27th ABCM International Congress of Mechanical Engineering
December 04-08, 2023, Florianópolis-SC, Brazil

COB-2023-0759
ANALYSIS OF STRESS CONCENTRATION IN PSEUDOELASTIC THIN SHEETS USING THE FINITE ELEMENT METHOD

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Abstract. *Shape Memory Alloys (SMA) are materials that have a strong thermomechanical coupling associated with phase transformation processes. These alloys have special characteristics in terms of their stress-strain-temperature behavior, making them attractive for use in various applications of interest in engineering, such as actuators, intelligent structures and mechanical vibration absorbers. When submitted to thermomechanical loadings, SMAs present a complex nonlinear behavior and require the use of specific methodologies to evaluate their performance and integrity. The design of mechanical components with complex geometries, involving geometric discontinuities such as notches and holes, has been treated through simplified mechanical design approaches involving stress concentration factors (SCF), both for static loads and during cycles of mechanical and/or thermal loads. The presence of plastic deformations increases the complexity of the analysis due to stress redistribution and non-linearities present, and mechanical design approaches involving elastoplastic stress concentration factors have also been used. Similarly, strains associated with the phase transformation process in shape memory alloys promote a stress field redistribution effect. In this work the phenomenon of stress concentration in thin sheets of pseudoelastic SMAs is investigated through numerical simulations using a model based on finite elements. The effect of the redistribution of the stress field resulting from the phase transformation process is investigated, with the objective of identifying specific concentration factors. The pseudoelastic behavior is modeled using a constitutive model for SMAs. The results show different behaviors from those observed in materials with elastic and elastoplastic behavior, indicating the importance of considering the effect of phase transformation during the loading mechanical components.*

Keywords*: Shape Memory Alloys, Pseudoelasticity, Stress Concentration, Finite Element Method, Modeling*

1. INTRODUCTION

Shape memory alloys (SMA) are classified as multifunctional materials, i.e., structural materials that meet additional functionalities, such as performance, sensing or electromagnetic sealing (Lagoudas, 2008). They are classified as multifunctional materials that convert non-mechanical signals (e.g. thermal, magnetic, optical, etc.) into mechanical response.

Considering the various types of shape memory alloys, nickel and titanium alloy has a wide application (Lecce, 2015) and is the target of this work. The shape memory effect occurs due material phase transformations. The present phases are austenite, a phase with a simpler and more orderly crystalline structure that is stable in a stress-free state at high temperatures; and martensite, with a more complex crystalline structure and stable in a stress-free state at low temperatures (Lagoudas, 2008).

The pseudoelastic behavior occurs at temperatures higher than the final austenite transformation temperature (A_f). When applying the load, reorientation to detwinned martensite occurs, and during the unloading there is a total reversal of the strain as well as the return to the austenite phase (Lagoudas, 2008).

The study of the stress concentration factors (SCF) in mechanical components with notches is a determining factor for the dimensioning of parts in relation to the initiation and propagation of cracks according to Simoes and Martínez-Pañeda (2021) and Hasan, Baxenavis (2019). The stress concentration also in has influence on damage behavior, according to Phillips et. Al (2019)

Some studies can be found on the literature that addresses this subject. Shariat *et al.* (2014) and Xiao *et al.* (2016) had performed experimental investigations on pseudoelastic plates with holes and semi-circular notches and they described the influence of the geometry discontinuity on the volumetric phase distribution when compared with unnotched plates. Zhu *et al.* (2014) had used an elastic-transformation-plastic constitutive model to simulate the interaction among holes in pseudoelastic models and compared the numeric results with the data obtained by the DIC (Digital Image Correlation) technique, the comparison of finite elements and DIC results is also approached by Romanowicz *et al.* (2020)

but in this case for elastoplastic materials and show the possibilities of such comparison. Phillips *et al.* (2017) studies the stress concentration in pseudoelastic round bars with different notch geometries, the results indicate that even with the same minimum cross section the large and small notches have very different results when compared with the smooth specimen due to the different transformation patterns. Hosseini and Ashrafi (2022) performed a stress concentration analysis in thin sheets with circular cavities to analyze the effects on phase transformation and plasticity. The authors also present the stress concentration factors along the load application considering the diameter and the number of holes. The present work presents an analysis of different approaches for the determination of stress concentration factors in pseudoelastic thin sheets.

2. STRESS AND STRAIN CONCENTRATION FACTORS

In the design of mechanical components, geometric discontinuities are responsible for the increase in stresses and strains when comparing to regions far from stress concentrations. As it can significantly affect the stress and strain fields in mechanical components, and consequently its mechanical integrity, it is important to consider the effects of geometric discontinuities in the mechanical design of components.

The classic approach adopted in machine design of mechanical components considers that far from the geometric discontinuities, such as holes, the mechanical component is in the elastic linear regime. Therefore, nominal stresses are calculated in places far from the geometric discontinuities using simple solid mechanics equations and a correction factor is applied to assess values of maximum stress near these regions.

In the elastic regime, the stress concentration factor (k_t) can be calculated as the ratio between the maximum stress (σ_{max}) and the nominal stress (σ_{nom}), a stress far from geometric discontinuity:

$$k_t = \frac{\sigma_{max}}{\sigma_{nom}} \quad (1)$$

In the case of the strain concentration factor, the calculation is made in a similar way to that of kt, that is, the rate between the maximum strain (ε_{max}) and the nominal strain (ε_{nom}):

$$k_\varepsilon = \frac{\varepsilon_{max}}{\varepsilon_{nom}} \quad (2)$$

There are several analytical solutions for concentration factors in linear elastic regime, as the Kirsh problem (Pilkey, Walter, 1997) , which studies the stress field in an infinite plate with a circular hole subjected to a uniaxial stress loading applied far from the hole.

The Kirsh analytic solution predicts a stress concentration factor equal to 3 associated to the maximum stress observed at the edge of the hole of an infinite linear-elastic plate. The value of k_t is different for finite plates, when considering the effect of the edge of the plate. Similar values of strain concentration are observed.

For elastoplastic regime that can be present near geometric discontinuities, the stress concentration factor is different from the one calculated for the linear-elastic regime. After the yielding, the stress concentration factor (k_t) decreases while the strain concentration factor (k_ε) decreases, as shown in Fig. 1. This behavior is associated with a process of redistribution of the stress field in the region near the geometric discontinuity.

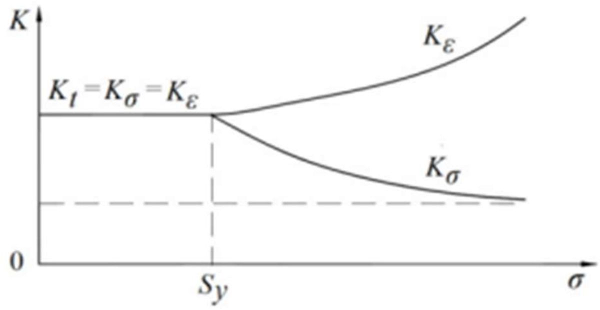


Figure 1 - Elastoplastic stress and strain concentration factors (Gonzáles, 2018)

For these situations, there are other forms to calculate the stress concentration factor. The Neuber rule (Young and Budynas, 2002) considers the stress concentration factor as the square root of the ratio between the product of the variations of the stress ($\Delta\sigma$) and the strain ($\Delta\varepsilon$) and the product of the variations of the nominal stress ($\Delta\sigma_n$) and nominal strain ($\Delta\varepsilon_n$):

$$k_t^2 = \frac{\Delta\sigma . \Delta\varepsilon}{\Delta\sigma_n . \Delta\varepsilon_n} \quad (3)$$

Eq 4 was used to calculate the equivalent strain of von Mises, which takes into account the principal components of strain. Thus, it is possible to study the stress concentration factor according to Neuber's rule.

$$\varepsilon = \frac{2}{3}\sqrt{\varepsilon_1^2 - \varepsilon_1\varepsilon_2 + \varepsilon_2^2} \tag{4}$$

As in the case of plastic strains that promotes the redistribution of the stress and strain fields near discontinuities, martensitic transformations present in regions close to the notch in pseudoelasticity components can significantly alter the stress and strain distribution in this region, affecting the stress concentration values, especially for cyclic loadings. Therefore, it is important to study how phase transformation affects stress concentration in pseudoelastic components.

3. CONSTITUTIVE MODEL

In this work, a constitutive three-dimensional model proposed by Auricchio (1996, 1997) that considers isothermal pseudoelasticity and the Drucker-Prager criterion to describe critical stresses for pseudoelastic transformation (Lagoudas 2008) is used to study the behavior of thin pseudoelastic plates with holes.

The formulation, implemented in Abaqus computational package (Abaqus, 2018), is based on the additive decomposition of the strains, where the total strain is found by the sum of the elastic and transformation components (Lecce 2015):

$$\Delta\varepsilon = \Delta\varepsilon^{el} + \Delta\varepsilon^{tr} \tag{5}$$

where $\Delta\varepsilon, \Delta\varepsilon^{el}$ and $\Delta\varepsilon^{tr}$ are the total, elastic and phase transformation strain increments, respectively.

The constitutive model considers that the elastic modulus and the poisson ratio under intermediate transformation conditions obey the rule of mixtures:

$$E = E_A + \varsigma(E_M - E_A) \tag{6}$$

$$\nu = \nu_A + \varsigma(\nu_M - \nu_A) \quad . \tag{7}$$

The typical uniaxial stress-strain curve for the constitutive model, with critical transformation stresses values (σ_{tL}^{S}, σ_{tL}^{E}, σ_{tU}^{S}, σ_{tU}^{E}) and transformation strain (ε^{L}) is presented in Figure 2.

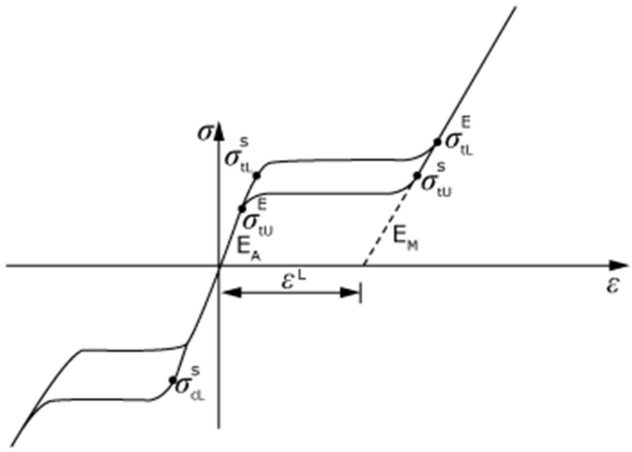


Figure 2- Typical uniaxial stress-strain curve of the constitutive model for pseudoelasticity (Abaqus, 2018).

Incremental phase transformation strain is calculated according to the rule:

$$\Delta\varepsilon^{tr} = \Delta\varsigma\frac{\partial G^{tr}}{\partial\sigma} \quad . \tag{8}$$

where G^{tr} is the transformation flow potential, given in the form of the Drucker-Prager model:

$$G^{tr} = q - p\,tan\,\psi \tag{9}$$

where p the hydrostatic pressure and q the of von Mises equivalent stress, defined as:

$$p = -\frac{1}{3} tr\sigma \tag{10}$$

$$q = \sqrt{\frac{3}{2} S : S} \tag{11}$$

The phase transformation surface (F^{tr}) is given by the Drucker-Prager equation and varies linearly with temperature:

$$F^{tr} = q - p \tan\beta \tag{12}$$

Typical critical transformation stress-temperature curves for the constitutive model are shown in Fig. 3. The critical transformation stresses and linear coefficients of temperature dependence of these curves are represented in the figure.

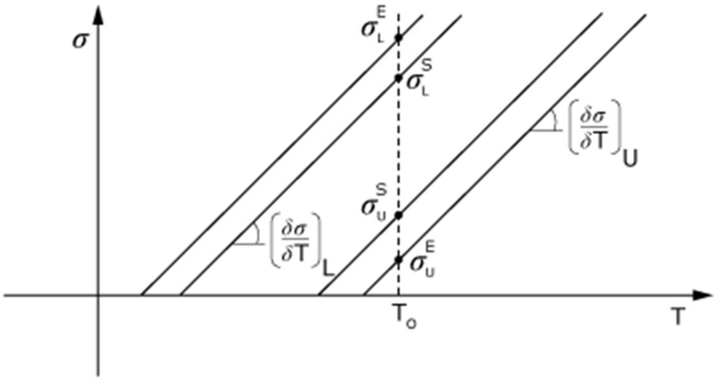


Figure 3- Typical critical transformation stress x temperature curve (Abaqus, 2018)

This constitutive model for isothermal pseudoelastic behavior has the possibility of including the hardening effect during the phase transformation process.

4. NUMERICAL MODEL

The aim of this chapter is to present the methodology adopted to study the effects of stress concentration in the stress and phase transformation fields for thin pseudoelastic shape memory alloys plates with holes under unidirectional traction. Numerical simulations using a constitutive pseudoelastic model implemented in finite element package Abaqus is used to study the geometries.

Initially, a comparison was made between experimental results developed with a pseudoelastic SMA wire and a one-dimensional finite element numerical model to calibrate the parameters of the constitutive model used. The parameters of the constitutive model used were calibrated using experimental data obtained by Adeodato *et al.* (2022) for a 0.9 mm wire pseudoelastic shape memory alloy at a temperature of 323 K. A wire with a diameter equal to 0.9 mm was modeled using truss elements. The wire is fixed in the left end and submitted at the right end to a loading-unloading cycle with maximum prescribed displacement of 11%. Figure 4 shows the mesh of the finite element model for the SMA wire. Table 1 shows the constitutive model parameters obtained after the calibration process. Figure 5 shows the experimental and simulated results, where a good agreement is observed.

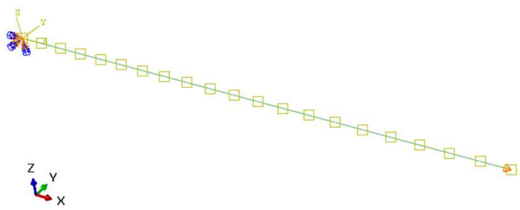


Figure 4- Finite element uniaxial model of a SMA pseudoelastic wire.

Table 1 - Input parameters of the constitutive model

E_A	ν_A	E_M	ν_M	ε_r	σ_{tL}^S	σ_{tL}^E	σ_{tU}^S	σ_{tU}^E	σ_{cL}^S	T_0	C_A	C_M
MPa		MPa			MPa	MPa	MPa	MPa	MPa	K	MPa/K	MPa/K
45700	0.3	17400	0.3	0.055	523.6	587.1	314.6	270.3	250	323	7	6.5

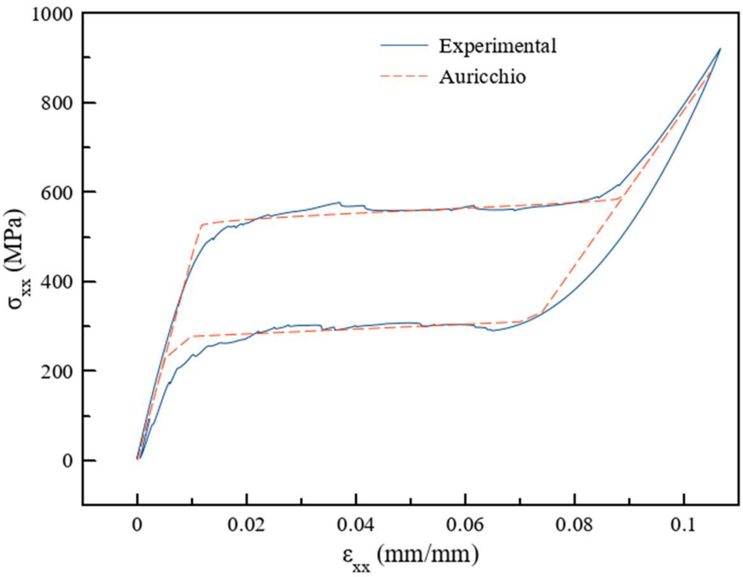
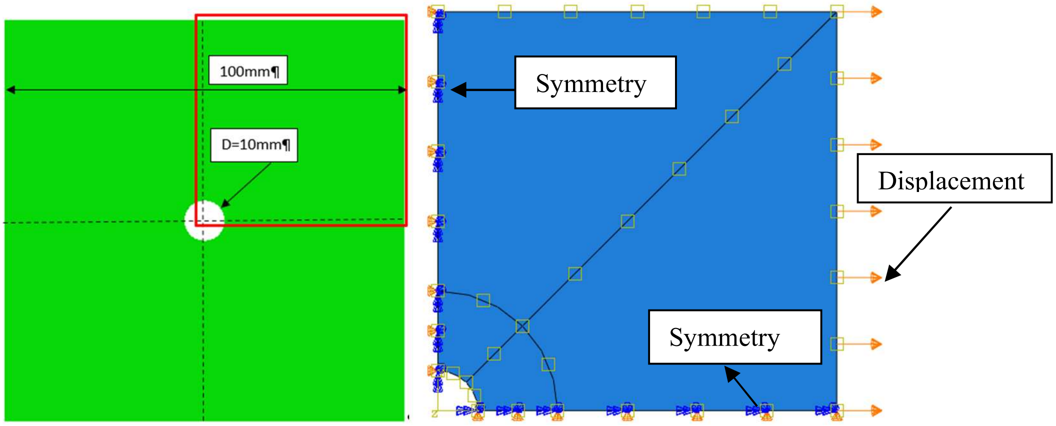


Figure 5- Numerical and experimental stress-strain curves for a pseudoelastic wire at a temperature of 323 K.

For study the effect of stress concentration in pseudoelastic shape memory alloys, a square plate with a width of 100 mm, 1mm of thickness and a circular hole of 10 mm diameter is considered. Figure 6 shows the geometry of the plate with a hole and the prescribed displacement loading in the horizontal direction. Symmetry conditions in the horizontal and vertical directions, and a plane stress condition are used in the numerical model.



(6) (b)

Figure 6- Model geometry (a) and analyzed region with boundary conditions and prescribed horizontal displacement loading (b)

An initial constant temperature condition of 323K is applied and a prescribed displacement loading and unloading stages with a maximum value of 10 mm applied at the right side, as shown in Fig. 6*b*, which corresponds to a nominal horizontal maximum strain of 20%.

It has been performed a sensitivity analysis for both the mesh refinement and load increment. The final mesh presented on Fig. 7, uses quadrilateral elements with 8 nodes per element (CPS8).

The load increment was set with a maximum step size of 1/500 (each step ends at 1) obtained after a convergence analysis. The model considers geometric nonlinearities associated with large displacements.

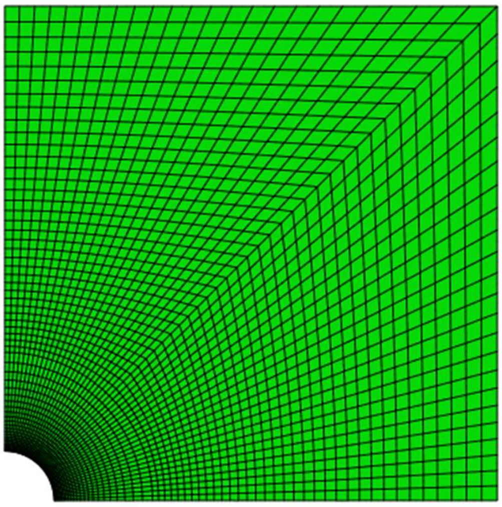


Figure 7- Mesh of the model obtained after a convergence analysis.

5. NUMERICAL RESUTS

Numerical results of the model of the plate with hole are presented in this section. The stress-strain curve at the stress concentrator at the hole (where maximum stress values are observed for a linear-elastic analysis) and in a far region of the plate are shown in Figure 8.

As the objective in this study is to assess the effect of stress concentration in the phase transformation evolution and, as consequence, in the stress redistribution, plastic strain is not considered and, therefore, the values of stress and strain considerably exceed the flow values associated to yielding (a yielding stress of 1.2 GPa is expected for this material).

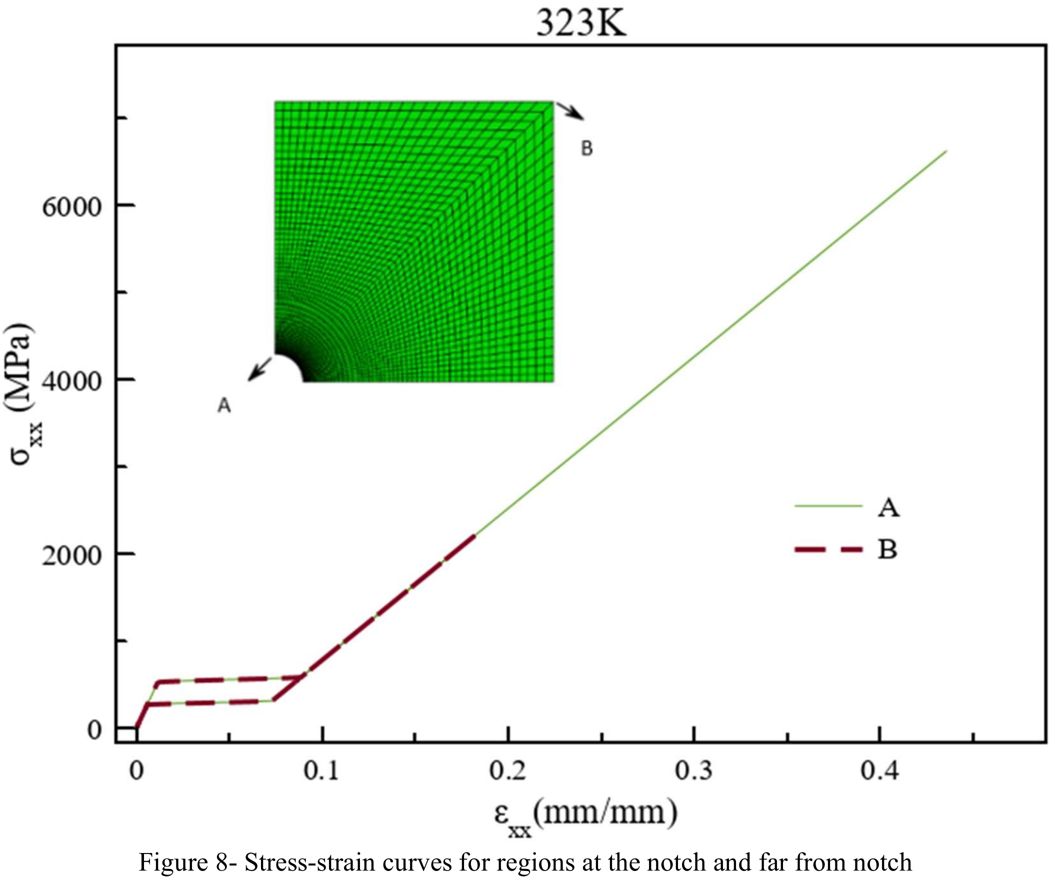
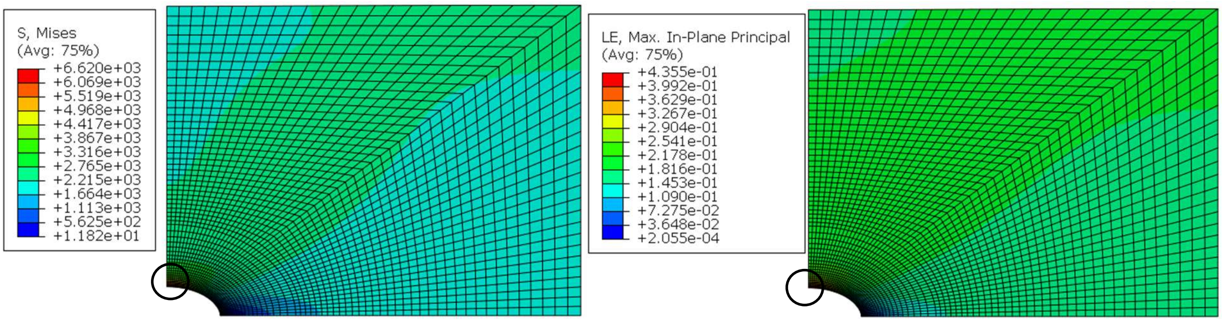


Figure 8- Stress-strain curves for regions at the notch and far from notch

The stress results, considering the geometric nonlinearity (NLG), show values of approximately 6.6 GPa, and there are no significant differences between von Mises stresses, the first principal direction, and the stresses in the horizontal direction. Figure 9 shows the von Mises equivalent stress and maximum principal strain distributions at the end of the loading stage. The region where the highest stress value occurs is identified with a circle in the figures.

Figure 9*a* shows that the highest stress value occurs at the notch at the end of load. Results shows a non-uniform stress distribution and the region near horizontal axis presents lower stress values than the rest of the piece, being the last regions to transform from austenite to martensite.

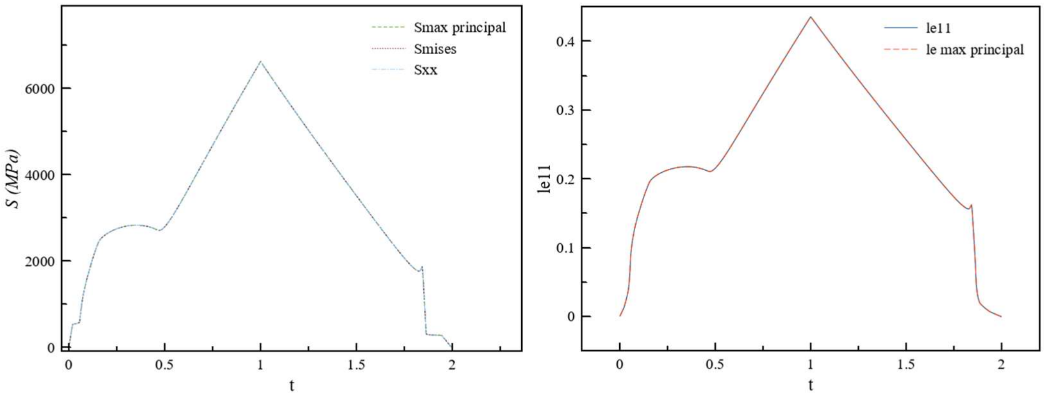
Figure 9*b* shows the maximum principal strain at the endo of load. As with stresses, near the notch occurs a much more expressive strain, and larger than in the region away from the hole. This region is identified in the figure with a circle.



(a) (b)

Figure 9- (a) von Mises equivalent stress (MPa) , (b) maximum principal strain (mm/mm)

The stress evolution for the maximum principal, horizontal direction and von Mises equivalent stress are shown in Figure 10*a*.The strain evolution for maximum principal and the horizontal direction are presented in Fig. 10*b*. All the results are obtained from the node of maximum stress, and components of stress and strain presents, respectively, similar values over the load increment.



(a) (b)

Figure 10- Stress (a) and strain (b) evolution.

The volumetric fraction of martensite at the end of loading stage is shown in Figure 11. It can be observed that the plate undergoes phase transformation in almost all regions. This is due to the high levels of stress and strain to which the plate is submitted. The region away from of the hole, is one of the last regions to be transformed due to the state of stress that occurs in this region.

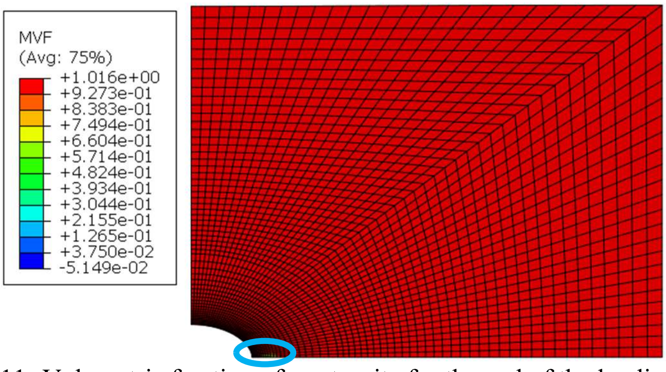


Figure 11- Volumetric fraction of martensite for the end of the loading stage.

The evolution of the volumetric fraction of martensite as a function of the load steps is presented in Figure 12 in 2 regions studied (: in the hole (A) and in a region far from the hole (B). There is a difference in the instant that the phase transformations take place, which influences the calculation of the stress concentration factor in the piece. Therefore, in a given instant of loading, the region where the stress concentration is calculated can present different amount of volumetric martensitic phase from regions far from this point.

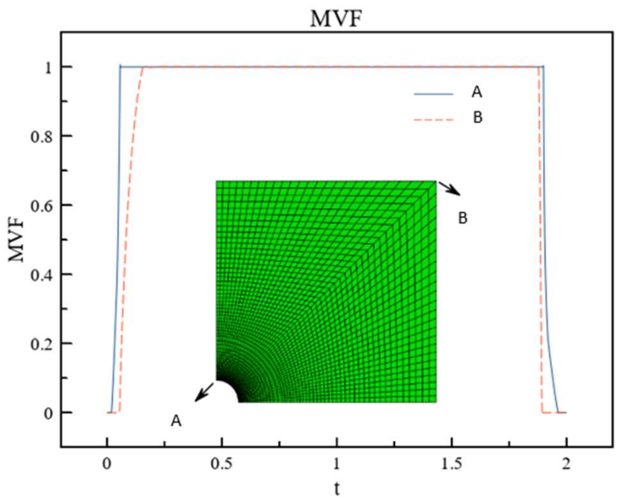


Figure 12- Volumetric fraction of martensite

Fig. 13 presents a comparison with different approaches for calculate the stress and strain concentration factors. The factors are calculated according to eq. 1 for k_t and with eq. 2 for k_ε, whereas the Neuber concentration factor is calculated using eq. 4. These factors are obtained for three components: horizontal direction, principal direction and equivalent component of von Mises.

The Fig. 13 also shows the distribution of the volumetric fraction of martensite at different moments of the loading, illustrating different phase distributions. Initially, in Fig. 14*a*, the plate is almost completely austenitic, the martentise transformation starts at the notch while most of the piece is still in the austenitic phase. In Fig.14*b*, it occurs that phase transformation also occurs in a region away from the notch, near the corner opposite to the notch. In Fig. 14*c*, a more pronounced presence of martensitic phase is observed, which influences the stress concentrations factors. Figure 14*d* shows that almost all regions have transformed to martensitic volumetric phase, and the SCF approaches the values observed in linear-elastic analysis, since the entire part almost has a homogeneous volumetric phase distribution with the presence of martensite and presents a linear-elastic behavior.

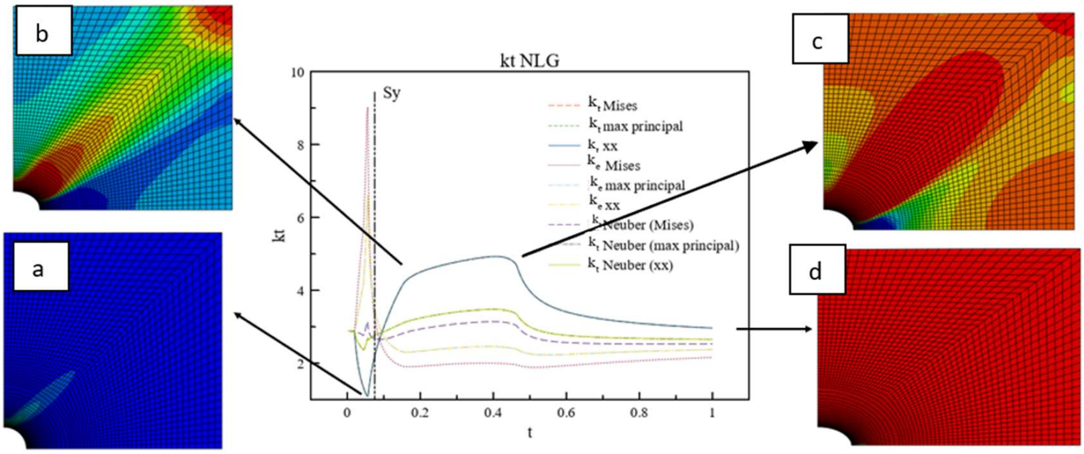


Figure 13 - Stress and strain concentration factors: *(a)* - the notch begins the transformation. *(b):* phase transformation also occurs in a region away from the notch. *(c)* - more pronounced presence of martensitic phase. *(d)* - almost all regions have transformed to martensitic volumetric phase

It is observed that for the stress concentration factors calculated with Eq. (1), all the component values coincide. For the strain concentration factors using Eq. (2), von Mises equivalent component presents values higher than the others. For the concentration factors calculated with the Neuber rule established by Eq. (3), the von Mises equivalent components presents a different curve from the others, due to the influence of the equivalent strain.

Therefore, Neuber stress concentration factors present the same values when considering the maximum principal and the horizontal direction components and have a different behavior from the von Mises equivalent component. The Neuber factor in the von Mises component offers an advantage since it can be used for multiaxial loading.

It is observed that the largest value of stress concentration occurs in approximately half of the loading stage, while the largest strain concentration factor occurs at the beginning of the martensite transformation, where the notch is already completely transformed, while the region far from the notch on the edge of the piece is not. However, the highest stress values occur at the end of loading. This problem can be mitigated with the use of Neuber stress concentration factor

Comparing the results of the linear elastic stress concentration with the pseudoelastic ones, it is possible to observe that the maximum stress concentration results in a pseudoelastic component has a higher value, so the results indicate that is not safe design a pseudoelastic component using the linear elastic SCF.

6. CONCLUSION

Stress concentration factor (SCFs) are widely used in the design of linear-elastic mechanical components with complex geometries, involving geometric discontinuities such as notches and holes. The presence of nonlinearities, as plastic strains or phase transformation present in shape memory alloys, promotes perturbations in the stress distribution altering values of SCFs estimated for linear-elastic mechanical components. Therefore, new methodologies must be used for these conditions.

In this work, a study of stress concentration factors in pseudoelastic plates is developed considering different approaches for calculate the SCF. Results indicate that it is not safe to calculate the stress concentration factor using traditional methodologies applied for linear-elastic mechanical components, as it can result in non-conservative values. The martensitic phase transformation affects the stress distribution field and as, a consequence, affects the stress concentration factor. In the present study, the use of the Neuber method with the von Mises component shows appropriate results. Stresses, strains and phase transformations obtained with the proposed methodology are compatible with the data found in the literature. However, experimental analysis are necessary to confirm the proposed methodology for SCF calculation for psudoelastic plates with discontinuities.

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8. ACKNOWLEDGEMENTS

The authors would like to acknowledge the support of the Brazilian Research Agencies CNPq, CAPES, and FAPERJ.

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