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COMPARATIVE ANALYSIS OF THE PERFORMANCE OF TMD AND KADAMPER VIBRATION ATTENUATORS

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Abstract. *Mechanical structures are commonly exposed to dynamic loads. The action of such loads, recurrently, can promote unwanted occupational effects in users and operators of industrial equipment and means of transportation, as well as in certain circumstances, can cause catastrophic structural failures. Currently, there has been an intensification of technical and scientific studies that aim to implement Dynamic Vibration Absorbers (DVA) for dynamic loads, which are generally classified into active and passive absorbers, according to the use or not of external energy, respectively, for their operation. The scope and general objective of this work is the comparative analysis of levels of vibration attenuation that two different systems, TMD (Tunned Mass Damper) and KDamper, provide in the structures to which they are coupled. TMD systems are widely used to attenuate vibrations in civil and mechanical structures, as well as targets for parametric optimization studies, where the best sets of stiffnesses and dampings are sought to minimize dynamic responses. Damping systems of the KDamper type however, have been relatively little explored compared to conventional damping systems, such as Tunned Mass Dampers. The use of a negative stiffness elastic element in the KDamper allows the inertial components used to attenuate the dynamic responses to reach higher kinetic energies than those that would be reached in TMD systems. Thus, providing greater dissipation of mechanical energies from vibrations and consequently greater attenuation of dynamic loads. The specific objectives of this work is the presentation of the main advantages and disadvantages using the two DVAs TMD and KDamper, by means of graphs and numerical comparisons, transfer functions and dynamic responses obtained by computational modeling. To evaluate the damping levels provided, both systems were coupled to a dynamic system and subjected to forced vibration loads, and their performances were compared. By the analysis of the dynamic response, it was concluded that the KDamper presented a performance superior to the TMD, having reduced the rms acceleration compared to the value obtained for the structure without any vibration absorber. Thus, the study presented evidence of the good behavior of the KDamper compared to the TMD even in a wide band around the resonance frequency used for the design of vibration absorber and in the case of a simple structure due to its intrinsic negative stiffness feature.*

Keywords: *KDamper, TMD, Passive Vibration Attenuation, Negative Stiffness*

1. INTRODUCTION

Vibrations with significant magnitudes in structures (civil, mechanical, naval, aeronautical, among others) pose risks of system failure due to fatigue, especially when the structures oscillate at a frequency close to the fundamental frequency of their components. One of the methods often adopted to mitigate the structural phenomena resulting from vibrations is the use of passive and active attenuation systems. One of the most commonly used systems is the Tuned Mass Damper (TMD). However, TMDs have limitations due to the requirement of installing relatively large inertias. Researchers have proposed modifications in the component arrangement of such dynamic vibration absorbers, as well as new systems, which eliminate the need for high inertias. Examples of such systems include the TMDi implemented by Smith (2002) and the KDamper implemented by Antoniadis et al. (2018).

The springs in the TMD provide the necessary restoring force to the system to counterbalance the excitation forces of the structure. The elastic potential energy stored in the compressed springs during vibratory oscillations is released as the mass oscillates in response to the movement of the structure. The springs, therefore, work to restore the mass to its equilibrium position, counteracting the vibrations and reducing their amplitude. The dampers, in turn, are responsible for dissipating the vibrational energy by converting it into heat. By dissipating this energy in the form of heat, the dampers help control the vibrations and reduce the detrimental effects they can have on the structure. The use of tuned mass is crucial for the performance of the TMD. By adjusting the mass to resonate at the same frequency as the structure, it is possible to achieve an optimized response from the system. This means that when an unwanted vibration occurs, the mass of the TMD oscillates in opposite phase to the movement of the structure, resulting in a significant reduction in vibration amplitude.

The KDamper system can perform better in attenuating dynamic loads, making structures safer. The KDamper system is a technology that utilizes a negative stiffness component in conjunction with a Tuned Mass Damper (TMD) device to improve the isolation and damping of structures. According to the study conducted by Sapountzakis (2017), KDampers offer superior isolation and damping capabilities compared to conventional TMDs. Furthermore, they provide a variety of applications that can be explored due to the nonlinear behavior of the negative stiffness element. Kostantinos (2021) believes that incorporating the nonlinear behavior of the inserted elastic elements can add greater robustness to structures. This means that the KDamper system can perform better in attenuating dynamic loads, making structures safer and more resilient.

The objective of this work is to perform a comparison between the vibration attenuation systems TMD (Tuned Mass Damper) and KDamper. Graphs and numerical comparisons of the transfer functions and dynamic responses of both systems will be used to analyze their characteristics and relative performances.

2. METHODOLOGY

The analyzed Tuned Mass Damper system, as shown in Figure 1, is composed of the following elements: mass, spring, and damper, with each of them present in one component. Thus, the considered TMD is of the conventional type, which is one of the four types of TMDs found in the literature (conventional TMD, bidirectional and homogeneous TMD, pendulum TMD, and tuned liquid column dampers).

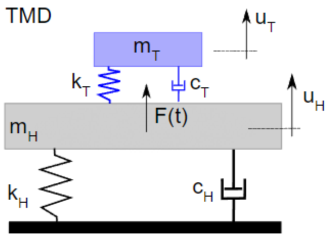


Figure 1. Schematic of a structure with TMD, Adapted from Sapountzakis (2017).

The KDamper system, similar to the TMD, does not have active components in its composition, as shown in Figure 2. This system, KDamper, is composed of a damping assembly that includes a negative stiffness element, in addition to the positive stiffness spring present in the TMD. This negative stiffness elastic component is grounded on the anchorage surface of the vibrating structure. The composition of the KDamper does not require any reduction in the stiffness of the structure in which the objective is to minimize the dynamic response, while providing more significant vibrational attenuation than TMD systems.

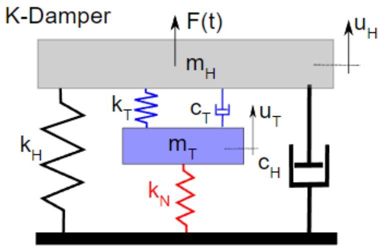


Figure 2. Schematic of a structure with KDamper, Adapted from Sapountzakis (2017).

2.1 Numerical Modeling of the Tuned Mass Damper (TMD)

In the system illustrated in Figure 1, with 2 degrees of freedom, the primary system (H) is coupled to the secondary system (T) to reduce vibration. An exciting force acts on the primary system, generating a dynamic response. By analyzing a 2-degree-of-freedom system and considering the equilibrium equations of motion one arrives at Equations 1 and 2. These equations describe the behavior of the system and provide insight into its response to external forces when in a balanced state.

$$m_T\ddot{u}_T(t) - k_T[u_H(t) - u_T(t)] - c_T[\dot{u}_H(t) - \dot{u}_T(t)] = 0 \qquad (1)$$

$$m_H\ddot{u}_H(t) + k_H u_H(t) + c_H\dot{u}_H(t) + c_T[\dot{u}_H(t) - \dot{u}_T(t)] + k_T[u_H(t) - u_T(t)] = F(t), \tag{2}$$

where the indices of the variables are defined with the letter H for those related to the primary system, and the letter T for those related to the secondary system, which is the TMD attenuator. The variables m, k and c correspond to the mechanical elements of the systems, representing mass, stiffness, and damping, respectively. Meanwhile, the variables u, $\dot{u}$ e $\ddot{u}$ represent the dynamic responses of the system, corresponding to displacement, velocity, and acceleration, respectively. Using harmonic excitation in the structure ($\mathrm{F(t)} = \mathrm{F_H e^{i\omega t}}$), the characteristic solution of the system is of periodic nature: $\mathrm{u_T(t)} = \mathrm{U_T e^{i\omega t}}$ and $\mathrm{u_T(t)} = \mathrm{U_T e^{i\omega t}}$. Thus, substituting into the previous equations, one obtains:

$$[-i\omega c_T - k_T]U_H + [-\omega^2 m_T + i\omega c_T + k_T]U_T = 0 \tag{3}$$
$$[-\omega^2 m_H + i\omega(c_H + c_T) + (k_H + k_T)]U_H + [-i\omega c_T - k_T]U_T = F_H \tag{4}$$

Some dimensionless variables are created, as indicated in Table 1, to facilitate the manipulation of the previous equations.

Table 1. Variables for simplification of equation manipulation.

$\gamma = \dfrac{m_T}{m_H}$	Mass ratio (mass of the DVA to the secondary system mass) (adm)
$\omega_H = \sqrt{k_H/m_H}$	Undamped modal frequency of the primary system (rad/s)
$\Omega = \omega/\omega_H$	Frequency ratio (Excitation frequency to structure mass) (adm)
ζ_H	Damping ration for the structure (adm)
$\omega_T = \sqrt{k_T/m_T}$	Undamped modal frequency for the TMD (rad/s)
$\beta = \omega_T/\omega_H$	Frequency ratio (undamped TMD modal frequency to structure undamped modal frequency) (adm)
ζ_T	Damping ratio for the TMD (adm)
$U_{H0} = F_H/k_H$	Static displacement of the structure (m)

To evaluate the absolute value of the final complex equation and obtain Equation (5), which defines the ratio of the displacement $\mathrm{U_H}$ of the structure to the static displacement $\mathrm{U_{H0}} = \mathrm{F_H/k_H}$ of the structure when there are no dynamic load actions, also known as the dynamic amplification function for structural displacements.

$$\left|\frac{U_H}{U_{H0}}\right| = \sqrt{\frac{(\beta^2 - \Omega^2)^2 + (2\Omega\beta\zeta_T)^2}{[(\beta^2 - \Omega^2) - \Omega^2\beta^2(1+\gamma) + \Omega^2(\Omega^2 - 4\beta\zeta_H\zeta_T)]^2 + 4[(\beta^2 - \Omega^2)\zeta_H + (1 - \Omega^2 - \Omega^2\gamma)\beta\zeta_T]^2}} \tag{5}$$

The optimal frequency of the TMD (as obtained by Den Hartog, 1956) can also be defined in Equation (6), according to the frequency of the structure and it is related to the mass ratio (γ) as.

$$f_{T,opt} = \frac{f_H}{1 + m_T/m_H} = \frac{f_H}{1+\gamma} \tag{6}$$

By graphically observing the displacement equations of the structure, it is possible to determine the factors that optimize the equation in terms of TMD damping, aiming to minimize the dynamic amplification function of Equation (7), resulting in:

$$\zeta_{T,opt} = \sqrt{\frac{\dfrac{3m_T}{m_H}}{8(1+\dfrac{m_T}{m_H})^3}} = \sqrt{\frac{3\gamma}{8(1+\gamma)^3}} \tag{7}$$

In Equation 7, it is possible to observe how the ratios between the mass of the TMD and the mass of the structure influence the optimal damping to be chosen for the TMD. In optimized situations, the ratio between them (γ) should be around 3 to 5%. Since a higher ratio results in greater attenuation achieved, this value should be limited by the extra mass and efforts to be added to the structure, as well as the maximum amplitude that the TMD can oscillate.

2.2 Numerical Modeling of the KDamper

Consider the system with 2 degrees of freedom, as indicated in Figure 2. Similar to the system indicated in Figure 1, the KDamper has a secondary system (T) coupled to a primary system (H) to attenuate the existing vibration. Similar to the TMD, the excitation force acts on the primary system as indicated. By analyzing a 2-degree-of-freedom system and considering the equations of motion for a system in equilibrium, we can arrive at the following Equations 8 and 9.

$$m_T\ddot{u}_T(t) + k_N u_T(t) - k_T[u_H(t) - u_T(t)] - c_T[\dot{u}_H(t) - \dot{u}_T(t)] = 0 \quad (8)$$
$$m_H\ddot{u}_H(t) + k_H u_H(t) + k_T[u_H(t) - u_T(t)] + c_T[\dot{u}_H(t) - \dot{u}_T(t)] = F(t), \quad (9)$$

where the indices of the variables are denoted by the letter H when related to the primary system, and index T when related to the KDamper device (secondary). Here, k_N represents negative stiffness ($k_N < 0$). In order to maintain the final static stiffness of the primary structure, it can be imposed that the equivalent final stiffness is preserved, such that:

$$k_{eq} = k_H + \frac{k_N k_T}{k_N + k_T}, \quad (10)$$

Using a harmonic excitation acting on the primary structure ($F(t) = F_H e^{i\omega t}$), the final solution of the system is also periodic in nature: $u_T(t) = U_T e^{i\omega t}$and $u_H(t) = U_H e^{i\omega t}$. Thus, by substituting the periodic excitation and responses into the previous equations, and recalling that $F_H = k_{eq}U_{H0}$, we arrive at:

$$[-i\omega c_T - k_T]U_H + [-\omega^2 m_T + i\omega c_T + k_T]U_T = 0 \quad (11)$$
$$[-\omega^2 m_H + i\omega c_T + (k_H + k_T)]U_H + [-i\omega c_T - k_T]U_T = F_H = k_{eq}U_{H0} \quad (12)$$

By solving Equation (11) for U_T and substituting into Equation (12), and performing some procedures to evaluate the absolute value of the final complex equation, as well as using the variables for simplification of the manipulation of the equations presented in Table 1, one arrives at the following Equation (13), which defines the ratio of displacement U_H of the structure to the static displacement ($U_{H0} = F_H/k_H$) of the primary structure without KDamper and without external dynamic load actions. This ratio is also referred to as the dynamic amplification function for the structure displacements.

$$\left|\frac{U_H}{U_{H0}}\right| = \sqrt{\frac{(\beta^2 - \Omega^2)^2 + (2\Omega\beta\zeta_T)^2}{[\Omega^2 - \Omega^2[1 + \beta^2 + (1+k)^2]\gamma\beta^2] + \beta^2 + (2\beta\zeta_T)^2\beta\Omega[(1 + \kappa^2\gamma\beta) - \Omega^2(1+\gamma)]}} \quad (13)$$

The transfer function indicated in Equation (12) exhibits two poles (peaks), resulting in two maximum values. According to Antoniadis et al. (2018), the optimal selection of KDamper parameters requires that both peaks be as small as possible and become equal to each other. As stated by Den Hartog (1956), this can be achieved by an optimization approximation. In the same paper, Antoniadis et al. (2018) presents this deduction and how the optimal parameters of KDamper can be choose. Once the mass ratio $\gamma = m_T/m_H$ and the relationship $\kappa = -k_N/k_T$ are defined, the optimal ratio $\beta_{otm} = \omega_T/\omega_H$ can be evaluated as indicated by Equation (13):

$$\beta_{otm} = \omega_T/\omega_H = \sqrt{\frac{1}{(1 + \gamma + \kappa\gamma)(1+\gamma) - \kappa^2\gamma}} \quad (14)$$

For the negative stiffness of KDamper, it can be chosen as $k_N = -\kappa\gamma\beta_{otm}^2 k_{eq}$, and for the usual spring stiffness of KDamper, one has $k_T = (1+\kappa)\beta_{otm}^2 k_{eq}$. Lastly, for the ideal damping ratio, it can be assumed as $\omega = \sqrt{\beta_{otm}}$.

2.3 Newmark Method for Solving Equations of Motion

For solving the system of time-differential equations, there are direct integration methods such as the Newmark method, the Houbolt method, and the Central Difference method, which are algorithms applied directly to the original system of equations. The Newmark method is the most efficient among implicit methods and is unconditionally stable. Using parameters $\delta = 1/2$ and $\alpha = 1/4$, it is also referred to as the Constant Average Acceleration method or the Trapezoidal Rule method (RAO, 2011).

Based on the assumption that acceleration varies linearly between two time instants, t and $t + \Delta t$, the method utilizes Equations (15) and (16) to determine the velocity and displacement vectors at the end of the interval $t + \Delta t$ for a system with multiple degrees of freedom (RAO, 2011).

$$\dot{\vec{x}}_{t+\Delta t} = \dot{\vec{x}}_t + [(1-\delta)\ddot{\vec{x}}_t + \delta\ddot{\vec{x}}_{t+\Delta t}]\Delta t \quad (15)$$

$$\vec{x}_{t+\Delta t} = \vec{x}_t + \Delta t \dot{\vec{x}}_t + [(0.5 - \alpha)\ddot{\vec{x}}_t + \alpha \ddot{\vec{x}}_{t+\Delta t}]\Delta t^2 \tag{16}$$

By expressing acceleration and velocity in terms of displacement and substituting them into the differential equation of motion, and solving for displacement, one arrives at Equation (19).

$$\vec{x}_{t+\Delta t} = \left[\frac{1}{\alpha(\Delta t)^2}\mathbf{M} + \frac{\delta}{\alpha\Delta t}\mathbf{C} + \mathbf{K}\right]^{-1}\left\{\vec{F}_{t+\Delta t} + \mathbf{M}\left(\frac{1}{\alpha(\Delta t)^2}\vec{x}_t + \frac{1}{\alpha\Delta t}\dot{\vec{x}}_t + \left(\frac{1}{2\alpha} - 1\right)\ddot{\vec{x}}_t\right) + \mathbf{C}\left(\frac{\delta}{\alpha\Delta t}\vec{x}_t + \left(\frac{\delta}{\alpha} - 1\right)\dot{\vec{x}}_t + \left(\frac{\delta}{\alpha} - 2\right)\frac{\Delta t}{2}\ddot{\vec{x}}_t\right)\right\} \tag{17}$$

One can summarize the application of Newmark's method as follows: with the known values of displacement ($\vec{x}_{t=0}$) and velocity ($\dot{\vec{x}}_{t=0}$) initially, i.e., when $t = 0$, the initial acceleration ($\ddot{\vec{x}}_{t=0}$) is determined using Equation (18). Then, with the appropriate values of the time step (Δt) and parameters α and δ, the displacement vector ($\vec{x}_{t+\Delta t}$) is calculated using Equation (17) and the respective mass, damping, and stiffness matrices. The calculation starts with the initial displacement at $t = 0$. Next, the acceleration ($\ddot{\vec{x}}_{t+\Delta t}$) and velocity ($\dot{\vec{x}}_{t+\Delta t}$) vectors are computed using Equations (19) and (20) provided below.

$$\ddot{\vec{x}}_{t=0} = \mathbf{M}^{-1}(\vec{F}_{t=0} - \mathbf{C}\,\dot{\vec{x}}_{t=0} - \mathbf{K}\,\vec{x}_{t=0}) \tag{18}$$

$$\ddot{\vec{x}}_{t+\Delta t} = \frac{1}{\alpha(\Delta t)^2}(\vec{x}_{t+\Delta t} - \vec{x}_t) - \frac{1}{\alpha\Delta t}\dot{\vec{x}}_t - \left(\frac{1}{2\alpha} - 1\right)\ddot{\vec{x}}_t \tag{19}$$

$$\dot{\vec{x}}_{t+\Delta t} = \dot{\vec{x}}_t + (1 - \delta)\Delta t\,\ddot{\vec{x}}_t + \delta\Delta t\,\ddot{\vec{x}}_{t+\Delta t} \tag{20}$$

In this work, a time step was adopted to ensure the accuracy of the numerical results $\Delta t = 1 \times 10^{-4}$s), obtained by gradually decreasing the time intervals' discretization until the numerical values of the Root Mean Square (RMS) did not change by at least 1%.

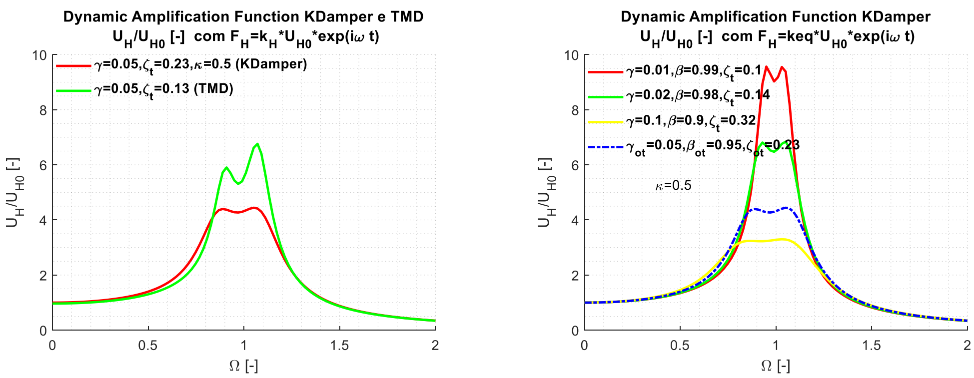
3. RESULTS AND DISCUSSIONS

This chapter will discuss the transmissibility and dynamic responses of TMD and KDamper systems, in order to identify which one exhibits more significant attenuation of vibration parameters (such as vertical displacement, velocity, and acceleration), under identical conditions of mass, stiffness, and damping, as well as dynamic loading conditions.

3.1 Comparison of the Dynamic Amplification Function of TMD and KDamper

By using the transfer function of the systems (TMD and KDamper), Equations (5) and (13) respectively, the vibration transmissibility can be illustrated graphically as a function of the excitation frequency ratio to the natural frequency of the structure. The relationship between transmissibility and frequency ratio is important for analysis, as it allows the identification of frequencies that are most detrimental to the structure and the magnitude of amplification of vibrational parameters. This information enables the preliminary design of elastic and inertia parameters to avoid resonance phenomena at predominant excitation frequencies.

Figure 2 shows the vibration amplification curves as a function of the frequency ratio, with varying parameters: damping ratio (ξ), mass ratio (γ), and ratio between the natural frequency of the attenuation system and the excitation force frequency (β).



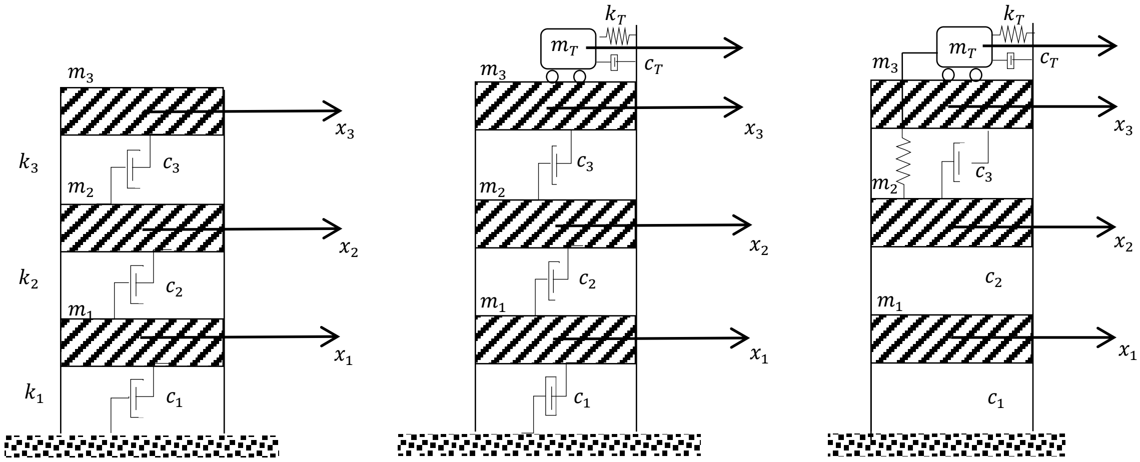
(a) (b)

Figure 3. (a) Dynamic Amplification Function for KDamper and TMD. (b) Dynamic Amplification Function for KDamper.

In the presence of resonance, when the ratio between the excitation frequency and the natural frequency of the structure is 1, vibration attenuation becomes crucial to ensure structural integrity and user's comfort. In this context, Figure 2 reveals that the KDamper attenuation system performs significantly better than the TMD, providing a relatively more pronounced reduction in the amplification of the structural dynamic response for excitation frequencies closer to the natural frequencies. These results indicate superior performance of KDamper compared to TMD, demonstrating that it is a suitable solution for vibration mitigation in applications where resonance is a critical or potential problem.

3.2 Numerical model of the structure used in the dynamic analysis.

For the analysis of the dynamic response, a 3-DoF (Degrees of Freedom) portal frame was used as the primary (H) structure, as shown in Figure 3(a). Initially, the dynamic response of the portal frame without any attenuation system was obtained. Then, the attenuation system was coupled with the third DOF of the portal frame, first using the TMD (Tuned Mass Damper), as shown in Figure 3(b), and subsequently using the KDamper, as shown in Figure 3(c). In all the analyzed cases, a sinusoidal excitation was applied, coinciding with the first natural frequency of the structure, to induce the largest dynamic responses due to the resonance phenomenon. The amplitude of this force was 1×10^6 N, of sinusoidal type, and it was applied to the structure in each of the 3 DOFs as $F_i(t) = 1 \times 10^6\, sen(2\pi f_1 t)$. The attenuation systems were tuned to this frequency (f_1). Once the dynamic responses of the three aforementioned configurations were obtained, for all degrees of freedom, the analysis was performed to determine which attenuation solution (TMD or KDamper) provided better levels of vibration energy dissipation.



(a) (b) (c)

Figure 4. (a) Original structure. (b) Structure with TMD (Tuned Mass Damper) damping system. (c) Structure with KDamper damping system.

3.2.1 System Equation of Motion

The structure used to compare the dynamic response of the TMD and KDamper systems is a 3-DOF (Degrees of Freedom) portal frame, with translational motion in the horizontal directions x_1, x_2 and x_3. The stiffness between the masses is obtained by summing the stiffnesses of the bars connecting them in parallel. This simplified model is chosen so that each mass has only horizontal translational motion and no rotational motion. Equation (23) represents the matrix form of the equation of motion for the original structure in free vibration, for the sake of simplifying the derivations. For the forced vibration case, the vector of identical sinusoidal excitation forces, as indicated in Section 4.2, will be added to the right-hand side of the equations.

Equation (23) represents the matrix form of the equation of motion for the original system.

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{23}$$

Once the TMD damping system is coupled to the top of the original portal (third DOF), the matrix equation of motion governing the structure becomes Equation (24) (with 4 DOFs). From now on, the new DOF will be referred to as DOF 4.

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \ddot{x}_4 \end{Bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 & 0 & 0 \\ -c_2 & c_2+c_3 & -c_3 & 0 \\ 0 & -c_3 & c_3+c_4 & -c_4 \\ 0 & 0 & -c_4 & c_4 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{24}$$

By adding an elastic element with negative stiffness to the fourth degree of freedom, the configuration of the system, previously a TMD, becomes a KDamper, as explained in the theoretical background. The matrix equation of motion governing the structure becomes Equation (25), where k_5 represents the negative stiffness.

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \ddot{x}_4 \end{Bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 & 0 & 0 \\ -c_2 & c_2+c_3 & -c_3 & 0 \\ 0 & -c_3 & c_3+c_4 & -c_4 \\ 0 & 0 & -c_4 & c_4 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2+k_3+k_5 & -k_3 & -k_5 \\ 0 & -k_3 & k_3+k_4 & -k_4 \\ 0 & -k_5 & -k_4 & k_4+k_5 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{25}$$

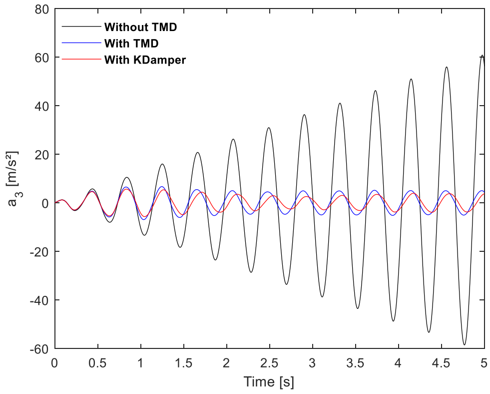
Here, approximate values of elastic and damping properties for reinforced concrete buildings are assumed, with a floor-to-ceiling height of 2.80m and a weight of 12 kN/m². Thus, the properties of the original structure are as follows: $m_1 = 8{,}40 \times 10^5[kg]$, $m_2 = 5{,}60 \times 10^5[kg]$, and $m_3 = 8{,}40 \times 10^5[kg]$. The modal damping ratios are defined as $\xi_1 = 1{,}22\times10^{-3}$, $\xi_2 = 1{,}96\times10^{-3}$ e $\xi_3 = 5\times10^{-3}$. The stiffnesses of the original structure are $k_1 = 9{,}90 \times 10^8[N/m]$, $k_2 = 8{,}46 \times 10^8[N/m]$, $k_3 = 6{,}12 \times 10^8[N/m]$. Based on the damping ratios, the corresponding damping values are $c_1 = 0{,}96 \times 10^5[N.s/m]$, $c_2 = 1{,}57 \times 10^5[N.s/m]$, $c_3 = 0{,}86 \times 10^5[N.s/m]$. Therefore, the damped natural frequencies for this original structure are $f_1 = 2{,}413$ Hz, $f_2 = 6{,}136$ Hz e $f_3 = 9{,}815$ Hz.

For the TMD attenuator, based on the previous optimal design, the following parameters were calculated: $m_4 = 1{,}12 \times 10^5[kg]$ (corresponding to 5% of the total mass of the original structure), stiffness $k_4 = 2{,}3354 \times 10^8[N/m]$, and damping $c_4 = 4{,}117 \times 10^5[N.s/m]$. For the KDamper system, assuming the same mass ratio as the TMD and a value of κ = 0.1, we obtain $k_5 = -2{,}76 \times 10^6[N/m]$, $k_4 = 3{,}042 \times 10^7[N/m]$, and the optimal damping of the KDamper as $c_4 = 3{,}565 \times 10^5[N.s/m]$. The subscripts 1, 2, 3, and 4 identify the degrees of freedom to which the parameters (mass, stiffness, and damping) are related, corresponding to the first, second, third, and fourth degree of freedom, respectively. While the first three degrees of freedom are associated with the structure where the dynamic vibration absorbers are attached, the fourth degree of freedom refers to that of the DVA themselves, as shown in Figure 3.

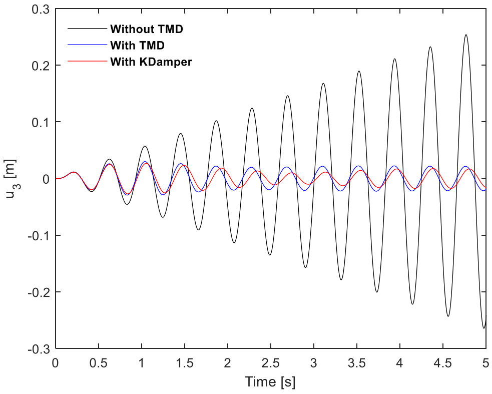
3.2.2 Dynamic Response of the system with vibration damper to a sinusoidal excitation at the 1st natural frequency of the structure.

Figures 4a and 4b depict the dynamic responses (displacements and accelerations) of the third degree of freedom of the structure. The total simulation time was T=30 s with a $\Delta t = 1\times10^{-4}$ s, but for better visualization of the graphs, they were limited to 5 s. The RMS values are calculated for the entire simulation time (T=30 s). The graph for vibration velocities has been omitted due to space constraints and its level of importance.

Once the dampers (TMD and KDamper) were coupled to the structure, it exhibited the following modal characteristics (damped frequencies and damping ratios): (a) for the coupled TMD $f_1 = 2{,}0409$ Hz, $f_2 = 2{,}686$ Hz, $f_3 = 6{,}152$ Hz and $f_4 = 9{,}817$ Hz; $\xi_1 = 5{,}681\times 10^{-2}$, $\xi_2 = 7{,}497\times10^{-2}$, $\xi_3 = 4{,}162\times10^{-3}$ and $\xi_4 = 5{,}227\times10^{-3}$; and (b) for the structure with the coupled KDamper: $f_1 = 2{,}101$ Hz, $f_2 = 2{,}846$ Hz, $f_3 = 6{,}158$ Hz e $f_3 = 9{,}814$ Hz ; $\xi_1 = 3{,}201\times10^{-2}$, $\xi_2 = 7{,}255\times10^{-2}$, $\xi_3 = 4{,}152\times10^{-3}$ and $\xi_4 = 5{,}217\times10^{-3}$.



(a)



(b)

Figure 5. (a) Accelerations of the third DOF (top) for the 3 evaluated structures. (b) Displacements of the third DOF (top) for the 3 evaluated structures

One can observe the occurrence of resonance in the structure (maximization of dynamic response amplitudes) when no damper is coupled. This is justified as the excitation frequency coincides with the fundamental frequency of the structure. The greater attenuation provided by the KDamper system compared to the TMD system can be observed, even though the absolute difference between the original and damped systems is not significant, the relative difference between the damped systems is relevant, reaching -38.28% for displacement and -38.75% for acceleration.

In Figure 5, the displacements in the TMD and KDamper are presented for a comparison between their values.

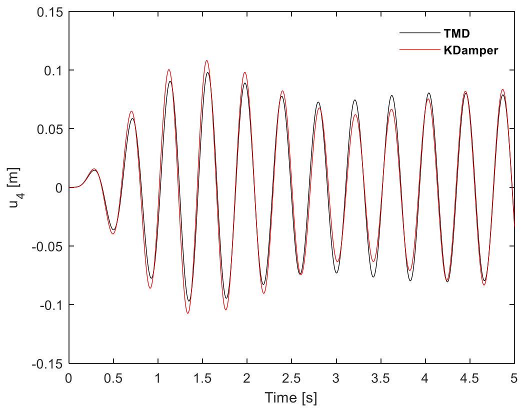


Figure 6. Displacements in the TMD and KDamper dampers.

It can be observed that the KDamper exhibits slightly higher values at some instances and lower values at others compared to the displacements of the TMD damper (maximum absolute values of the order of 10 cm). However, as shown later in RMS values, they are still lower than those of the TMD. In Table 2, the average RMS values of acceleration, velocity, and displacement are recorded for each degree of freedom. It is noted that DOF 4 was not considered in the original system since it does not have a damper.

Based on the presented Table 2, it can be concluded that the KDamper attenuation system, when compared to the TMD, exhibited a good ability to reduce accelerations, velocities, and displacements in all DOFs. The relative difference between the two systems shows that the KDamper provided significantly lower values of dynamic responses in all DOFs of the original structure. When comparing each system individually with the original structure, the KDamper was able to reduce the RMS accelerations in DOF 3 by up to 98.03%, while the TMD achieved reductions of 97.24%.

Table 2 - RMS values of acceleration, velocity, and displacement for each degree of freedom.

GDL	**Original Structure**	**Structure with TMD**	**Structure with KDamper**	**Relative Difference 100×(KDamper - TMD)/ TMD**	**Relative Difference 100×(TMD – Orig.)/ Orig.**	**Relative Difference 100×(KDamper – Orig.)/ Orig.**
Aceleração R*MS* (m/s²)						
1	45,21	1,2983	0,7953	-38.75%	-97.13%	-98.24%
2	87,92	2,4746	1,5912	-35.70%	-97.19%	-98.19%
3	128,42	3,5510	2,4996	-29.61%	-97.23%	-98.05%
4	-	12,6986	12,4231	-2.17%	-	-
Velocity RMS (m/s)						
1	2,99	0,0855	0,0525	-38.60%	-97.14%	-98.24%
2	5,82	0,1632	0,1054	-35.44%	-97.19%	-98.19%
3	8,49	0,2343	0,1656	-29.33%	-97.24%	-98.05%
4	-	0,8371	0,8198	-2.06%	-	-
Displacement RMS (m)						
1	$1{,}967\times10^{-1}$	0,0056	0,0035	-38.28%	-97.13%	-98.23%
2	$3{,}825\times10^{-1}$	0,0108	0,0070	-35.07%	-97.19%	-98.17%
3	$5{,}586\times10^{-1}$	0,0154	0,0110	-28.93%	-97.24%	-98.03%
4	-	0,0553	0,0543	-1.93%	-	-

In terms of direct comparison between the dampers, the KDamper showed a maximum reduction gain of 38.75% in acceleration compared to the TMD for the 1st DOF. For the 3rd DOF, this maximum reduction reached 28.93%.

However, overall, the KDamper consistently outperformed the TMD in any DOF, whether it was displacement, velocity, or acceleration.

In Figure 6, the FFTs (Fast Fourier Transforms) of the acceleration signals (original, original with TMD, and original with KDamper) are presented. The FFT curves represent the magnitude of acceleration by frequency. The black curve corresponds to the original structure, the blue curve corresponds to the structure without the TMD damper, and the red curve corresponds to the structure with the KDamper damper. From the analysis of Figure 6, it can be observed once again the better performance of the KDamper in most of the indicated frequencies.

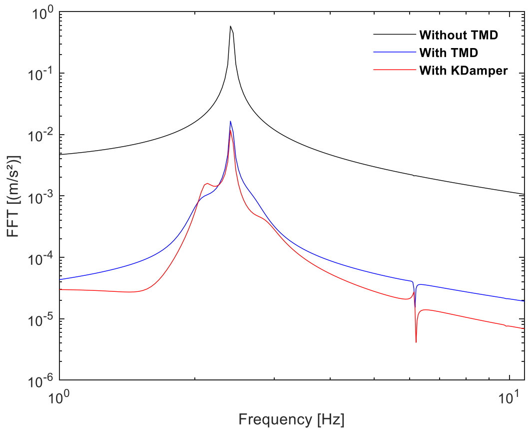


Figure 7. FFT curves of the accelerations for the analyzed structures.

4. CONCLUSION

The analyses conducted in the respective study demonstrate a significant relative difference in vibration attenuation levels in structures using the KDamper and TMD systems. The results from the amplification function and dynamic responses converge to indicate that the KDamper is more effective in reducing acceleration, velocity, and displacement values, which is crucial for ensuring the safety and comfort of individuals using structures subjected to vibrations.

The superior performance of the KDamper compared to the TMD is due to the incorporation of an elastic element with negative stiffness, which reduces the equivalent static stiffness of the attenuation system. This characteristic makes the KDamper system more advantageous in vibration reduction and allows for the use of significantly smaller inertial components than those used in TMDs for the same amount of energy dissipation. With this innovation, the KDamper system can be implemented in a more compact manner than TMDs in a variety of structures, such as buildings, bridges, wind towers, among others.

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