optimization flow performed in the determination of AKF uncertainty matrices.

The adopted procedure consists of two optimization steps. In the first step, each mono-objective function is individually optimized to obtain corresponding maximum and minimum values. Then, these optimized values are integrated into the multi-objective optimization process to solve the general estimation problem. This process involves setting the limits of the design variables and searching for viable values for the matrices P, Q_A and R_A. This search, in turn, aims to reduce the AKF forecast error and the measured data, making the model more accurate and reliable.

Altogether, two mono-objective functions are considered in the procedure, and all of them are based on the use of the R^2 metric, as follows:

$$f_i = \sum_{j=1}^{n} 1 - \left|R^2(y_j, \hat{y}_j)\right| \tag{15}$$

where y represents the measurement values (sensors), $\hat{y}$ the values to be compared (AKF estimates), and n the number of signals to be compared simultaneously in the same *i-th* mono-objective function.

The first mono-objective function of the optimization process directly compares the displacement values predicted by the AKF with the data measured at the positions of the physical sensors. In turn, the second mono-objective function also evaluates displacements but focuses on virtual displacements estimated from the forces predicted by the AKF algorithm. These virtual displacements are obtained through a numerical integration process, in which the forces predicted by the AKF are used together with the numerical model.

As a result, it is expected that the configuration of the error covariance matrices will be improved, which will result in more accurate estimates of the rotor state through the AKF algorithm. This becomes especially relevant when prior knowledge of covariance matrices is limited or imprecise.

4. USE CASE

In this contribution, a numerical case study was carried out to evaluate the developed approach. The study involved the analysis of a rotor model composed of two hard disks with a diameter of 150mm and a thickness of 20mm, each weighing 2.65kg, and roller bearings. For this, the numerical model of the system was developed using the finite element method and the open-access library ROSS (Rotordynamics Open-Source Software), which is implemented in Python.

In the modeling process, a flexible shaft with a length of $1000mm$ was used, which was divided into up to 37 Timoshenko beam elements, as shown in Figure 3. In carrying out this division, the following mechanical properties were considered for the shaft flexible: modulus of elasticity $E = 210.98GPa$, density $\rho = 7850\, kg/m^3$ and Poisson coefficient $\nu = 0.29$.

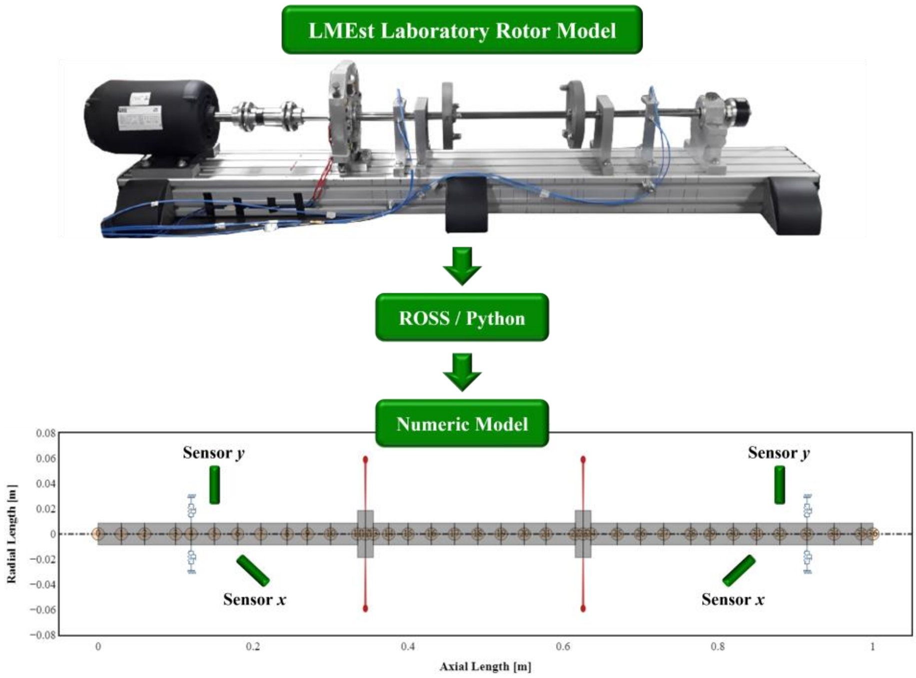


Figure 3. Modeling of the experimental bench using FEM and the ROSS library.

As shown in Figure 3, hard disks D_1 and D_2 are located at nodes #12 and #23, while roller bearings M_1 and M_2 are at nodes #4 and #33, respectively. Furthermore, adequate coupling modeling at nodes 0 and 1 was carried out to ensure a more accurate representation of the rotor, considering the mass contribution of the coupling in the system.

Subsequently, to simulate a possible condition for monitoring the machine under study, virtual proximity sensors were strategically positioned along the rotor shaft. Nodes #5 and #32 were selected as reference positions for monitoring. In other words, only measurement data from nodes #5 and #32 in horizontal and vertical directions will be considered experimental data in applying the AKF.

The stiffness and damping coefficients applied to the bearings in the numerical model were derived through a process of adjusting the peaks of the FRFs (Frequency Response Functions) using real monitoring data obtained from the instrumented rotor bench at the Structural Mechanics Laboratory - Prof. José Eduardo Tannús Reis (LMEst/UFU) from the Federal University of Uberlândia (UFU). The procedure adopted for this analysis was conducted according to the instructions detailed by Cavallini Jr (2013).

Table 1, therefore, presents the stiffness and damping coefficients used in the finite element model of this work.

Table 1. Stiffness and damping coefficients are used in the rotor numerical model.

	Stiffness Coefficients				**Damping Coefficients**			
	k_{xx}	k_{xy}	k_{yx}	k_{yy}	c_{xx}	c_{xy}	c_{yx}	c_{yy}
M_1	$3.925209 \cdot 10^8$	0	0	$2.572044 \cdot 10^9$	20.19760	0	0	11.46349
M_2	$3.881098 \cdot 10^9$	0	0	$3.515525 \cdot 10^9$	20.29810	0	0	8.457755

Next, a virtual condition of faulty operation due to rotor unbalances was implemented to test the developed virtual sensing method. This step was crucial to verify the methodology's effectiveness in practical situations, as the main objective is to reduce machine vibration amplitudes caused by fault conditions. Therefore, the conducted study is feasible as it aligns perfectly with the proposed practical utilization of the vibration analysis and rotor performance improvement methodology.

Two external input forces were numerically calculated (according to equation 16) and then applied to the rotor hard disks to address this issue. Each disk received a set of input values (forces) x and y, considering the data in Table 2.

$$\begin{cases} F_x = m_u \cdot d \cdot \Omega_{rad/s}^2 \cdot \cos\left(\Omega_{rad/s} \cdot t + \frac{\phi\pi}{180}\right) \\ F_y = m_u \cdot d \cdot \Omega_{rad/s}^2 \cdot \sin\left(\Omega_{rad/s} \cdot t + \frac{\phi\pi}{180}\right) \end{cases} \quad (16)$$

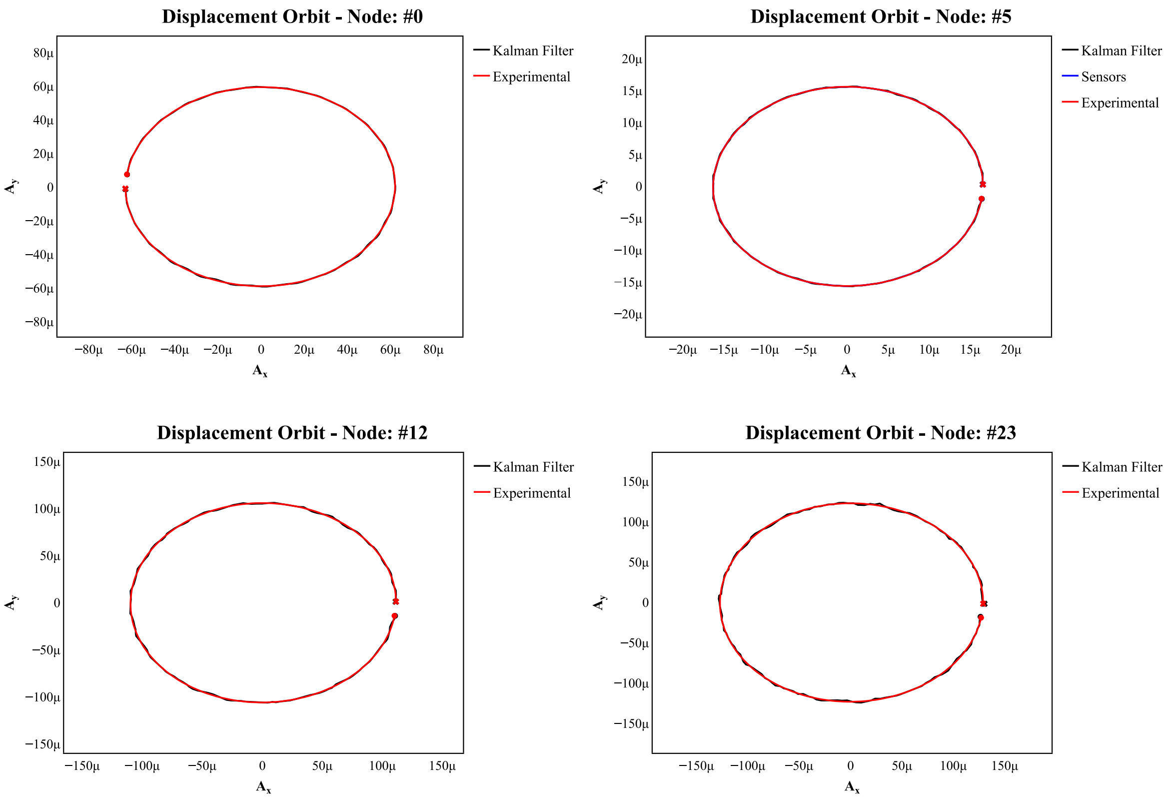
where F_x represents the unbalancing force imposed in the x (horizontal) of the disk direction and F_y the unbalancing force imposed in the y (vertical) of the disk direction. The speed machine is given by Ω in *rad/s*, while the mass (m_u), the radial distance (d) and the angular position (ϕ) of the unbalance are given according to Table 2.

Table 2. Numerical unbalance parameters.

Nodal Position	**Ω [*rad/s*]**	**M_u [*kg*]**	**ϕ [*degree*]**	**d [m]**
D_1 [Node #12]	87.96	0.007	70	0.068
D_2 [Node #23]	87.96	0.007	30	0.068

After identifying the unbalance forces, the vibration amplitudes (numerical) were estimated. These amplitudes represent the measurements of the physical sensors for the AKF and are also used as a reference for comparison with the prediction results. The EF numerical model is used in conjunction with the run_time_response ROSS function to perform these initial vibration estimates. Next, the multi-objective optimization process described in Section 3 is applied, where the covariance matrices of the P, Q and R errors are adjusted, and the AKF estimates are obtained.

That said, Figure 4 presents a comparison between the orbits predicted by the Augmented Kalman Filter (black signs), the orbits of the sensors used in the algorithm (blue signs), and the expected numerical orbits (red signs) for the main points critical parts of the rotor (extreme nodes, bearings/sensors and disks).



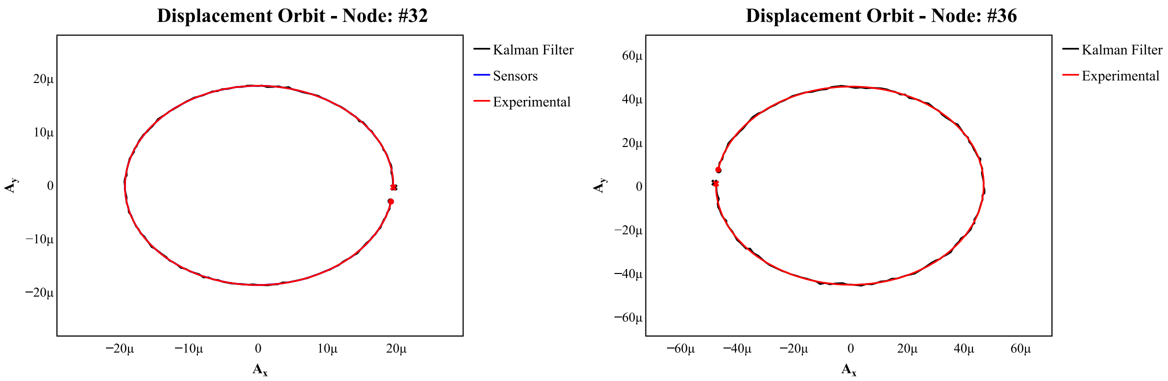


Figure 4. Comparison of the AKF predicted orbits and the expected ones.

Note that the prediction values obtained were close to the expected values in all critical points of the rotor, thus demonstrating the effectiveness of the proposed methodology regarding the virtual sensing of rotating machines. Furthermore, in Figure 5, it is possible to observe that the maximum prediction error did not exceed $11\mu m$ in the worst estimation region (prediction delay), which reinforces the previous report.

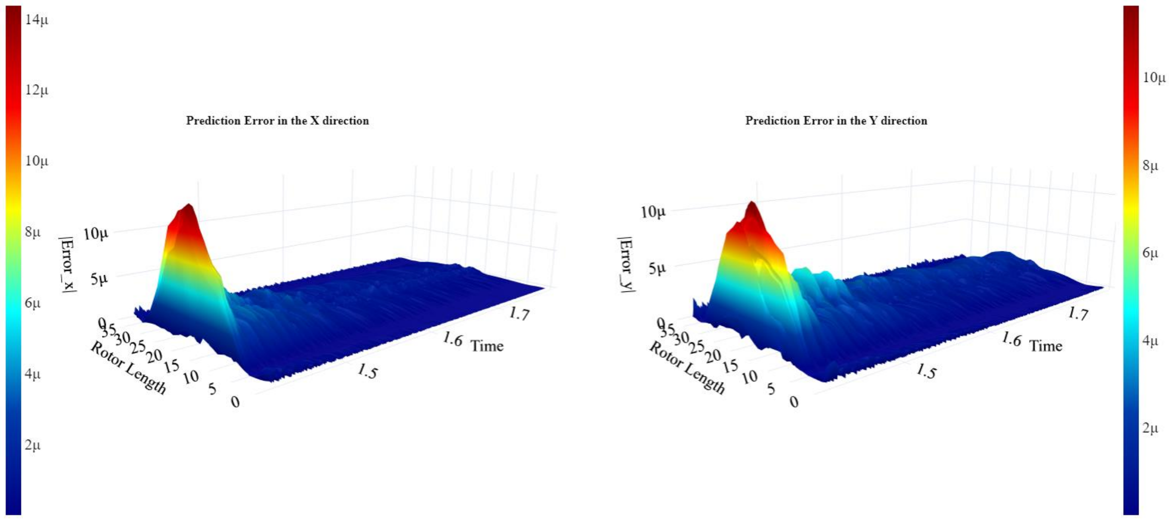


Figure 5. Prediction error achieved by AKF in this case study.

5. CONCLUSIONS

In the present work, an alternative for the virtual sensing of rotating machines was proposed, using the Augmented Kalman Filter algorithm in conjunction with finite element models derived from the ROSS rotordynamics library. Additionally, multi-objective optimization processes, such as differential evolution and compromise technique, were employed to determine the uncertainty matrices of the AKF.

Subsequently, a numerical case study was conducted using a rotor model to evaluate the effectiveness of the method. In this study, a virtual unbalance condition, representative of real cases, was applied. The construction of the AKF was based only on data from sensor regions close to the bearings. However, the vibration amplitudes of the entire rotor were estimated virtually using virtual sensors.

Considering all the aspects discussed, it is concluded that the approach is efficient, with a maximum prediction error of less than $11\mu m$ for the worst estimation region (prediction delay). Therefore, it can be applied to experimental cases.

Future work intends to investigate the methodology in real monitoring cases using the experimental setup depicted in the case study, which is already in the assembly process at the LMEst/UFU laboratory.

6. ACKNOWLEDGEMENTS

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