

COBEM-2023-1064

COMPARATIVE STUDY OF MULTIAXIAL FATIGUE LIFE PREDICTION METHODOLOGIES BASED ON CRITICAL PLANE APPROACHES: A REVIEW OF MAXIMUM VARIANCE METHOD (MVM) AND OPTIMIZATION ALGORITHMS

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Abstract. This study aims to evaluate methodologies for predicting multiaxial fatigue life based on random loading using critical plane approaches and optimized search algorithms for the identification of critical planes. Various multiaxial fatigue studies assert that most mechanical components and structures are often under multiaxial stress states, which, when caused by dynamic loads, can lead to the initiation and propagation of fatigue cracks. For instance, aircraft loads in flight, wind loads in wind turbines, wave action in offshore environments, liquid flow, and industrial pipe vibrations. Therefore, multiaxial fatigue life prediction has evolved with new strategies to ensure reliability and safety in mechanical designs. Additionally, the development of an efficient life prediction algorithm can optimize design time by providing preliminary numerical simulations of multiaxial fatigue failure before experimental and service operational tests. The literature review also addresses topics related to random multiaxial fatigue, such as stochastic processes and simulated signal reconstruction techniques based on standard deviation, spectral characteristics, and Power Spectral Density (PSD) reference functions of the original signal. Initially, the critical planes are mapped by the algorithm based on their variances using the Maximum Variance Method (MVM), in which normal and shear stresses act as driving forces, and gradient-based optimizers perform the selection of the final critical plane. Then, decoupled damage model approaches are applied for numerical life prediction based on the selected critical plane, material mechanical characteristics, and spectral characteristics of the signal. In the time domain, the Rainflow cycle counting method combined with the Palmgren-Miner damage accumulation rule, the Serensen-Kogayev correction, and the Modified Wöhler Curves Method (MWCM) to calibration of the model when the driving force is the shear stress. In the frequency domain, spectral analysis is performed using the Rayleigh distribution in the damage model, which can be combined with the Serensen-Kogayev correction. Finally, based on the results obtained, we can state that the methodologies used proved to be efficient in predicting multiaxial fatigue life with numerical estimates that were very close to the fatigue life of the material. For this reason, this work will contribute significantly to the application of reliable and low computational cost methodologies in multiaxial fatigue analysis, being an important step to guarantee the safety and reliability of mechanical structures subjected to cyclic loads.

Keywords: multiaxial fatigue, life prediction, Maximum Variance Method, driving forces, decoupled damage model

1. INTRODUCTION

In mechanical structures used in aerospace, automotive, and offshore applications, cyclic loads with some level of randomness are encountered, which makes loading extremely complex and requires a specific strategy for fatigue life prediction. This is due to the fact that the multiaxial fatigue behavior of materials poses challenges in characterizing a loading cycle and its corresponding amplitude when the real loading varies randomly over time (Papadopoulos, 2001; Susmel *et al.*, 2009; Araújo *et al.*, 2011; Gates and Fatemi, 2017; Luo *et al.*, 2020; Lima, 2021; Ferreira *et al.*, 2022b,a).

Different multiaxial fatigue criteria have been proposed to predict failure under complex loading conditions, categorized into stress-based, strain-based, and energy-based criteria (Karolczuk and Macha, 2005; Lima, 2021). When considering the influence of random multiaxial loads on fatigue, extensive discussions have been conducted in the literature regarding the estimation of the critical plane position with stress-based. This concept is commonly described by the mean values of director cosines associated with equivalent stress. Among the stress criteria that employ the critical plane approach, three notable methods are frequently discussed: the weight functions method, the variance method (or method of equivalent stress variance), and the method of fatigue damage accumulation (Macha, 1989b; Łagoda and Macha, 1994; Łagoda *et al.*, 2005a). Focusing on the second method mentioned, the analysis of fatigue involves reducing the three-dimensional stress state to an equivalent stress or driving force. In 1989, Macha proposed the equivalent normal stress, which was defined as a linear combination of the axial and shear components of the stress tensor acting on a spe-

cific material plane (Bedkowski and Macha, 1987; Macha, 1989b,a; Łagoda *et al.*, 2005a, 2019). Subsequently, in the mid-2009, Susmel introduced the concept of the critical plane as the material plane with the maximum variance of shear stress (Susmel, 2009; Susmel *et al.*, 2009; Susmel, 2010; Louks *et al.*, 2014; Luo *et al.*, 2020).

When dealing with random multiaxial loads, the estimation of the critical plane is a topic of discussion in the literature with different strategies involving driving forces. The variance method is one approach that reduces a three-dimensional stress state to an equivalent stress or driving force. This article focuses on three critical plane approaches using the variance method and optimization algorithms for multiaxial fatigue life prediction. The goal is to develop an efficient and versatile algorithm that closely predicts fatigue life based on input stress histories, allowing for preliminary numerical simulations and reducing costs associated with experimental testing.

2. FATIGUE MODELS WITH MAXIMUM VARIANCE METHOD

In the context of multiaxial fatigue analysis, the Maximum Variance Method (MVM) provides an approach for searching the critical plane based on the assumption that the critical plane coincides with the material plane where the maximum variance of specific driving forces, such as normal stress, equivalent stress, or shear stress, is observed, as shown in Fig. 1. The application of MVM involves the use of the covariance matrix $[C]$ in both the time domain, represented by stress histories of each component of the stress tensor, and the frequency domain, represented by Power Spectral Density (PSD) or Cross Power Spectral Density (Cross PSD) spectra (Łagoda *et al.*, 2005b; Niesłony and Macha, 2007; Lima, 2021; Ferreira *et al.*, 2022b,a). Additionally, the plane search process is performed by sweeping the angles of the director cosines to determine the maximum variance of a given driving force selected from the failure criterion applied on the critical plane found by the MVM search or post-MVM optimization algorithms.

When the critical planes coincide in both the time and frequency domains, another approach can be utilized, taking advantage of the critical plane search performed in the time domain. Upon identifying the driving force of the failure criterion, this stress can be transformed into a power spectrum in the frequency domain and applied to the spectral damage accumulation model with the moments and spectral parameters presented in Section 2.4. In Fig. 1, damage accumulation models are discussed in Sections 4 and 5, among them: Palmgren–Miner damage accumulation rule and spectral damage (with or without Serensen–Kogayev correction) and Modified Wöhler Curves Method for calibration using shear stress.

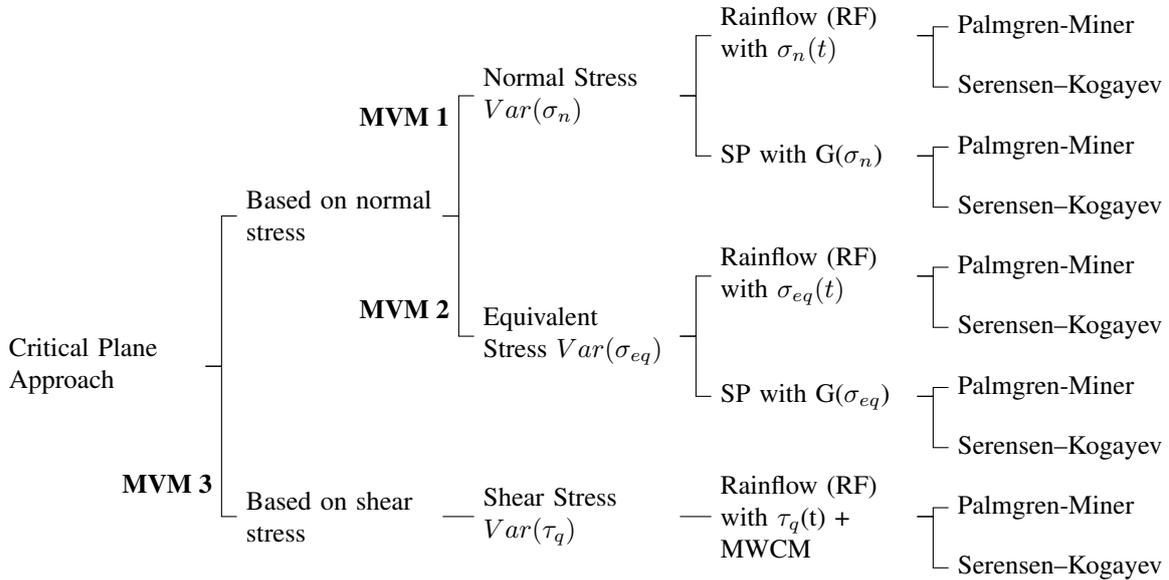


Figure 1: Approaches employed by MVM and sequential techniques to determine multiaxial fatigue life prediction (RF: Rainflow | SP: Spectral Analysis | MWCM: Modified Wöhler Curves Method).

2.1 Normal stress

The criterion of maximum normal stress on the critical plane indicates that fatigue failure is attributed to the normal stress acting on that material plane where the maximum variance of normal stress occurs. The critical plane, which is perpendicular to the mean direction, corresponds to the orientation along which the maximum principal stress σ_1 occurs (Macha, 1989a; Łagoda *et al.*, 2005a). The normal vector is defined by Eq. (1):

$$\mathbf{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{bmatrix} \quad (1)$$

Based on the transformation equation of normal stress in any plane, Eq. (2) can be rewritten as a linear combination of the vector of direction cosines \mathbf{d}^n :

$$\sigma_n(t) = \begin{bmatrix} n_x & n_y & n_z \end{bmatrix} \begin{bmatrix} \sigma_{xx}(t) & \tau_{xy}(t) & \tau_{xz}(t) \\ \tau_{yx}(t) & \sigma_{yy}(t) & \tau_{yz}(t) \\ \tau_{zx}(t) & \tau_{zy}(t) & \sigma_{zz}(t) \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \sum_{k=1}^6 d_k^n \sigma_k(t) \quad (k = 1, \dots, 6) \quad (2)$$

where:

$$\mathbf{d}^n = \begin{bmatrix} d_1^n \\ d_2^n \\ d_3^n \\ d_4^n \\ d_5^n \\ d_6^n \end{bmatrix} = \begin{bmatrix} n_x^2 \\ n_y^2 \\ n_z^2 \\ 2n_x n_y \\ 2n_x n_z \\ 2n_y n_z \end{bmatrix} \quad \text{and} \quad \boldsymbol{\sigma}(t) = \begin{bmatrix} \sigma_{xx}(t) \\ \sigma_{yy}(t) \\ \sigma_{zz}(t) \\ \tau_{xy}(t) \\ \sigma_{xz}(t) \\ \sigma_{yz}(t) \end{bmatrix} \quad (3)$$

2.2 Shear stress

According to the maximum critical shear stress criterion, fatigue failure is influenced by the shear stress present in the critical plane. The Maximum Variance Method (MVM) determines the critical plane as the plane where the material encounters the greatest magnitude of shear stress τ_α along the direction of maximum variance (Susmel, 2009, 2010; Ferreira *et al.*, 2022b). To determine the stress in the direction \mathbf{q} of maximum shear stress amplitude $\tau_q(t)$, given by Eq. (5), it is necessary to project the stress state in this specific direction using the following approach:

$$\mathbf{e}_a = \begin{bmatrix} \sin \phi \\ -\cos \phi \\ 0 \end{bmatrix} \quad \mathbf{e}_b = \begin{bmatrix} \cos \theta \cos \phi \\ \cos \theta \sin \phi \\ -\sin \theta \end{bmatrix} \quad \mathbf{q} = \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \begin{bmatrix} \cos \alpha \sin \phi + \sin \alpha \cos \theta \cos \phi \\ -\cos \alpha \cos \phi + \sin \alpha \cos \theta \sin \phi \\ -\sin \alpha \sin \theta \end{bmatrix} \quad (4)$$

$$\tau_q(t) = \begin{bmatrix} q_x & q_y & q_z \end{bmatrix} \begin{bmatrix} \sigma_{xx}(t) & \tau_{xy}(t) & \tau_{xz}(t) \\ \tau_{yx}(t) & \sigma_{yy}(t) & \tau_{yz}(t) \\ \tau_{zx}(t) & \tau_{zy}(t) & \sigma_{zz}(t) \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \sum_{k=1}^6 d_k^q \sigma_k(t) \quad (k = 1, \dots, 6) \quad (5)$$

where:

$$\mathbf{d}^q = \begin{bmatrix} d_1^q \\ d_2^q \\ d_3^q \\ d_4^q \\ d_5^q \\ d_6^q \end{bmatrix} = \begin{bmatrix} n_x q_x \\ n_y q_y \\ n_z q_z \\ n_x q_y + n_y q_x \\ n_x q_z + n_z q_x \\ n_y q_z + n_z q_y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \left[\sin(\theta) \sin(2\phi) \cos(\alpha) + \sin(\alpha) \sin(2\theta) \cos(\phi)^2 \right] \\ \frac{1}{2} \left[-\sin(\theta) \sin(2\phi) \cos(\alpha) + \sin(\alpha) \sin(2\theta) \sin(\phi)^2 \right] \\ -\frac{1}{2} \sin(\alpha) \sin(2\theta) \\ \frac{1}{2} \sin(\alpha) \sin(2\phi) \sin(2\theta) - \cos(\alpha) \cos(2\phi) \sin(\theta) \\ \sin(\alpha) \cos(\phi) \cos(2\theta) + \cos(\alpha) \sin(\phi) \cos(\theta) \\ \sin(\alpha) \sin(\phi) \cos(2\theta) - \cos(\alpha) \cos(\phi) \cos(\theta) \end{bmatrix} \quad (6)$$

2.3 Application of MVM

Applying the method of maximum variance for the normal stress σ_n , given by Eq. (7), and the shear stress resolved in the direction of maximum variance τ_q , given by Eq. (8), we obtain:

$$Var[\sigma_n(t)] = Var \left[\sum_{k=1}^6 d_k^n \sigma_k(t) \right] = \sum_{k=1}^6 \sum_{l=1}^6 d_k^n d_l^n Cov[\sigma_k(t), \sigma_l(t)] = (\mathbf{d}^n)^T [C] \mathbf{d}^n \quad (7)$$

$$Var[\tau_q(t)] = Var \left[\sum_{k=1}^6 d_k^q \sigma_k(t) \right] = \sum_{k=1}^6 \sum_{l=1}^6 d_k^q d_l^q Cov[\sigma_k(t), \sigma_l(t)] = (\mathbf{d}^q)^T [C] \mathbf{d}^q \quad (8)$$

where the covariance matrix is given from the variances V_k and covariances $C_{k,l}$ of the stress tensor components:

$$[C] = \begin{bmatrix} V_x & C_{xx,yy} & C_{xx,zz} & C_{xx,xy} & C_{xx,xz} & C_{xx,yz} \\ C_{yy,xx} & V_y & C_{yy,zz} & C_{yy,xy} & C_{yy,xz} & C_{yy,yz} \\ C_{zz,xx} & C_{zz,yy} & V_z & C_{zz,xy} & C_{zz,xz} & C_{zz,yz} \\ C_{xy,xx} & C_{xy,yy} & C_{xy,zz} & V_{xy} & C_{xy,xz} & C_{xy,yz} \\ C_{xz,xx} & C_{xz,yy} & C_{xz,zz} & C_{xz,xy} & V_{xz} & C_{xz,yz} \\ C_{yz,xx} & C_{yz,yy} & C_{yz,zz} & C_{yz,xy} & C_{yz,xz} & V_{yz} \end{bmatrix} \quad Cov[\sigma_k(t), \sigma_l(t)] = V_k = Var[\sigma_k] \text{ for } k = l \quad (9)$$

2.4 MVM in frequency domain

The stress state tensor can be described by a matrix of power spectral density (PSD) functions, $\mathbf{G}(f)$ when converted to time domain, as shown in Eq. (10) (Łagoda *et al.*, 2005a, 2019; Niesłony and Macha, 2007).

$$\mathbf{G}(f) = \begin{bmatrix} G_{11}(f) & G_{12}(f) & G_{13}(f) & G_{14}(f) & G_{15}(f) & G_{16}(f) \\ G_{21}(f) & G_{22}(f) & G_{23}(f) & G_{24}(f) & G_{25}(f) & G_{26}(f) \\ G_{31}(f) & G_{32}(f) & G_{33}(f) & G_{34}(f) & G_{35}(f) & G_{36}(f) \\ G_{41}(f) & G_{42}(f) & G_{43}(f) & G_{44}(f) & G_{45}(f) & G_{46}(f) \\ G_{51}(f) & G_{52}(f) & G_{53}(f) & G_{54}(f) & G_{55}(f) & G_{56}(f) \\ G_{61}(f) & G_{62}(f) & G_{63}(f) & G_{64}(f) & G_{65}(f) & G_{66}(f) \end{bmatrix} \quad (10)$$

where G_{kl} is components of matrix of power spectral density functions or one-side power spectral density function.

The application of the MVM in the frequency domain occurs in an analogous way to the MVM in the time domain, with the identification of the maximum value of the variance of the power density function of the equivalent stress (Łagoda *et al.*, 2005a, 2019; Niesłony and Macha, 2007; Lima, 2021):

$$Var[G_{eq}(f)] = \sum_k^6 \sum_l^6 d_k^n d_l^n Var(G_{kl}) \quad \text{where} \quad Var(G_{kl}) = \Re \left[\int_0^\infty G_{kl}(f) df \right] \quad (k,l=1,\dots,6) \quad (11)$$

Other parameters of interest for the realization of the fatigue lifetime in the frequency domain are the spectral moments Eq. (12) (Łagoda *et al.*, 2005a; Niesłony and Macha, 2007; Mršnik *et al.*, 2013; Ferreira *et al.*, 2022a).

$$m_o = Var(G_{eq}) \quad m_2 = \Re \left[\int_0^\infty f^2 G_{eq}(f) df \right] \quad m_4 = \Re \left[\int_0^\infty f^4 G_{eq}(f) df \right] \quad (12)$$

The expected number of oscillations through the zero level with a positive slope in a time unit (also known as the crossover rate), the number of local peaks, the coefficient of irregularity and parameter of spectrum width are determined respectively by Eq. (13), Eq. (14), Eq. (15) and Eq. (16) (Łagoda *et al.*, 2005a; Niesłony and Macha, 2007; Mršnik *et al.*, 2013; Lima, 2021; Ferreira *et al.*, 2022b,a):

$$N_o^+ = \sqrt{\frac{m_2}{m_0}} \quad (13)$$

$$M^+ = \sqrt{\frac{m_4}{m_2}} \quad (14)$$

$$\alpha_f = \frac{N_o^+}{M^+} \quad (15)$$

$$\zeta = \sqrt{1 - \alpha_f^2} \quad (16)$$

3. NUMERICAL APPROXIMATION

Optimization algorithms play a fundamental role in several areas of science and engineering, allowing the optimization of mathematical solutions found for complex problems. These algorithms are widely used in various fields, such as machine learning, process optimization, and mathematical modeling. In this work, these algorithms are used to optimize the critical planes found during the plan search by MVM. Among the methods used, we have:

- Gradient ascent (GA) method:

$$u_{n+1} = u_n + \kappa \nabla f(\theta^n, \phi^n, \alpha^n) \quad \text{where} \quad d^n = \nabla f(\theta^n, \phi^n, \alpha^n) \quad (17)$$

$$u_{n+1} = \begin{bmatrix} \theta^{n+1} \\ \phi^{n+1} \\ \alpha^{n+1} \end{bmatrix} \quad \text{and} \quad u_n = \begin{bmatrix} \theta^n \\ \phi^n \\ \alpha^n \end{bmatrix} \quad (18)$$

- Gradient ascent with momentum (GA+momentum) method:

$$u^n = u^{n-1} + \kappa \nabla f(\theta^{n-1}, \phi^{n-1}, \alpha^{n-1}) \quad \text{where} \quad d^{n-1} = \nabla f(\theta^{n-1}, \phi^{n-1}, \alpha^{n-1}) \quad (19)$$

$$u^{n+1} = u^n + \kappa \nabla f(\theta^n, \phi^n, \alpha^n) + \gamma(u^n - u^{n-1}) \quad \text{where} \quad d^n = \nabla f(\theta^n, \phi^n, \alpha^n) \quad (20)$$

$$u_{n+1} = \begin{bmatrix} \theta^{n+1} \\ \phi^{n+1} \\ \alpha^{n+1} \end{bmatrix} \quad \text{and} \quad u_n = \begin{bmatrix} \theta^n \\ \phi^n \\ \alpha^n \end{bmatrix} \quad \text{and} \quad u_{n-1} = \begin{bmatrix} \theta^{n-1} \\ \phi^{n-1} \\ \alpha^{n-1} \end{bmatrix} \quad (21)$$

- Conjugate gradient (CG) method:

$$u^{n+1} = u^n + \kappa \nabla f(\theta^n, \phi^n, \alpha^n) \quad \text{where} \quad d^n = \nabla f(\theta^n, \phi^n, \alpha^n) \quad (22)$$

$$d^{n+1} = \nabla f(\theta^{n+1}, \phi^{n+1}, \alpha^{n+1}) + \left[\frac{\|\nabla f(\theta^{n+1}, \phi^{n+1}, \alpha^{n+1})\|}{\|\nabla f(\theta^n, \phi^n, \alpha^n)\|} \right]^2 \cdot d^n \quad (23)$$

$$u^{n+2} = u^{n+1} + \kappa d^{n+1} \quad (24)$$

$$u_{n+2} = \begin{bmatrix} \theta^{n+2} \\ \phi^{n+2} \\ \alpha^{n+2} \end{bmatrix} \quad \text{and} \quad u_{n+1} = \begin{bmatrix} \theta^{n+1} \\ \phi^{n+1} \\ \alpha^{n+1} \end{bmatrix} \quad \text{and} \quad u_n = \begin{bmatrix} \theta^n \\ \phi^n \\ \alpha^n \end{bmatrix} \quad (25)$$

where κ is the increment rate of gradient ascent method (equivalent to adopted numerical tolerance), n denotes the solution calculated in the n th step, $n+1$ for the subsequent step of n and $n+2$ for the subsequent step of n to be taken in the direction proportional to the gradient of a function. And, the stopping criterion considered in this approach was $\|\nabla f(\theta, \phi)\| \leq \varepsilon$ or $\|\nabla f(\theta, \phi, \alpha)\| \leq \varepsilon$.

To simulate, a numerical tolerance of $\varepsilon = 10^{-4}$, a gradient ascending method increment rate equal to $\kappa = 10^{-6}$ and a usual value of $\gamma = 0.9$ to the moment rate of the gradient ascent method were used. Generally, values between 0 and 1 are used for the moment rate of the gradient ascent method (Susmel, 2010; Snyman, 2005).

4. DAMAGE MODEL

With regard to structural integrity, the estimation of accumulated damage in the time domain is performed by application of the cycle counting method, specifically the Palmgren-Miner damage accumulation hypothesis, given by Eq. (26). According to this hypothesis, fatigue failure occurs when the accumulated damage reaches a critical value given by a material damage parameter D_{crit} . In other words, a constant D_{crit} represents the sensitivity of the mechanical component to accumulated damage for fatigue life for failure.

$$D(T_o) = \sum_{j=1}^m \frac{n_j}{N_j} = \sum_{j=1}^m d_j \quad (26)$$

The fatigue life prediction and the accumulated material damage obtained by the numerical observation time can be associated with the fatigue life prediction and the critical damage for fatigue failure. The relationship between life predictions can be established using the Eq. (27):

$$\frac{T_{RF}}{D_{crit}} = \frac{T_o}{D(T_o)} \implies T_{RF} = \frac{T_o D_{crit}}{D(T_o)} \quad (27)$$

Another more adequate way of evaluating damage accumulation in stresses with variable amplitude would be the Serensen-Kogayev hypothesis. From the Serensen-Kogayev correction, a new fatigue life equation was obtained in Eq. (28) (Łagoda *et al.*, 2005a; Niesłony and Macha, 2007; Karolczuk and Blacha, 2011; Lima, 2021; Ferreira *et al.*, 2022b):

$$T_{RF} = D_{crit} \left(\frac{T_o b_{SK} A}{\sum_{i=1}^j n_i \sigma_{ai}^m} \right) \quad \text{for} \quad \sigma_{ai} \geq a_{SK} \sigma_{af} \quad (28)$$

where b_{SK} is the Serensen-Kogayev coefficient. Being calculated by:

$$b_{SK} = \frac{\sum_{i=1}^j \sigma_{ai} t_i - a_{SK} \sigma_{af}}{\sigma_{a,max} - a_{SK} \sigma_{af}} \quad (29)$$

where t_i is a frequency of occurrence for observation time T_o , A and m are characteristic parameters of the S-N curve and a_{SK} is a coefficient for include amplitudes below the fatigue limit. The value of 0.5 was adopted for a_{SK} in the references (Łagoda *et al.*, 2005a; Niesłony and Macha, 2007). The σ_{af} and τ_{af} are respectively fatigue limits for bending and torsion, $\sigma_{a,max}$ is maximum stress amplitude for a loading block of bending, and $\tau_{a,max}$ is maximum stress amplitude for a loading block of torsion.

$$t_i = \frac{n_i}{\sum_{i=1}^k n_i} \quad \sigma_{amax} = 3,73 \sqrt{Var[\sigma_n(t)]} \quad \text{or} \quad 3,73 \sqrt{Var[\sigma_{eq}(t)]} \quad \tau_{amax} = 3,73 \sqrt{Var[\tau_q(t)]} \quad (30)$$

To estimate fatigue life using the spectral method, an essential first step is to make an assumption about the characteristics of the stress spectrum, such as whether it is wideband or narrowband (Łagoda *et al.*, 2005a; Niesłony and Macha,

2007; Lima, 2021; Ferreira *et al.*, 2022b). In the context of the spectral method, the probability density function of stress amplitude is often modeled using a Rayleigh distribution. By incorporating the Rayleigh distribution into the fatigue life estimation process, engineers and researchers can analyze the statistical properties of stress amplitudes and make predictions about the expected fatigue life of a component or structure. Thus, the accumulated damage estimate by the spectral method can be calculated as follows:

$$D(T_o) = \frac{N_{T_o}(2m_0)^{m/2}}{A} \Gamma\left(\frac{m+2}{2}\right) = \frac{M+T_o}{A} (2m_0)^{m/2} \Gamma\left(\frac{m+2}{2}\right) \quad (31)$$

The life prediction by spectral analysis T_{SP} can be calculated by Eq. (32) and Eq. (33) with Serensen-Kogayev correction (Łagoda *et al.*, 2005a; Niesłony and Macha, 2007; Lima, 2021; Ferreira *et al.*, 2022b):

$$T_{SP} = \frac{D_{crit}A}{M+(2m_0)^{m/2} \Gamma\left(\frac{m+2}{2}\right)} \quad (32)$$

$$T_{SP} = \frac{D_{crit}b_{SK}A}{M+(2m_0)^{m/2} \Gamma\left(\frac{m+2}{2}, \frac{a_{SK}^2\sigma_{af}^2}{2m_0}\right)} \quad (33)$$

where Serensen–Kogayev coefficient in the spectral approach b_{SK} is calculated in the frequency domain from the incomplete gamma function given by $\Gamma(w_1, w_2) = \int_{w_2}^{\infty} e^{-t} t^{w_1-1} dt$:

$$b_{SK} = \frac{\sqrt{2m_0} \Gamma\left(\frac{3}{2}, \frac{a_{SK}^2\sigma_{af}^2}{2m_0}\right) \exp\left(\frac{a_{SK}^2\sigma_{af}^2}{2m_0}\right) - a_{SK}\sigma_{af}}{\sigma_{a,max} - a_{SK}\sigma_{af}} \quad (34)$$

The maximum amplitude of normal and shear stress for a loading block is calculated respectively by $\sigma_{a,max} = 3,73\sqrt{m_0(\sigma_n(t) \text{ or } \sigma_{eq}(t))}$ and $\tau_{a,max} = 3,73\sqrt{m_0(\tau_q(t))}$.

5. RESULTS AND COMPARISONS OF METHODOLOGIES WITH AVAILABLE DATA IN THE LITERATURE

The methodologies developed for critical plane identification generally have low computational cost and have signal reconstruction as a starting point. Through a Gaussian simulation process, it is possible to recover an approximate signal in the time domain that characterizes the real loading that occurred during a mechanical test from the spectral density function. For numerical simulation, the sampling frequency was 500 Hz, and the observation time was 800 s. The time discretization was 0.002 s, and the frequency discretization was approximately 0.122 Hz.

Numerical results were obtained based on the mechanical properties and dimensions of specimens in experimental multiaxial fatigue tests with 18G2A steel (Łagoda *et al.*, 2005a; Niesłony and Macha, 2007; Ferreira *et al.*, 2022a), a type of structural steel used for civil construction and welded structures.

In addition to normal and shear stress, another driving force was tested based on the modification of the maximum normal stress criterion from the equivalent stress σ_{eq} projected by the critical plane, calculated by Eq. (35) (Łagoda *et al.*, 2005a; Niesłony and Macha, 2007).

$$\sigma_{eq}(t) = \sigma_{xx}(t) n_x^2 + 2 n_x n_y \left(\frac{\sigma_{af}}{\tau_{af}}\right) \tau_{xy}(t) \quad (35)$$

And finally, decoupled damage model approaches are applied for numerical life prediction. In the time domain, the cycles are counted and identified by the classic Rainflow method. Later, the Palmgren-Miner damage accumulation rule is used to account for the accumulated damage and determine the numerical prediction of the material's life, also allowing the use of the Serensen-Kogayev correction (Łagoda *et al.*, 2005a; Niesłony and Macha, 2007). For shear stresses, the damage model needs to be calibrated by the Modified Wöhler Curves Method (MWCM) before applying the damage accumulation rule. In the frequency domain, cumulative damage by spectral analysis is performed using the Rayleigh distribution in the Palmgren-Miner damage accumulation rule, which can also be combined with the Serensen-Kogayev (Łagoda *et al.*, 2005a; Niesłony and Macha, 2007) correction. The spectral properties of the signal, such as spectral moments and crossover rates, are obtained to calculate cumulative damage. The damage parameter should be modified to 0.3 based on a set of experimental data obtained for bending and torsion in refs. (Niesłony and Macha, 2007; Ferreira *et al.*, 2022a). The fatigue life predictions in the time and frequency domains were compared with the experimental life from Fig. 2. The abbreviation “RF” represents the numerical prediction of life obtained by the time-domain damage model, while the abbreviation “SP” in Fig. 2 represents the prediction numerical value of life obtained by the frequency domain damage model.

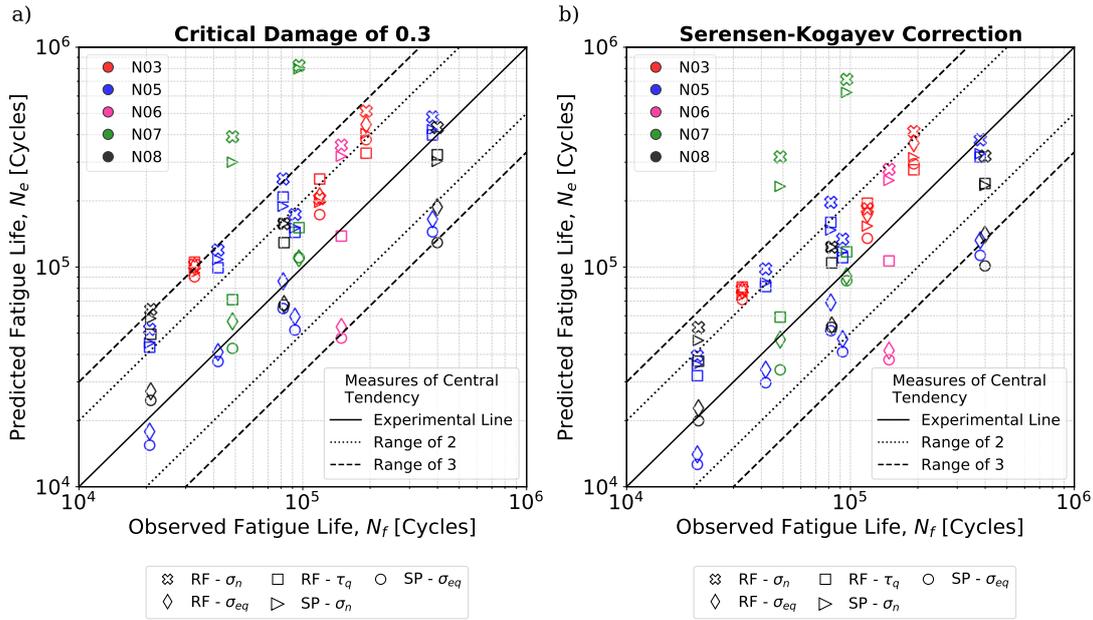


Figure 2: Comparison between the lives observed in the fatigue tests performed with 18G2A steel and the numerical estimates (a) by the fatigue life predictions obtained for damage parameters equal to 0.3 and (b) by the fatigue life predictions obtained by Serensen-Kogayev correction.

Processing times are a measure of time that reflects the computational cost of a numerical simulation. In order to evaluate the computational efficiency of the algorithm, processing times were recorded for different cases: without correction (N03), with unitary correction (N05 and N06) and with correction equal to 0.5 (N07 and N08). Fig. 3 presents the average value of the computational times of the gradient ascent method (GA method), gradient ascent method with momentum (GA+Momentum method), and conjugate gradient method (CG method). The simulations were performed on a machine with an Intel Processor Core i7-8750H 8th Generation and with 24 GB of RAM memory (8GB DDR4 2666 MHz of factory and plus 16 GB of expansion).

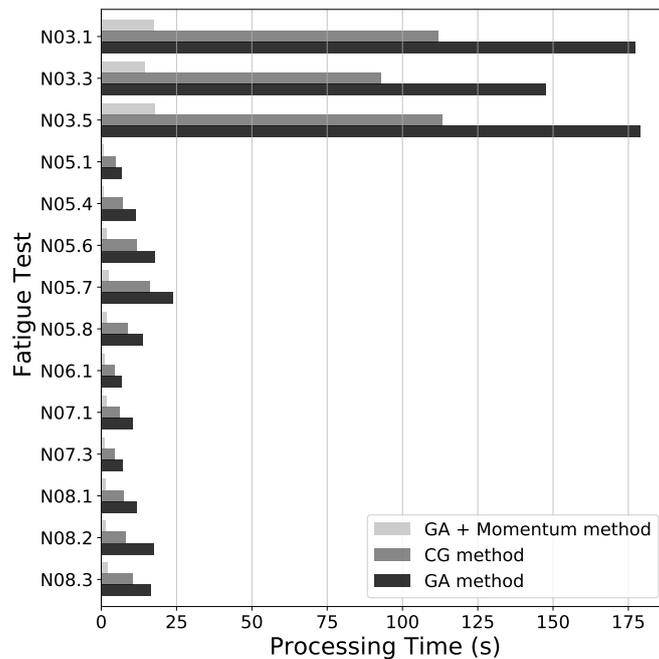


Figure 3: Comparison of computational cost between gradient methods.

The proposed algorithms based on the MVM showed good efficiency and quickly in identifying the orientation of the material plane that is subjected to the maximum variance of a given driving force, presented in Fig. 1. Among its advantages we have that: the critical planes can be determined directly by the variance and covariance of the loading

histories present in the covariance matrix $[C]$; provide a physical interpretation of the material plane of crack nucleation and the influence on the quantification of material damage; and can be easily adapted with optimization algorithms.

The signal reconstruction showed satisfactory results in which the largest error recorded between the variance of the original signal and the variance of the simulated signal was equal to 3.7% for one of the cases N07. Based on the results obtained in Fig. 2, it was observed that the fatigue life prediction model using the normal stress $\sigma_n(t)$ and shear stress τ_q showed better performance when applying the Serensen-Kogayev correction. In comparison, adjustments to the material damage parameters to 0.3 resulted in less satisfactory outcomes, particularly in the case of the normal stress, which exhibited a non-conservative behavior in this parameter adjustment strategy. The fatigue life prediction model based on equivalent stress yielded satisfactory results in both strategies. Certain frequency domain cases (N05.8, N06.1, and N08.3) exhibited a conservative behavior, exceeding the 3-factor band, which was expected due to the utilization of the Serensen-Kogayev correction. The predicted fatigue life models in the time domain for a specific driving force closely aligned with the corresponding frequency domain predictions, confirming the expected agreement between the two domains. This consistency between time and frequency approaches serves as an important validation in the numerical simulations.

6. CONCLUSIONS

Computers have become indispensable tools for engineers and researchers in various stages of the development process, from mechanical component design to numerical simulations for finite element analysis and fatigue life prediction. Based on the results obtained in this study, it is expected to serve as a solid foundation for future research and improvements in multiaxial fatigue life prediction methodologies. After analyzing and discussing the results, some important conclusions can be deduced from this study:

- Regarding the prediction of life by the damage models, the spectral approach did not show non-conservative results. Most of the results were within the band of 3 and some cases had predictions below the experimental life, which indicates a conservative behavior, as shown in Fig. 2
- In a general way, Serensen-Kogayev correction presented more results within band 3 than the Palmgren-Miner damage accumulation rule with correction of the damage parameter to 0.3, which favors its use mainly in simulated signals like the ones in this work.
- As shown in Fig. 3, gradient ascent method with momentum showed better computational efficiency compared to the other two gradient methods (gradient ascent method and conjugate gradient method). An improvement in the speed of the search process is observed with the inclusion of an impulse term in the iterative process.

7. ACKNOWLEDGEMENTS & FUNDING

Acknowledgements: The authors would like to acknowledge the laboratory support of the University of Brasilia (UnB) and the Master's Program of Mechanical Sciences (PCMEC-UnB).

Funding: The authors would also like to acknowledge the financial support of Transmissoras Brasileiras de Energia (TBE) for the project entitled "Machine Learning and Multiaxial Fatigue for Overhead Cables: Manufacturing of Device and Development of Android Application for Monitoring and Calculation of Remaining Life of Transmission Lines". This project has been funded in the context of the Research and Development Program of the Brazilian National Agency of Electric Energy (ANEEL). J. L. A. Ferreira, J. A. Araújo, and C. R. M. Silva (contracts 307122/2021-2, 311607/2021-7, and 307769/2022-4) are also grateful for the support provided by CNPq. The administrative and financial support of FINATEC is also acknowledged.

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