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# EVALUATING THE VISCOELASTIC LOAD SHARE OF PORCINE KNEE LIGAMENTS

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***Abstract.*** *An analytical model of load share of porcine knee ligaments, statically loaded, is presented. The proposed model includes the viscoelastic behavior of the ligaments, which can be considered an evolution, in comparison with the most literature available models, based on an elastic constitutive approach. The proper mechanical description of the knee ligaments relies on the right modeling of their constitutive relationships. To accomplish this primary objective, a series of relaxation experimental tests were conducted using the following porcine knee ligament specimens: lateral collateral ligament (LCL), anterior cruciate ligament (ACL), posterior cruciate ligament (PCL) and medial collateral ligament (MCL). The results of these relaxation experimental tests were processed by software, using Schapery's equations. The Schapery's model constants, thus obtained, were used to describe the constitutive relationships of the knee porcine ligaments, which are applied in the proposed load share model. Preliminary results show that although load share rearrangement after the viscoelastic phase was quite discrete, the remaining load capacity in the ligaments can drop to less than half of their initial value. This fact can, probably, explain why operated knee ligaments, which are tight just after surgical operations, can build an unstretched state, in a couple of weeks.*

***Keywords:*** *knee ligaments, analytical model, viscoelasticity, load share.*

## 1. INTRODUCTION

Viscoelasticity is the property exhibited by materials that simultaneously display viscous and elastic behavior. This concept must be utilized in the description of the mechanical characteristics of knee ligaments. It has been observed that non-linear viscoelastic models adequately capture the realistic behavior of ligaments. Provenzano *et al.* (2002) examined the ability of the non-linear Schapery's and the Modified Superposition Method to estimate the stress behavior of ligaments during relaxation. Their investigation concluded that both models successfully achieved this objective. Subsequently, Blandford (2017) employed the same models to simulate the relaxation behavior of the MCL, producing satisfactory results for both approaches.

Zheng *et al.* (1998) introduced an analytical model of the knee joint to estimate the forces exerted on the knee during exercise. Muscle forces were estimated by considering electromyographic activities observed during exercise and maximum voluntary isometric contraction (MVIC). In a related study, Peña *et al.* (2006) presented analytical and Finite Element Models (FEM) approaches, providing comprehensive results, combining the principal stresses developed in the knee ligaments. Incorporating the effect of viscous behavior into the description of the mechanical properties of knee ligaments can be highly beneficial in modeling knee performance. Bernardes *et al.* (2005) aimed to determine biomechanical parameters for modeling the human knee joint by incorporating extensive exercises and utilizing videofluoroscopy images, thereby enabling the assessment of viscous behavior results.

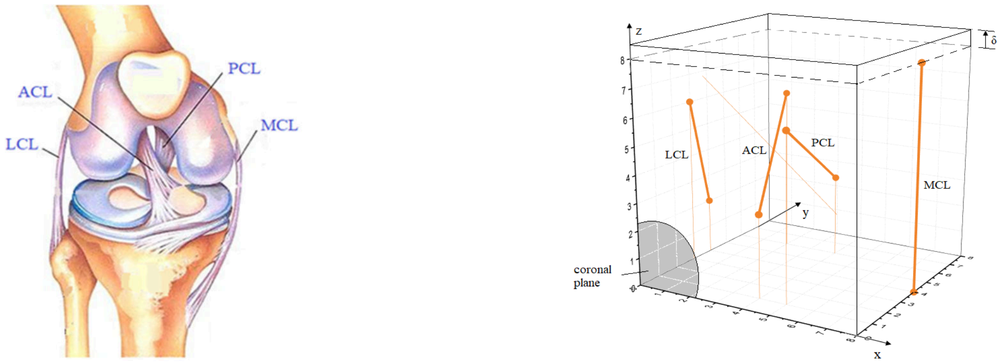
This paper presents a detailed mechanical characterization of statically loaded porcine knee ligaments under an imposed vertical force that induces a vertical displacement. The study examines the relationships between angles, displacements, strains, forces, and stresses involved in the load sharing among the knee ligaments. Furthermore, the viscoelastic nature of the knee ligament material is taken into consideration.

Despite numerous articles addressing the viscoelasticity of soft tissues, including those mentioned in this Introduction section, which could be used as a reference, the direct comparison between them and the proposed model is not feasible, due to significant differences in ligament sources. Thus, this article compares the proposed model with a previous model proposed by Silva *et al.* (2020) from the same research group, which represents a simpler model, which not consider the viscoelastic effect.

## 2. ANALYTICAL MODEL

The proposed analytical model focuses on analyzing load sharing among knee porcine ligaments under an initially imposed vertical force of 100 N. Additionally, it is assumed that this force induces a rapid step-like vertical displacement in the knee, followed by a period of constant displacement, simulating a relaxation test. This analytical model represents an improvement over the previous model Silva *et al.* (2020), which was only elastic. The proposed analytical model incorporates continuous variations in both ligament lengths and angles, encompassing the viscoelastic behavior by adopting Schapery's nonlinear approach. Additionally, the assumption is made that the cross-sectional area remains constant, and the utilization of the small strains' theory is acceptable. It is noteworthy that this is a limitation of the model.

The proposed analytical model employs solid mechanics theory to estimate the load distribution among four knee ligaments, as depicted schematically in Fig. 1.a: the lateral collateral ligament (LCL), anterior cruciate ligament (ACL), posterior cruciate ligament (PCL), and medial collateral ligament (MCL). It should be noted that in Fig. 1.b, the ligaments are represented schematically, maintaining their relative lengths, angles, and relative positions.




(a) (b)

Figure 1. Left knee: (a) view from behind of knee's ligaments (adapted from vitalitychiropractor.com.au) and (b) schematic representation of the geometry of the 1D ligament model submitted to a vertical force that induces a vertical displacement, adapted from Silva *et al.* (2020).

Fig. 2 provides a schematic representation of one ligament before and after the elastic loading was applied.

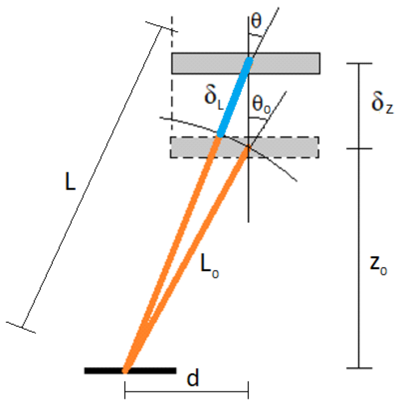



Figure 2. Ligament geometrical characterization.

The geometric relations, for each ligament, are shown:

$$L = L_0 + \delta_L \qquad z = z_0 + \delta_z \qquad L^2 = z^2 + d^2 \tag{1}$$

$$\sin(\theta_0) = {}^{d}/_{L_0} \qquad \cos(\theta_0) = {}^{z_0}/_{L_0} \qquad \tan(\theta_0) = {}^{d}/_{z_0} \tag{2}$$

$$\sin(\theta) = {}^{d}/_{L} \qquad \cos(\theta) = {}^{z}/_{L} \qquad \tan(\theta) = {}^{d}/_{z} \tag{3}$$

where, $L_0$, $L$, $z_0$, $d$, $\theta_0$, and $\theta$ represent, respectively, the ligament's initial and final length, the height, the projection in the *x-y* plane axis, and the initial and final angle of a ligament, with the vertical axis $z$.

The equations for a final ligament angle $\theta$, the ligament displacement variation $\delta_L$ (in blue color in Fig. 2), and the displacement variation projection in the vertical direction $\delta_z$ are shown next.

$$\theta = \arctan\left[\frac{L_0 \cdot sin(\theta_0)}{L_0 \cdot cos(\theta_0) + \delta_z}\right] \tag{4}$$

$$\delta_L = \sqrt{[L_0 \cdot \cos(\theta_0) + \delta_z]^2 + [L_0 \cdot \sin(\theta_0)]^2} - L_0 \qquad \delta_z = \sqrt{(L_0 + \delta_L)^2 - [L_0 \cdot \sin(\theta_0)]^2} - L_0 \cdot \cos(\theta_0) \tag{5}$$

Moreover, the model uses the following vertical equilibrium condition, Eq. (6), and vertical compatibility equation, Eq. (7):

$$\sum_1^4 F_n \cdot \cos(\theta_n) = P_z \tag{6}$$

$$\delta_z = \delta_{nz} = \delta_{1z} = \delta_{2z} = \delta_{3z} = \delta_{4z} \tag{7}$$

Where, $P_z$ is knee vertical force, and $\delta_{nz}$, $\theta_n$ and $F_n$ are, respectively, the vertical displacement variation, the angle, and the force in ligament *n*. Note that the ligaments are renamed as follows: n = 1 for LCL, n = 2 for ACL, n = 3 for PCL, and n = 4 for MCL. This paper utilizes Schapery's nonlinear viscoelastic model to describe the force-displacement and stress-strain relationships. Numerical methods are utilized for all calculations, due to the implementation of the model.

### 2.1 Schapery's model

The non-linear Schapery's model (1969, 1997, 2000) employs the Boltzmann superposition method, incorporating thermodynamic concepts, to establish a non-linear strain-stress relationship. While the model allows for the consideration of temperature effects, it is not considered in this paper as the relaxation tests were conducted under laboratory constant temperature conditions.

A significant drawback of Schapery's model is its reliance on the stress value during the equilibrium condition, particularly as time tends to infinity. This dependence assigns a limitation because the accuracy of the numerical model hinges on accurately determining the equilibrium state in experimental tests. To achieve this equilibrium state, it is often necessary to test for lengthy periods, sometimes spanning several hours, as observed in numerous research studies. In this study, it was not possible to spend several hours on each test due to limitations in ligament specimens and laboratory availability. Blandford (2017) proposed an alternative approach to address this limitation by utilizing curve fitting techniques. Yet, in this study, the approach utilized was based on Provenzano *et al.* (2002), which uses all data points of the stress-time curve instead of a subset. Silveira (2021) showed that, for the ligament specimens used in this research, the Provenzano approach resulted in a more accurate numerical model compared to the method proposed by Blandford (2017), as evidenced by direct comparison with experimental data.

Schapery (1969) established a non-linear stress-strain relation $\sigma(\varepsilon, t)$ for the creep case and stated that a corresponding equation can be formulated for the relaxation case, Eq. (8). Furthermore, it is assumed that the strain ε is applied instantaneously, like a step function, and the ramp time is not considered as in prior studies of Schapery (1969), Provenzano *et al.* (2002), Duenwald *et al.* (2009), and Blandford (2017).

$$\sigma(\varepsilon, t) = h_e(\varepsilon) \cdot G_e \cdot \varepsilon(t) + h_1(\varepsilon) \cdot \int_0^t \Delta G\big(\rho(t) - \rho(\tau)\big) \cdot \frac{d[h_2(\varepsilon) \cdot \varepsilon(\tau)]}{d\tau} d\tau \tag{8}$$

where $\rho(t) = \int_0^t \frac{dx}{a_\varepsilon[\varepsilon(x)]}$ and $\rho(\tau) = \int_0^\tau \frac{dx}{a_\varepsilon[\varepsilon(x)]}$

where $\Delta G$ and $G_e$ represent, respectively, the transient relaxation function and the relaxation function at equilibrium. The reduced time function is represented by $\rho(t)$ and $\rho(\tau)$, and it relies on the material constant $a_\varepsilon$. This constant is associated with both strain and effect of temperature and depends on entropy and free energy of the system. The variables $h_e$, $h_1$ and $h_2$ are material constants and are linked to both strain and Helmholtz free energy. Note that the small strains' theory is utilized.

According to Schapery (1969), Eq. (8) can be rewritten by setting $h_1(\varepsilon)$ and $a_\varepsilon(\varepsilon)$ equal to 1, as tissue tests exhibit low stress, and the temperature variation is not considered for relaxation. The Eq. (9) development is available in Silveira (2021).

$$\sigma(\varepsilon,t) = h_e(\varepsilon)\cdot G_e\cdot\varepsilon(t) + \int_0^t \Delta G(t-\tau)\cdot\frac{d[h_2(\varepsilon)\cdot\varepsilon(\tau)]}{d\tau}d\tau \tag{9}$$

According to Provenzano *et al.* (2002) and Blandford (2017), when the strain remains constant throughout the test, $\varepsilon(t) = \varepsilon_i$, so the Eq. (9) can be reformulated as follows.

$$\sigma(\varepsilon_i,t) = h_e(\varepsilon_i)\cdot G_e\cdot\varepsilon_i + h_2(\varepsilon_i)\cdot\varepsilon_i\cdot\Delta G(t) \tag{10}$$

Therefore, a simplified equation for relaxation with constant strain was derived, leading to improved curve fitting and numerical implementation. The transient relaxation function ΔG in soft tissues, such as knee ligaments, is often described using a power law due to the logarithmic correlation observed between time and stress during relaxation, as suggested by previous studies of Duenwald *et al.* (2009); Provenzano *et al.* (2002); Blandford (2017).

$$\Delta G(t) = C\cdot t^{-n}, \tag{11}$$

where *C* represents the material's stiffness to relaxation at the initial time and *n* represents the relaxation rate, which can be interpreted as the slope on a log-log plot of stress versus time *t*. By applying Eq. (11) in Eq. (10), Eq. (12) can be derived, which is commonly employed in various studies and served as the basis for obtaining Schapery's constants through curve fitting in this research.

$$\sigma(\varepsilon_i,t) = h_e(\varepsilon_i)\cdot G_e\cdot\varepsilon_i + h_2(\varepsilon_i)\cdot\varepsilon_i\cdot C\cdot t^{-n} \tag{12}$$

According to Blandford, to enhance the accuracy of the curve-fitting process, the following equation can be employed for both variables:

$$h_e(\varepsilon) = a\cdot e^{b\cdot\varepsilon} + c\cdot e^{d\cdot\varepsilon} \qquad h_2(\varepsilon) = f\cdot e^{i\cdot\varepsilon} + j\cdot e^{m\cdot\varepsilon} \tag{13}$$

where *a*, *b*, *c*, *d*, *f*, *i*, *j*, and *m* are dimensionless constants determined with curve fitting. Those variables are analogous to the same presented in Blandford (2017) and Silveira (2021).

Furthermore, due to the complexity of the stress-strain relationship, it was necessary to perform numerical calculations, as presented below.

### 2.2 Numerical implementation

Given the complexity of the proposed model, a software application was developed in C# programming language to handle all the necessary calculations. It generates a CSV file containing the time evolution of each result: ligaments' stress, ligaments' force, ligaments' vertical force, ligaments' load share, and knee vertical force. Best practices of Object-Oriented Programming were used to develop resilient, performant, and maintainable software, catering to its frequent use. Furthermore, the decision not to utilize commercial solutions was motivated by the wish for enhanced control and flexibility in the numerical calculation methodologies. Although the code is not publicly available, interested individuals can request the authors to access it. See Silveira (2021) for a quite complete description of the developed software.

In terms of the numerical implementation, specific parameters were determined. The time range was established from the time step to 1200 s. It does not start at zero since it was considered that, in the initial time, t = 0 s, the forces were still not applied to ligaments. The time step was 0.1s, consistent with previous studies of Silveira (2021); Silveira *et al*. (2022); Rodarte *et al*. (2021); and Rodarte (2022). The initial calculation step, which estimated the knee displacement induced by the applied force, considered a maximum acceptable error of 1%, a maximum of $10^6$ iterations, and the time was set as 0.1s, equal to the time step. This time was chosen because Schapery's model does not yield satisfactory results when the time value tends to zero, and it was not possible to achieve convergence for the displacements of knee and ligaments under these conditions. The time step, the maximum acceptable error, and the maximum of iterations were obtained from benchmark tests that evaluated execution time, computational resources used (CPU, memory, and disk), accuracy of results, and simplicity (for example: instead of using 0.868% of maximum error, it was used 1%).

The analysis began with the estimation of the knee displacement induced by the applied force. This was achieved by solving the system of equations presented in Eq. (14), using the bisection method, for the *n* ligaments. This numeric method requires an initial and final value. The initial value was set as zero and the final value (approximately 66.02 mm) was estimated by calculating the maximum vertical displacement resulting from the imposed vertical force ($P_z$ = 100N) considering only a single ligament on the knee. Moreover, it was required for the calculations, around 27k iterations.

$$\begin{cases} \delta_z = \delta_{1z} = \delta_{2z} = \delta_{3z} = \delta_{4z} \\ \theta_n = \arctan\left[\dfrac{L_{n0} \cdot sin(\theta_0)}{L_{n0} \cdot cos(\theta_0) + \delta_{nz}}\right] \\ \delta_{nL} = \sqrt{[L_{n0} \cdot \cos(\theta_{n0}) + \delta_{nz}]^2 + [L_{n0} \cdot \sin(\theta_{n0})]^2} - L_{n0} \\ \varepsilon_n = \dfrac{\delta_{nL}}{L_n} \\ \sigma_n = \sigma_n(\varepsilon_n, t) \\ F_n = \sigma_n \cdot A_n \\ \sum_1^4 F_n \cdot \cos(\theta_n) = P_z \end{cases} \tag{14}$$

After calculating the knee displacement, the same system of equations (Eq. 14) is employed to determine all ligaments results of angle, vertical displacement, displacement, strain, force, vertical force, load share of the ligaments, and knee vertical force. During this step, iteration is performed solely over time from 0.1s to 1200s.

## 3. RESULTS

It is important to note that the geometrical and elastic properties, presented in Table 1 of porcine ligaments, can exhibit significant variability, as observed in previous studies. Factors such as the age of the porcine specimens, diseases, ligament preparation techniques (including freezing and sampling methods), and test strain rates can contribute to the quite scattered data.

The lengths and cross-sectional areas of the ligaments were determined by direct measurements of the ligament specimens. The Young's modulus of the ligaments and Schapery's model constants were obtained from experimental data Rodarte *et al*. (2021); Rodarte, (2022). Table 2 shows the constants for Schapery's model results.

Table 1. Knee porcine ligaments' geometrical and elastic properties.

| Variable | Ligament | | | |
|---|---|---|---|---|
| | LCL | ACL | PCL | MCL |
| $A_n$ mm² | 63.62 | 63.62 | 63.62 | 20 |
| $L_n$, mm | 38 | 26 | 30 | 37 |
| $\theta_n$, rad | 0.357 | 0.812 | 0.644 | 0.233 |
| $E_n$, MPa | 4.4 | 5.8 | 25.1 | 3.2 |

Table 2. Constants for Schapery's model results.

| Variable | Ligament | | | |
|---|---|---|---|---|
| | LCL | ACL | PCL | MCL |
| $C$ | -7.949 | -1.309 | -24.31 | -0.905 |
| $n$ | -0.050 | -0.148 | -0.054 | -0.120 |
| $G_e$, MPa | 16.03 | 10.61 | 47.87 | 4.75 |
| $a\ [h_e(\varepsilon)]$ | 12.57 | 37.79 | 2.483 | 18.33 |
| $b\ [h_e(\varepsilon)]$ | -126.7 | -175.6 | -56.91 | -153.2 |
| $c\ [h_e(\varepsilon)]$ | 0.944 | 0.876 | 0.351 | 0.764 |
| $d\ [h_e(\varepsilon)]$ | -9.088 | -2.81 | 14.95 | 2.15 |
| $f\ [h_2(\varepsilon)]$ | 39.01 | 2139 | 4.103 | 937500 |
| $i\ [h_2(\varepsilon)]$ | -166.1 | -291.7 | -68.16 | -513.2 |
| $j\ [h_2(\varepsilon)]$ | 1.017 | 0.685 | 0.288 | 0.698 |
| $m\ [h_2(\varepsilon)]$ | -10.91 | -1.194 | 16.26 | 4.831 |

Figs. 3. a-f presents the results of the four ligament types for the final displacement, the final strain, the initial force, the final force, the final stress, and the final angle for both the elastic model Silva *et al.* (2020) and the proposed model. The elastic model of Silva *et al.* (2020) serves as an initial approximation for the proposed model. The knee vertical induced displacement for elastic Silva *et al.* (2020) and the proposed model are, respectively, 1.31 mm and 1.78 mm.

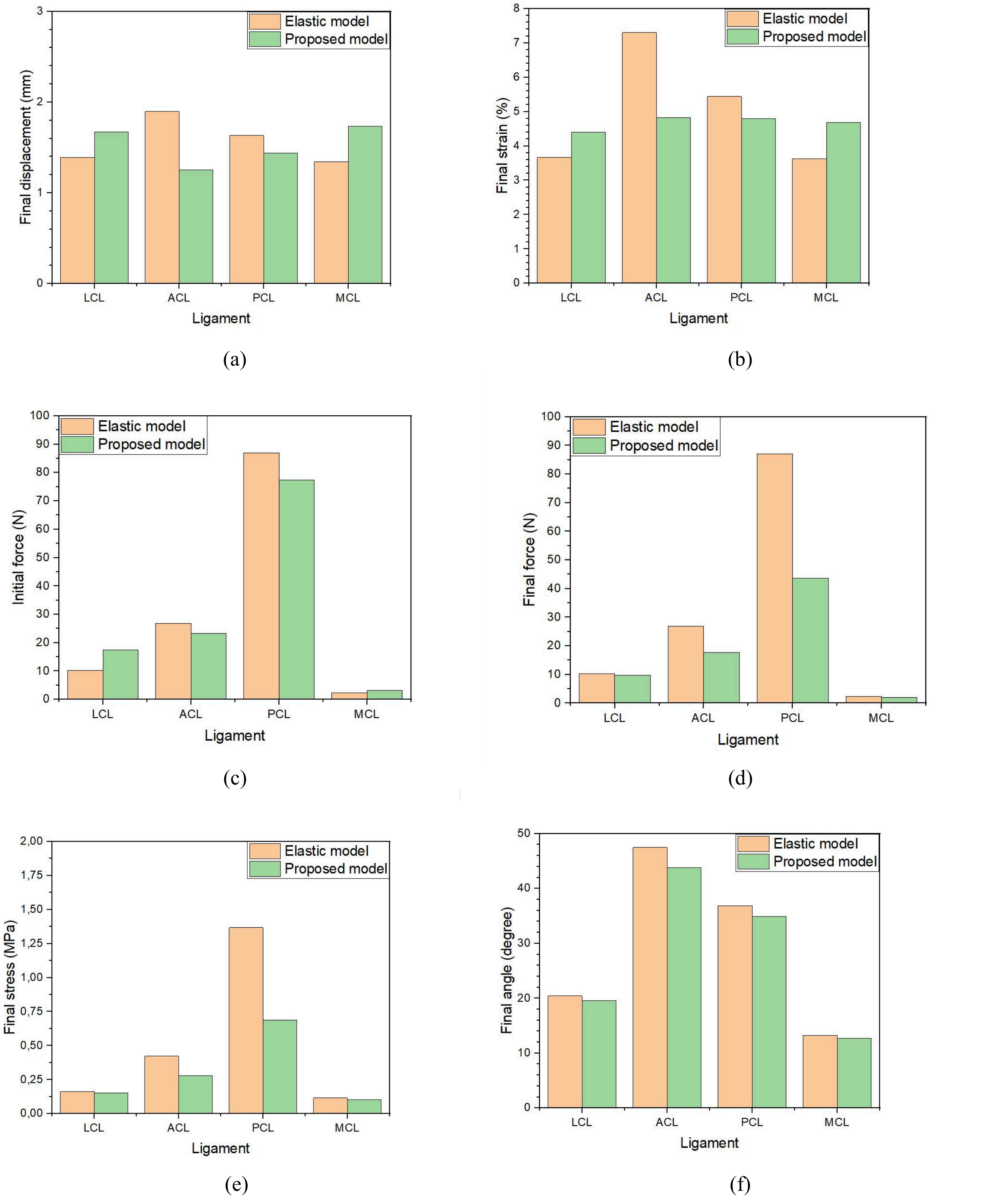



Figure 3. Outputs variables for the elastic model Silva *et al.* (2020) and proposed model: (a) final displacement, (b) final strain, (c) initial force, (d) final force, (e) final stress, and (a) final angle.

While the final angle in Fig. 3.f showed some similarities between the elastic model Silva *et al.* (2020) and the proposed model, significant differences were observed in the final force in Fig. 3.d, and final stresses in Fig. 3.e. The elastic model of Silva *et al.* (2020), which lacks consideration of viscous effects, performed poorly in capturing these aspects of soft tissues like porcine knee ligaments. Therefore, it can be concluded that the elastic model, and similar approaches, are not suitable for properly modeling such tissues due to their inability to account for the prevailing viscous effects.

Table 3 presents the percentage of force reduction in the proposed model, representing the decrease in force capacity (remaining load capacity) from the initial state (before considering viscoelastic effects) to the final state (after accounting for viscoelastic effects).

Table 3. The force capacity decreases in the proposed model along the porcine knee ligament axis.

| Variable | Ligament | | | |
|---|---|---|---|---|
| | LCL | ACL | PCL | MCL |
| Initial force (N) | 17.55 | 23.33 | 77.42 | 3.25 |
| Final force (N) | 9.77 | 17.76 | 43.69 | 2.08 |
| Force decrease (%) | 44.32 | 23.89 | 43.56 | 36.08 |

It is important to emphasize that the forces presented in Table 3 correspond to the forces along the axis of each ligament. To calculate the knee vertical force, Eq. (6) should be utilized with the values provided in Tables 1 and 3. Table 4 presents the results for the load share calculations at the initial time (before the ligaments are influenced by viscous effects) and the load share at the final time (once the ligaments have undergone viscous effects).

Table 4. Load share between porcine knee ligaments at the beginning and the end of relaxation tests in the vertical direction.

| Variable | Ligament | | | |
|---|---|---|---|---|
| | LCL | ACL | PCL | MCL |
| Initial shared force (%) | 16.53 | 16.84 | 63.46 | 3.17 |
| Final shared force (%) | 15.38 | 21.41 | 59.83 | 3.39 |

Although the percentage of ligament load sharing did not change significantly, as indicated in Table 4, there was a notable reduction in the total vertical force capacity (remaining load capacity) of the ligaments from 100N to 59.86N when comparing the elastic model Silva *et al.* (2020) and the proposed model.

## 4. CONCLUSIONS

A 1D load share model was developed to analyze the load distribution in porcine knee ligaments. This model operates in a three-dimensional space, considering the viscous phenomenon. The proposed approach incorporates Schapery's equations and experimental constants to capture the viscoelastic behavior and determine the load share among the knee ligaments.

The results obtained from the proposed model exhibit significant differences if compared to the previous, simpler, elastic model Silva *et al.* (2020). These differences highlight the fundamental importance of incorporating the viscoelastic phenomenon when modeling soft tissues, such as porcine knee ligaments.

The load sharing among porcine ligaments, both before and after the development of the viscous phenomenon, exhibited a significant decrease in their force resistance capacity (remaining load capacity). Specifically, the resisting forces for LCL and PCL experienced a reduction of approximately 44% from their initial force-resistant values, while the reduction was slightly less pronounced in the case of ACL and PCL.

Two major conclusions can be drawn from the study: a) the load distribution among the four knee porcine ligaments is non-homogeneous, with the PCL being the most important ligament; b) the significant reduction in the ligaments' ability to withstand loads can be attributed to the viscoelastic effects.

This research targets the healthcare professionals in the field of knee health, such as knee surgeons and physiotherapists. It provides valuable quantitative information regarding knee ligaments, complementing their huge qualitative knowledge.

## 5. ACKNOWLEDGEMENTS

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