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Vibration Mitigation and Energy Harvesting with Bistable Resonators in Metamaterial Beams

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Abstract. *Metamaterials are designed engineering systems that have qualities that are not seen in conventional materials. They may offer excellent energy absorption, acoustic insulation, negative compressibility, and other properties. Engineers are using these materials in various fields, including spatial, medical, and military applications. This study focuses on designing a metamaterial-based structure capable of simultaneously attenuating mechanical waves and generating low-power electrical energy. By converting vibrations into electricity, this structure eliminates the need for batteries in sensors or other embarked devices, reducing environmental impact, and can also supply energy to active control. The proposed structure consists of a cantilever beam with resonators equipped with piezoelectric material to convert kinetic energy into electricity. These oscillators induce local resonance, establishing a bandgap region where elastic waves cannot propagate. Linear and nonlinear resonators with bistable potential are considered. Their performances are studied and compared. Numerical simulations are conducted, and the results show that nonlinearity reduces resonance peaks and widens the bandgap for vibration attenuation. In terms of power generation, nonlinearity expands operating bandwidth and optimizes the amount of recovered energy. The study concludes that nonlinear resonators are essential to improve both vibration attenuation and energy harvesting. Additionally, the use of bistable resonators opens exciting possibilities for developing novel configurations that can further enhance the performance of the system.*

Keywords: *Metamaterial, Vibration Attenuation, Vibration energy harvesting, Bistable system, Nonlinear dynamics*

1. INTRODUCTION

Combining vibration reduction of a structure and energy recovery for powering embedded electronic devices has emerged as a promising concept in structural engineering. This approach holds the potential to mitigate harmful vibrations, while this energy can be taken advantage of to convert into electricity to power various devices, e.g. sensors and actuators. Although this idea has been studied in the past decade by researchers such as Ali and Adhikari (2013) and Brennan *et al.* (2014), the integration of energy harvesting and vibration attenuation has focused primarily on the conventional setup of vibration absorbers and energy harvesters. However, in these cases, a single resonator has limitations in effectively suppressing vibration and harvesting energy simultaneously. To address this challenge, recent advances have explored the design of structures inspired by metamaterials with different mechanisms, to achieve a system that fulfills both objectives.

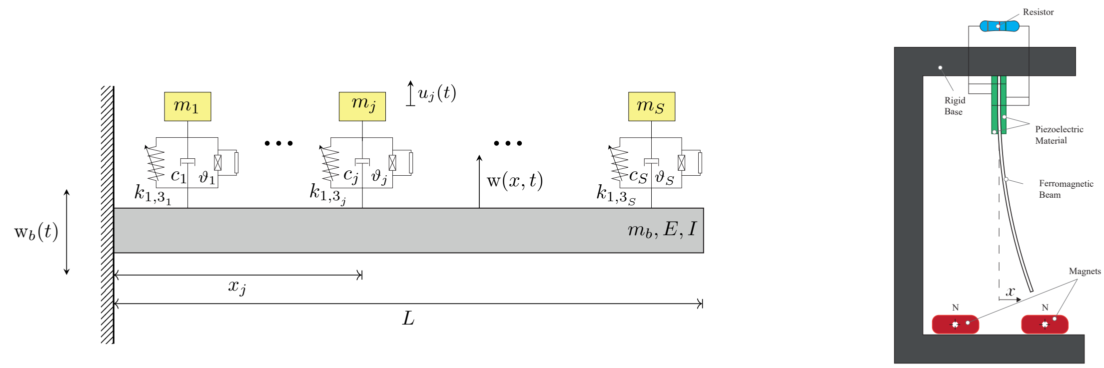
Metamaterials are designed systems that have been engineered to possess unique properties that are not usually found in conventional materials (Hussein *et al.*, 2014). They are typically composed of artificial structures known as metastructures, which are carefully designed in periodic or non-periodic arrangements. These arrangements allow for effective control over the material's parameters, creating properties that do not occur naturally or are challenging to achieve in practice. Photonic crystals (Casadei *et al.*, 2012) and locally resonant structures Liu *et al.* (2000) are examples of such kinds of smart systems. Other metamaterial properties can be seen in Misseroni *et al.* (2016) and Sanders *et al.* (2021).

Locally resonant metastructures, in particular, are notable for their ability to store and transfer energy due to the presence of resonant elements, whether mechanical or electromechanical. These structures are effective at absorbing vibrations and can accommodate vibration energy harvesters. Sugino and Erturk (2018) developed a comprehensive electromechanical modeling framework for two representative systems. Through modal analysis, they explored the frequency response and electrical energy maps for different electrical resistances. The study demonstrated that useful energy could be harvested without significantly compromising the remarkable vibration attenuation achieved within the bandgap. Vasconcellos *et al.* (2022) examined a discrete metastructure incorporating both linear and nonlinear absorbers, highlighting the advantages of nonlinear oscillators in optimizing the system. Chen *et al.* (2019) investigated a plate metastructure with linear resonators, while Lu *et al.* (2019) proposed a plate model incorporating nonlinear resonators.

In this study, the focus is on developing a metamaterial beam that can effectively attenuate vibrations while simultaneously generating electrical energy. The primary goal is to create a metastructure that incorporates nonlinear bistable resonators, which can form optimal bandgaps and address the challenges of vibratory energy harvesting. The host structure chosen for this purpose is a cantilever beam and is modeled as a continuous model using the Euler-Bernoulli beam theory. Multiple local resonators are strategically distributed within the host structure to function as mechanical vibration absorbers. These resonators also feature piezoelectric patches that serve as energy collectors. Each resonator is designed with a nonlinear restoration force (bistability) to widen the operating frequency range, enhancing the system's efficiency in real-world environments. Numerical simulations are conducted to explore the advantages and potential obstacles associated with utilizing bistability for simultaneous vibration mitigation and energy harvesting. A comparison between the metastructure and linear resonators is included to show the higher performance when nonlinear resonators are taken into account.

2. METASTRUCTURE WITH BISTABLE RESONATORS

The proposed system consists of a cantilever beam with bistable resonators and piezoelectric inserts coupled as shown in Figure 1a. The beam has length L, is fixed at one end, and is free to vibrate at the other end. It has a mass of m_b and flexural stiffness of EI. Each local resonator with m_j mass is a bistable oscillator ($j = 1, \ldots, S$, where S is the number of resonators) connected to the beam through linear stiffness $k_{1j} < 0$, cubic stiffness $k_{3j} > 0$ and a coefficient of damping c_j. Furthermore, in each resonator, a piezoelectric material with electromechanical coupling ϑ_j connected in a resistive circuit is introduced. Each mass illustrated in Figure 1a is equivalent to a system shown in Figure 1b.



(a) (b)

Figure 1: Illustration of the cantilever beam with piezo-magneto local resonators. (a) is the metastructure with a schematic representation of bistable resonators with piezoelectric coupling, and (b) is the bistable oscillator where the mass is a ferromagnetic beam equipped with piezoelectric patches under a magnet field.

The system dynamics is modeled using a continuous beam model based on Euler-Bernoulli beam theory with discrete masses coupled along its length. In this way, the total transverse displacement of the beam under base excitation at a position x and time t is given by

$$\mathrm{w}_t(x,t) = \mathrm{w}_b(t) + \mathrm{w}(x,t)\,, \tag{1}$$

where w_b is the base displacement, and w is the beam relative displacement concerning its reference configuration. Modal damping of the beam, c, is also considered. The governing equations for this electromechanical system excited by base motion are given by

$$m_b \frac{\partial^2 \mathrm{w}}{\partial t^2}(x,t) + c\frac{\partial \mathrm{w}}{\partial t}(x,t) + EI\frac{\partial^4 \mathrm{w}}{\partial x^4}(x,t) - \sum_{j=1}^{S}\left(c_j \dot{u}_j(t) + k_{1_j} u_j(t) + k_{3_j} u_j^3(t) - \vartheta_j v_j(t)\right)\delta(x - x_j) = -m_b \ddot{\mathrm{w}}_b(t)\,, \tag{2}$$

$$m_j\left(\frac{\partial^2 \mathrm{w}}{\partial t^2}(x_j,t) + \ddot{u}_j(t)\right) + c_j \dot{u}_j(t) + k_{1_j} u_j(t) + k_{3_j} u_j^3(t) - \vartheta_j v_j(t) = -m_j \ddot{\mathrm{w}}_b(t)\,, \quad j = 1, 2, \ldots, S\,, \tag{3}$$

$$C_{p,j}\dot{v}_j(t) + \mathcal{Y}_j v_j(t) + \vartheta_j \dot{u}_j(t) = 0\,, \quad j = 1, 2, \ldots, S\,, \tag{4}$$

where $\delta(.)$ denotes the Dirac's delta function, $C_{p,j}$ is the effective piezoelectric capacitance, $\mathcal{Y}_j$ is the admittance of the resistive circuit, $u_j(t)$ is the j-resonator displacement, and $v_j(t)$ is the voltage in the function of the time generated in j-resonator.

Neglecting nonlinear modal interactions since the beam modes are well separated, an expansion using the modal shapes of the cantilever beam without the resonators, $\phi_r(x)$, is used as shown below

$$\mathrm{w}(x,t)=\sum_{r=1}^{N}\eta_r(t)\phi_r(x)\,, \tag{5}$$

where N is the number of modes in the expansion, $\eta_r(t)$ is the modal displacement of the r-th mode of the cantilever beam, and $\phi_r(x)$ is the mass-normalized modal shapes of the beam given by the following equation

$$\phi_r(x)=\frac{1}{\sqrt{m_bL}}\left[\cos\left(\frac{\lambda_r x}{L}\right)-\cosh\left(\frac{\lambda_r x}{L}\right)+\frac{\sin\lambda_r-\sinh\lambda_r}{\cos\lambda_r+\cosh\lambda_r}\left(\sin\left(\frac{\lambda_r x}{L}\right)-\sinh\left(\frac{\lambda_r x}{L}\right)\right)\right]\,, \tag{6}$$

where λ_r is the wavelength for r-th mode.

Using the orthogonality conditions of the modal forms and replacing Eq. 5 in Eq. 2 and Eq. 3, obtain

$$\ddot{\eta}_r(t)+2\zeta_r\omega_r\dot{\eta}_r(t)+\omega_r^2\eta_r(t)-\sum_{j=1}^{S}\left(c_j\dot{u}_j(t)+k_{1_j}u_j(t)+k_{3_j}u_j^3(t)-\vartheta_j v_j(t)\right)\phi_r(x_j)=-m_b\ddot{\mathrm{w}}_b(t)\int_{x=0}^{L}\phi_r(x)dx\,, \tag{7}$$

$$m_j\left[\sum_{r=1}^{N}\ddot{\eta}_r(t)\phi_r+\ddot{u}_j(t)\right]+c_j\dot{u}_j(t)+k_{1_j}u_j(t)+k_{3_j}u_j^3(t)-\vartheta_j v_j(t)=-m_j\ddot{\mathrm{w}}_b(t),\quad j=1,2,\ldots,S\,, \tag{8}$$

such that ω_r is the natural frequency of the cantilever beam and ζ_r is its damping ratio for the r-th mode.

Equations 7, 8 and 4 form a $N+2S$ system of coupled second-order ordinary differential equations that can be written in matrix form as

$$[\boldsymbol{M}]\,\ddot{\boldsymbol{X}}(t)+[\boldsymbol{C}]\,\dot{\boldsymbol{X}}(t)+[\boldsymbol{K}]\,\boldsymbol{X}(t)+[\boldsymbol{G}(\boldsymbol{X}(t))]\,\boldsymbol{X}(t)+[\boldsymbol{\vartheta}]\,\boldsymbol{V}(t)=\boldsymbol{F}(t)\,, \tag{9}$$

$$[\boldsymbol{C_p}]\dot{\boldsymbol{V}}(t)+[\boldsymbol{\mathcal{Y}}]\boldsymbol{V}(t)+[\boldsymbol{\vartheta}]\dot{\boldsymbol{X}}(t)=0\,, \tag{10}$$

where $\boldsymbol{X}=[\eta_1\;\eta_2\;\ldots\;\eta_N\;u_1\;u_2\;\ldots\;u_S]^T$ is the displacement vector; $\boldsymbol{V}=\left[\boldsymbol{0}_{N+S}\;v_1\;v_2\;\ldots\;v_S\right]^T$ is the voltage vector, in way that $\boldsymbol{0}$ is a zero vector with a size of $N+S$; $\boldsymbol{F}(t)$ is the force vector; $[\boldsymbol{M}]$, $[\boldsymbol{C}]$, $[\boldsymbol{K}]$, $[\boldsymbol{G}]$, $[\boldsymbol{\vartheta}]$, $[\boldsymbol{C_p}]$, $[\boldsymbol{\mathcal{Y}}]$, are the mass, damping, linear stiffness, nonlinear stiffness, electromechanical coupling, piezoelectric capacitance and electrical admittance matrices, respectively.

3. NUMERICAL RESULTS

The model equations are numerically integrated using the Runge-Kutta method. All resonators are considered identical and equally spaced. The mechanical system parameters were assigned values based on Xia *et al.* (2020), while the electrical and electromechanical properties were assigned values based on Erturk and Inman (2008). The values of the resonator parameters were designed to absorb vibration at the second natural frequency of the beam. These parameter values are presented in Tab. 1.

In this approach, the focus is on utilizing a sinusoidal sweep-up within a specified frequency range centered around the second natural frequency of the beam. This sweep is applied to investigate both the transmissibility and the energy harvesting performance. The transmissibility involves calculating the ratio between the velocity at the top of the beam and the velocity at its base. The energy harvesting performance is assessed by calculating the sum of the root mean square of the voltage across the resistor circuit of each resonator. Firstly, the proposed system is studied when the local resonators are linear ($k_{3_j}=0$) and further when they are nonlinear with bistability ($k_{3_j}\neq 0$).

3.1 Linear local resonators

In the initial investigation, we focus on a system where the local resonators are linear. In this case, the nonlinear stiffness (k_{3_j}) is set to zero, while the linear stiffness (k_{1_j}) is assigned a positive value. Figure 2 illustrates the transmissibility (represented by the blue line) of the beam coupled with linear resonators to a piezoelectric system. It also shows the total mean squared voltage generated (depicted by the orange line). The blue dashed line represents the transmissibility of the beam without the presence of local resonators, i.e., the cantilever beam. The gray band depicted in the figure indicates the region where the transmissibility is lower than one, known as the bandgap region. Within this region, wave propagation under the structure is inhibited. By coupling the linear resonators to the host structure, the large motion observed in the second resonance peak of the beam (represented by the dashed line) is effectively mitigated. Moreover, within the bandgap region, a voltage peak is observed. Although this peak is lower than the energy generation observed in other

Table 1: Physical and geometrical parameters for the electromechanical system.

Beam length, L	889 mm
Beam width, b	31.75 mm
Thickness of the beam, h	2.6 mm
Young's modulus of the beam, E	69 GPa
Mass of the beam, m_b	218 g
Mass of the resonator, m_j	36 g
Linear stiffness, k_{1_j}	-63.451 N/m
Cubic stiffness, k_{3_j}	634509 N/m^3
Beam modal damping, ζ_r	0.002
Resonator damping ratio, ζ_j	0.02
Effective piezoelectric capacitance, $C_{p,j}$	43 nF
Electromechanical coupling, ϑ_j	-4.57 mN/V
Electrical admittance, $\mathcal{Y}_j$	1/ 5kΩ
Number of modes, N	10
Number of resonators, S	7

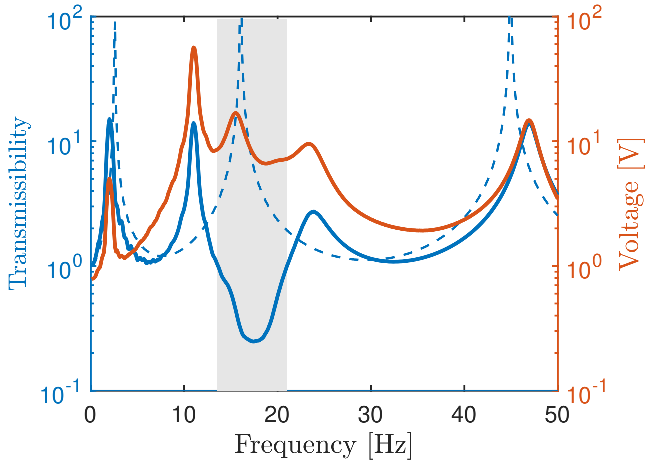


Figure 2: Transmissibility (blue line) and root mean square voltage (orange line) of a cantilever beam equipped with linear resonators coupled to a piezoelectric energy harvesting circuit under amplitude of excitation of 0.3 g. The transmissibility of the cantilever beam without resonators (blue dashed line) and the bandgap region (gray area) are also displayed. The left axis corresponds to the transmissibility, while the right axis represents the voltage.

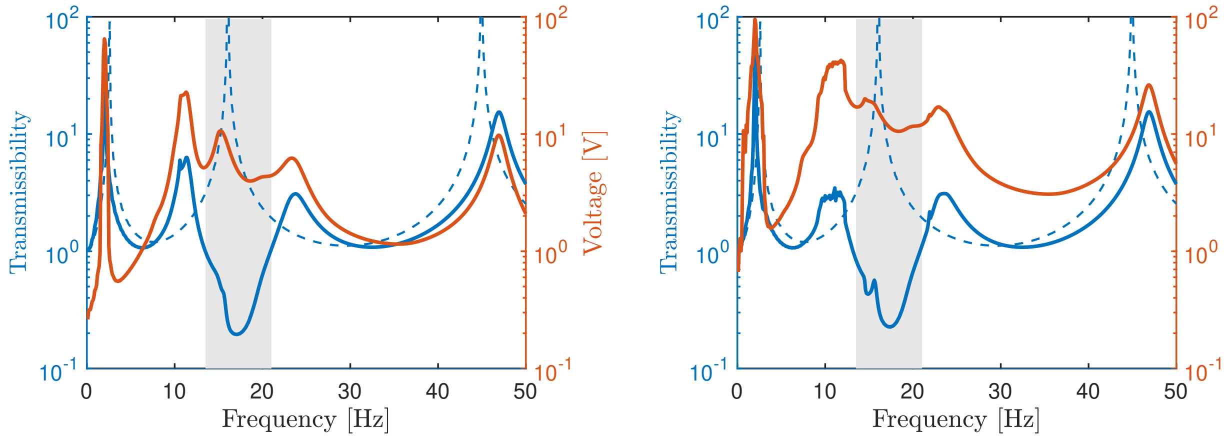
resonance peaks, it indicates the possibility of simultaneously generating energy and attenuating vibration. Furthermore, this result demonstrates that the presence of the energy conversion system does not adversely affect vibration attenuation.

The presented result highlights a significant limitation of the linear configuration, specifically that voltage generation is only feasible within narrow frequency ranges. Consequently, slight deviations in the excitation frequency can greatly impact the effectiveness of energy harvesting. This limitation also applies to the bandgap region, where energy harvesting is constrained to a narrow frequency range. To overcome this challenge and achieve a broader frequency range for energy harvesting, researchers have proposed the introduction of bistability as a promising alternative. Studies conducted by Cottone *et al.* (2009), Erturk *et al.* (2009) and Norenberg *et al.* (2023) have investigated the concept of bistability and its potential to address this limitation, opening up possibilities for more efficient broadband energy harvesting. This avenue is now investigated.

3.2 Nonlinear local resonators

To address the limitations of narrow frequency range voltage generation and vibration mitigation, the nonlinearity of the local resonators is taken into account. By incorporating nonlinear elements into the system, such as bistable oscillators, the potential for broadband energy harvesting and vibration mitigation can be enhanced because it can exhibit nonlinear behavior, excitation amplitude-dependence, and large amplitude vibrations across a wide frequency band. In addition, the system becomes more complex, presenting multiple solutions, high sensitivity to initial conditions, and chaotic behavior.

In Figure 3, the blue line represents the transmissibility of the beam equipped with nonlinear resonators and piezoelectric harvesting systems, while the orange line represents the total root mean square voltage generated. Additionally, the dashed blue line shows the transmissibility of the cantilever beam without resonators, and the gray area indicates the bandgap region. The figure includes two intensities of excitation acceleration: $0.3g$ (Figure 3a) and $0.8g$ (Figure 3a), where g denotes the gravity acceleration. It provides an investigation at different levels of nonlinearity because as the excitation amplitude increases, the nonlinearity behavior also becomes more pronounced. In the bandgap region, there is a peak in energy generation. Specifically, when the excitation is set to $0.8g$, the peak becomes higher and flatter. This feature is beneficial for power generation because it increases the operating band compared to the system with linear resonators. Moreover, the voltage peak within the bandgap region is also higher when the excitation intensity is $0.8g$. This demonstrates that nonlinearity offers an additional benefit to harvesting performance. Additionally, the presence of the energy harvesting system has no impact on the bandgap region itself. From the perspective of vibration attenuation, the inclusion of nonlinear local resonators effectively reduces the resonance peaks around the bandgap region, improving the vibration attenuation. Consequently, this proposed system exhibits superior performance in terms of both energy generation and vibration attenuation when compared to the linear system.



(a) excitation amplitude of 0.3g

(b) excitation amplitude of 0.8g

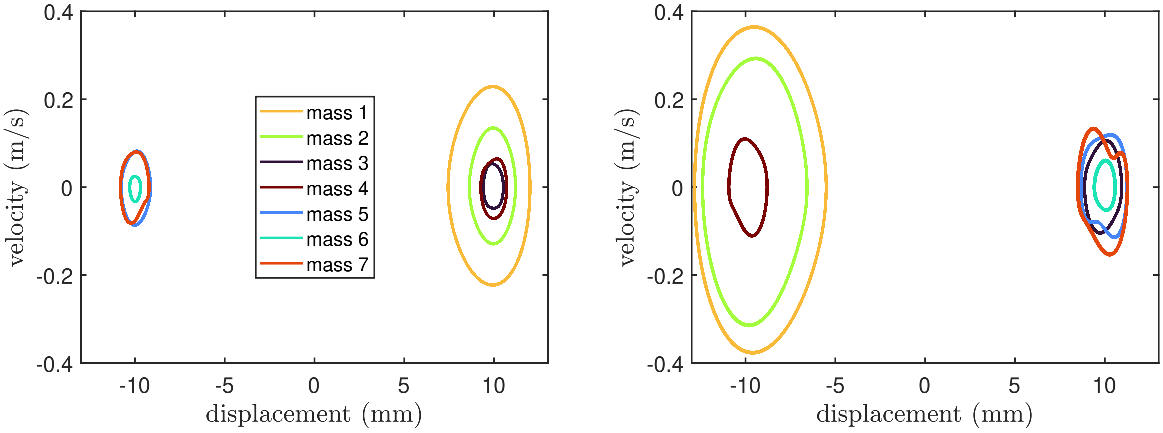
Figure 3: Transmissibility (blue line) and root mean square voltage (orange line) of a cantilever beam equipped with nonlinear resonators coupled to a piezoelectric energy harvesting circuit. The transmissibility of the cantilever beam without resonators (blue dashed line) and the bandgap region (gray area) are also displayed. The left axis corresponds to the transmissibility, while the right axis represents the voltage.

Table 2 shows a comparison performance of vibration attenuation (width of the bandgap and minimum attenuation factor) and power generation (maximum voltage and voltage bandwidth) between the system with linear and nonlinear resonators. While the nonlinear system shows slightly better performance than the linear system for the intended objectives, it is important to note that generating energy within the bandgap frequency range might not be the most favorable scenario. Interestingly, it is observed that the voltage peaks generated outside the bandgap region are higher than the peak within the absorption band.

Table 2: Comparison of vibration attenuation and energy recovered during bandgap between linear and nonlinear system ($0.3g$ and $0.8g$ base excitation).

	Linear	Nonlinear	
		0.3g	0.8g
Bandgap width	7.4 Hz	7.8 Hz	8 Hz
Minimum attenuation factor	0.24	0.19	0.21
Generation bandwidth (-3 dB/peak)	1.8 Hz	1.7 Hz	3.4 Hz
Maximum voltage	16.8 V	10.6 V	19.9 V

This situation can be explained by observing the dynamic behavior of the resonators by the phase portrait in Figure 4. These graphs represent displacement by velocity in a steady state when the excitation amplitude is $0.3g$ and $0.8g$, and the excitation frequency coincides with the voltage peak on the bandgap region. The state-space relationship described forms orbits, where higher orbits correspond to greater energy of motion in the oscillator. The phase portrait observation reveals that all masses are confined to a stable equilibrium point, exhibiting limited motion and oscillating at low amplitudes. This behavior is not ideal for energy harvesting purposes, as the system has the potential to oscillate at larger amplitudes in a bistable condition. Additionally, the results indicate that mass 1, located closest to the base of the beam, experiences higher motion amplitudes compared to the other masses. As the resonators move towards the free end of the beam, the displacement decreases. This suggests that the amplitude of the wave propagating along the beam attenuates progressively along its length. Notably, at mass 6, where its position coincides with the fixed modal point of the beam at this particular frequency, the system demonstrates the lowest energy orbit. Furthermore, when comparing excitation amplitudes of $0.3g$ and $0.8g$, the resonators exhibited a higher energy orbit when the nonlinearity was more pronounced (at $0.8g$). This finding highlights the significance of the nonlinearity feature in enabling greater energy recovery.



(a) excitation amplitude of $0.3g$ (b) excitation amplitude of $0.8g$

Figure 4: Phase portrait of each resonator for a base amplitude equal to (a) 0.3g and (b) 0.8g when the excitation frequency coincides with the voltage peak on the bandgap region, 15.3 Hz.

Based on these findings, it can be inferred that resonators have the ability to absorb a greater amount of energy from the host system and harvest more energy when they undergo high-amplitude motions in bistable conditions. Consequently, utilizing an appropriate configuration of bistable resonators can lead to a substantial increase in energy generation. Moreover, this approach opens up new possibilities to contribute not only to energy harvesting but also to vibration absorption simultaneously.

4. CONCLUSION

The conclusion of this work is that the use of metamaterials can offer a solution to two problems simultaneously: attenuating incident mechanical waves and generating low-power electrical energy. The proposed structure consists of a cantilever beam equipped with resonators that generate electricity using the piezoelectric effect. The resonators induce the phenomenon of local resonance and establish a bandgap region where elastic waves are forbidden to propagate. The study compared the performance of bifunctional systems based on metamaterials with linear and nonlinear resonators with bistable potential. The numerical simulations showed that nonlinearity decreases the resonance peaks around the attenuation band and increases the bandgap width. For power generation, nonlinearity increased the operating bandwidth and demonstrated a capability to optimize the total amount of recovered energy. Therefore, the nonlinearity of the resonators is fundamental to improving the performance of both vibration attenuation and energy harvesting. Furthermore, the use of bistable resonators presents exciting prospects for developing novel configurations aimed at further improving the system's performance.

5. ACKNOWLEDGEMENTS

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