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THE GENERALIZATION AND ANALYSIS OF THE STUSSI MODEL FOR SEVERAL FATIGUE DAMAGE PARAMETERS

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Abstract. *In structures subject to cyclic stresses, mechanical failures, and damage cause unacceptable losses in performance, resulting in unexpected stops and corrective maintenance, which increase the cost of maintenance and reduce the reliability of the structures or components. In this situation, the fatigue life prediction of these structures, both for low and high cycle fatigue, is indispensable, especially if the element is subject to cracks or the effects of mean stress. The non-linear function proposed by Stussi is a geometrical approach for stress fatigue cycles. However, the analysis of only the stress parameter may not consider the effects of mean stress or crack initiation, which parameters such as Smith-Watson-Topper and strain includes. Because of this, this study proposes the generalization of the Stussi model for different fatigue damage parameters, such as stress, deformation, and the Smith-Watson-Topper parameter. This study uses the fatigue data of several steels. And with the experimental data, we compared the results with the conventional models for the damage parameters and with another generalized model. The generalization of the models shows good predictions to the materials in our study for the high and low-cycle regions.*

Keywords: *Fatigue, Stüssi model, Full-life, Model Generalization*

1. INTRODUCTION

Typically engineering structures are subject to different types of cyclic loading that can cause fatigue failure, so predicting the fatigue life of these components/structures is critical to ensure reliability during operation and structural integrity (Barbosa *et al.*, 2020; Correia *et al.*, 2017a; Mayorga *et al.*, 2017; Zhou *et al.*, 2023). However, this situation is aggravated when there is a meaningful component in stress, strain, cracks in the material, or the presence of mean stress, which anticipates the fatigue failure to what was predicted (Apetre *et al.*, 2015; Barbosa *et al.*, 2020; Caiza and Ummenhofer, 2020; Mayorga *et al.*, 2017).

Furthermore, the construction of fatigue life prediction models is subject to several difficulties, as to the range of the prediction for both low fatigue cycle and high fatigue cycle (Caiza and Ummenhofer, 2020; Canteli *et al.*, 2022; Correia *et al.*, 2017b), the use of different parameters in addition to the usual stress model (Correia *et al.*, 2017a), the presence of the component mean stress/strain (Apetre *et al.*, 2015; Barbosa *et al.*, 2020; Canteli *et al.*, 2022), or the probabilistic behavior of materials used in components subjected to stress and strain cycles (Apetre *et al.*, 2015; Barbosa *et al.*, 2020; Caiza and Ummenhofer, 2011, 2020; Mayorga *et al.*, 2017). As pointed out by Ince and Glinka (2014), even though several parameters of fatigue damage are been studied during the last decades, most of them are limited to special cases of loading, material, cycle, and one damage parameter. In general, damage parameters are classified into three categories: stress-based, generally applied to high-cycle and infinite-life regimes with plastic deformation neglected; strain-based, generally applied at low cycle, even though can be applied both to low and high cycle; and finally based on the energy that relates strain energy to stress (Ince and Glinka, 2014).

The Kohout-Věchet (KV) model was first presented in Kohout and Vechet (2001), which proposed a fatigue life prediction model using the stress that covers both the very-low cycle region and the very high fatigue cycle region. In the work of Correia *et al.* (2017a) a new generalization of the Kohout-Věchet fatigue model for different damage parameters such as stress parameter, Smith-Watson-Topper, Walker-like stress/strain, energy-based parameter for uniaxial conditions and among others is discussed. The generalization proposed in Correia *et al.* (2017a) uses the work of Karunananda *et al.* (2012), which applied the Kohout-Věchet model first for the strain damage parameter.

Another model generalization is show by Correia *et al.* (2017b), generalizing the probabilistic fatigue model of Castillo and Fernández-Canteli. These probabilistic models are highlighted because they can incorporate different sources of uncertainty, such as material properties, microstructures, or geometric characteristics of components. A large part of the fatigue models is deterministic, which requires a probabilistic evaluation for project application purposes. Because of this need, many authors have proposed several models of probabilistic fatigue, with emphasis on the fatigue model of Castillo and Fernández-Canteli, based on the Weibull model for constant stresses for different stress levels, this model has also extended to the strain parameter.

In the context of model generalization and fatigue damage parameters, our objective is to generalize the Stüssi model for fatigue damage parameters, such as stress, strain, and the Smith-Watson-Topper. We evaluate how the generalization of this full-life stress-based model can predict each approach, compared with the conventional models and with the generalized Kohout-Věchet model.

2. STÜSSI MODEL

The nonlinear Stüssi function is a stress-based full-life model proposed based on the geometric principle of the material behavior during its life (Caiza and Ummenhofer, 2018). Therefore, this model must estimate the fatigue threshold and the ultimate tensile strength to fit in the low and high-cycle regions (Barbosa, 2019). The model is given by the following equation,

$$\sigma = \frac{\sigma_u + \alpha N_f^{\beta}\sigma_\infty}{1 + \alpha N_f^{\beta}}, \tag{1}$$

where σ is the stress, σ_u is the ultimate tensile strength, σ_∞ is the fatigue limit, N_f is the number of cycles to failure, and α, β are the geometric parameters of the material. To estimate this geometric parameters, Eq. (1) is rewritten as

$$\alpha N_f^{\beta} = \frac{\sigma_u - \sigma}{\sigma - \sigma_\infty}, \tag{2}$$

taking the logarithm in Eq. (2) leads to

$$log(N_f) = \frac{1}{\beta} log\left(\frac{\sigma_u - \sigma}{\sigma - \sigma_\infty}\right) - \frac{1}{\beta} log(\alpha). \tag{3}$$

The Eq. (3) can be written as a linear equation denoting $Y = log(N_f)$, $X = \frac{\sigma_u - \sigma}{\sigma - \sigma_\infty}$, $A = 1/\beta$ and $B = 1/\beta log(\alpha)$, as follow,

$$Y = XA + B, \tag{4}$$

where the parameters A and B can be estimated from a linear regression and then the geometric parameters α and β.

3. GENERALIZATION

The generalization of the Stüssi model is made similar to the generalization Kohout Věchet model. The Kohout-Věchet model is a full-life stress-based approach and was generalized by Correia *et al.* (2017a) based on the hypothetical ultimate strain requirement for the damage parameters proposed by Karunananda *et al.* (2012). Its original form is written as,

$$\sigma(N) \equiv a\left[\frac{(N_f + B)C}{N_f + C}\right]^b \equiv \sigma_\infty\left(\frac{(N_f + B)}{N_f + C}\right)^b \equiv \sigma_1\left(\frac{(1 + B/N_f)}{1 + C/N_f}\right)^b. \tag{5}$$

where B and C are the number of cycles corresponding to the intersection of the tangent line of finite life region and the horizontal asymptote of the ultimate tensile strength and the horizontal asymptote of the fatigue limit, seen in Fig. (1).

These parameters are given by,

$$C = 10^7 \frac{1 - \gamma_{kv}}{\gamma_{kv} - \beta_{kv}}, \tag{6}$$

$$B = \beta_{kv}\gamma_{kv}, \tag{7}$$

where β_{kv} and γ_{kv} are,

$$\beta_{kv} = \left(\frac{\sigma_u}{\sigma_\infty}\right)^{1/b}, \tag{8}$$

$$\gamma_{kv} = \left(\frac{\sigma_c}{\sigma_\infty}\right)^{1/b}, \tag{9}$$

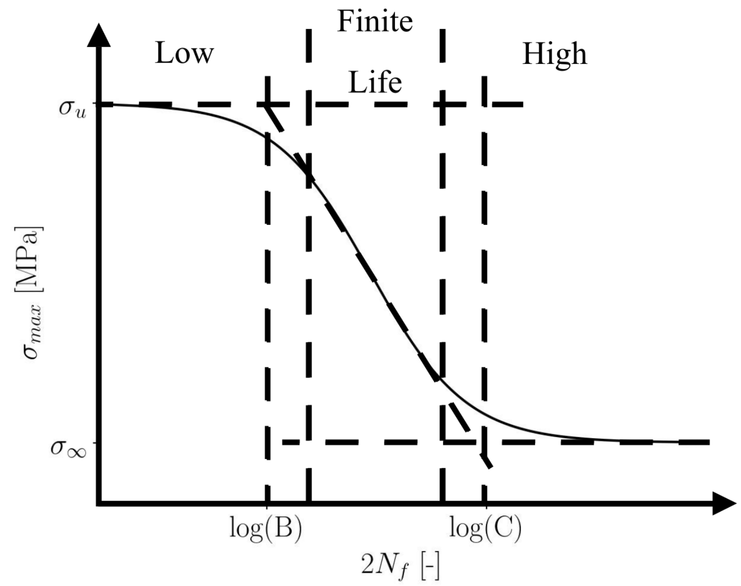


Figure 1. Schematic representation of the full-life stress-life curve

σ_c is the fatigue limit for a predefined number of cycles, usually 10^7. As proposed by Correia *et al.* (2017a) the Eq. (5) was generalized for several damage parameters (ψ), such as stress, strain, and energy-based parameters in the uniaxial loading condition. The generalized equation is given by,

$$\psi(N_f) \equiv \psi_e \left(\frac{N_f + N_u}{N_f + N_e}\right)^{b'} \equiv \psi^{ULCF} \left(\frac{(N_f + N_u)N_e}{N_f + N_e}\right)^{b'} \equiv \psi^{UHCF} \left(\frac{(1 + N_f/N_u)}{1 + N_f/N_e}\right)^{b'}. \quad (10)$$

Where ψ_e is the fatigue limit damage parameter, ψ_{ULCF} is the ultimate damage parameter for the low-cycle fatigue regime, ψ_{UHCF} is the ultimate damage parameter for the high-cycle fatigue regime, N_u and N_e are the number of cycles corresponding to the intersection of the tangent line of finite life region and the horizontal asymptote of the ultimate high cycle fatigue damage parameter and the horizontal asymptote of the fatigue limit damage parameter, seen in Fig. (1).

In this context, the stüssi model, Eq. (1), can be rewritten as,

$$\psi = \frac{\psi^{ULCF} + \alpha_\psi N_f^{\beta_\psi} \psi_e}{1 + \alpha_\psi N_f^{\beta_\psi}}, \quad (11)$$

where α_ψ and β_ψ are the geometrical parameters of the fatigue damage parameter. To identify these new geometric parameters the linear Eq. (3) is generalized as shown by,

$$log(N_f) = \frac{1}{\beta_\psi} log\left(\frac{\psi^{ULCF} - \psi}{\psi - \psi_e}\right) - \frac{1}{\beta_\psi} log(\alpha_\psi) \quad (12)$$

4. DAMAGE PARAMETERS AND GENERALIZATION

This section will show an overview of the damage parameters (stress, strain, and Smith-Watson-Topper), the conventional models, and the generalization for each parameter. The monotonic and fatigue properties of each material utilized in this study are shown in Tab. (1). For all material data used in this work, the fatigue test applied was strain-controlled and with a strain ratio of -1.

Table 1. Monotonic ($N_f = 0.5$) and fatigue ($N_f = 10^6$) damage parameters for each material

Materials	σ_u [MPa]	σ_e [MPa]	ϵ_u [%]	ϵ_e [%]	SWT_u [-]	SWT_e [%]
TTStE32	558.0	277.87	2	0.13	11.6	0.29
StE690	872.0	466.49	5.5	0.22	47.96	1.03
StE460	682.0	332.97	2	0.17	13.64	0.56
S355	732	266.33	4	0.16	29.28	0.43

4.1 Stress Approach

The stress approach is the analysis of the stress life curve (S-N curve), where the conventional model, Basquin, is suitable for high cycle fatigue, in which the strain is predominantly within the elastic range. The Basquin model is given by,

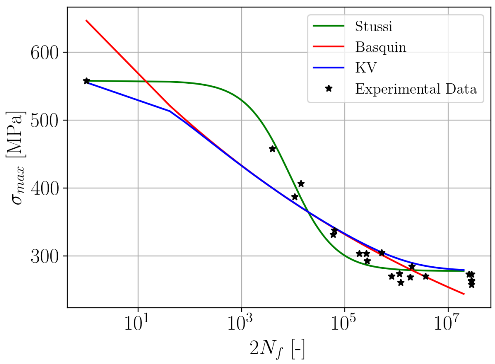
$$\sigma(2N_f) = a(2N_f)^b, \tag{13}$$

where a and b are material constants, N_f is the number of cycles to failure, and $2N_f$ is the number of reversals to failure. These material constants can be estimated by a linear regression using a linear model from the log of Eq. (13) of the experimental data. For this regression, we used only the high cycle region until the fatigue limit, in which any data point after 10^6 cycles wasn't used in the regression, seen in Tab. 2. The parameters for the Kohout-Věchet and Stüssi models were obtained using the ultimate tensile strength, fatigue limit, and the geometric regression from the data, Eq. (3), also seen in Tab. 2. The behavior of each model tested for the stress parameter is seen in Fig. (2).

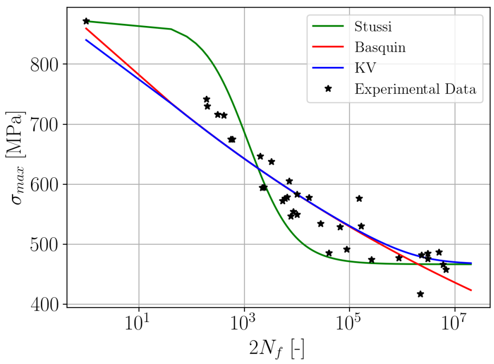
Table 2. Stress approach parameters for the Basquin, Kohout-Věchet and Stüssi models

Materials	a	b	C	B	α	β
TTStE32	646.4	-0.057	2146456	12.67	0.00011	0.99984
StE690	859.54	-0.042	1999999.99	0.71	0.00084	0.99828
StE460	811.25	-0.061	1999999.99	16.9	0.00016	0.99984
S355	667.27	-0.063	1999999.99	0.23	0.00048	0.99933

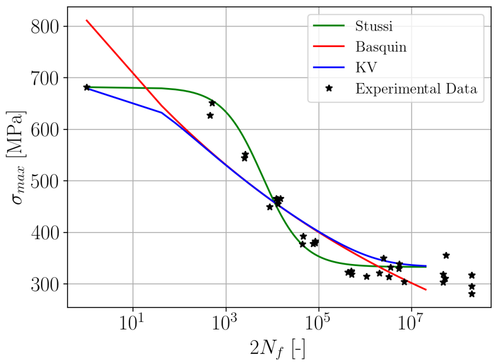
The full-life models, KV and Stüssi, give good predictions for the low and high cycle regimes, since these models captured the sigmoid behavior of the data, predicting the monotonic region and the high cycle region. Although, for the steel StE690, the Stüssi model didn't predict very well the finite life region, as for the KV and Basquin, it approximates a mean line from the data. Also, the Basquin model for most materials in the low cycle region does not accurately predicts the monotonic region. And after 10^6 cycles, the Basquin model predicts lower values not taking into account the fatigue limit behavior of each material in which the elements don't fail or the experiment time runout.



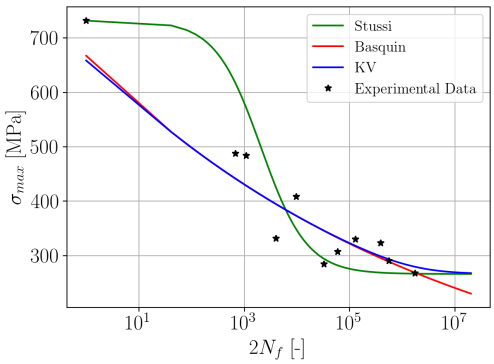
(a) TTStE32



(b) StE690



(c) StE460



(d) S355

Figure 2. Stress fatigue damage approaches

4.2 Strain Approach

For the strain damage parameter is often used the Coffin-Manson model,

$$\epsilon_a = \epsilon_a^e + \epsilon_a^p = \frac{\sigma'_f}{E}(2N_f)^b + \epsilon'_f(2N_f)^c \tag{14}$$

where ϵ_a is the total strain amplitude, ϵ_a^e is the elastic strain amplitude, ϵ_a^p is the plastic strain, σ_f is the fatigue strength coefficient, E is the elastic modulus, b is the fatigue strength exponent, ϵ'_f is the fatigue ductility coefficient, and c is the fatigue ductility exponent. These parameters of the Coffin-Manson are estimated by taking into account that for the low cycle region, the main strain is plastic deformation, so Eq. (14) can be summarized in,

$$\epsilon_a = \epsilon_a^p = \epsilon'_f(2N_f)^c \tag{15}$$

and for the high cycle region, the main strain is elastic deformation, so the Eq. (14) can be summarized in,

$$\epsilon_a = \epsilon_a^e = \frac{\sigma'_f}{E}(2N_f)^b. \tag{16}$$

And similar to the stress approach, any data point after 10^6 cycles wasn't used in the regression for the elastic and plastic parts of the Coffin-Manson model. In the strain approach, the Stussi model and the KV model are written following Eq. (11) Eq. (10),

$$\epsilon_a(N_f) = \frac{\epsilon^{ULCF} + \alpha_\epsilon N_f^{\beta_\epsilon}\epsilon_e}{1 + \alpha_\epsilon N_f^{\beta_\epsilon}}, \tag{17}$$

$$\epsilon_a(N_f) \equiv \epsilon_e \left(\frac{N_f + N_u}{N_f + N_e}\right)^{b'} \equiv \epsilon^{ULCF}\left(\frac{(N_f + N_u)N_e}{N_f + N_e}\right)^{b'} \equiv \epsilon^{UHCF}\left(\frac{(1 + N_f/N_u)}{1 + N_f/N_e}\right)^{b'}, \tag{18}$$

where ϵ_e is the fatigue limit strain, ϵ^{ULCF} is the strain for the ultimate low cycle region which is equal to ϵ'_f or estimated from a single generic power law, the same power law can be estimate b' which is considered an adjustment parameter (Correia *et al.*, 2017a), and ϵ^{UHCF} is the ultimate monotonic strain. All the parameters for the strain approach are shown in Tab. (3) and were estimated based on the monotonic and fatigue data from Tab. (1). In Fig. (3) is seen the Coffin-

Table 3. Strain approach parameters for the Coffin-Manson, Kohout-Věchet and Stüssi models

Materials	ϵ'_f	σ_f/E	b	c	N_e	N_u	α_ϵ	β_ϵ	ϵ^{ULCF}	b'
TTStE32	0.228	0.00456	-0.092	-0.467	8746054	0.136	0.00088	0.99854	1.6	-0.15
StE690	0.813	0.00442	-0.053	-0.661	494438	0.338	0.0064	0.99111	4.3	-0.22
StE460	0.384	0.00534	-0.093	-0.522	9251898	1.2	0.00052	0.9993	2	-0.15
S355	0.415	0.00425	-0.082	-0.606	220205	0.19	0.0157	0.97714	4.3	-0.26

Manson and the full-life models with each respective prediction. For the conventional model for the strain approach, is observed an overshoot value for the monotonic strain. And for the full-life models, only the Stüssi model was able to predict accurately the monotonic region, but the behavior of most of the materials evaluated, between 10^3 and 10^6 cycles, this model gave a conservative prediction lowering the strain value in which the materials could work. Unlike the KV model, that for this region predicted very well the behavior of the material.

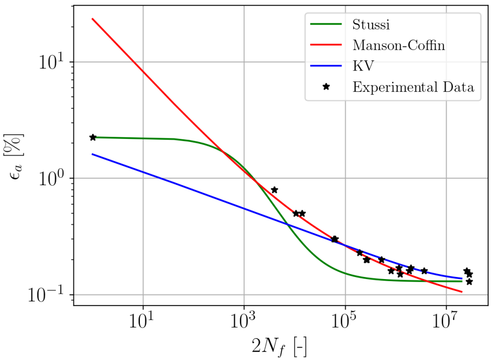
4.3 Smith-Watson-Topper (SWT) approach

The SWT approach proposed by Smith (1970) take into the account both analysis of stress and strain combining the plastic and elastic behavior into the stress. Also, it can account for the mean stress effects, which updates the Coffin-Manson model given by the following equation,

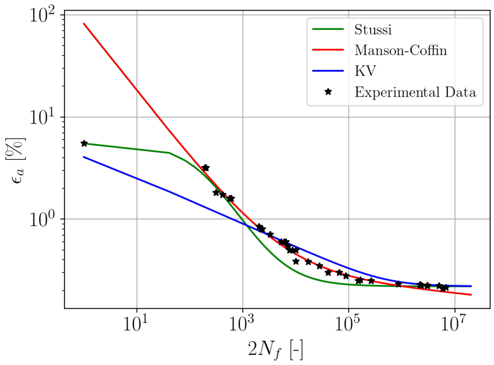
$$SWT(N_f) = \sigma_{max}\epsilon_a = \frac{(\sigma'_f)^2}{E}(2N_f)^{2b} + \epsilon'_f\sigma'_f(2N_f)^{b+c}, \tag{19}$$

Each parameter of the SWT model was estimated by the stress and strain approach seen in Tab. (2) and Tab. (3). With the monotonic and fatigue stress and strain, the SWT damage parameter we can estimate and see its results in Tab. (1). Using the same assumptions of Correia *et al.* (2017a) and Karunananda *et al.* (2012) for the other damage parameters shown in this study the full life models, Stüssi and KV for this parameter, can be rewritten as follow,

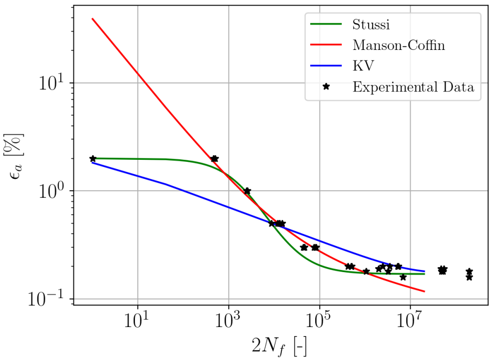
$$SWT(N_f) = \frac{SWT^{UHCF} + \alpha_{SWT}N_f^{\beta_{SWT}}SWT_e}{1 + \alpha_{SWT}N_f^{\beta_\epsilon}}, \tag{20}$$



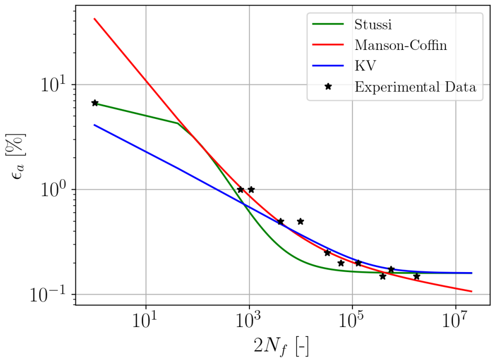
(a) TTStE32



(b) StE690



(c) StE460



(d) S355

Figure 3. Strain fatigue damage approaches

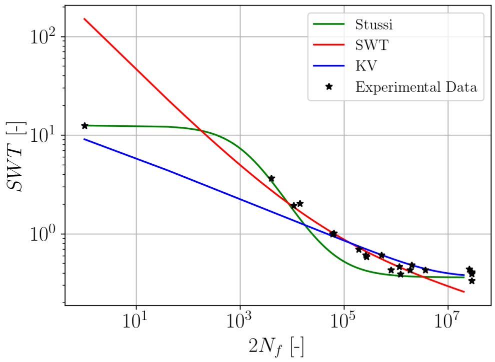
$$SWT(N_f) \equiv SWT_e \left(\frac{N_f + N_u}{N_f + N_e}\right)^{b'} \equiv SWT^{ULCF} \left(\frac{(N_f + N_u)N_e}{N_f + N_e}\right)^{b'} \equiv SWT^{UHCF} \left(\frac{(1 + N_f/N_u)}{1 + N_f/N_e}\right)^{b'}, \quad (21)$$

where SWT^{UHCF} is the ultimate high-cycle SWT, which corresponds to the product between the monotonic strain (ϵ_u) and stress (σ_u). SWT^{ULCF} is the ultimate low cycle SWT, which corresponds to the plastic component of the SWT, Eq. (19), and b' is the adjustment parameter estimated in the same way for the strain approach. All these parameters are seen in Tab. (4).

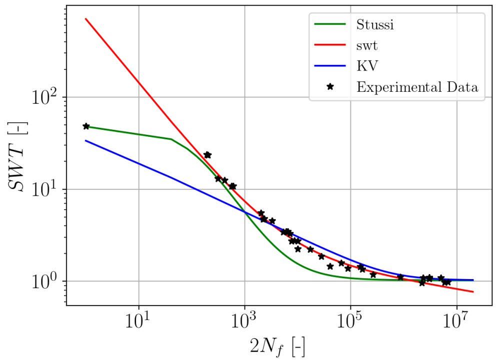
Table 4. SWT approach parameters for Kohout-Věchet and Stüssi models

Materials	N_e	N_u	α_{SWT}	β_{SWT}	SWT^{ULCF}	b'
TTStE32	5838036	0.277	0.0007	0.99907	9.58	-0.21
StE690	547616	0.349	0.0102	0.98568	36.38	-0.27
StE460	6686751	1.32	0.00019	0.99987	14.46	-0.2
S355	282774	0.203	0.0283	0.96037	28.74	-0.33

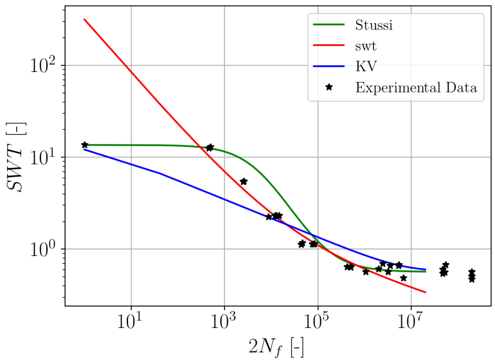
The behavior of the models for the SWT damage parameter is shown in Fig. (4). The results for the SWT parameter have similar behavior os the strain parameter due to the absence of mean stress in the fatigue test. For all the materials tested, the Stüssi model was able to predict the monotonic and fatigue life regions. The SWT model, Eq. (19), made good predictions for the finite life region (10^3 to 10^6). Unlike the full-life models that made non-conservative predictions, for some materials such as StE460.



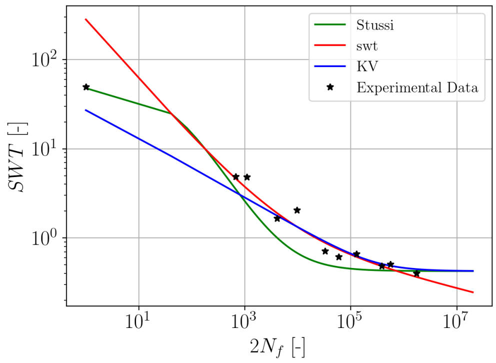
(a) TTStE32



(b) StE690



(c) StE460



(d) S355

Figure 4. SWT fatigue damage approaches

5. FINAL REMARKS

This work has analyzed the generalization of the full-life stress-based Stüssi model for other fatigue damage parameters and compared it to the conventional models and with the generalized KV model. The generalized model was tested using different steels, and the fatigue data, were strain-controlled tests with a strain ratio of -1. All the parameters for the models were calculated with the cyclic data until 10^6 cycles. The results show that for all the materials and fatigue damage parameters tested the Stüssi model was successful and was able to predict the monotonic and infinity life region, even for the SWT parameter that is best used in the presence of mean stress. This generalization is an enhancement tool for the fatigue assessment in materials covering the high and low cycle regions.

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