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# A NEW APPROACH USING TRANSMISSIBILITY AND KERNEL PRINCIPAL COMPONENT ANALYSIS TO DETECT DAMAGE FOR A NONLINEAR STRUCTURE WITH UNCERTAINTIES

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**Abstract.** Structural health monitoring (SHM) has emerged as a promising tool for detecting and managing damage in structures and systems. However, SHM becomes more challenging when uncertainties and the intrinsic nonlinear behavior of systems and structures are considered. This is because nonlinear phenomena can be mistakenly identified as damage when classical SHM techniques, which rely on linear metrics, are employed. Therefore, this study aims to address this issue by detecting early-stage damage in systems with nonlinear behavior while considering operational and environmental uncertainties. To convert these vast amounts of data into meaningful information, approaches based on the Statistical Pattern Recognition (SPR) paradigm are utilized. In this regard, a proposed method employs measures of transmissibility obtained from an experimental nonlinear beam over multiple days of measurement as a consolidated output. Kernel Principal Component Analysis (KPCA) is then utilized to extract and classify features. For the feature extraction phase, the KPCA algorithm is used to reduce the dimensionality of the transmissibility measurements and for the feature classification phase a new transmissibility should be mapped onto the high kernel space. The vibration data utilized in the study were acquired from an experimental setup consisting of a beam constructed by connecting Lexan layers. To simulate the propagation of damage with nonlinear quadratic behavior in the response, a breathing crack was intentionally introduced into the structure. Furthermore, to introduce additional cubic nonlinearities, two fixed steel masses were attached to the free end of the beam, which interacted with a magnet. The results of this study demonstrate the effectiveness of the proposed approach in damage detection, outperforming traditional Principal Component Analysis (PCA) techniques. This approach presents a promising solution for detecting damage in complex systems characterized by multiple nonlinearities and uncertainties. By improving the reliability and efficiency of structural health monitoring, it can find broad applications across various industries.

**Keywords:** Structural health monitoring (SHM), Nonlinear behavior, Transmissibility measurements, Kernel Principal Component Analysis (KPCA)

## 1. INTRODUCTION

With data based on vibration signals, this article addresses the problem in which it is desired to detect damage in its initial state in structures that present intrinsic nonlinearity, that is, in its healthy condition, with the presence of uncertainties, operational variability, and the type of damage produces in the structure a nonlinear behavior of a different nature from the initial.

In general terms, damage can be defined as changes introduced into a system that negatively affect the current or future performance of that system; it is the variation of a physical parameter. The concept is that damage is not significant without comparing two different states of the system, one of which is assumed to represent the initial state and is often undamaged (Farrar *et al.*, 2001).

The process of detecting damage to mechanical, aerospace, and civil systems and structures, based on characteristics and statistical analysis of damage, is called Structural Health Monitoring (SHM) (Farrar and Worden, 2007) and aims to identify damage before it reaches a critical state and avoid false indications of damage.

However, the problem of SHM increases when considering the intrinsic nonlinear behavior of systems and structures because nonlinear phenomena can be confused with damage when classical SHM techniques are based on linear

metrics (Bornn *et al.*, 2010).

Over the past three decades, SHM has been a promising tool in bridge management activities as it potentially allows for condition assessments to reduce uncertainty in the planning and design of maintenance activities, as well as increase service performance and operation safety (Figueiredo and Brownjohn, 2022).

SHM was introduced in the context of the statistical pattern recognition (SPR) paradigm to handle large amounts of data and perform damage identification automatically. The general idea has been transforming massive data obtained from monitoring systems and numerical models into meaningful information. The proposed method is divided into two phases according to this paradigm: feature extraction and feature classification, where the machine learning (ML) algorithm plays a vital role (Figueiredo and Brownjohn, 2022).

Frequency response is a type of feature used in vibration-based damage detection (Maia *et al.*, 2003). These frequency response signals are used to generate damage indicators (D) that reveal the structural condition of monitored structures (Avci *et al.*, 2021). However, the limitation of using frequency response signals in real-world SHM solutions is that measuring the excitation forces is necessary. This limitation arises because measuring the excitation forces can be difficult and expensive in real-world applications. Therefore, an alternative approach is to use transmissibility measurements as features, which have an output-only nature (Sampaio *et al.*, 2016). This means that only the response signals need to be measured, making it a more practical solution for real-world SHM applications. Therefore, this study aims to address this issue by detecting early-stage damage in systems with nonlinear behavior while considering operational and environmental uncertainties.

## 2. THE DAMAGE DETECTION METHOD

At this first moment, it is important to note that the training data consists of a learning beam of the transmissibility measurements from the undamaged condition. The test data comprises all the measurement sets, including those used during the training phase. A variance percentage of 0.99 was utilized, and the threshold was determined based on a 95% cutoff from the learning beam data.

For the stage of generation of damage-sensitive features, Transmissibility was chosen. Transmissibility measurements are defined as relationships between motion responses and motion reference responses. In practice, it is often convenient to obtain the transmissibilities without knowing the excitation forces (Santos *et al.*, 2019). Equation (1) represents the transmissibility function between two points  $\mathbf{i}$  and  $\mathbf{j}$  on a structure at a given frequency  $\omega$ ,

$$\mathbf{T}_{\mathbf{i},\mathbf{j}}(\omega) = \frac{\mathbf{S}_{\mathbf{i},\mathbf{j}}(\omega)}{\mathbf{S}_{\mathbf{j},\mathbf{j}}(\omega)}, \quad (1)$$

where the numerator of the equation,  $\mathbf{S}_{\mathbf{i},\mathbf{j}}(\omega)$ , represents the cross-spectral density between the response at point  $\mathbf{i}$  and the excitation at point  $\mathbf{j}$  and denominator of the equation,  $\mathbf{S}_{\mathbf{j},\mathbf{j}}(\omega)$ , represents the auto-spectral density of the excitation at point  $\mathbf{j}$ . The transmissibility function is useful for detecting structural damage, as changes in the structural properties can affect the transmissibility function at different frequencies.

Generally, a measure of Transmissibility can be estimated in several ways for a random input and considering a stationary stochastic process. The most common option is to use only the output.

Machine learning algorithms based on kernels have been widely applied to detect damage in structural health monitoring (Khoa *et al.*, 2014). In the feature extraction phase, KPCA will be used to reduce the dimensionality of the transmissibility measurements (Schölkopf *et al.*, 1998). Using kernel functions in conjunction with the traditional linear Principal Component Analysis (PCA) employs the kernel trick, which allows the mapping of features to high-dimensional spaces. Instead of specifying the mapping  $\phi$ , a Gaussian kernel, Eq. (2) will be employed, which can provide room for adding nonlinear features to the traditional PCA, resulting in kernel PCA (KPCA) (Reynders *et al.*, 2014),

$$\mathbf{K}(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2\sigma^2}\right), \quad (2)$$

where the equation represents a kernel function, denoted by  $\mathbf{K}$ , which takes two input vectors,  $\mathbf{x}_i$  and  $\mathbf{x}_j$ , and outputs a scalar value. The kernel function is the exponential of the negative squared Euclidean distance between the input vectors, divided by twice the square of a parameter  $\sigma$ . The Euclidean distance between two vectors,  $\mathbf{x}_i$  and  $\mathbf{x}_j$ , is the square root of the sum of the squared differences between their corresponding elements. The parameter  $\sigma$  controls the width of the kernel function and is often referred to as the bandwidth parameter.

## 3. LABORATORY STRUCTURE

### 3.1 Description of the structure and data sets

The applicability of the SHM-SPR paradigm and the proposed method was previously verified in an experimental laboratory configuration of a beam set. This structure obtains structural responses by simulating various operational

and environmental conditions and damage. The beam set presents nonlinear behavior even in healthy condition. It is exposed to a type of damage that causes the structure to present a nonlinear behavior different from the initial one. In addition, the uncertainties associated with data variation are considered in the methodology's application. This experiment was conducted by the SHM laboratory of the University of California San Diego with the collaboration of SHM Lab UNESP/Ilha Solteira (Villani *et al.*, 2020).

### 3.2 Experimental setup

The experimental configuration used is shown in Fig. 1. The monitored structure is formed by a free-set beam constructed from the bonding of four thin beams of Lexan (Polycarbonate), of 2.4 mm x 24 mm x 240 mm each, to emulate a damage spread described below. Two masses of steel are fixed to the free top of the beam and interact with a magnet, generating a nonlinear behavior in the system's response, even in the reference condition due to the added magnetic potential.

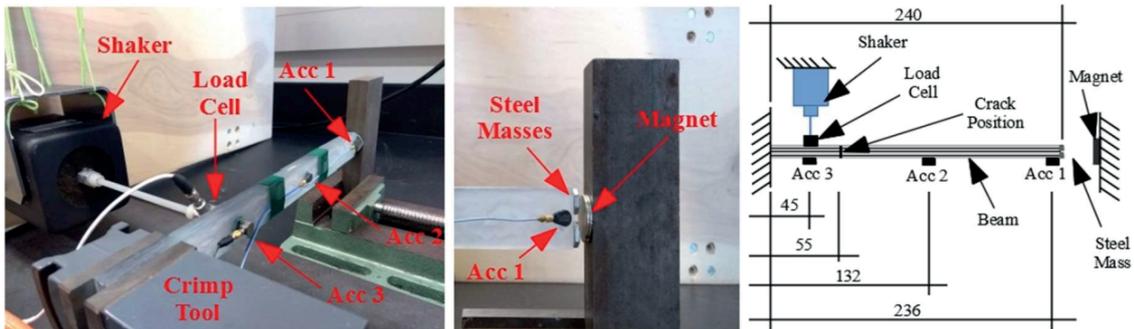


Figure 1. Experimental setup.

The data acquisition system configuration consists of the following equipment: A NI CompactDAQ Chassis Acquisition System (NI cDAQ-9178); AC Series Sound and Vibration Input Modules (NI-9234); AC Series Voltage Output Module (NI-9263). Electrodynamic Transducer Labworks Inc. (ET-132); MB Dynamics amplifier (SL500VCF); PIEZOTRONICS Load Cell PCB (208C02). PIEZOTRONICS Accelerometer PCB (352C22).

The electrodynamic transducer excites the structure with different signals and considers two inputs (low – 1 V (RMS) and high – 6 V RMS). Accelerometers (Acc) measure the output data positioned near the free end (Acc1) and the electrodynamic transducer (Acc3) of the beam because the authors are interested in the transmissibility measurements between accelerometers 1 and 3.

The input signal analyzed is the voltage signal applied to the electrodynamic transducer. As a single-input/single-output (SISO) model is considered, this pair of signals is sufficient to identify and monitor structural integrity. All acquisition parameters, signs considered, and equipment used were the same in the experiments, considering the different structural conditions.

The damage enforced to the beam is meant to mimic a systemic breathing crack. In this study, four beams were constructed. Two beams were undamaged (Baseline condition - BC), referred to as the learning beam and the test beam, while the other two were intentionally damaged (Damaged condition - DC). The first damaged condition, labeled as damage condition 1, was constructed with three intact thin beams and one cut-thin beam. The second damaged condition, labeled as damage condition 2, was constructed with two intact thin and two cut-thin beams. These damaged beams were specifically prepared for applying the damage detection methodology.

### 3.3 Intrinsic nonlinear behavior

This experimental configuration presents nonlinear behavior even in the reference condition without the presence of damage. The results obtained during the application of the stepped sine test considering two input levels when a low-level input signal (1 V RMS) is applied, the output signal shows a linear characteristic for both up-sweep and down-sweep input. However, when the input signal is of a high level (6 V RMS), the nonlinear phenomenon can be seen with the presentation of a jump. The frequency-time diagram of the system response considers a high input level of a chirp signal in the region of the first mode of the structure. The presence of a cubic harmonic in the response confirms the nonlinear characteristic of the response caused by the interaction between the steel mass at the end of the beam and the magnet (Villani *et al.*, 2020). The distance between the magnet and steel masses varied during the tests from 2 to 3.5 mm. The tests were repeated on different days to obtain 200 experimental tests for each beam constructed. The purpose of varying the distance and conducting multiple tests was to introduce data variation and uncertainties in the experimental setup. However, two measurements were discarded for each beam, resulting in 198 measurements per beam, totaling 792 observations.

#### 4. RESULTS AND DISCUSSION

In this section, the proposed method is demonstrated by application and its set of measurements obtained by monitoring a polycarbonate beam under different structural conditions. The methods based on the KPCA and PCA algorithms are employed, and their damage detection performances are compared.

The variation of the system response during the tests and transmissibility measurements is shown in the Fig. 2. The results consider the first mode of vibration of the learning beam, test beam, beam in damage condition 1, and beam in damage condition 2, measured at accelerometers 1 and 3 for a high-level input of 6 V.

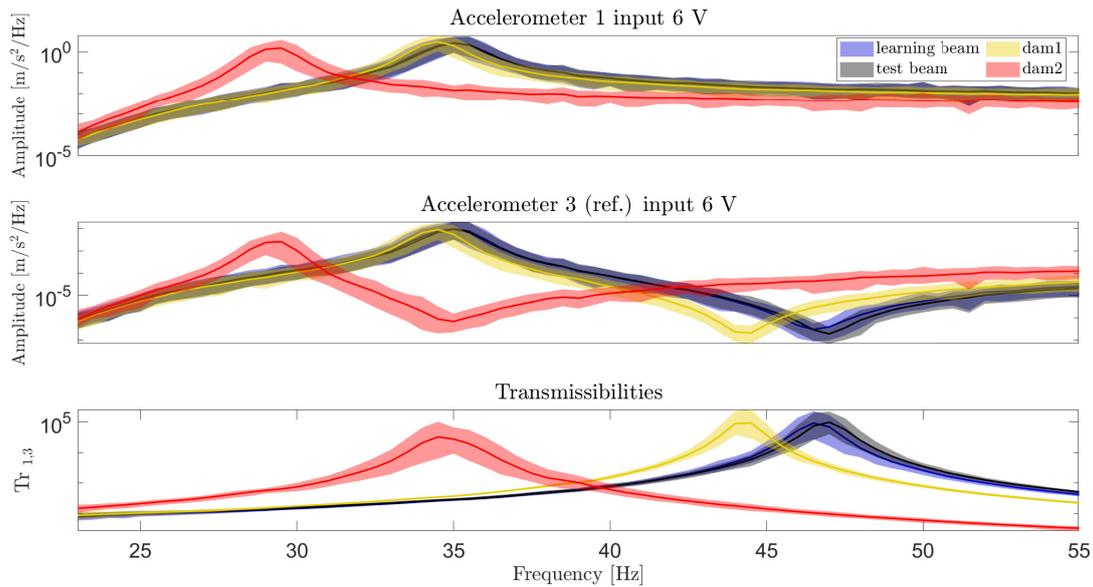


Figure 2. Frequency spectrum of accelerometer 1 and 3 responses and Transmissibility for a high-level input signal.

The frequency responses of the learning and test beams are practically overlapped, as the structures are similar and in a healthy condition. The frequency responses of accelerometer 1 correspond to the resonance frequencies of the beams. It can be observed that when the extent of damage is severe, it becomes easier to classify the structural condition.

The frequency response of the beam in damage condition 2 is shifted to the left, away from the responses of the healthy condition beams. However, visually, it is not evident to distinguish between damage condition 1 and the reference condition. The frequency response for damage condition 1 is very close to the frequency responses of the reference condition beams. This result demonstrates how challenging it is to detect the presence of the in its initial state, damages, even when considering data variation described in the experimental setup.

In the frequency responses of accelerometer 3, damage condition 1 is also not visually detectable. However, the presence of anti-resonance in the frequency response makes the detection of damage condition 1 visible. At this stage, the transmissibility measurements were obtained by dividing the signals from accelerometer 1 by the respective signals from accelerometer 3 for each beam.

The damage detection performances of approaches based on the PCA and KPCA algorithms are evaluated in Fig. 3.

Methods based on the KPCA and PCA algorithms are employed, and their damage detection performances are compared. In this scenario, we observe the damage indicators derived from the KPCA and PCA algorithms and a threshold set for a significance level of 5% on the learning data. The PCA and KPCA detected damage condition 1. However, the type 1 error classified in the PCA comprises all the test beam data, beam in the undamaged condition, BC (Test).

The performances of the algorithms are summarized in Tab. 1, wherein an overall analysis the KPCA has better damage detection performance than the PCA.

Table 1. Number and percentage of Type I and Type II errors.

Algorithm	Type I	Type II	Total
PCA	208 (52.53%)	0 (0.00%)	208 (26.26%)
KPCA	33 (8.33%)	0 (0.00%)	33 (4.17%)

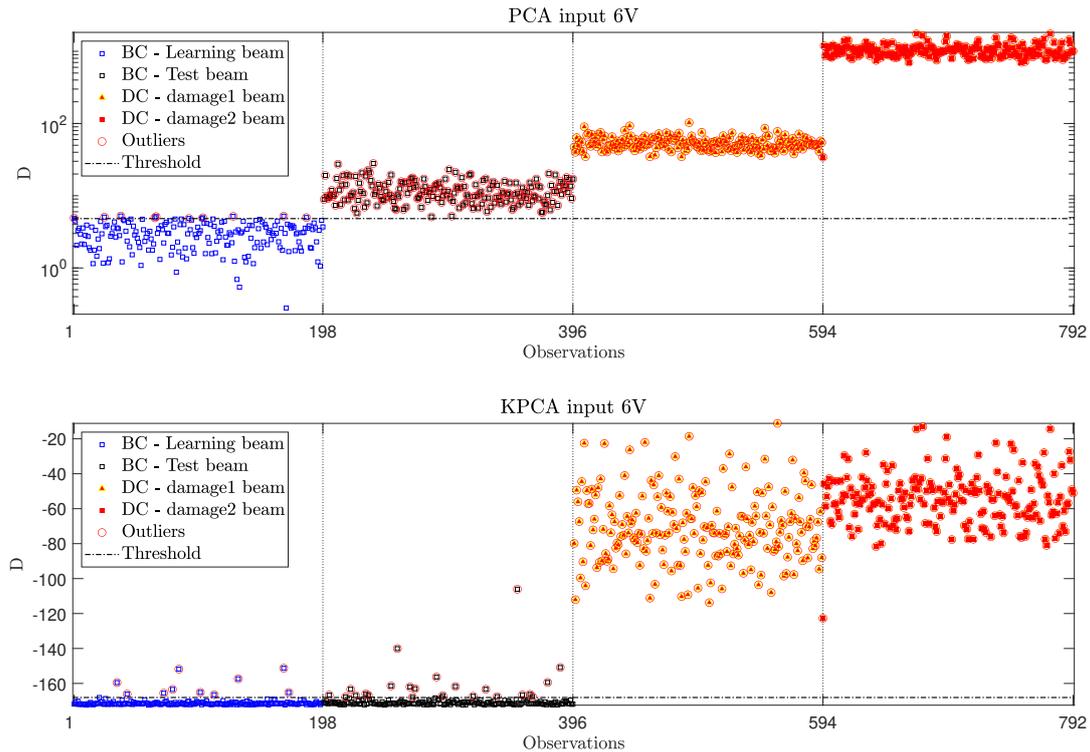


Figure 3. Damage detection performance based on the PCA (top) and KPCA (bottom) algorithms.

## 5. CONCLUSION

This study aimed to detect the damage in its initial state using transmissibility measurements. The steps of feature extraction and classification were based on the SPR paradigm to reduce the dimensionality of Transmissibility via KPCA and generate a damage indicator that establishes a level of damage for each new measurement of Transmissibility.

In conclusion, the result of this study demonstrates the effectiveness of the proposed approach in damage detection. The strategy of using only the frequency response signal presented in this study showed an advantage in that the transmissibility measurements are processed by the KPCA so that an appropriate dimensionality is achieved, outperforming traditional PCA techniques. This approach presents a promising solution for detecting damage in complex systems characterized by multiple nonlinearities and uncertainties. It can find broad applications across various industries by improving the reliability and efficiency of structural health monitoring.

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