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COMPARATIVE ANALYSIS OF THE ROTORDYNAMIC COEFFICIENTS OF A PLAIN SEAL

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Abstract. *Mechanical seals are components installed in rotating machines in order of reducing the leakage flow of the lubricating fluid by restricting the space between the stator and rotor and, consequently, increasing the frictional forces between the fluid and the walls. Therefore, most studies related to this topic are dedicated to minimizing the leakage flow of the lubricating fluid, that is, few works in the literature have analyzed the behavior of the dynamic coefficients of the seals. One of the works dedicated on Analysis of Rotordynamic Coefficients of plain seals was developed by the Rotating Machinery and Controls (ROMAC) Industrial Research Program, with a model based on empirical relations between the shear stress acting on a wall and the relative velocity of a fluid concerning this wall, using equations in axial and circumferential directions for the stator and rotor. Then, after a perturbation analysis, in which a displacement and an angular velocity are imposed to the rotor center, a set of zeroth order equations and a set of 1st order equations are obtained and solved along the length of the seal and for various perturbation angular velocities to determine the pressure distributions, due to the 1st order perturbation, for each perturbation angular velocity and along the length of the seal. In this way, the pressure is integrated along the area of the seal in contact with the lubricating fluid and the rotor response forces to imposed displacements, velocities, and/or accelerations are obtained. Since dynamic coefficients are related to response forces, dynamic coefficients are determined using a curve fit. The objective of this work is to develop a computational algorithm to determine the rotordynamic coefficients of a plain seal and compare the obtained results with those determined by the model. The results obtained by using the developed algorithm are very close to the reference (ROMAC), with some exceptions, since the linear system solved in the curve fitting is very sensitive, and, small changes in the vector of the independent terms may cause significant variations in the results obtained for the dynamic coefficients of the seal.*

Keywords: *Rotordynamic Coefficients, Perturbation Analysis, Curve fit.*

1. INTRODUCTION

In the context of the dynamics of rotating machines, obtaining dynamic coefficients for seals is extremely important, for these coefficients models how the element influences the dynamic system of rotating machines.

Many seal studies are aimed at optimizing the ability to stop fluid leakage, as that is the function of this element. However, few of these studies are about the behavior of its dynamic properties due to the complexity of the flow. Therefore, in order to determine the dynamic properties of a seal, the Rotating Machinery and Controls (ROMAC) Industrial Research Program developed models to provide such information.

Through (Hirs, 1973) empirical relationship, equations in axial and circumferential coordinates are developed for the stator and rotor. Then, after a perturbation analysis, a set of zeroth order equations and a set of 1st order equations are obtained and solved for pressure distribution and integrated along the seal area to obtain the rotor response forces. The dynamics coefficients are obtained after a curve fit.

Therefore, a program was developed by the authors, based on the ROMAC's work, with the objective of carrying out a comparative analysis between the results obtained by the ROMAC and the program developed by the authors in MATLAB enviroment.

2. MATHEMATICAL MODEL

Table 1 shows a symbol list to aid the development of mathematical model equations.

According to (Hirs, 1973), it is possible to empirically relate the shear stress (τ') acting on the walls of the stator and rotor with the relative velocity of the fluid between the walls (u), through Eq. 1.

$$\frac{\tau'}{\frac{1}{2}\rho u^2} = n\left[\frac{2\rho h}{\mu}\right]^m \tag{1}$$

Variables m and n are empirical parameters estimated through experimental data, ρ is the density of the lubricating fluid, μ is the dynamic viscosity and the film thickness is h. In addition the shear stress acting on the two surfaces separated by the fluid film can be written as a function of a pressure gradient (Eq.2).

$$\tau' = -\frac{1}{2}h\frac{dp}{ds} \tag{2}$$

To take into account the drag due to the movement of the walls, the fictitious gradient $\frac{dp_d}{ds}$ is inserted in Eq. 2. Since the coefficients m and n depend on the roughness of the surfaces, which are different for the stator and rotor, the surfaces were analyzed separately in a way that the pressure gradient contributes positively to the analysis on the stator surface and negative on the rotor surface.

The analysis on both surfaces was performed in axial (z) and circumferential (θ) directions by replacing Eq. 2 into Eq. 1. Then, the resulting equations are summed according to their respective directions and the two remaining equations combined with the Continuity Equation are written in non-dimensional form using the term $\left(\frac{1}{\rho\bar{c}(r\omega)^2}\right)$. With this, the necessary equations for perturbation analysis are obtained (Allaire and Imlach, 1986).

$$\begin{aligned}-H\frac{\partial P}{\partial Z} = {} & \frac{n_{sz}}{2}\left(\frac{l}{\bar{c}}\right)U_Z\left(U_Z - U_2\right)R_a^{m_{sz}}\left[\left(1-\frac{U_2}{U_Z}\right)^2 + \left(\frac{U_\theta}{U_Z}\right)^2\right]^{\frac{m_{sz}+1}{2}} + \\ & + \frac{n_{rz}}{2}\left(\frac{l}{\bar{c}}\right)U_Z\left(U_Z - U_1\right)R_a^{m_{rz}}\left[\left(1-\frac{U_1}{U_Z}\right)^2 + \left(\frac{U_\theta - 1}{U_Z}\right)^2\right]^{\frac{m_{rz}+1}{2}} + \\ & + H\left[\frac{\partial U_Z}{\partial \tau} + U_Z\frac{\partial U_Z}{\partial Z} + \frac{U_\theta}{R}\frac{\partial U_Z}{\partial \theta}\right]\end{aligned} \tag{3}$$

$$\begin{aligned}-\frac{H}{R}\frac{\partial P}{\partial \theta} = {} & \frac{n_{st}}{2}\left(\frac{l}{\bar{c}}\right)U_\theta U_Z R_a^{m_{st}}\left[\left(1-\frac{U_2}{U_Z}\right)^2 + \left(\frac{U_\theta}{U_Z}\right)^2\right]^{\frac{m_{st}+1}{2}} + \\ & + \frac{n_{rt}}{2}\left(\frac{l}{\bar{c}}\right)U_Z\left(U_\theta - 1\right)R_a^{m_{rt}}\left[\left(1-\frac{U_1}{U_Z}\right)^2 + \left(\frac{U_\theta - 1}{U_Z}\right)^2\right]^{\frac{m_{rt}+1}{2}} + \\ & + H\left[\frac{\partial U_\theta}{\partial \tau} + U_Z\frac{\partial U_\theta}{\partial Z} + \frac{U_\theta}{R}\frac{\partial U_\theta}{\partial \theta}\right]\end{aligned} \tag{4}$$

$$\frac{\partial H}{\partial \tau} + \frac{\partial\left(HU_Z\right)}{\partial Z} + \frac{1}{R}\frac{\partial\left(HU_\theta\right)}{\partial \theta} = 0 \tag{5}$$

The subscripts rz, sz, rt and st indicate that the empirical parameters m and n vary according to the surface and the direction (axial or circumferential) and $R_a = \frac{2\rho u_z h}{\mu}$.

2.1 Perturbation Analisys

In order to solve Eq. 3, Eq. 4 and Eq. 5, a perturbation was imposed. In this way, H, U_Z and U_θ can be written as a sum of a steady-state flow part with a rotor concentric to the seal and a first order variation in relation to the steady state part, which corresponds to the a disturbance of the rotor to its concentric position to the seal (Allaire and Imlach, 1986).

$$H = \bar{H} + \hat{H} \tag{6}$$

Table 1. Symbol List.

Symbol	Description
$\bar{c}$	Radial Clearance
C	Direct Damping Coefficient
$\tilde{C} = \frac{C}{\pi r \Delta p T}$	Non-Dimensional Direct Damping Coefficient
c	Cross-Coupled Damping Coefficient
$\tilde{c} = \frac{c}{\pi r \Delta p T}$	Non-Dimensional Cross-Coupled Damping Coefficient
τ'	Shear Stress
h	Fluid Film Thickness
$H = \frac{h}{\bar{c}}$	Non-Dimensional Fluid Film Thickness
K	Direct Stiffness Coefficient
$\tilde{K} = \frac{K}{\pi r \Delta p}$	Non-Dimensional Direct Stiffness Coefficient
k	Cross-Coupled Stiffness Coefficient
$\tilde{k} = \frac{k}{\pi r \Delta p}$	Non-Dimensional Cross-Coupled Stiffness Coefficient
l	Seal Length
M	Direct Mass Coefficient
$\tilde{M} = \frac{M}{\pi r \Delta P T^2}$	Non-Dimensional Direct Mass Coefficient
m	Cross-Coupled Mass Coefficient
$\tilde{m} = \frac{m}{\pi r \Delta P T^2}$	Non-Dimensional Cross-Coupled Mass Coefficient
p	Pressure
Δp	Pressure Drop
$P = \frac{p}{\rho (r\omega)^2}$	Non-Dimensional Pressure
r	Shaft Radius
$R = \frac{r}{\bar{c}}$	Non-Dimensional Shaft Radius
r_0	Radial Perturbation
$R_0 = \frac{r_0}{\bar{c}}$	Non-Dimensional Radial Perturbation
t	Time
$T = \frac{l}{r\omega}$	Auxiliary Variable for Time Non-Dimensionalization
τ	Non-Dimensional Time
θ	Circumferential Coordinate
u_z, u_θ	Fluid Velocity
$U_z, U_\theta = \frac{u_z}{\omega r}, \frac{u_\theta}{\omega r}$	Non-Dimensional Fluid Velocity
ζ	Entrance Loss Coefficient
ρ	Fluid Density
μ	Fluid Dynamic Viscosity
ω	Shaft Speed
Ω	Perturbation Shaft Speed
x	Cartesian Coordinate
y	Cartesian Coordinate
z	Axial Coordinate
1	Subscript referring to the stator
2	Subscript referring to the rotor
$Z = \frac{z}{l}$	Non-DimensionalCartesian Coordinate
$bs0z$	Auxiliary symbol to suppress large terms
$br0z$	Auxiliary symbol to suppress large terms
$bs0t$	Auxiliary symbol to suppress large terms
$br0t$	Auxiliary symbol to suppress large terms
$A1Z$	Auxiliary symbol to suppress large terms
$A2Z$	Auxiliary symbol to suppress large terms
$A3Z$	Auxiliary symbol to suppress large terms
$A1T$	Auxiliary symbol to suppress large terms
$A2T$	Auxiliary symbol to suppress large terms
$A3T$	Auxiliary symbol to suppress large terms

$$U_Z = \bar{U}_Z + \hat{U}_Z \tag{7}$$

$$U_\theta = \bar{U}_\theta + \hat{U}_\theta \tag{8}$$

The superscript $(-)$ represents the steady-state part (Order 0) and the superscript $(\wedge)$ represents the perturbation (Order 1).

For Zeroth Order Equations, some simplifications can be done. The perturbation terms and terms with time derivatives must be neglected, since in steady-state there are no perturbations and time variations. As the rotor is concentric to the seal, the non-dimensional fluid film thickness is $H = 1$, since for this case $h = \bar{c}$. This also implies that changes in axial velocity (U_Z) in the Z and θ directions are zero, as well as the variations of pressure (P) and circumferential velocity (U_θ) in the θ direction. Therefore, the Zeroth Order Continuity Equation becomes singular and the set of Zeroth Order Equations becomes complete (Eq.9 and Eq.10).

$$\frac{\partial \bar{P}}{\partial Z} + \frac{\bar{U}_Z}{2}\left[\left(\bar{U}_Z - U_2\right)\cdot bs0z + \left(\bar{U}_Z - U_1\right)\cdot br0z\right] = 0 \tag{9}$$

$$\frac{\partial \bar{U}_\theta}{\partial Z} + \frac{1}{2}\left[\bar{U}_\theta \cdot bs0t + \left(\bar{U}_\theta - 1\right)\cdot br0t\right] = 0 \tag{10}$$

For First Order Equations, the neglected terms in Zeroth Order Equations were analyzed, with the exception of second or higher order terms, and time derivatives for Zeroth Order terms (Allaire and Imlach, 1986). Since U_θ and P does not vary with θ, just as U_Z does not vary along both Z and θ, and neglecting skewing effects $\left(\frac{\partial \hat{H}}{\partial Z} = 0\right)$, the First Order Equations were obtained (Eq. 11, Eq. 12 and Eq. 13).

$$\frac{\partial \hat{P}}{\partial Z} = -A1Z \cdot \hat{H} - A2Z \cdot \hat{U}_\theta - A3Z \cdot \hat{U}_Z - \left[\frac{\partial \hat{U}_Z}{\partial \tau} + \bar{U}_Z \frac{\partial \hat{U}_Z}{\partial Z} + \frac{\bar{U}_\theta}{R}\frac{\partial \hat{U}_Z}{\partial \theta}\right] \tag{11}$$

$$\frac{1}{R}\frac{\partial \hat{P}}{\partial \theta} = -A1T \cdot \hat{H} - A2T \cdot \hat{U}_\theta - A3T \cdot \hat{U}_Z - \left[\frac{\partial \hat{U}_\theta}{\partial \tau} + \bar{U}_Z \frac{\partial \hat{U}_\theta}{\partial Z} + \frac{\bar{U}_\theta}{R}\frac{\partial \hat{U}_\theta}{\partial \theta}\right] \tag{12}$$

$$\frac{\partial \hat{U}_Z}{\partial Z} + \frac{1}{R}\frac{\partial \hat{U}_\theta}{\partial \theta} = -\left[\frac{\hat{H}}{\partial \tau} + \frac{\bar{U}_\theta}{R}\frac{\partial \hat{H}}{\partial \theta}\right] \tag{13}$$

2.2 Dynamic Coefficients

The dynamic coefficients can be related to the rotor response forces to a perturbation by the Eq. 14 (Allaire and Imlach, 1986).

$$-\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \begin{bmatrix} K & k \\ -k & K \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} + \begin{bmatrix} C & c \\ -c & C \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} M & m \\ -m & M \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} \tag{14}$$

The xy plane is perpendicular to the axial direction (z).

First Order Equations can be related with the dynamic coefficients, as the rotor response forces are obtained by integrating the perturbation pressure distribution $\hat{P}$ along the seal area (Eq. 15 and Eq. 16).

$$F_x = -rl\rho\left(r\omega\right)^2 \int_0^1 \int_0^{2\pi} \hat{P}\cos\theta \, d\theta dZ \tag{15}$$

$$F_y = -rl\rho\left(r\omega\right)^2 \int_0^1 \int_0^{2\pi} \hat{P}\sin\theta \, d\theta dZ \tag{16}$$

The term $rl\rho\left(r\omega\right)^2$ was used in reason of the pressure and seal length non-dimensional forms.

3. SOLUTION OF EQUATIONS AND DYNAMIC COEFFICIENTS DETERMINATION

Determining the rotordynamic coefficients requires integrating the seal's pressure field along seal's area. For this, it is necessary the First Order Equations solutions. And the First Order Equations solutions requires the Zeroth Order Equations solutions.

3.1 Zeroth Order Equations

The resolution of Zeroth Order Equations considers two boundary conditions. A first boundary condition imposes a constant axial velocity $\left(\bar{U}_Z\right)$ along the length of the seal. While the other boundary condition is the circumferential velocity value at the seal inlet $\left(\bar{U}_\theta\left(0\right)\right)$, which is known. Then, from $\bar{U}_\theta\left(0\right)$ value, $\bar{U}_Z$ is found, which is constant throughout the seal's length. After that, $\bar{U}_Z$ value is used to find $\bar{U}_\theta$ along the seal length. In an iterative process has been used, which from an initial estimate to $\bar{U}_Z$, a correction has been applied in order to determinate the value that will be used in the next iteration, where the correction value is function of Zeroth Order Equations and their partial derivatives with respect to $\left(\bar{U}_\theta\right)$ and $\left(\bar{U}_Z\right)$. The process was repeated until the value used for the correction be less than a previously determined tolerance or the maximum number of iterations be exceeded (Allaire and Imlach, 1986).

3.2 First Order Equations

For First Order Equations solution, a sinusoidal solution is assumed for $\hat{H}$, $\hat{P}$, $\hat{U}_Z$ and $\hat{U}_\theta$.

$$\hat{H} = H_C\cos\theta + H_S\sin\theta \tag{17}$$

$$\hat{P} = P_C\cos\theta + P_S\sin\theta \tag{18}$$

$$\hat{U}_Z = U_{ZC}\cos\theta + U_{ZS}\sin\theta \tag{19}$$

$$\hat{U}_\theta = U_{\theta C}\cos\theta + U_{\theta S}\sin\theta \tag{20}$$

After a series of mathematical manipulations, the time dependence was eliminated and a new set o First Order Equations is obtained.

$$\frac{\partial P_1}{\partial Z} = -\left[A1Z - i\bar{U}_Z\Gamma\right]\cdot R_0 - \left[A2Z + i\frac{\bar{U}_Z}{R}\right]\cdot U_{\theta 1} - \left[A3Z + i\Gamma\right]\cdot U_{Z1} \tag{21}$$

$$\frac{\partial U_{\theta 1}}{\partial Z} = -\left[\frac{A1T}{\bar{U}_Z}\right]\cdot R_0 - \left[\frac{A2T + i\Gamma}{\bar{U}_Z}\right]\cdot U_{\theta 1} - \left[\frac{A3T}{\bar{U}_Z}\right]\cdot U_{Z1} + \left[i\frac{1}{R\bar{U}_Z}\right]\cdot P_1 \tag{22}$$

$$\frac{\partial U_{Z1}}{\partial Z} = -iR_0\Gamma + i\frac{U_{\theta 1}}{R} \tag{23}$$

$$R_0 = \left|H_C + iH_S\right| \tag{24}$$

$$P_1 = \left|P_C + iP_S\right| \tag{25}$$

$$U_{Z1} = \left|U_{ZC} + iU_{ZS}\right| \tag{26}$$

$$U_{\theta 1} = |U_{\theta C} + iU_{\theta S}| \tag{27}$$

In order to eliminate the R_0 dependence of the following complex solution, witch is function of Z and ΩT only, Eq. 28 was considered.

$$\begin{Bmatrix} U_{Z1} \\ U_{\theta 1} \\ P_1 \end{Bmatrix} = R_0 \cdot \begin{Bmatrix} F1C + iF1S \\ F2C + iF2S \\ F3C + iF3S \end{Bmatrix} \tag{28}$$

That yields a set of 6 unknowns and 6 equations.

For the new set of equations, there are 3 boundary conditions (Eq. 29, Eq. 30 and Eq. 31), that can be extended according to the new set of 6 equations.

$$P_1(1) = 0 \tag{29}$$

$$U_{\theta 1}(0) = 0 \tag{30}$$

$$U_{Z1}(0) = -\frac{P_1(0)}{\left[\bar{U}_Z\,(1+\zeta)\right]} \tag{31}$$

Starting from an initial value for each of the 6 unknowns, an iterative process was used to determine the solution of the First Order Equations, where, at each iteration, the value of the unknowns along Z was determined by backward or forward substitutions and then corrected for the difference between their current value and their value before backward or forward substitutions. The process was repeated until the absolute value of all corrections be less than a previously determined tolerance or the maximum number of iterations be exceeded (Allaire and Imlach, 1986).

3.3 Dynamic Coefficients Determination

For the dynamic coefficients determination, initially, Eq. 14 is written in non-dimensional form.

$$-\frac{1}{\pi r \cdot r_0 \Delta p}\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \begin{bmatrix} \tilde{K} & \tilde{k} \\ -\tilde{k} & \tilde{K} \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} + \begin{bmatrix} \tilde{C} & \tilde{c} \\ -\tilde{c} & \tilde{C} \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} \tilde{M} & \tilde{m} \\ -\tilde{m} & \tilde{M} \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} \tag{32}$$

Knowing that the imposed perturbation had angular velocity Ω and displacement r_0, it is possible to write the displacements x and y, as well as their time derivatives,in terms of both Ω and r_0. Then, Eq. 32 was analysed at time $t = 0$, when the Cartesian and Polar coordinate systems are coincident (Allaire and Imlach, 1986).

$$-\frac{1}{\pi r \cdot r_0 \Delta p}\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \begin{bmatrix} \tilde{K} & \tilde{k} \\ -\tilde{k} & \tilde{K} \end{bmatrix}\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} + \begin{bmatrix} \tilde{C} & \tilde{c} \\ -\tilde{c} & \tilde{C} \end{bmatrix}\begin{Bmatrix} 0 \\ 1 \end{Bmatrix} + \begin{bmatrix} \tilde{M} & \tilde{m} \\ -\tilde{m} & \tilde{M} \end{bmatrix}\begin{Bmatrix} -1 \\ 0 \end{Bmatrix} \tag{33}$$

And after some mathematical manipulations, the dynamic coefficients are presented in two second order polynomials (Eq. 34 and Eq. 35).

$$\tilde{K} + \tilde{c}\,(\Omega T) - \tilde{M}\,(\Omega T)^2 = \frac{2}{c_d}\left(\frac{l}{\bar{c}}\right)\left(\frac{1}{\bar{U}_Z}\right)^2 \int_0^1 F3S\,dZ \tag{34}$$

$$-\tilde{k} + \tilde{C}\,(\Omega T) + \tilde{m}\,(\Omega T)^2 = \frac{2}{c_d}\left(\frac{l}{\bar{c}}\right)\left(\frac{1}{\bar{U}_Z}\right)^2 \int_0^1 F3S\,dZ \tag{35}$$

After the perturbation, the rotor operates in steady state, that is, with a rotational speed ω. Therefore, the velocity Ω imposed by the perturbation is such that $0 \leq \Omega \leq \omega$, and the First Order Equations must be solved according to this variation (Allaire and Imlach, 1986).

Ω assumes different values in the interval $0 \leq \Omega \leq \omega$, dividing it in smaller intervals with the same length, and rotordynamic coefficients are determined through curve fitting. Consequently, ΩT assumes different values in the interval $0 \leq \Omega T \leq \frac{l}{r}$.

The numerical integration of the pressure distribution along the seal was calculated using Simpson's $1/3$ Rule. And instead of using a Gaussian Elimination in the linear system solution for the curve fir, like in ROMAC's work, an approach through matrix inversion in MATLAB environment was applied.

4. COMPARATIVE ANALYSIS

In order to validate the results obtained by the authors, a plain seal example in ROMAC's work is used. Table 2 shows the input data for this example.

Table 2. Example Data Input.

Symbol	Value
l	$2,604 \cdot 10^{-2}\,[m]$
$\bar{c}$	$1,778 \cdot 10^{-4}\,[m]$
r	$1,143 \cdot 10^{-1}\,[m]$
Δp	$-1,379 \cdot 10^{7}\,[Pa]$
ω	$523,599\,[rad/s]$
m_{rz}	$-0,25$
m_{rt}	$-0,25$
m_{sz}	$-0,25$
m_{st}	$-0,25$
n_{rz}	$0,079$
n_{rt}	$0,079$
n_{sz}	$0,079$
n_{st}	$0,079$
μ	$1,951 \cdot 10^{-4}\,[Pa \cdot s]$
ρ	$913,437\,\left[kg/m^3\right]$
ζ	$0,1$
N	100
$relax$	$0,05$
fac	$0,01$

Different stoppage criteria is used for equations. For Zeroth Order Equations the used tolerance is 10^{-3} , while for First Order Equations the tolerance is 10^{-4}. Using higher tolerance values leads to compromised results for the rotrdynamic coefficients. Table 3 shows the rotordynamic coefficients obtained in the present work and comparisons with values obtained with ROMAC's model.

Table 3. Dynamic Coefficients Comparison.

Dynamic Coefficients	Obtained Results	ROMAC	Relative Error $(\%)$
$\tilde{K}\,[-]$	$-24,3232$	$-24,3255$	$0,0096$
$\tilde{k}\,[-]$	$-2,9058$	$-2,9013$	$0,1560$
$\tilde{C}\,[-]$	$-25,5144$	$-25,5025$	$0,0464$
$\tilde{c}\,[-]$	$-0,4646$	$-0,4683$	$0,8040$
$\tilde{M}\,[-]$	$-2,0398$	$-2,1304$	$4,4529$
$\tilde{m}\,[-]$	$-0,0011$	$-0,1143$	$100,96$

Non-dimensional values were used in the comparison due to the large difference in magnitude of the dimensional values.

From Table 3 it is observed that the greatest differences occur for the cross-coupled coefficients $(\tilde{c}, \tilde{m})$ and for the direct mass coefficient $\left(\tilde{M}\right)$. This occurs due to the sensitivity of linear systems to be solved in curve fittings. That is, small variations on independent terms vector, which is function of First Order Equations solutions, can result in significant differences for rotordynamics coefficients, mainly for the coefficients referring to the quadratic terms on curve fittings, that is, the mass coefficients. These differences may be even proportionally greater for coefficients with a low non-dimensional absolute value.

Figures 1 and 2 shows a comparison of second order polynomials obtained in the present work with ROMAC's results. Those polynomials are evaluated for ΩT values in the interval $0 \leq \Omega T \leq \frac{l}{r}$.

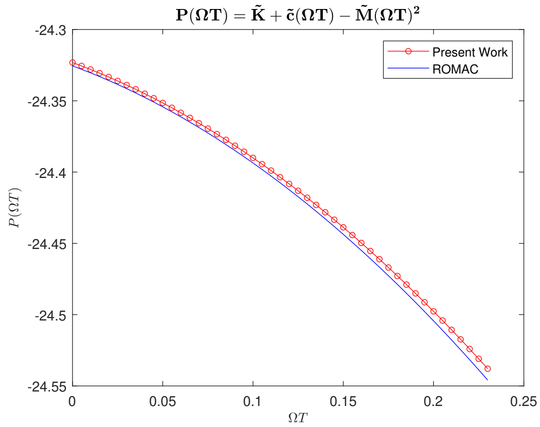

Figure 1. Curve fitting polynomial $P(\Omega T) = \tilde{K} + \tilde{c}(\Omega T) - \tilde{M}(\Omega T)^2$.

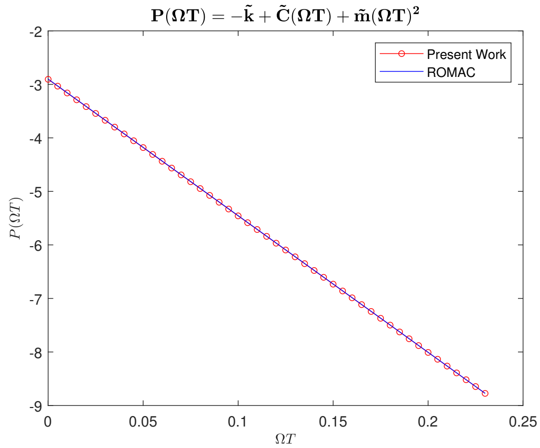

Figure 2. Curve fitting polynomial $P(\Omega T) = -\tilde{k} + \tilde{C}(\Omega T) + \tilde{m}(\Omega T)^2$.

Despite the mass coefficients values differences observed in Table 3, the adjusted polynomials assume very close values in the interest interval, proving that small variations on the independent terms vector of the curve fittings can compromise the values of rotordynamic coefficients, even if the curve fitting results are identical curves in the interest interval.

5. CONCLUSION

As proposed, a comparative analysis was performed. The obtained results are very close to reference results, despite some significant differences due to the sensitivity of the linear system to be resolved by adjustment.

So, it is possible to optimize results through the solution of the First Order Equations or through another way of determining the pressure field to be integrated into the along the seal area, as any difference in the value of pressure distribution integration can significantly change the values of dynamic coefficients.

Thus, a possible future application of this project may involve the analysis of the dynamic coefficients of more complex seals, such as labyrinth seal and honeycomb seal, for example.

6. ACKNOWLEDGEMENTS

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