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STUDY OF THE FIRST MODE OF VIBRATION OF TRANSMISSION LINE CABLES VIA RAYLEIGH'S METHOD AND ELASTIC LINE EQUATION

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Abstract. *The study of the vibration of cables is a relevant topic for the Brazilian and world industry. The first mode is considered of particular importance because it is especially influenced by the initial deformed configuration of the cable when subjected to the acceleration of gravity. Cables are extraordinarily slender and eminently flexible structural elements by their own nature. Systems formed by cables need to be tensioned to allow its build. The traction force modifies their natural frequencies by introducing the portion of geometric stiffness into the total stiffness of the system. In determining these frequencies, analytical models are particularly desired because they are traditional processes in the field of mathematical analysis. In this sense, the main aspect of this work is the proposition of an analytical solution for the first natural mode of vibration that considers the traction in the cable. In a first approach, the approximate elastic line equation is considered as a shape function. The validation of the analytical solution is performed in comparison with modal analysis, of non-linear characteristics, by using finite element method. The object of study is a conductor cable of 18.30 mm in diameter, 13.385 m long, subjected to a traction force of 15.86 kN.*

Keywords: *analytical solution, overhead cables, mode shapes, natural frequency, approximate elastic line equation.*

1. INTRODUCTION

Throughout their useful life, overhead cables of power transmission and communication lines are subjected to climatic conditions whose effects can activate their vibration modes. The study of the vibration of these cables is a relevant topic for the Brazilian and world industry. The first mode is considered of particular importance because it is especially influenced by the initial deformed configuration of the cable when subjected to the force of gravity. Under these conditions, its configuration is subordinated to the geometric and material properties of the element itself, potentially including its viscoelastic behavior, if any. Therefore, it is legitimate to infer that these mechanical systems are intrinsically non-linear, from both geometric and material point of view.

Cables are extraordinarily slender and eminently flexible structural elements by their own nature, which is why their balance can only be found in the deformed configuration of the system. The magnitude of the traction force to which they are subjected in operation modifies their natural frequencies by introducing the portion of geometric stiffness (Wahrhaftig, 2022a) into the total stiffness of the system (Wahrhaftig, 2022b). In determining these frequencies, analytical models are particularly desirable because they constitute traditional processes in the field of mathematical analysis, in addition to allow obtaining a simple equation to solve the problem under study.

In this sense, the main aspect of this work is the proposition of an analytical solution for the calculation of the first natural mode of vibration that considers the tension in the cable. The elaboration of this solution is based on the Rayleigh method. In a first approach, as a shape function for the application of the method, the approximate elastic line equation is considered. In this case, the solution can be obtained by using traditional methods of the solid mechanics, being possible to find a closed-form equation for the problem. The validation of the analytical solution is performed in comparison with modal analysis, of non-linear characteristics, from computational modeling based on the finite element method (FEM). The object of study is a conductor cable, with 18.30 mm in diameter, 13.285 m long, subjected to a traction force of 15.86 kN.

2. GENERAL ASPECTS OF THE PROBLEM UNDER STUDY

Throughout history, the cable has proved to be an essential and versatile structural element, having been used in both civil and military areas. Its origin dates to the dawn of civilization when it was used in the development of weapons and war artifacts. Outside the military context, its use was crucial in the construction of more complex civil, mechanical, and naval engineering structures. More recently, it has become an essential component in network engineering in the telecommunications sector. In the post-modern world, cables are indispensable for establishing high-speed communication, allowing data to be transmitted across the world efficiently and instantly. Cables are flexible and slender structural elements. Due to these characteristics, it is necessary to apply an axial traction to enable the construction of mechanisms constituted by them. The application of a tensile force changes the natural frequencies of vibration of cables since it adds a geometric component to the total stiffness of the system. For the determination of the natural frequencies of cables, analytical models are highly desirable as they are traditional methods in the field of mathematical analysis.

In this field of study, Main and Jones (2007) evaluated the use of dampers to dissipate the vibration of cables, and highlighted the importance of modal forms when they oscillate around their equilibrium position, particularly referring to the specific mode of vibration that they needed to dampen. For this, they developed an exact analytical solution based on the dynamic stiffness method. The study was conducted on the free vibration modes of a tensioned beam element that had vibration dampers. It could be observed that the use of dampers eliminated the oscillations corresponding to a certain vibration mode of importance.

Ceballos and Prato (2008) developed an analytical expression for calculating the natural frequencies of cables considering flexural stiffness and rotational restrictions in the form of elastic springs applied at their ends. With that, they concluded that the estimate of the axial force to be applied to the cable is independent of the axial stiffness and the curvature that the cable assumes under the action of gravity.

According to Dias, Penteado Neto and Tonetti (2018), cables represent an important cost for the implementation of a transmission system. For this reason, studies are needed for enabling an increase in the useful life of these components. The mentioned authors indicated the wind as the main agent responsible for the wear of transmission lines. For cables subjected to the action of the wind, the drastic increase in traction used to reduce the amplitude of vibration diminishes the useful life of the system, they concluded. To avoid premature wear of the cables, the insertion of damping systems along the line was suggested.

Oliveira et al. (2019) pointed out that a substantial number of accidents in transmission lines occur at wind speeds lower than the design speed. This may be related to the non-linearity imposed by the system geometry, which ends up resulting in a change in the sizing parameters used in the transmission line design.

According to Dua, Clobes and Hobbel (2015), the forces due to the wind are quite complex when related to the cables of transmission lines. This occurs, to a substantial extent, due to the geometric non-linearity of these systems and the turbulence associated with the average wind speed. They used modeling by the finite element method to evaluate towers and power lines. The authors performed a dynamic analysis of nonlinear characteristics and compared the results found with the pseudo-static wind forces obtained according to the European Code EN-50341 (2017). In that study, the researchers concluded that the procedures of the referred standard underestimated the behavior of the cables under wind action and that the use of vibration dampers would be a viable solution to improve the behavior of these structures, avoiding excessive vibrations.

Golebiowska, Dutkiewicz and Usewicz (2017) stated that distinct aspects can influence the amplitudes and frequencies of vibrations of systems formed by cables under ambient excitation. Among these are the weather conditions, terrain shape, and mechanical properties of the line. They stressed that it is extremely important to reduce the vibration amplitudes, as they can cause the destruction of the cable, interrupting the transmission network.

Zhang, Zhao and Li (2018) pointed out that conductor cables made of steel-reinforced aluminum are the most used in power transmission lines. These are elements fundamentally characterized by their lightness. When exposed to the effects of wind and heavy rain, they can present strong vibrations, which can cause damage to these elements and risk of transmission failures.

For high voltage overhead transmission lines, Zhang, Zhao and Li (2018) performed a non-linear dynamic analysis based on two-dimensional equations to represent cable vibration. For this, numerical simulations and theoretical studies were performed showing that, when they were in vibration induced by the action of the wind, there was resonance between the excitation and the transmission lines. The authors were able to observe that these transmission lines had a vibration mode, with a non-linear characteristic, which, when mobilized, increased the amplitude of vibration and also the tension in the cable, potentiating the possibility of rupture.

In their study, Onunka and Ojo (2018) used the MATLAB software to perform a finite element analysis to find a closed equation that could provide the vibration frequencies induced by the wind in high voltage transmission lines. The authors admitted, for the cable, the equation of motion compatible with a beam element and obtained results close to those of experimental data, validating the adoption of the beam model in the representation of cables.

3. MATHEMATICAL DEVELOPMENT BY THE RAYLEIGH METHOD

3.1 Characteristics of the structural model

The analyzed system consists of a conductor cable having the following parameters: apparent modulus of elasticity, E, of 2010820773.656 N/m^2; equivalent diameter, d, of 18.30 mm; area, A, given by $\pi d^2/4$, equivalent to 2.63022×10^{-4} m^2; area moment of inertia, I, defined by $\pi d^4/64$, equivalent to 5.50521×10^{-9} m^4; span length, L, of 13.385 m; and specific weight, γ, of 30301.13 N/m³ (acceleration of gravity, g, considered equal to 9.807 m/s^2). The cable is subjected to the action of gravity and, therefore, under the action of its own weight. The cable is pulled by a force, P, of 15.86 kN, as shown in Figure 1, which is a representation of a simply supported beam B-C; B is a pinned support as in Wahrhaftig (2018), and C is a roller support as Darwich et al. (2010); f indicates frequency; and $\bar{m}$ is the mass per unit length, equal to 0.813 kg/m, obtained doing $A\gamma/g$. The element characterized by these parameters represents an overhead cable of transmission lines. The motivation for studying that type of system relies in the fact that these elements are subject to wind action. Depending on certain conditions, the wind flow can enter in resonance with their natural frequencies of vibration which in some moment can become the amplitude of vibration potentially dangerous for the system safety.

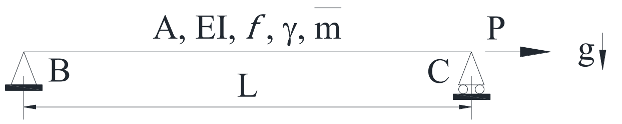


Figure 1. Cable basic parameter description.

3.2 Definition of the approximate elastic line equation

The definition of the approximate elastic line equation is a preliminary step to the determination of the fundamental natural frequency of vibration. In approaching the present problem, only the first mode is contemplated because the shape equation obtained by the elastic line refers to this mode of vibration, which is compatible with the deformed configuration of the cable when subjected to the gravitational field of the earth.

The approximate elastic line equation describes the deformation of beam elements subjected to transverse loads to its axis, being adequate to represent the deformation of a cable simply supported at its ends. The theoretical basis to be adopted resembles the traditional path of the solid mechanics, applied to the problem of a beam subjected to uniformly distributed loading, q, here of 7.97 N/m, obtained by multiplying the specific weight, γ, by the cable area, A. In the case of cables without other forces applied, this loading originates from their own weight. Figure 2 represents the static model of a cable modeled as a beam element regarding the vertical equilibrium of a rigid body, and containing the parameters needed to define the equation of the elastic line. The elastic line of a beam is the locus of points on its axis when bent.

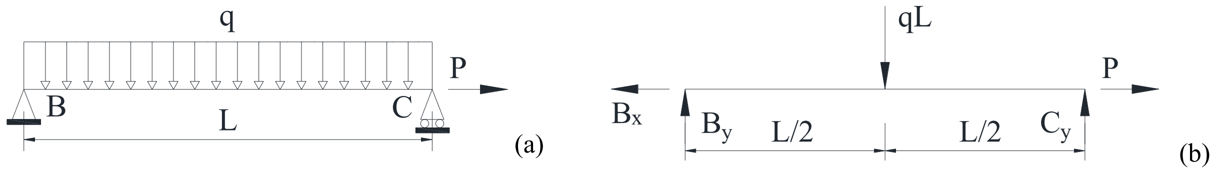


Figure 2. Static model of a cable modeled as a beam: (a) loading diagram; (b) free body diagram.

It is easy to realize that the horizontal equilibrium is ensured by the horizontal reaction, B_x, of the pinned support B. In its turn, the vertical balance is guaranteed by the two vertical reactions B_y and C_y. Using the equilibrium condition of rotation of rigid body by means of the sum of moments in relation to the support C, where the resultant moment must be equal to zero, it is possible to calculate the reaction B_y, of the support B, with the objective of finding the two vertical reactions of structure support. The structure is symmetric, so B_y is equal to C_y, and this is defined in Eq. (1). Anticlockwise moments are taken as positive (Keighley and Doyle, 1998).

$$+\circlearrowleft \sum M_C = 0 \therefore -B_y L + qL\frac{L}{2} = \therefore B_y = \frac{qL}{2} = C_y \ . \tag{1}$$

After calculating the vertical reactions of the supports, it is possible to formulate the problem to have its equations as a function of the variable x defined through a section α located from the left side of the beam, as indicated in Figure 3(a), where x is the independent variable of the problem, being valid throughout its domain, that is, from 0 to L. It is considered that the force P does not affect the bending moment equation.

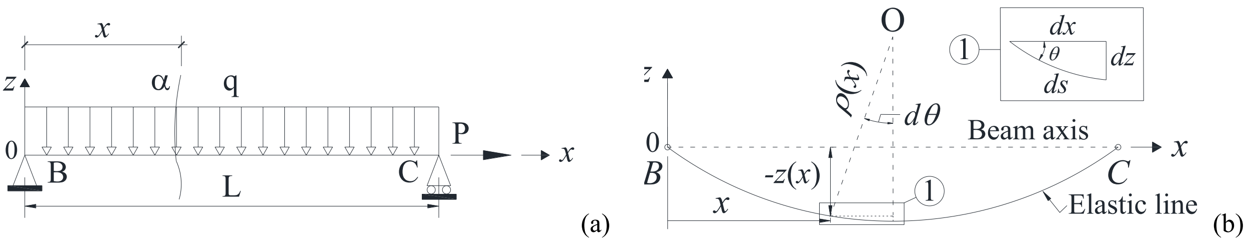


Figure 3. Definition of parameters of the model: (a) undeformed configuration, and (b) bent configuration.

In this sense, it is possible to establish the equation of the bending moment, $M(x)$, by summing the moments produced for all the forces situated to the left side of an arbitrary section α, as shown in Eq. (2):

$$+\circlearrowleft\perp\circlearrowleft \sum M_\alpha = M(x) = \frac{-qx^2}{2} + \frac{qL}{2}x \ . \tag{2}$$

A positive moment, as indicated by Rončević et al. (2019), is that one which put the concavity of the elastic line turned up, i.e., produces negative displacements considering the xz plane, as shown in Figure 3(b), where O and ρ are the center and the radius of curvature of the bent member, respectively and $z(x)$ is the displacement function of the elastic line. The relation moment-curvature for any deformation regime is given by Eq. (3):

$$\frac{M(x)}{EI} = \frac{1}{\rho(x)} \ . \tag{3}$$

For problems of small rotations and displacements, the relationship between the vertical displacements of the beam axis with the corresponding bending moment, is given by Eq. (4), which was found by considering the following path (x is omitted when convenient to compact notation):

$$ds = \rho d\theta \rightarrow \frac{1}{\rho} = \frac{d\theta}{ds}. \text{ As } ds \cong dx \text{ and } \theta \cong tg\theta = \frac{dz}{dx} \rightarrow \frac{d\theta}{ds} = \frac{M}{EI} \rightarrow \frac{d\left(dz/dx\right)}{dx} = \frac{M}{EI} \ \therefore\ z''(x) = \frac{M(x)}{EI} \tag{4}$$

Integrating the bending moment equation with respect to x, as shown in Eq (5), it is possible to find the equation for the rotation angle of a cross-section in a position x, as described in Eq. (6):

$$\int M(x) = z'(x) = -\frac{qx^3}{6} + \frac{qLx^2}{4} + C_1 \ \therefore \tag{5}$$

$$z'(x) = \frac{1}{EI}\left(\frac{-qx^3}{6} + \frac{qLx^2}{4} + C_1\right), \tag{6}$$

where E and I are, respectively, the apparent modulus of elasticity (as it is applicable to the current case) and the area moment of inertia of the section. Therefore, the product of these two parameters corresponds to the known bending stiffness product EI. In previous equations, the prime indicates derivative (Leibniz notation). Integrating Eq. (6), it is possible to find the deflection equation for the system, $z(x)$, that is the elastic line of the cable, as established in Eq. (7):

$$z(x) = \frac{1}{EI}\left(\frac{-qx^4}{24} + \frac{qLx^3}{12} + C_1 x + C_2\right). \tag{7}$$

It is worth mentioning that when performing the integrations of the previous equations, two constants of integration are found, C_1 and C_2, whose respective values must be determined by using the boundary conditions of the problem. One is that which establishes the zero deflection in the support position, i.e., when $x = 0$, $z(0) = 0$; and the other is that which defines for $x = L/2$ the rotation of the section also is equal to zero, meaning that the tangent to the curve at that position is horizontal, i.e., $z'(L/2) = 0$. Therefore, $C_2 = 0$, and $C_1 = -qL^3/24$. Simplifying Eq. (7) after inserting the constants of integration, Eq. (8) is found in following form:

$$z\left(x\right) = -\frac{qx}{24EI}\left(x^3 - 2Lx^2 + L^3\right). \tag{8}$$

Equation (8) is the expression for the approximate elastic line to be taken as the shape function for calculating the natural frequency of vibration of the cable. The equation for $z(x)$ is a function that describes how the cable is deformed under the action of a uniformly distributed loading, which has the same direction of the acceleration of gravity. The deflection presented by the cable obeying Eq. (8) with the parameters mentioned in the previous item can be seen in Figure 4.

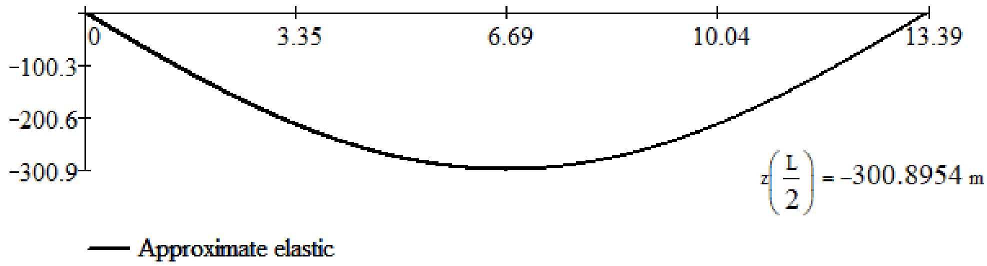


Figure 4. Analytical calculation of the cable deflection (values in "m").

3.3 Determination of the natural frequency of vibration

From the shape equation defined by the elastic line, according to Eq. (8), it is possible to calculate the frequency of the first mode of vibration of the cable through the Rayleigh's method. For this, it is necessary to determine the stiffness constants to bending, k_0, and the geometric parcel, k_G, to define the total stiffness of the system, K, as well as the generalized mass, M. The first one fundamentally depends on the behavior of the material, and the second one depends on the applied axial force. Rayleigh (1877) employed a technique for solving vibration problems using variational calculus with linear approximation to the basic functions chosen to represent the deformation of the system. The objective was to minimize a special class of analytic functions that satisfied the boundary conditions of the problem. It should be mentioned that the Rayleigh's method is the theoretical basis of the finite element method (FEM). Although both the Rayleigh's method and FEM are based on the energy minimization principles, the shape function adopted in the former is applicable to the entire domain of the problem, while in FEM, the interpolation or shape functions are valid for the subdomains, that is, the individual elements.

The use of the Rayleigh's method presupposes the adoption of a shape function for the considered vibration mode, whose amplitude in the generalized coordinate is unitary, and which obeys the boundary conditions of the problem. In the present case, this means to be one in the middle of the beam length, and zero in the support positions. Thus, the equation of the elastic line obtained by Eq. (8) naturally meets boundary conditions of the problem, but it needs to be normalized. The normalization of the Eq. (8) is reached by employing Eq. (9). The normalized form of the elastic line for the problem under study is given by Figure 5.

$$\beta\left(x\right) = \frac{z\left(x\right)}{z\left(L/2\right)}. \tag{9}$$

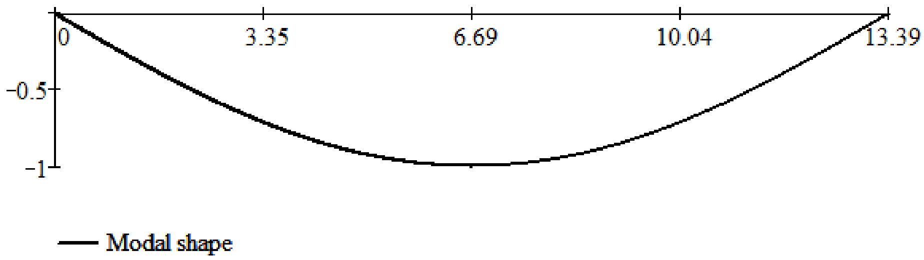


Figure 5. Normalized modal shape (horizontally values in "m").

Therefore, according to Clough and Penzien (2003), the flexural stiffness constant can be calculated through Eq. (10):

$$k_0 = \int_0^L EI\left(\beta''(x)\right)^2 dx \,, \tag{10}$$

and the geometric stiffness constant by Eq. (11):

$$k_G = P\int_0^L \left(\beta'(x)\right)^2 dx \,. \tag{11}$$

The total stiffness of the system, K, is established by summing the bending stiffness, Eq. (10), with geometric one, Eq. (11), in terms of Eq. (12):

$$K = k_0 + k_G \,. \tag{12}$$

The distributed mass, $\bar{m}$, along the length of the cable is found from Eq. (13), which is used to determine the generalized cable mass, M, as shown in Eq. (14):

$$\bar{m} = \frac{q}{g} \,, \tag{13}$$

$$M = \int_0^L \bar{m}\beta(x)^2 dx \,. \tag{14}$$

The angular frequency, ω, of the cable is obtained by the square root of the ratio between the total stiffness, K, in Eq. (12), by the generalized mass, M, in Eq. (14), according to the indicated in Eq. (15):

$$\omega = \sqrt{\frac{K}{M}} \,. \tag{15}$$

Therefore, the natural frequency, f, in Hertz, is given by Eq. (16):

$$f = \frac{\omega}{2\pi} \,. \tag{16}$$

Considering what was previously established, the adoption of the Rayleigh's method allows finding a closed-form analytical equation relatively easy to use, Eq. (17), to determine the natural frequency, in Hz, of the first vibration mode of the cable, which includes the effect of a traction force on it:

$$f = \frac{0.381}{\pi}\left(\frac{168EI + 17PL^2}{\bar{m}L^4}\right)^{1/2} \,. \tag{17}$$

4. FEM SOLUTION

The analytical results were evaluated through a modal analysis, of non-linear characteristics, using FEM. The individual elements in the computer modeling were beam elements, each one containing six degrees of freedom, as indicated in Figure 6. In Figure 6, x and z are the local axes of the element; l, u, and v are the length of the element and the longitudinal and transverse displacements of the axis; u_1, v_1, θ_1, u_2, v_2, θ_2 are the displacements related to the translational and rotational movement of nodes 1 and 2, respectively (Wahrhaftig et al., 2021).

The FEM solution is obtained by solving Eq. (18), where ω are the eigenvalues and ϕ the eigenvectors.

$$\left(\left[K\right]-\omega^2\left[M\right]\right)\phi=0\,. \tag{18}$$

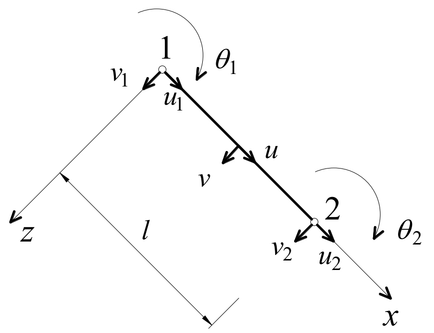


Figure 6. Individual beam element with 6 degrees of freedom.

In FEM, the shape functions are polynomial equations valid for the given subdomain, i. e., the finite elements. This means that, while the analytical procedure presented in the previous section provides a single form for the entire domain of the problem, the finite element formulation establishes that the interpolation functions are applicable to the domain of each one of the elements individually.

According to Oden (2013) and Kurrer (2018), the FEM is a specific advance in structural engineering, which occurred in the field of solid mechanics, based on the principle of virtual works (PTV). It is important to mention that the FEM is a discretization technique that represents continuous systems and their numerical approximations from differential equations. The FEM is rooted in the shape functions implicit in the variational methods of Rayleigh (1877) and Ritz (1909), and in the weighted residuals method of Galerkin (1915). Therefore, the FEM can be understood as a modification of the Rayleigh-Ritz technique, without loss of generality.

In Eq. (18), [M] is the generalized mass matrix, and [K] is the total stiffness matrix, which includes the bending and geometrical parts. All these parameters are given by Eqs. (19) to (20). The frequency, in Hz, is found similarly to what is presented in Eq. (16). Particularly, the typical stiffness matrices of a beam element are as shown next. The bending and geometric stiffnesses are given respectively by:

$$\left[k_0\right]=E\begin{bmatrix} \frac{A}{l} & 0 & 0 & -\frac{A}{l} & 0 & 0 \\ & \frac{12I}{l^3} & \frac{6I}{l^2} & 0 & -\frac{12I}{l^3} & \frac{6I}{l^2} \\ & & \frac{4I}{l} & 0 & -\frac{6I}{l^2} & \frac{2I}{l} \\ & & & \frac{A}{l} & 0 & 0 \\ & symmetric & & & \frac{12I}{l^3} & -\frac{6I}{l^2} \\ & & & & & \frac{4I}{l} \end{bmatrix},\ \left[k_g\right]=\frac{P}{l}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & \frac{6}{5} & \frac{l}{10} & 0 & -\frac{6}{5} & \frac{l}{10} \\ & & \frac{2l^2}{15} & 0 & -\frac{l}{10} & -\frac{l^2}{30} \\ & & & 0 & 0 & 0 \\ & symmetric & & & \frac{6}{5} & -\frac{l}{10} \\ & & & & & \frac{2l^2}{15} \end{bmatrix}, \tag{19}$$

and mass matrix by:

$$\left[M\right]=\frac{\rho Al}{420}\begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ & 156 & 22l & 0 & 54 & -13l \\ & & 4l^2 & 0 & 13l & -3l^2 \\ & & & 140 & 0 & 0 \\ & symmetric & & & 156 & -22l \\ & & & & & 4l^2 \end{bmatrix}. \tag{20}$$

It is important to consider that the computational analysis was elaborated under the consideration of geometric nonlinearity, with the assumption of large displacements. This means that the equilibrium is established in the deformed configuration of the system. To perform the modal analysis, in accordance with Eq. (18), a geometric non-linear static processing was preliminary performed to provide the final stiffness of the system. After this step, the modal analysis was conducted considering the stiffness resulting from that processing. The normalized modal form obtained by computational modeling built and processed in SAP2000 (2022) is represented in Figure 7 In total, 50 beams elements were considered.

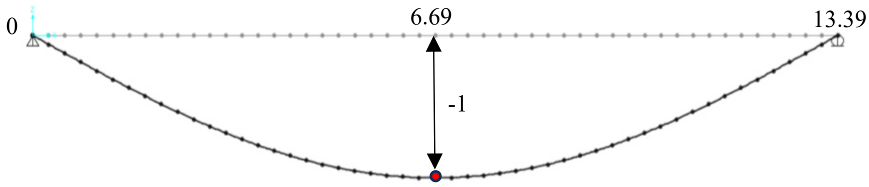


Figure 7. FEM modal shape of the 1st mode of vibration.

On the other hand, the deformed cable is shown in Figure 8, obtained under the assumption of geometric linearity.

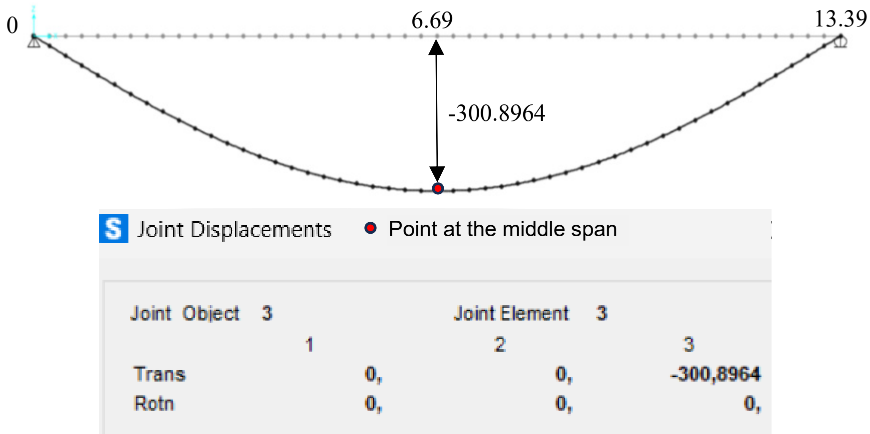


Figure 8. FEM deflection of the cable (value in “m”).

It is of interest to emphasize that the modal computational analysis through finite-element method was performed under the condition of geometric non-linearity. That hypothesis was used to define the modal shape and the natural frequency of the first mode of vibration. However, the deformed configuration of the cable was obtained linearly. The cable natural frequency, therefore, was calculated analogously to that of the analytical method with consideration of a traction force acting on the structural element. Nevertheless, in the present approach, the force is not still considered actuating on the structural displacements, i.e., only its effect on the stiffness is computed.

It is important to mention that, in the hypothesis of large displacements, all equilibrium equations are written in the deformed configuration of the structure, which can demand a reasonable number of iterations in the computational processing. Therefore, the hypothesis considered in this work is the most rigorous procedure among those that could have been used for obtaining the modal frequency and the shape of vibration via computational modelling. That hypothesis is the most adequate for the analysis of cables because they can present large deformations when subjected to the action of gravity. An analysis based on large displacement automatically captures the existing non-linearity if it is important to the analyzed problem. Although large displacement and large rotation effects are modeled, all strains are assumed to be small.

5. RESULTS AND DISCUSSION

The comparison between deflections and modal forms of the analytical and computational solutions can be seen in Figure 9 and Figure 10, respectively.

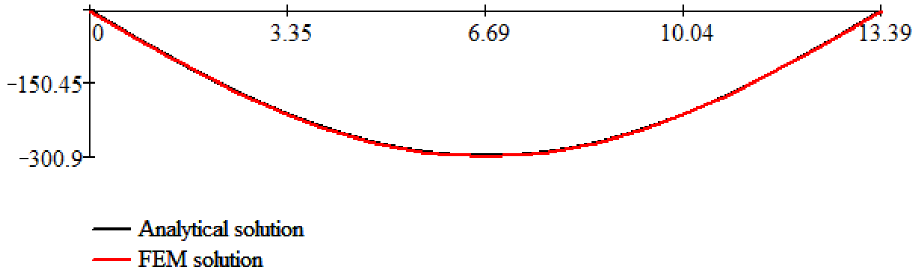


Figure 9. Analytical and FEM deflections (values in “m”).

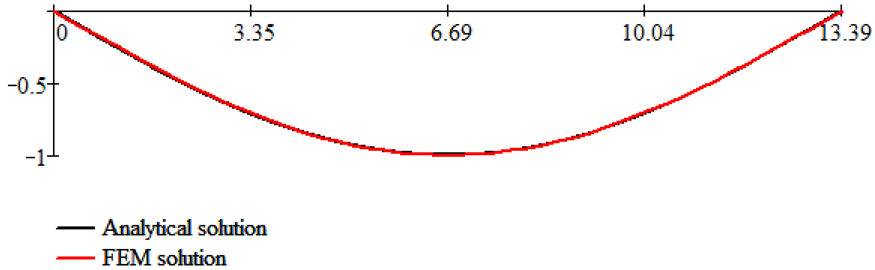
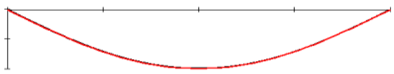


Figure 10. Analytical and FEM modal shapes (horizontally values in “m”).

The results of the analytical solution and computational modeling by FEM are summarized in Table 1, where z_{max} is the maximum deflection found and f is the frequency of the first mode of vibration.

Table 1. Obtained results by analytical solution and FEM.

Analytical solution	**FEM simulation**	**Difference (%)**
 z_{max} = 300.8954 m f = 5.218864 Hz	 z_{max} = 300.8964 m f = 5.218502 Hz	 0.0003 0.0069

6. CONCLUSIONS

In this work, an analytical solution based on Rayleigh's method was used to calculate the natural frequency of vibration of a structural element typically used in overhead cables of power transmission lines. For the implementation of the analytical solution, the approximate equation of the elastic line was assumed. With that assumption, the Rayleigh's method allowed obtaining a simple, closed-form equation for calculating the frequency of the first mode of vibration.

The results from the analytical solution were evaluated through computational modeling using the finite element method (FEM). In it, a modal analysis of non-linear geometric characteristics was performed, whose processing was based on the hypothesis of large displacements. In the modal analysis by MEF, the effect of a traction force acting on the cable was also included. In the end, it could be verified that the results of the analytical solution are in line with those produced by the computational modeling as for the natural frequency as to the description of the first mode of vibration. At the same time the deformation of the structural system was calculated, in a linear fashion, having been verified a good agreement in this aspect by both mathematical processes, analytical and FEM.

It is important to note that the use of a traction force changes the vibration conditions of the structural element because it modifies its stiffness. However, changes in the modal shapes were not noted because of it. Future studies will be conducted to make it possible to include the viscoelastic behavior of the cable material, as well the use of the exact equation for elastic line and the effect of the axial force on the vertical displacements of the structure.

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