27th ABCM International Congress of Mechanical Engineering
December 04-08, 2023, Florianópolis-SC, Brazil

COB-2023-0034
EXPERIMENTAL ANALYSIS AND NUMERICAL-ANALYTICAL DISCUSSION ABOUT THE YOUNG´S MODULUS ON CANTILEVER BEAM UNDER STATICAL ELASTIC STRESS

Marcelo Acacio De Luca Rodrigues
Vila Companhia Fabricadora de Skate
marcelo.lean.engenharia@gmail.com

Bruno Fernandes de Holanda
Saneamento Basico do Estado de São Paulo – SABESP
Fernandes.holanda@hotmail.com

Abstract. *This article aims to describe an experimental procedure to be used on strength of materials or solid mechanics classroom to measure the Young´s Modulus as a direct constant obtained from a cantilever beam fixed at one end, loaded with a range of static deflection force. This experimental procedure is suggested to graduate students to obtain experimental results which are compared with classical analytical modelling and with finite element analysis (FEM). The geometric details of the structure are also measured and modelled by the students. The main goal of this article is to develop the perception of interconnectivity among experimental results, the classical modelling and the FEM analysis. Another important point is a worthy discussion about the results deviation which must be emphasized whenever the values have different magnitude or tendency. The writing skill of students are also worked out. It is easily observed that engineering courses are reducing the long-term workload and this experimental procedure converges to a reasonable professional qualification. A good agreement was found between the Young modulus obtained on bending of a cantilever beam and the classic value assumed for structural steels.*

Keywords: *Young Modulus, cantilever beam, cognitive skills, deflection, engineering courses*

1. INTRODUCTION

The different practical activities on laboratory classes have promoted a significant increase on cognitive skills for engineering graduate courses in the last few decades. However, the reduction at the course load for both theoretical and practical classes or even the exclusion of the subject haven´t brought on the development of laboratory practices. The subjects Strength of Materials and Solid Mechanics are considered theoretical topics in most of the engineering graduate programs, but the strictly necessary physical and mathematical foundations added to the intrinsic difficulty of the subject itself have pointed out the need of laboratory practical activities.

Nowadays an effective subject crossing and integration of different topics in engineering courses is sought despite the actual reduced course loads and a required wide professional qualification. It means that several topics are connected in one single laboratory activity which must fulfill the reduction of the load. Materials science and design, Calculus and even Classical Mechanics are related with the Strenght of Materials and it suggests that a laboratory practical activity must help on the understanding of all these topics coupled.

It is important to point out an unavoidable increase of the application of numerical results based on the Finite Element Method on engineering courses. However, it is also observed that some programs are not offering a reasonable foundation of this computational numerical method and most students are about to apply it on product design without the complete skill for understanding some important details as the constitutive material model or even the mesh refining concept further than the simulation itself.

Another important consideration of this article is to present a scientific routine which can be taught in the classroom and fully used on design planning which connects analytical results with numerical and experimental observation. This connection must emphasize the all the results must be presented and discussed with no tendency for attending a good or a bad expectation. Science must highlight a result as it is instead of following a tendency or expectation which unfortunately leads the designer to wrong conclusions.

Finally, this investigation has no intention to substitute the classical tensile test for measuring the Young Modulus for metalic materials. However, considering the several times when this three steps method for investigating the Young Modulus using a simple Cantilever beam submitted to an external static load, it has been possible to consider that the values measured on the experimental procedure and the finite element modelling are in a very good agreement with the analytical development based on classical equation derived from linear elastic differential equation proposed by strength of material subjects on engineering courses.

2. BIBLIOGRAPHICAL REVIEW

2.1 The three steps method for teaching hybrid topics on Strenght of Material

The actual workload of engineering courses is decreasing on every reform plainly authorized by the authorities on engineering in Brazil. Some important topics are not discussed the way it is suggested to be. And some others were fully eliminated. There are important aspects of teaching Strength of Material and Structural Analysis that are not presented anymore. The reasons are the lack of time, applicability to local professional situations and finally the interests of these students. So, it's useful for the engineering courses to develop a method to overcome these difficulties.

This method of teaching can be considered as an active development. Firstly, the students are invited or even challenged to solve a simple problem of bending or any other topic based on theoretical foundations. It is important that theoretical classes occur before the two next steps. In some cases, the theoretical aspects can be presented along the practical class when the experimental procedure in fact takes place. It's useful for these students to get theoretical classes as close as possible to experimental development. The teacher will always remind the main equations and concepts during the experimental procedure. Another important detail that must be justified by the teacher is the objective of this study not only for the topic Strength of Material but all the scientific structured method that can be applied elsewhere.

The second step is to use commercial software to solve the same theoretical problem already discussed. Several aspects of the numerical method of solution are presented during the use of the software. The teacher must be aware that it's not a finite element method class. On the other hand, it's an application for this method. It differs. from a specific class of finite element method because the main point is to compare the results between the theoretical development and the software application. All the discussion is instigated by the teacher to emphasize the convergence or not of both results. The teacher must conduct the finite element method analysis with some variations in the details of geometry that helps the students to understand further than one exercise already solved theoretically.

Finally, the students develop an experimental procedure to support a complete discussion within the method. All practical procedures have an advantage compared to theoretical class because of the real physical observation of the problem. A cantilever beam is easy to imagine or sketch. The real structure when submitted to a load condition will offer a complete interpretation of a variety of terms and details discussed before with easier understanding. The student must be concerned that the easier it becomes on practical development just because the theoretical foundation has been presented respectably. Sometimes this class can be the same classroom with a few materials. The teacher must ask the students to perform. On the beginning the teacher leads one or two step of the practical method, but after that the students must execute. It is important to students to realize during the execution what kind of practical errors can happen on the execution specifically on measurement, and what could be improved to achieve a proper result which represents the scientific truth of the experiment.

There are some other skills on parallel that are important for the students like writing a report, creating a presentation with graphs and equations and working on groups of students to interact and conclude about the study. The teacher must emphasize this active method and prepare all the detailed structure of the three steps in advance. Every time a new idea to increase the efficiency of the method is found during the class, it must become a part of the method next time. It is suggested about twelve hours divided into four classes to conclude this activity. Table 1 presents the holistic consideration of the method.

At the end, it is important to evaluate all the execution and formalize the results because it can be used as previous examples and review for future experimental development. This methodology is a solution to the problem of the decrease of workload but it goes further than university activity. The university must prepare the students to think and solve the problems with a minimal scientific perspective. It is seen in practical professional application of Engineering that some scientific routines are wrongly considered a waste of time. And only the numerical solution is applied. This mistake is overcome when the engineer does confidently the foundation for solving a problem considering at least two of these three steps. The culture at regular actual industry is having no time for building a reasonable scientific answer for a technical problem. And because of that these three steps method try to unity and emphasize how important is the discussion among three aspects around a problem: theoretical, numerical and experimental.

Table 1 – Holistic Interpretation of the three steps method

Step	Detail	Comment
1	Objective	Fully justified and explained. Interactions with other topics on engineering course must be pointed. Indicate the importance. Share the expectations. Define an evaluative mechanism.
	Theoretical development	Execute in classroom. Indicate books to support a self-study. Emphasize the details from other engineering topics. Solve the same problem on the three steps.
2	Numerical analysis	Ask the students to practice some tutorial previously. Use the university structure as much as possible. Apply commercial solvers.

	Computer application	Consider to ask the students to model the geometry previously. Teach every detail of the software routine and explain the classical terms of the FEM method. Create the simulation among the students. Remember it is not a FEM class but a class for an specific application.
3	Experimental procedure	Do the assembly and perform an introductory routine. Ask students to work on different functions during the execution. Discuss the possible errors of execution.
	Report writing	Justify the need to express the ideas in a professional way. Suggest all the computational medias and models to help them to express the scientific routine and results.
	Conclusive achievement	Collect all the reports. Indicate the goals. Discuss the errors of the majority and praise the importance of all the routine.

2.2 The Young Modulus and the linear consideration behaviour

It's easily observed that ductile metals stressed under linearly increasing loads indicate a direct response of strain up to a region where this linear behavior of the relation between stress and strain ends. According to Souza (1974), this region is unstable and a lower and an upper yielding limit can be measured on tensile test. However, the Young modulus is a constant value and its determination on a simple tensile test without an historic cumulative strengthening or hardening will result in the same constant value.

Hooke's law establishes that the relation between strain and stress is the constant value derived from trigonometrical relation under the yielding limit (Souza, 1974), also called Young modulus (E). It is expressed as the ratio of strain (ε) as function of stress (σ). Equation 1 presents the mathematical foundation of the linear ductile behavior of metals up to the yielding region.

$$E = \lim_{\Delta \to 0} \frac{\Delta\sigma}{\Delta\varepsilon} \tag{1}$$

It is common to use the Young modulus as E not only for the tensile or compression, but it is also used on bending without correction, based on the Poisson coefficient (Beer et al, 1982).

There are two important aspects related with the linear behavior of the relation with leads to Young modulus: isotropy and homogeneity (Boresi et. Al., 1993).

A material is said to be elastically isotropic if it characterizes elastic constants invariant under any rotation of coordinates. If the material properties are identical for every point in a member and invariant under translation, it is considered homogeneous. (Boresi et. Al., 1993, Timoshenko, 1934).

If an elastic member is composed of an isotropic material, the strain energy tensile depends only on the principal strains (ε) since for isotropic materials the elastic constants are invariant under arbitrary rotation (Boresi et. al,1993).

An important aspect after discussing the Young modulus is the use of mathematical shape of the function which relates stress and strain. It is the assumption of a constitutive model for the numerical solution used on finite element method.

At the end, it is impossible to neglect the temperature or the strain rate influences on the Young modulus evaluation. On the linear perfectly elastic analysis of the structure both influences are considered invariant.

Metallurgical influences are also important, and it implies statistical modelling to assume the correct value of E. The hardening or strengthening changes the value of the Young modulus. Thermal treatment can also be applied to increase stiffness of the material (Souza, 1974, Callister, 2008). A higher value of the Young modulus indicates a more rigid material and a lower value a flexible one (see equation 1). Steels are considered ductile metals with a linear elastic behavior under the yielding region. The value of the Young modulus is round 200 GPa.

The bending cantilever considered in this investigation applied the A36 Structural Steel (ASM, 2005; NBR 7007, 2007). The figure 1 presents the relation between stress and strain simply obtained on tensile test for different structural steels. The test conditions are usual and the value of E differs rarely among all the samples. It justifies the assumption of 200 up to 210 GPa as the value of Young Modulus for steel. The figure 1 illustrates that although the metallurgical composition and the yielding limit are different, the slope of the relation stress and strain for different grades of steels indicates the same value of E.

The bending of a beam suggests that a transversal load may causes shearing and deflection effects. The equations related on section 2.3 consider only the loads and do not relate the normal or shear stresses with the vertical displacemet or the slope of the beam. However it is exactly the same effect, since the moment o inertia of the cross section and the distance to the neutral axis are known.

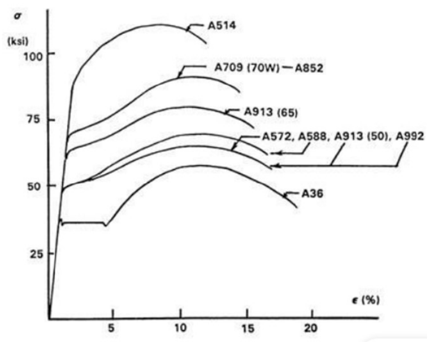

Figure 1: $\sigma - \varepsilon$ relation for different structural steel grandes (ASM, Vol.1, 2005)

2.3 The analytical approach

The suggested structure for this investigation is a single fixed cantilever beam loaded under a static force (*F*) at the free end presented in figure 2. A self-body weight is converted into an effect along the total length as a weight of unit volume (*q*) is also considered.

There are two systems of solution considered for the theoretical approach. The first considers a differential equation of the center line as an elastic line and all the displacements and stress are obtained as a solution of this differential equation (Timoshenko, 1934, Gere, 2003, Beer at al, 1982).

Another system is considered contemporary and wildly applied on engineering courses nowadays. It means to use equations derived from elastic line differential equation. In this case. These equations are found on Strength of Material books, handbooks and class notes everywhere. These are standardized models of structures followed by equations to solve it. Considering this investigation as a discussion of a method to be used on classroom with many different topics on engineering, the suggestion is to use these equations instead of using the elastic line differential equation.

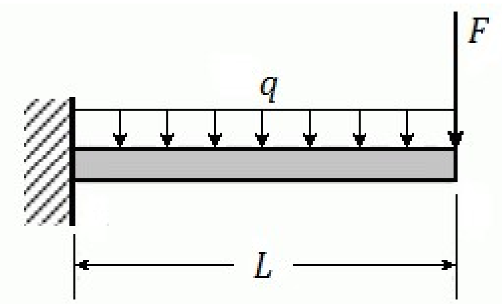

Figure 2: The Cantilever beam submitted to an external force (*F*) and a linearly distributed load (q).

The restricted end is ideally assumed as a total restriction of displacement meanwhile, the free end provides a plane state of distortion.

A pure bending is considered along the total length where the free end presents a geometrical distortion with vertical displacement (δ) and a slope (θ).

It is assumed that the complete length (*L*) has the same moment of inertia (*I*). The material is homogeneous, perfectly elastic and isotropic with a constant value of Young modulus (*E*).

Even though the common assumption of the self-body weight converts the total mass (m_b) onto a singular load at the centroidal position of the body, this consideration results in a different distorted profile, since the load at the centroidal position would submit the body on a bending moment which a free end beside the load do not deflect accordingly to the effect easily observed when the self-body weight affects the structure. So, the self-body weight is converted into a linear distributed load which provides a progressive increase of the bending moment according with the mass distribution.

The static load added at the free end is a simple mass directly converted onto a load by the second law with local gravity (*g*) as 9,816 m/s².

According, to Gere (2003) and Beer et al (1982), the vertical displacement (δ_F) and the slope (θ_F) since an external load *F* is applied in the free of a a cantilever beam end can calculated as:

$$\delta_F = \frac{FL^3}{3EI} \tag{2}$$

$$\theta_F = \frac{FL^2}{2EI} \tag{3}$$

Based on the same authors, the effects of the linearly distributed load (q) are:

$$\delta_q = \frac{qL^4}{8EI} \tag{4}$$

$$\theta_q = \frac{qL^3}{6EI} \tag{5}$$

The conversion of the self-body weight in linearly distributed load is:

$$q = \frac{m_b.g}{L} \tag{6}$$

The total of superimposed effects are:

$$\delta_T = \delta_F + \delta_q \tag{7}$$

$$\theta_T = \theta_F + \theta_q \tag{8}$$

And manipulating the equations 7 and 8 to obtain de experimental Young Modulus for the total vertical displacement ($E_{\delta exp}$) and slope ($E_{\theta exp}$) respectivly:

$$E_{\delta_{exp}} = \frac{L^3}{I\delta_T}\left(\frac{F}{3} + \frac{q.L}{8}\right) \tag{9}$$

$$E_{\theta_{exp}} = \frac{L^2}{2I\theta_T}\left(F + \frac{q.L}{3}\right) \tag{10}$$

According to Beer at al (1982), the distorted configuration assumed after the loading condition is showed on figure 3. It is important to conclude some geometrical relations after the loading. A curvature radius (ρ) is considered as constant along the total length and the curvature (κ) is:

$$\kappa = \frac{1}{\rho} = \frac{M}{EI} \tag{11}$$

The equations 12, 13 and 14 are geometrical considerations directly extracted from the trigonometrical description of the distorted configuration, where C is the length of total circumference described by the radius ρ.

$$C = 2\pi\rho \tag{12}$$

$$\bigwedge_{L=\gamma}^{C=2\pi} \therefore\ \gamma = \frac{2\pi L}{C} \tag{13}$$

$$\bigwedge_{\theta\perp\psi}^{\psi+\gamma+90°=180°} \therefore \bigvee_{-\gamma}^{180°-\gamma} = \theta \tag{14}$$

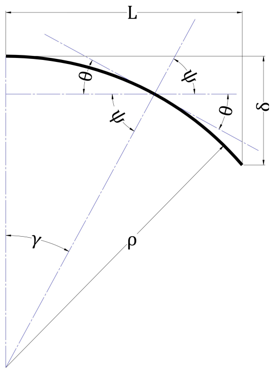

Figure 3: Distorted configuration and trigonometric relations.

2.4 The finite element method for basic systems on classroom

The Finite Element Method (FEM) is a numerical technic for solving differential equations (Buchanan, 1994) under boundary conditions which indicates that the structure can be simplified in a large number of small elements connected by them nodal points. The linear elastic one-dimension static structures are initial problems to be studied with FEM (Bathe, 1996) on classroom even with some analytical solution classically formulated by different authors.

The complexity of the structures is simplified into an easier problem when the elements are modelled separated, and the matrixial notation allows the coupling of the elements and results on the total structure behavior. One-dimension problems are the most common example to be discussed on classroom to illustrate the relation on equation 15, where $[F]$ is a vector of forces applied on the nodal points, {K} is a stiffness matrix related to geometrical and material properties of the structure and {u} is a vector of nodal displacements.

$$[F] = \{K\}[u] \tag{15}$$

The presentation of the solution for equation 15 is in FEM class. Equation 15 is the simplest way to introduce the FEM method, and this is the base for understanding how transient problems with higher complexity are solved. The details for solving it by computer are not discussed in this method because the application with software is done to help students with all the simulation routine.

There are several softwares with different solvers (Bathe, 1996). However, two details can be discussed during this step of the method: Mesh refining and material constitutive model. The simulation is supposed to converge or not to the analytical result if the mesh has a considerable number of elements which the convergence saturates, and no difference is achieved between analytical and numerical results. Finer mesh takes longer time to process. Coarse mesh increase the convergence error (Bathe, 1996). Isoparametric elements are the initial consideration for both fine or coarse mesh (Solidworks, 2010). Figure 4 shows to details of mesh.

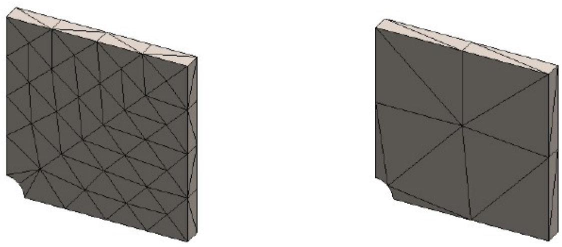

Figure 4 – Mesh detail, fine (left) and coarse (right) (Solidworks, 2010)

The constitutive model is related with how the stiffness matrix influences the answer. The assumption of perfectly linear elastic material is reasonable for A36 structural steel according to figure 1. It means that all the analysis under the yielding point considers a elastic response of the material.

The students must be able to manipulate not only the simulation routine on the but also discuss the numerical answer related to both results theoretical and experimental and consider it enough, correct and reliable for the pupouse of design analysis of the structure.

2.5 Practical approach of Structure analysis

Strength of materials and solid mechanics are considered theoretical topics on engineering courses. However, the difficulty of the topics suggests laboratory practices to support the students to understand complex concepts. Not all the concepts. Are discussed. Oh, laboratory classes. But. Those aspects. Which? Integrates. The theory results. And. The real structure behavior.

It is important to consider that students nowadays have less time on practical activities at university. And every time a practical activity is suggested a part of these students are aware and join in. By other hand some others give little importance to that. The social interpretation of the importance is not the main goal of the relation between these students. Some students intended to conclude the course with as less as possible among other students.

The practical activity takes place once and the audience shall be organized to participate. The results are measured and modelled by the students and the presentation of that must be tuned with the previous two steps of the method. Charts, tables and descriptions must be produced after the three steps.

No extra practical activity is asked. The outside class duty is to consolidate all the information and give it back to the responsible as soon as possible.

A final discussion to conclude all the routines for the three steps is the correct way to conclude the method when the final considerations are collected.

3. EXPERIMENTAL PROCEDURE

The experimental procedure is based on three steps method (see section 2.1). The first step is to discuss an analytical model (see section 2.3) which must be the most correct description of the physical set up to be observed and measured. The second step is to develop a finite element modelling with all the assumptions as close as possible to the analytical model (see section 2.4). The last step is the execute an experimental activity (see section 2.5) where the students will effectively develop a physical model and submit it to condition previously suggest by the analytical and FEM modelling.

3.1 Theoretical-Analytical solution

The analytical problem is to obtain vertical displacement and slope of a cantilever beam as shown in figure 2. Detailed data are on table 2. The Young modulus in this case is assumed to be 200 GPa. The sequence of equations 2 to 8 are used to obtain the analytical curve shown in figure 5. As a result, obtained in the direct use of the equations the answer is the vertical displacement and slope. The regression factor shall be the unit because it is a linear behavior for both results. The students are advised to organize these results on charts and tables.

Table 2 – Analytical inputs

Total Length - L (mm)	590
Moment of inertia of the cross section - I (mm^4)	50,809
Weight of unit volume - q (N/mm)	0,00449
Mass – m_b (g)	270

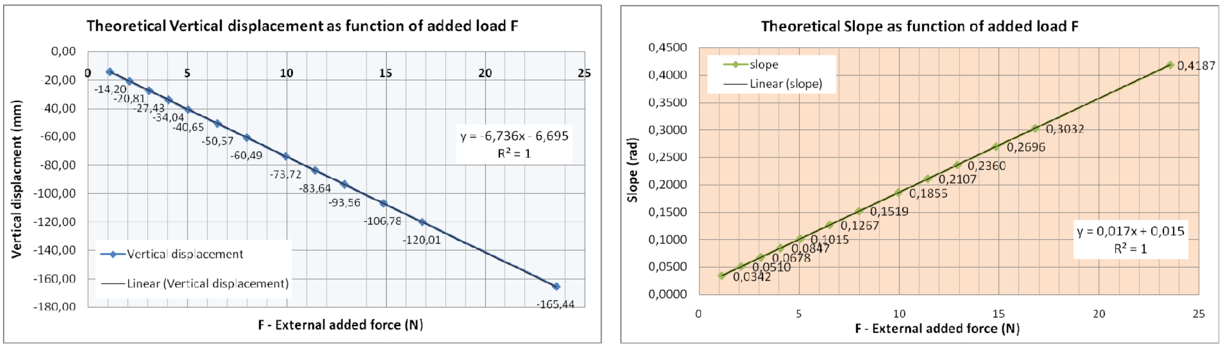

Figure 5 – Theoretical results according to equations 2 to 8.

The set of equations from 2 to 8 are the theoretical description derived from the differential elastic line equation and indicate a linear behavior between the force (F) and the displacement and the slope. It justifies the correlation factor R equals a unity.

3.2 Finite element method solution

The finite element analysis was performed on Solidworks (Solidworks, 2010) software applying the same input parameters described on table 2. The structure was submitted to twelve load steps from 2,09N to 23,56N without linear increase among the load steps. The loads are following the same steps applied on the experimental procedure. A standard fine mesh with isoparametric elements with 4 nodal jacobian points were considered. The triangle was the shape of the elements with the side between 0,082mm or 1,646mm. Figure 6 presents the initial configuration while figure 7 and the final distorted configuration. Figure 8 concludes the numerical analysis considering 200 GPa for Young Modulus. The simulation type is static and constitutive material is isotropic linear and perfectly elastic.

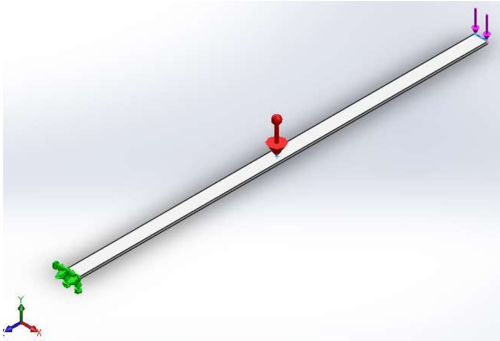

Figure 6 – Initial configuration of the structure. Purple arrow is the external load. Red arrow is the self body weight.

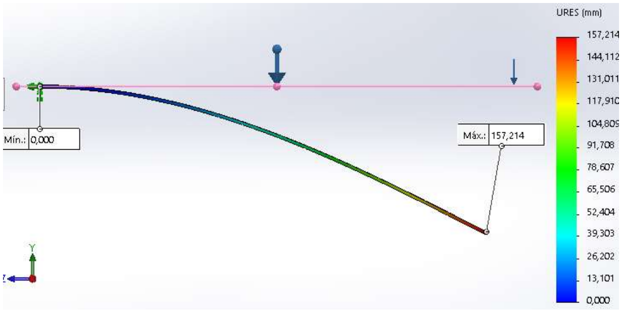

Figure 7 – Final configuration of the structure. It is possible to affirm that the curvature κ is not constant.

The FEM analysis for vertical displacement plainly agrees with the linear elastic behaviour justified by the correlation factor as a unity.

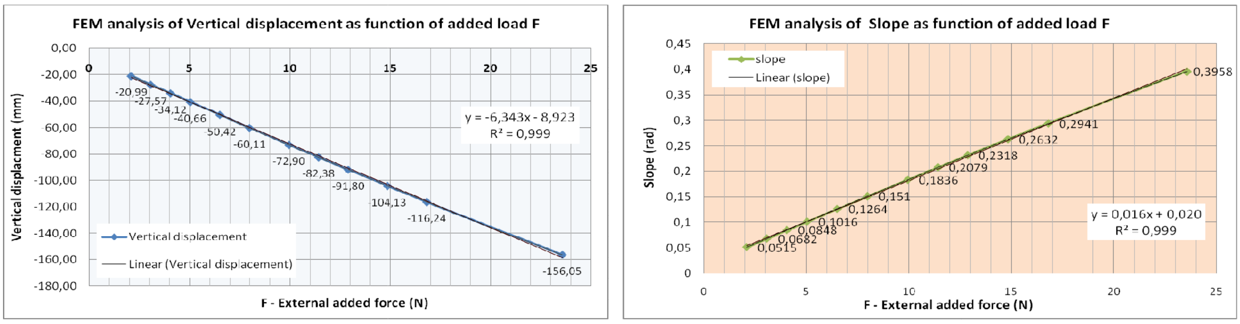

Figure 8 – vertical displacement and slope with FEM analysis

3.3 Experimental procedure

The experimental procedure was performed by students and teacher together. Figure 9 illustrates the structure and the measurement system. It is observed that the vertical displacement was measured by a fixed rule. A single cantilever beam fixed on one end and the opposite side is free. The load steps were from 2,09N to 23,56N without linear increase among the load steps. A narrow pointer was taken as the reference beside the vertical scale. A cable and a hook were applied to support the added masses. The hook mass begins the evaluation. Each step was unloaded before adding a new mass. This detail is to certify that no plasticization took place. The structure was a sample of 700mm bar of ASTM A36 structural steel with a cross section of ¾" x 1/8” as received from the market for producing welded structures. The bending length was 590 mm.

The experimental slope was not measured but calculated according to equations 12 to 14. The Young modulus considered for calculating the experimental slope was those obtained from the vertical displacement according to equation 9.

Figure 9 – Experimental set up and measurement procedure performed by students.

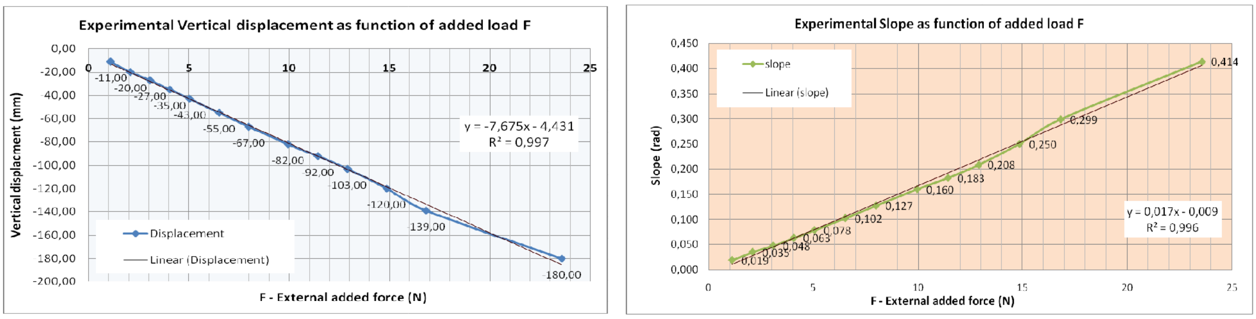

Figure 10 – Experimental results after measurement. The R coefficient close to unity indicates linear behavior.

4. RESULTS AND DISCUSSION

The next figures present the results compiled altogether for the three steps method and the experimental Young modulus evaluation. It is possible to affirm based on both graphs at figure 11 that a good agreement is reached as a linear elastic behavior. The error is lower than 5% to every step load on figure 11. All the correlation factors are close to unity which means acceptable convergence. The experimental slope was calculated using the experimental Young modulus obtained from the vertical displacement with an average equal to 191,94 GPa (coefficient of variation equals 2,86 %).

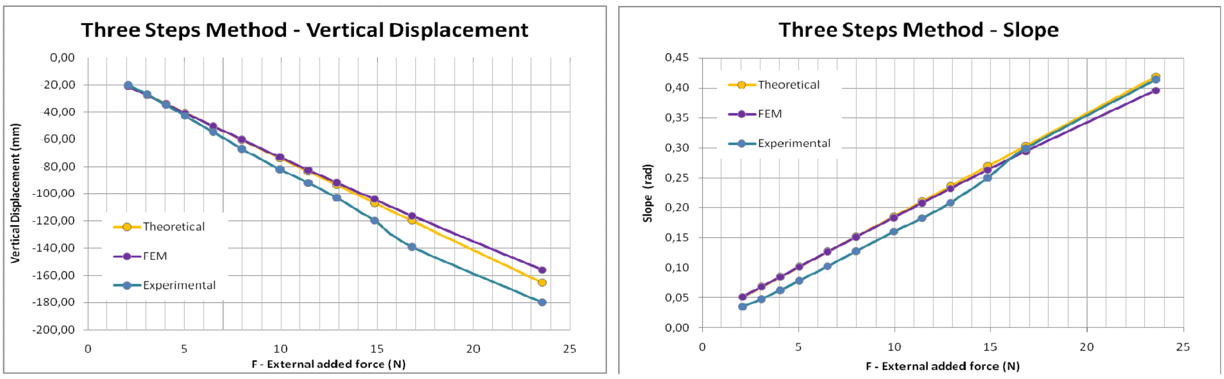

Figure 11 – Final results of the three steps method.

Figures 12 and 13 present the Experimental Young modulus obtained from vertical displacement and slope respectively (equations 9 and 10). The low correlation factor is easily justified because the *E* is constant and no inclination results in a non-existent correlation. A lower R indicates a better agreement. The experimental Young modulus for the slope has an average value of 250,57 GPa (coefficient of variation equals 4,65%).

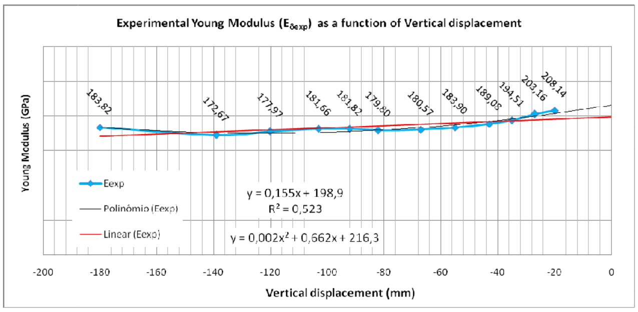

Figure 12 – Experimental Young Modulus for vertical displacement.

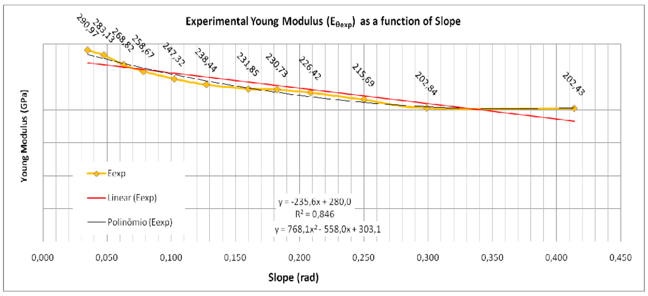

Figure 13 – Experimental Young Modulus for slope.

5. CONCLUSION

It is possible to affirm that the experimental Young Modulus obtained from a bending of cantilever beam has a good convergence considering the vertical displacement because a close but lower value was obtained. Despite it is lower (191,94 GPa < 200 GPa) the engineer must compensate the geometry in order to attend the desired operational condition of the structure. The Young modulus obtained from the slope has a convergence upper than the usual value (250,57 GPa > 200 GPa). Every time a comparison between the two values of experimental *E*, the lowest must be chosen because it suggests the increase of momentum of inertia which results in a higher stiffness. A safer design is reached when the geometry increases to satisfy a lower Young Modulus.

Another important observation is that the curvature κ in not constant (figure 7). Both FEM and experimental analysis showed that distorted beam is not a perfect radius. As close as the length comes to the fixed side the profile seems to be a radius and as far as the length comes to the free side the beam is not deformed equally.

A slope measurement could help to define $E_{\theta exp}$ instead o calculating that based on the $E_{\delta exp}$. Even though the *E* is the same value, there is a difference between $E_{\theta exp}$ and $E_{\delta exp}$. This difference is also justified by irregular distorted profile because the slope is different because of the non constant curvature.

The theoretical superimposed results from equations 7 and 8 are in total agreement with FEM (Figure 11).

The three steps method has proved its efficiency on teaching for several times and contributes to develop the interest and application of classical concepts of Strength of Materials and Solid Mechanics.

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