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A comparative analysis of online and offline methods for the secondary path estimation in active noise control applications using the Fx-LMS algorithm

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Abstract. *The proliferation of HVAC (Heating, Ventilation, and Air Conditioning) systems in our homes, industry, and daily lives has led to a growing problem of acoustic noise pollution. Passive and active noise control methods have been developed to address this issue. Active noise control (ANC) is an effective technique that uses control algorithms and destructive interference principles to attenuate noise that is difficult and expensive to control using passive methods. The Filtered-x Least-Mean Square (Fx-LMS) algorithm is a widely used ANC model that requires a secondary path model to estimate the sound filtering process between the control speaker and its reference microphone. Secondary path estimation can be performed using either online or offline methods. Online methods estimate the parameters in real-time during system operation, while offline methods estimate the model parameters beforehand or afterward using input/output data. This paper presents experimental results for the Fx-LMS algorithm with online and offline secondary path estimation methods in an acoustic duct. The experimental setup involves a tubular duct with a primary speaker at one end and an open end. The primary speaker generates the noise to be controlled, while the control speaker for generating the anti-noise is placed at an intermediate location on the tube. Two microphones measure the reference signal, and another monitor the signal error and the controller's performance. The ANC algorithm is implemented on a dSpace dS1104 platform. The study compares the control algorithm's complexity, computational cost, and performance for both an online and an offline method. The results provide quantitative and qualitative information about the strengths and weaknesses of the different designs, which can help identify the best approach for each application depending on specific requirements.*

Keywords: *Fx-LMS, Active noise control, Secondary Path , Online and Offline estimation, Acoustic Duct.*

1. INTRODUCTION

With the widespread and increasing use of HVAC (Heating, Ventilation, and Air Conditioning) systems over the years, noise pollution has become a more evident problem, especially regarding health problems created by uncontrolled sound levels (Berglund *et al.*, 1999). An active noise control (ANC) technique efficiently attenuates acoustic low-frequency unwanted noises using a secondary source anti-noise wave, which has the opposite phase concerning measured acoustic noise. These approaches are generally used where passive methods are either ineffective or tend to be very expensive or bulky. (Hassanpour and Davari, 2009; Shi *et al.*, 2023)

Due to its efficiency in canceling low-frequency noise, ANC applications have been an object of research in numerous fields, from noise-canceling headphones, chimneys, and electronic mufflers (Kuo and Morgan, 1999) to the control of thermoacoustic instabilities in combustion systems (Annaswamy and Ghoniem, 1995).

ANC systems must have adaptation capabilities since the environment and noise source characteristics can vary with time, frequency content, amplitude, phase, and other nonstationary noise properties. For this reason, adaptive filters are commonly used due to their capacity to adjust coefficients to minimize an error. The most common form of adaptive filter uses the least-mean-square (LMS) algorithm (Kuo and Morgan, 1999). An early computational model of a duct cancellation system based on adaptive filter theory was developed by Burgess (1981).

Figure 1 represents the block diagram of an LMS ANC system. Where $P(z)$ is called the primary path and models the acoustic response from the reference noise source to the error sensor (usually a microphone), and $S(z)$, called the secondary path, represents the acoustic response of the anti-noise wave to the error sensor $e(n)$.

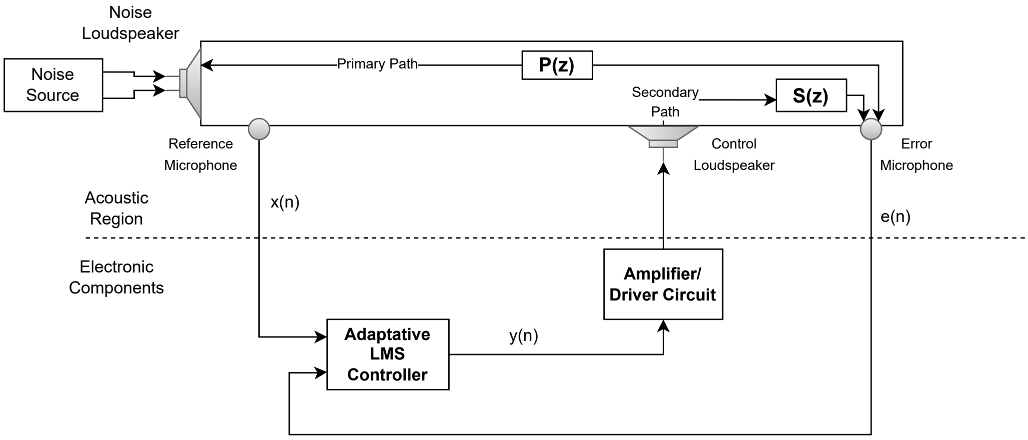

Figure 1. Simplified block diagram of an experimental LMS ANC system.

The influence of the secondary path on the control system may cause instability in the standard LMS algorithm. An extended version of the control plant called Fx-LMS is utilized to address this instability issue, which requires an estimation of the secondary path (Hassanpour and Davari, 2009).

The secondary path estimation can be achieved through offline or online approaches within the context of the ANC models. Offline estimation involves performing the estimation before the system's operation when the primary noise is absent. Conversely, online estimation entails estimating the secondary path during system operation. Primary noise is typically persistent in real-world scenarios, and operational/environmental conditions, such as humidity, temperature, etc., can exhibit temporal variations. Consequently, online modeling techniques may become necessary to guarantee the convergence of the Fx-LMS algorithm within the system (Zhang *et al.*, 2001).

Studies, such as the one presented by Chang *et al.* (2016), propose that if the attributes of S(z) remain time-invariant, ANC systems can employ offline modeling methods to estimate S(z). In this context, classical system identification techniques can be applied to a pre-sampled dataset to obtain an estimated secondary path model, denoted as Ŝ(z). This identified model is subsequently utilized to implement the Fx-LMS algorithm.

Online secondary path modeling often requires injecting additional random noise (generally white noise) as a training signal for accurate estimation. Increasing the amplitude of white noise improves modeling accuracy and convergence rate but at the expense of higher residual noise (Hassanpour and Davari, 2009). Researchers have also explored alternative online approaches that do not require additional noise injection, to address the secondary path estimation problem, such as the implicit estimation method proposed in the study by Zhou and DeBrunner (2007).

As far as the authors review of the existing literature have gone, no prior research that closely parallels the scope and focus of the present study, was found. In this particular context, the primary contribution of this paper lies in offering a comparative analysis regarding the noise attenuation capabilities and computational costs associated with two distinct approaches for modeling the secondary path in an acoustic duct. The first approach revolves around an offline state space model of the secondary path. In contrast, the second approach entails the utilization of an online secondary path system estimator based on Eriksson's method (Eriksson and Allie, 1989).

2. Fx-LMS ALGORITHM IN ANC SYSTEMS

The LMS algorithm can be characterized as a gradient descent algorithm, which iteratively updates the filter weights by incorporating a fraction of the negative gradient of the error surface. The error surface represents the error criterion plotted as a function of the filter weights to calculate an "improved" set of filter weights (Hansen, 2001). In a closed-loop adaptive filtering system, the filter weights or coefficients are iteratively adjusted to minimize the error between the filter output and the desired signal. The error represents the disparity between the actual and target signals (Nuñez, 2005).

The block diagram of the Fx-LMS algorithm is shown in Fig.2. Where, $x(n) \equiv$ Reference Signal (Reference Microphone), $d(n) \equiv$ Primary Noise (generated by the noise loudspeaker), $y(n) \equiv$ Controller Signal ($y'(n)$ is the actual signal emitted by the control loudspeaker), $e(n) \equiv$ Error Signal (Error Microphone), $W(z) \equiv$ Digital Finite Impulse Response Filter, $P(z) \equiv$ Primary Path, $S(z)$ Secondary Path and $\hat{S}(z) \equiv$ Secondary Path Estimate.

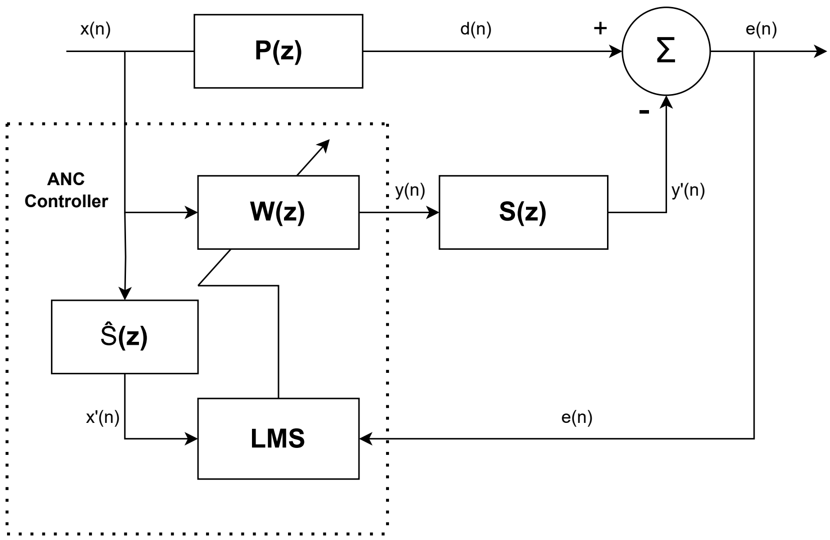

Figure 2. Block diagram of an Fx-LMS ANC system with offline secondary path estimation Kuo and Morgan (1999)

The placement of the secondary-path transfer function following the digital filter controlled by the LMS algorithm is shown in Fig. 2. The error signal is expressed as Eq. 1:

$$\mathrm{e}(n) = d(n) - y'(n) \tag{1}$$

The control signal is then calculated by multiplying the LMS calculated weights coefficient by the entrance signal and making a convolution of the model with the secondary path in Eq. 2 (Kuo and Morgan, 1999).

$$y'(n) = S(n) * [W^T(k)\mathbf{x(n)}] \tag{2}$$

The coefficients update equation for the Fx-LMS algorithm implemented can finally be expressed in terms of Eq. 3 (MathWorks, 2016):

$$W(k) = \alpha W(k-1) + f(\mathbf{x(n)}, e(n), \mu) \tag{3}$$

where, $\mu(n)$ is the adaptation step, which can be constant or adaptive, directly influencing the algorithm's stability and convergence as explained by Nuñez (2005). Here, N is the filter length, and α is the leakage factor.

The function $f(\mathbf{x}(n), e(n), \mu)$ depicts the coefficient update operation. In the literature, various update functions have been proposed. In this study, the normalized version, known as Normalized LMS or Fx-nLMS, was employed, as described by Eq. 4 (MathWorks, 2016):

$$f(\mathbf{x(n)}, e(n), \mu) = \mu \mathrm{e}(n) \frac{\mathbf{x(n)}^*}{\varepsilon + \mathbf{x(n)}^H \mathbf{x(n)}} \tag{4}$$

where ε is just a small floating point number to avoid potential numerical instability.

3. METHODOLOGY

The ANC algorithm was implemented with Matlab R2019/Simulink and ported to a dSpace dS1104 platform acting as the controller board.

Pure tone sinusoids with five different frequency values (120Hz, 160Hz, 240Hz, 360Hz, and 480Hz) were applied to the online and offline algorithms to verify the noise attenuation performance of both models. To guarantee plane waves inside the acoustic duct of the bench, it is necessary to establish a ceiling for the applied frequencies, calculated with the relation shown in Eq. 5 of Gerges (2000):

$$f_c = \frac{1.84c}{2\pi d^2} \tag{5}$$

where c is the velocity of the sound, and d is the diameter of the duct. Assuming $c = 343m/s^2$ and $d = 0.15m$, the experiment's cut-off frequency (f_c) is 669 Hz. A Sampling rate of 2400 Hz was chosen since it is approximately four times f_c.

3.1 Experimental Bench

The experimental bench is shown in Figure 3. The bench belongs to the LabNVH from UnB-FGA and has been used for research on active noise control. The system comprises a PVC plastic duct with a total length of 3.50 meters and a diameter of 0.15 meters. At one end of the duct, a loudspeaker is used as the primary noise source, and the other is used as the canceling loudspeaker to generate a proper anti-noise signal. A commercial power amplifier drives the loudspeakers.

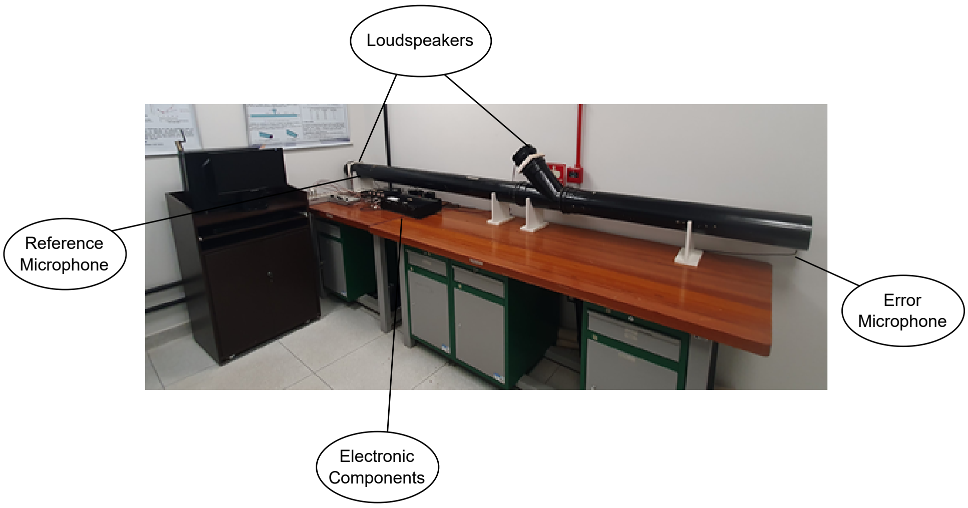

Figure 3. Experimental ANC Bench

Two microphones measure the reference and error signals. After passing the microphone's amplifier circuit, the measured signals go into an anti-aliasing filter. The filtered signal feeds into the DSpace® controller board, which acts as the ANC controller. The duct is sealed properly at all openings, except for the end of the path, to lower background noise. Table 1 presents the bench commercial components specifications.

Table 1. Bench Components Specification

	Component	**Details**
1	Loudspeaker	Hurricane, Class CM465 Quadriaxial, 6.5 inches in size, with a maximum power of 65 W RMS, a frequency response ranging from 80 Hz to 20 kHz, and an impedance of 4Ω
2	Power amplifier	18.0 Mark Audio MK1200 bi-channel, class AB amplification stage, an output power of 75 W RMS per channel, signal-to-noise ratio greater than 80 dB, frequency response between 20 Hz to 20 kHz, input impedance higher than 30 kΩ, and voltage gain ranging from -90 to 0 dB.
3	Microphones	Low-impedance electric microphones, a current consumption of 0.5 mA, a signal-to-noise ratio of 40 dB, and a maximum sound pressure level of 120 dB.
4	Pre-amplifiers	Behringer Tube Ultragain Mic100, the frequency response in the 10 Hz to 40 kHz range, with variable gain from +26 to +60 dB and output adjustment from $-\infty$ to +10 dB.
5	Controller board	Dspace DS1104 R&D and Controller Board

3.2 Offline Model

For the offline secondary path model identification process, an activation signal was applied to the control loudspeaker while the noise speaker was deactivated. The signals used for identification were the activation signal generated (input) and the signal obtained from the error microphone (output). In each experiment, the sampling frequency used was 2.4 kHz, and the duration of each experiment was 6 seconds. A white noise-type signal was used to excite the system. This signal is the most commonly used input signal for offline system identification because it provides uniformly-distributed spectral density at all frequencies. The data obtained were exported to Matlab software, and each data set was divided into two parts: the first 3 seconds for the identification process and the last 3 seconds for the model validation process.

The state space parameters were estimated using Matlab's System Identification toolbox with the following settings:

- The state-space parameterization was set to "Canonical parameterization".
- The state-space estimation method was the "Prediction Error Minimization (PEM)" algorithm. and
- The estimation weightings were assigned as "Simulation".

The model's accuracy was assessed by comparing its response to the measured output using the same input signal. The degree of agreement between the model's response and the measured output is represented by the fit percentage, where a value of 100 signifies a perfect match, while 0 indicates a poor fit. Table 2 displays the fit percentages for various model orders identified using white noise as the input signal. Although the 80-order model yielded the highest fit percentage, it is important to highlight that the 40-order model strikes a favorable balance between estimation quality and model complexity. By reducing the model order by half, the 40-order model showcases a mere 2.26% decrease in fit. Therefore, the 40-order model was chosen as the reference offline model (da Silva *et al.*, 2023).

Table 2. Fit Percentage for different model orders from da Silva *et al.* (2023)

Model Order	**Fit(%)**
80	89.40%
60	88.76%
40	87.14%
20	80.07%

3.3 Online Model

Eriksson's online secondary path estimation method was employed as the online model. Since it has low computational cost and implementation complexity when compared to the existing improved methods whom require a third adaptive filter (Akhtar *et al.*, 2006). The block diagram of the control algorithm as created by Eriksson and Allie (1989) is illustrated in Fig. 4:

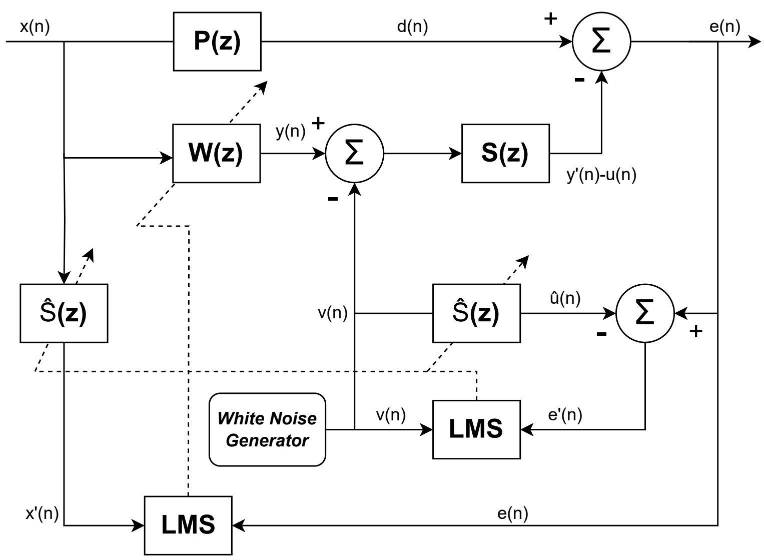

Figure 4. Online Fx-LMS - Eriksson Method

The algorithm employs a dual LMS operation in which the first iteration serves the same purpose of estimating the primary path, while the second LMS iteration utilizes added White Noise to estimate the secondary path during the system's operation.

Table 3 exhibits the designated parameters employed in the calibration process of the control model. For the offline model, the identical parameters as LMS 1 were employed, except for μ, which was assigned a value of 0.04. All parameters were adjusted by trial and error.

Table 3. Erikkson Model Coefficients

Eriksson Model LMS Coefficients	**Filter Length**	μ	α	**Excitation Signal RMS**
LMS 1	70	2e-08	0.999	-
LMS 2	40	1e-08	1	0.02

4. RESULTS AND DISCUSSION

Upon subjecting both models to a series of pure tones within the experimental setting and fine-tuning the controller parameters for each approach, Fig. 5 exhibits the temporal response and the frequency spectrum of both models measured in the error microphone. Neighboring the 160 Hz pure tone. A notable discrepancy between the two models becomes evident, with the offline model showcasing superior performance regarding the time response. This frequency represents the most significant disparity found between the two models, highlighting a clear advantage of the offline approach at this particular point.

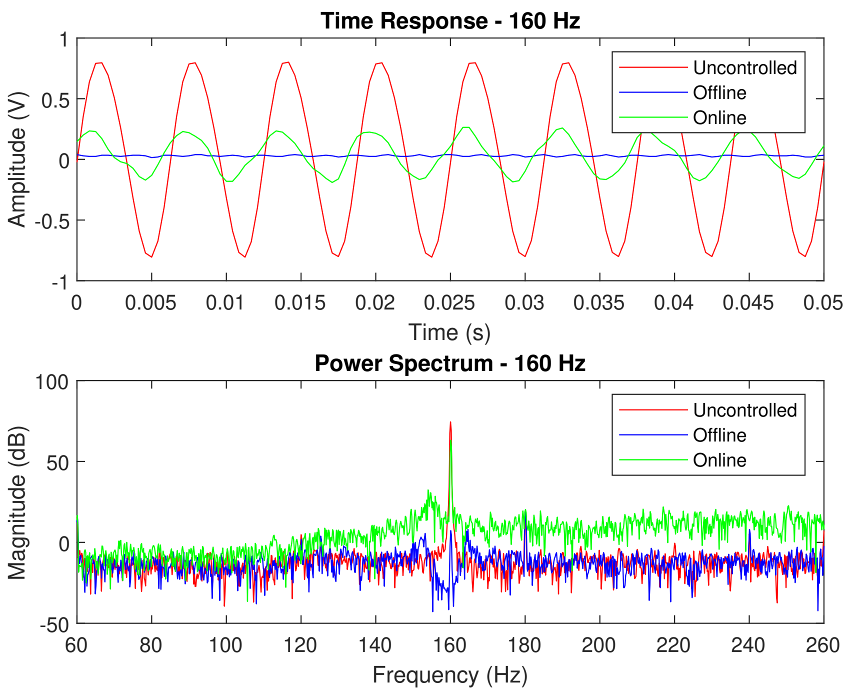

Figure 5. Model Response to a 160 Hz Pure Tone

Figure 6 illustrates a general response of the controllers to 240 Hz sinusoidal excitation of the system. Similar results were obtained to 120 and 360 Hz signals.

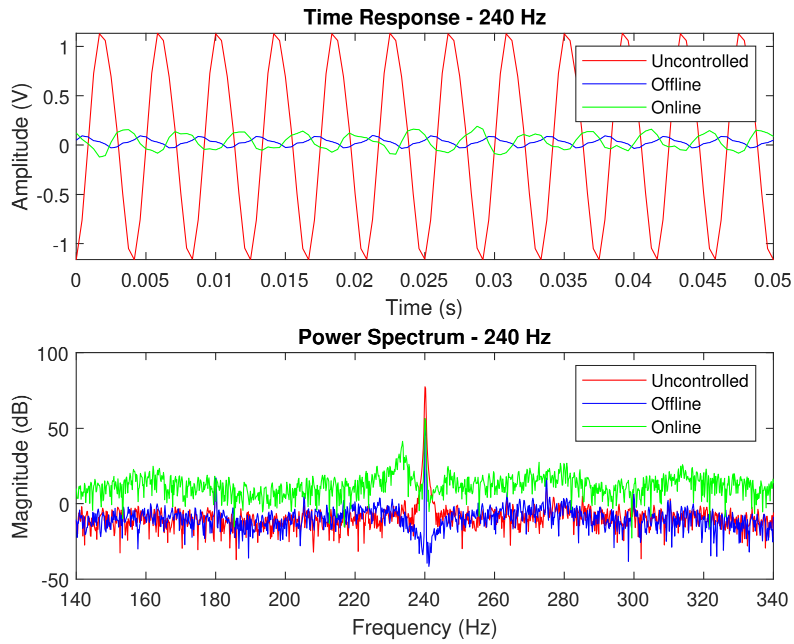

Figure 6. Model Response to a 240 Hz Pure Tone

While a visual inspection of the time response suggests similar outcomes for both controllers, a closer examination of the power spectrum reveals a notable distinction. Specifically, the online model exhibits a discernible presence of white noise injected into the system output.

Around 480 Hz (Fig. 7), it is worth remembering that the inherent acoustic duct structure can decrease the noise spectrum's peak by approximately 20 dB. Consequently, active noise control (ANC) becomes nearly unnecessary in this scenario. However, the white noise introduced by Eriksson's model actually increases the spectrum peak, resulting in almost no signal attenuation.

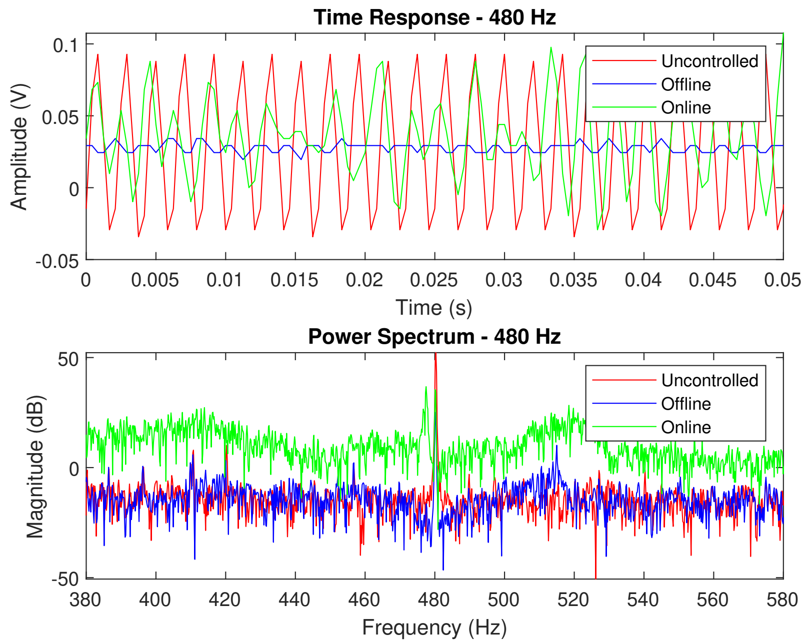

Figure 7. Model Response to a 480 Hz Pure Tone

Table 4 summarizes the performance results of both models along some frequencies of pure tones injected. In this table, it is possible to overview the results obtained in each model.

Table 4. Comparative of Pure Tones Attenuation Capabilities Between the Models

Signal -> Power Spectrum Peak(dB)/ Time Response (RMS)	**Uncontrolled**		**Offline ANC**		**Online ANC**	
	dB	**RMS**	**dB**	**RMS**	**dB**	**RMS**
120 Hz	71.707	0.427	58.814	0.103	62.348	0.144
160 Hz	74.454	0.570	51.746	0.030	63.145	0.154
240 Hz	77.476	0.818	52.562	0.048	56.443	0.087
360 Hz	73.108	0.453	51.886	0.036	53.702	0.066
480 Hz	52.208	0.053	51.221	0.028	54.372	0.043

The computational time of both models was measured and extracted for analysis. The corresponding results are illustrated in Fig. 8. The graphic demonstrates that the offline method exhibits a computational time approximately one-third shorter than the online method. This outcome aligns with expectations, as the offline model does not demand the recalculation of secondary path coefficients, leading to reduced computational requirements.

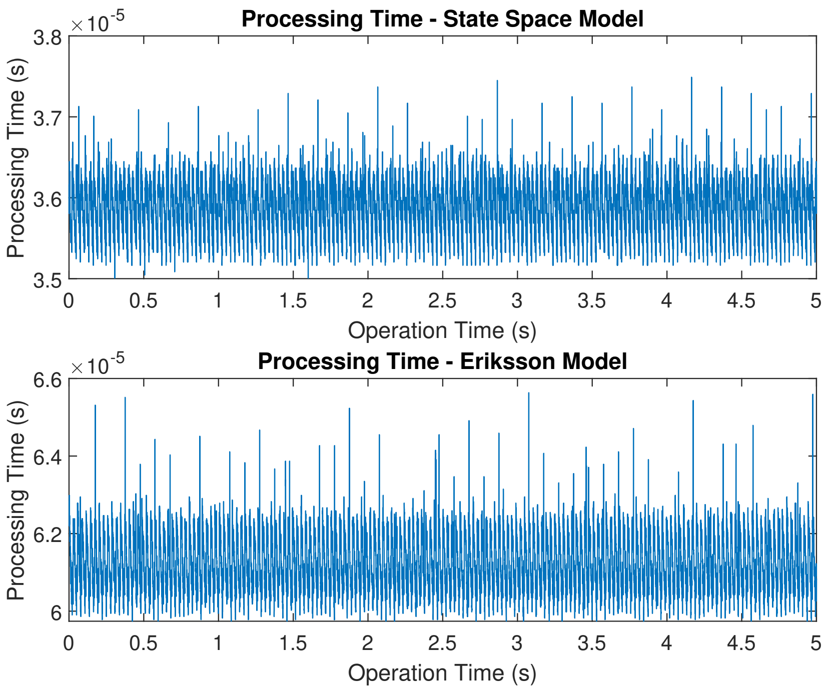

Figure 8. Processing Time

5. CONCLUSIONS

After comparing the online and offline secondary path identification models, it becomes apparent that the offline method outperforms the online approach regarding attenuation capabilities and processing time. The offline model proves to be a powerful tool in accurately characterizing the secondary path in a system.

It's worth mentioning that employing more intricate online approaches can improve the system's results, but it comes with the trade-off of requiring more computational power.

However, it is important to note that the effectiveness of the offline identification method relies on the assumption that system properties remain unchanged over time. This implies that any variations in the system's characteristics or environmental conditions may undermine the accuracy of offline models.

Nevertheless, despite this limitation, the offline method presents a compelling choice in situations where the system's properties remain stable or where the addition of noise to the system is problematic. Further research will explore different experimental scenarios to define the consistency and conditions of validity of the offline model.

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