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FATIGUE OF WELDED ELEMENTS USING THE STRESS GRADIENT FACTOR

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Abstract. *The objective of this study is to develop the Stress Gradient Factor (K_{gr}) based on the Stress Intensity Factor (SIF) to estimate the service life through Strain-Based Fracture Mechanics (SBFM), of welded and treated specimens by high-frequency mechanical impact (HFMI). Weight Functions (WF) analyses are conducted to calculate the stress intensity factors for both smooth and notched specimens. An algorithm for calculating the inverted boundary in front of the crack using the 2D point load weight function is defined. The model described in this work can estimate the fatigue of welded parts and cruciform welded joints treated with HFMI under conditions of variable loading, considering an initial semi-elliptical crack growing in two directions using the K_{gr} equations. The numerical analysis presented here to estimate the life of welding fatigue is compared with experimental data for 350W steel, conducted at the University of Waterloo, Canada. Subsequently, a statistical analysis is performed using the Monte Carlo method with both found and simulated data to validate the model.*

Keywords: *Stress Gradient Factor, Welded structures, Mechanical Impact treatment, Strain-Based Fracture Mechanics.*

1. INTRODUCTION

As technology advances and research progresses in the field of engineering, the utilization of metals in civil construction is on the rise. Along with this increase, various failures occur, necessitating prevention and protection measures to extend the lifespan of these materials to the maximum while minimizing costs. In Great Britain, during the Industrial Revolution, numerous accidents related to railway construction were documented, as knowledge of Fracture Mechanics was extremely limited at the time (Ghahremani et al., 2015). Although studies in this field began, industries did not show much concern about understanding the causes of accidents and machinery breakdowns, often resulting from mechanical failures due to repeated stress applications and varying loads.

Research in this area gained momentum with the occurrence of structural collapses that led to significant material losses and loss of human lives. One such example is the collapse of the Silver Bridge suspension bridge, constructed in 1928 and spanning 682 meters, connecting the cities of Point Pleasant (West Virginia) and Gallipolis (Ohio) in the United States. This tragic event occurred on December 15, 1967, resulting in 46 fatalities. According to Barsom and Rolfe (1999), unforeseen high-stress concentrators in the design were the primary cause of the structure's fatigue-related failure.

Stress concentrations can arise from various factors such as notches, contact with other objects, temperature gradients, and welded components. Precisely determining the stress concentration factor allows for the application of classical fracture mechanics principles to calculate the service life of welded components. Multiple approaches have been analyzed to accurately determine local stresses with the Stress Concentration Factor. In cases where the stress state varies within the concentration area, a method capable of accounting for these variations must be adopted. The impact of stress concentration on welded components significantly reduces their fatigue strength and service life, making it essential to quantify these effects in their design and analysis. Structural design uses the term Stress Gradient Factor (SGF) to account for the impact of stress concentrators.

The concept of Stress Gradient Factor (SGF) is based on the Elastic Stress Concentration Factor (ESCF) initially proposed by El Haddad et al. (1981). Originally, ESCF was introduced to account for the increase in Stress Intensity Factors (SIF) caused by notches or failures, and it was later applied to the fatigue analysis of structural components by Dabayeh et al. (1996). El Haddad et al. (1979) extended these concepts to Strain-Based Fracture Mechanics (SBFM) under elastoplastic conditions.

Ghahremani et al. (2015) applied SBFM in the fatigue analysis of welding joints under Variable Amplitude Loads (VAL), referring to ESCF as the "Modified Stress Concentration Factor" (MSCF) and denoting it as K_p. To provide a clearer notation, Rêgo et al. (2018) proposed unifying the terminology as the Stress Gradient Factor (K_{gr}). This term reflects its role in modifying a reference SIF to account for the effects of stress gradients induced by notch tips. In general, the I SIF mode can be calculated by Eq. (1).

$$K_I = \sigma\sqrt{\pi a}\Pi f_i \tag{1}$$

Where f_i quantify effects that affect the SIF (for instance, the effects of bifurcated cracks). The SIF can be calculated by Eq. (2).

$$K_I = \underbrace{\sigma\sqrt{\pi a}\cdot\eta\cdot f(a/w)}_{\Delta K_{I(ref)}}\cdot K_{gr}(a/w) \tag{2}$$

Where $K_{I(ref)}$ is the SIF of a reference geometry, a semi-infinite strip loaded in mode I. In other words, $K_{gr}(a/w)$ is defined by Eq. (3).

$$K_{gr}(a/w) = \frac{K_I}{K_{I(ref)}} \tag{3}$$

Where K_I is computed using the approach described by Miranda et al. (2019). The SGF corrects the SIF to consider notch-induced stress gradient effects that decrease as the crack tip moves away from the notch tip (or from any other features that induce stress gradients, like concentrated loads, residual stresses induced by welding or by plasticity, temperature gradients, etc.).

While it is possible to obtain K_{gr} analytically or empirically, most practical calculations require the use of numerical methods. In their work, authors such as Miranda, Antunes, et al. (2019) utilize the concept of weight functions to calculate SIFs by analyzing stress distributions on crack faces, which helps determine the frictional fatigue and obtain the K_{gr} factor. Rajan and Walbridge (2021) address in studies that many technologies can modify the residual stresses and the K_{gr} value in welded elements. Ghahremani (2015) has carried out improvement studies such as High Frequency Mechanical Impact (HFMI) treatment and it has proven to be attracting more and more attention due to its low cost and being a reliable method to extend the fatigue life of welded metal bridge structures.

Regular testing and inspection of welds on steel structures can help prevent performance fatigue of cyclically loaded structures. However, as they become increasingly expensive and time-consuming, it is often necessary to rely on modeling and simulation to assess the service life of welded joints, as shown in the works included by Ranjan and Walbridge (2021). Modeling does not completely replace the need for practical tests, but it helps quickly in estimating fatigue life. The present work aims to perform a numerical analysis using K_{gr} to estimate the service life of elements with welds and HFMI treatment using classical theories of fracture mechanics based on deformations, in addition to sample modeling using the Weight Functions (WF).

2. MODEL DESCRIPTION

The linear elastic fracture mechanics model can be adapted to account for non-linearity due to material deformation, as demonstrated in studies conducted by Khalil and Topper (2003). The effective relationship between crack growth rate and its lifespan is described by Eq. (4), commonly known as the Paris-Erdogan law of growth, which has been modified to incorporate Stress Intensity Factors (SIF) ranges referred to as ΔK_{th}. This factor is utilized to predict the stress state near the crack tip induced by either remote loading or residual stress, as explained by Anderson (2005).

$$da/dN = C\left(\Delta K_{eff} - \Delta K_{th}\right)^m, \tag{4}$$

The constants C and m are determined through the evaluation of non-propagating crack growth rates from tests performed on a compact specimen exposed to stress. By integrating these parameters over the crack depth range, from a_i to a_c , the fatigue life can be calculated using Eq. (5).

$$N = \int_{ai}^{ac} da/C\left(\Delta K_{eff} - \Delta K_{th}\right)^m, \tag{5}$$

The effective Stress Intensity Factor (SIF) range, ΔK_{eff}, which takes into account the influences of crack closure, is determined by Eq. (6).

$$\Delta K_{eff} = K_{max} - MAX(K_{OP}, K_{min}), \tag{6}$$

The terms K_{max} and K_{min} represent the Stress Intensity Factors (SIF) corresponding to the maximum and minimum stress (or strain) levels within a specific load cycle. K_{OP}, on the other hand, represents the SIF associated with the crack opening stress level during that cycle. Equation (7) is employed to compute each of these SIF values.

$$K = \gamma E\varepsilon\sqrt{(\pi(a + a_0))}, \tag{7}$$

The term γ represents a correction factor that takes into account factors such as the shape of the crack, the presence of a free surface on one side of the crack, and the finite thickness of the plate. However, it does not consider the non-uniform distribution of stresses along the crack path, which is addressed separately. The modulus of elasticity is denoted as E, a represents the depth of the crack below the surface and ε represents the local strain at this particular depth. Additionally, El Haddad et al. (1979) introduced the constant term a_0, as presented in Eq. (8), to account for small effects related to initial cracking.

$$a_0 = (\Delta K_{th}/\Delta\sigma_e)\,1/\pi, \tag{8}$$

$\Delta\sigma_e$ represents the fatigue limit of steel. To calculate the stresses and strains for each load cycle, a Ramberg-Osgood (cyclic) material model is employed, as described in Eq. (9). This model is used to simulate non-linear material behaviors under cyclic loading and relies on input material constants, including H_c (hardening coefficient) and h_c (hardening exponent of the stabilized cyclic curve).

$$\Delta\varepsilon = (\Delta\sigma/E) + (\Delta\sigma/2H_c)^{\frac{1}{h_c}}, \tag{9}$$

Sandor (1972) and Dowling (2007) employed the *ε-N* method to determine the calculation of fatigue life (*N*). This method frequently utilizes Neuber's Eq. (10) to establish the correlation between the nominal stress, $\Delta\sigma_n$, and the changes in strain and stress, $\Delta\varepsilon$ and $\Delta\sigma$, respectively, induced at the root of a notch or stress concentration site

$$\Delta\sigma\Delta\varepsilon = (K_t\Delta\sigma_n)^2/E, \tag{10}$$

In this context, K_t represents the (theoretical) stress concentration factor, which relates nominal stress-strains (σ_n and ε_n) to elastoplastic stress-strains (σ and ε) at the weld root. El Haddad, et al. (1979) prefer to use the acronym K_f, which stands for fatigue concentration factor, in their formulation. To characterize the cyclic elastoplastic behavior, fatigue tests are conducted, controlled by the strain range $\Delta\varepsilon$ with zero mean stress and strain. When subjected to cyclic loads, materials undergo a transient phase in which they may remain stable, soften, or harden. However, after a few cycles, the behavior stabilizes. The stabilized $\Delta\sigma\Delta\varepsilon$ curves are referred to as stabilized hysteresis loops (Rêgo et al., 2018).

The process of determining the life, N, involves iterative steps to solve for the Neuber tolerance and the hysteresis loop. This allows for the determination of stress and strain at the site of failure.

3. THE GENERAL WEIGHT FUNCTION

The Weight Function Method (WFM) is one of several techniques used to calculate Stress Intensity Factors (SIF). Ingraffea (2004) conducted a comprehensive exploration of Computational Fracture Mechanics methods, covering historical and state-of-the-art approaches and delving into various representations of cracking processes. These approaches are categorized into two groups: geometrical and non-geometrical. Geometrical methods can be further divided into Constrained methods (which involve prescribed, analytical geometry and known solutions) and Arbitrary methods (including mesh-free, adaptive Finite Element Method (FEM)/Boundary Element Method (BEM), lattice, particle, and atomistic approaches). The WFM is classified as a Known Solution Method and is used to compute 2D Stress Intensity Factors (SIF), followed by the Stress Gradient Factor (SGF).

Glinka and Reinhardt (2000) outlined essential steps that should be followed before determining the Stress Intensity Factor (SIF) using the weight function approach. These steps include:

- Calculating the stress distribution along a specified plane through linear elastic analysis.
- Applying the obtained stress distributions to the surface of the crack. Select an appropriate weight function from a predefined set of options.
- Integrate the product of the stress function and the selected weight function over either the length or the surface of the crack.

The weight function used to calculate the stress intensity factor depends on the geometry of the cracked body. For a 2D crack, the stress intensity factor can be determined using Eq. (11), which involves integrating the weight function and the stress field over the crack area Ω_c. This equation provides a means of quantifying the stress intensity factor for the given crack geometry and loading conditions.

$$K_A = \iint_{\Lambda} \sigma(x,y) w_A(x,y,A,\Omega_c)\, dx\, dy, \tag{11}$$

Where the term A represents an arbitrary point at the crack front, Ω_c, where the stress intensity factor is calculated, and $w_A(x,y,A,\Omega_c)$ represents different weight functions for different crack geometries.

Oore and Burns (1980) introduced a general weight function designed for planar cracks with arbitrary shapes, based on their analysis of SIF solutions across various crack geometries. The proposed weight function relies on two key parameters the distance (ρ) between the load point $P(x_p, y_p)$ and a specific point $A(x_A, y_A)$ along the crack front, where the SIF calculation is required, and the line integral along the crack contour (Ω_c) in Eq. (12).

$$w_A(x_p, y_p) = \sqrt{2} \Big/ \left(\pi \rho^2 \sqrt{\oint_{\Omega_c} d\,\Omega_c / l^2}\right), \tag{12}$$

The weight function for an embedded circular crack in an infinite body can be obtained precisely by solving the line integral in previous equation, however, calculating the line integral for more intricate geometries poses a nontrivial challenge. To address this, Glinka and Reinhardt (2000) introduced a geometrical interpretation of the line integral that simplifies the computations. Instead of evaluating the line integral, it can be replaced by the length of the inverted crack contour, Γ_C, relative to the position of the point load. Consequently, the point-load weight function for an embedded crack can be expressed in a more straightforward manner, as shown in Eq. (13).

$$w_A(x_p, y_p) = \sqrt{2} / (\pi \rho^2 \sqrt{\Gamma_c}), \tag{13}$$

To compute the inverted contour, Γ_C, a contour is discretized by a set of linear segments, C_i. For each linear segment, the inverted contour, $\Delta\Gamma_{Ci}$, is displayed in Eq. (14).

$$\Gamma_c = \sum_{i=1}^{n} \Delta\Gamma_{ci}, \tag{14}$$

Figure 1 shows the geometry representation of inverted boundary of a segment $\vec{C}$ considering a point $\vec{p}$. This Figure is used to the algorithm to compute Γ_C that is presented in Figure 3, based on the description of other work (Jankowiak et al., 2009).

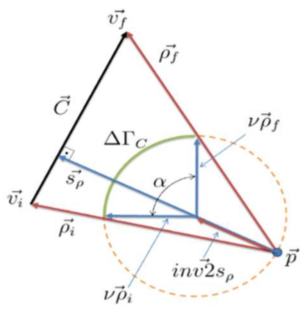

Figure 1. Definition of boundaries for cross section with a surface crack.

The effect of outer boundaries can be added using an inverted idea of a contour that is similar to the inverted crack contour. Glinka and Reinhardt (2000) presented an approach that considers the finite boundaries of the cracked body, denoted as Ω_B, to incorporate the influence of the external boundaries. The external contour needs to be inverted relative to the point A, where the calculation of the SIF is required. As a result, the universal point-load weight function for surface, edge, and embedded cracks in both infinite and (semi) finite bodies can be expressed in Eq. (15).

$$w_A(x_p, y_p) = \frac{\sqrt{2}}{\pi \rho^2} \frac{\sqrt{\Gamma_c + \Gamma_B}}{\Gamma_c}, \tag{15}$$

However, this last equation does not contemplate all different contour geometries in real crack components. For the propose of shows an algorithm to compute SIF, the boundaries on a cross section are classified as represented in Figure 2:

- Crack front (CF). The boundary of crack tip, the interface between the cracked and uncracked regions of a material.
- Crack edge (CE). The boundary of crack free surface of the material.
- Boundary edge (BE). The boundary of material surface that there is any crack.

In Figure 2, the crack surface is discretized in small triangles to represent each stress. Considering that the crack front is discretized in j^{th} segments, the K_{Ij} for a point in the center of this segment is given by presents in Eq (16).

$$\mathrm{K}_{Ij} = \sum_{i=1}^{n} P_i \left(\frac{\sqrt{2}}{\pi\rho^2}\right)\left(\frac{\sqrt{\Gamma_{Ci} + \Gamma_{CEj} + 2.3\Gamma_{BEj}}}{\Gamma_{Ci}}\right), \tag{16}$$

Where n is the number of triangles, P_i is the force for each triangle considering the stress distribution on triangle center (x_i, y_i) at $\sigma(x_i, y_i)$ in Eq. (16), Γ_{Ci} is the inverted boundary of crack front for the triangle center, Γ_{CEj} is the inverted boundary of crack edge for the j^{th} crack segment center, Γ_{BEj} is the inverted boundary of boundary edge for the j^{th} crack segment center, and ρ is the distance between the center of i^{th} triangle and the j^{th} crack segment center.

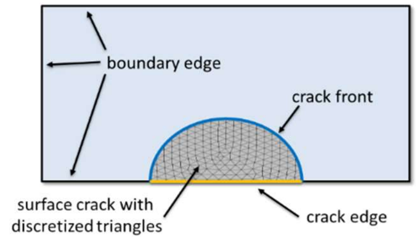

Figure 2. Definition of boundaries for cross section with a surface crack.

Figure 3 depicts an algorithm for computing the inverted boundary at the crack front using the 2D point load weight function. The algorithm takes as input the list of all segments of a boundary (L) and a specified preference point for calculating the inverted boundary. It outlines the steps and procedures required to determine this inverted boundary, which is a crucial component in various fracture mechanics analyses.

```
Algorithm 1: Inverted boundary
  Data:
    L, List with all segments of a boundary
    p⃗, reference point to compute inverted boundary
  Result: Γ_C inverted boundary
  Γ_C ← 0
  foreach segment s_i in L do
      v⃗_i ← get initial vertex of segment s_i
      v⃗_f ← get final vertex of the segment s_i
      C⃗ ← v⃗_f − v⃗_i
      ρ⃗_i ← v⃗_i − p⃗
      ρ⃗_f ← v⃗_f − p⃗
      N⃗ ← { −C_y ; C_x }
      n⃗ ← N⃗/||N⃗||
      d ← ρ⃗_i · n⃗
      s⃗_ρ ← n⃗ · d
      inv2s⃗_ρ ← s⃗_ρ · 0.5/(s⃗_ρ · s⃗_ρ)
      invρ⃗_i ← ρ⃗_i/(ρ⃗_i · ρ⃗_i)
      invρ⃗_f ← ρ⃗_f/(ρ⃗_f · ρ⃗_f)
      νρ⃗_i ← invρ⃗_i − inv2s⃗_ρ
      νρ⃗_f ← invρ⃗_f − inv2s⃗_ρ
      α ← arccos((νρ⃗_i · νρ⃗_f)/(||νρ⃗_i|| · ||νρ⃗_f||))
      ΔΓ_C ← ||inv2s⃗_ρ|| · α
      Γ_C ← Γ_C + ΔΓ_C
  end
```

Figure 3. Algorithm to compute inverted boundary.

Figure 4 presents an algorithm for calculating KI on the crack front using the 2D point load weight function. This algorithm takes as input the following data: List of segments of the crack front (L_{CF}), List of segments of the crack edge (L_{CE}), List of segments of the boundary edge (L_{BE}), and Triangle mesh data on the surface of the crack (M_{BE})

The algorithm processes this input data and produces a list of Stress Intensity Factors (SIF) denoted as L_{KI}. These SIF values are calculated at the midpoints of the segments along the crack front.

Algorithm 2: Compute K_I on crack front

Data:

L_{CF}, list with all segments of the crack front
L_{CE}, list with all segments of the crack edge
L_{BE}, list with all segments of the boundary edge
M_{BE}, mesh with all triangles on the surface crack

Result:

L_{KI} list of K_I compute on the middle of each segment in the list L_{CF}

foreach *triangle* t_i *in the mesh* M_{BE} **do**
  $\vec{c_i} \leftarrow$ get center of triangle t_i
  $\Gamma_{Ci} \leftarrow$ Inverted boundary $(L_{CF}, \vec{c_i})$
  $area \leftarrow$ get area of triangle t_i
  $\sigma \leftarrow$ get stress tension in $\vec{c_i}$
  $P_i \leftarrow area \cdot \sigma$, result force in the center of triangle t_i
end
foreach *segment* s_j *in the list* L_{CF} **do**
  $\vec{cs} \leftarrow$ get center of segment s_j
  $\Gamma_{CE} \leftarrow$ Inverted boundary $(L_{CE}, \vec{cs})$
  $\Gamma_{BE} \leftarrow$ Inverted boundary $(L_{BE}, \vec{cs})$
  $K_{Ij} \leftarrow 0$, init K_I for segment s_j
  foreach *triangle* t_i *in the mesh* M_{BE} **do**
    $\vec{\delta} \leftarrow \vec{c_i} - \vec{cs}$
    $\rho^2 \leftarrow \vec{\delta} \cdot \vec{\delta}$
    $K_{Ij} \leftarrow K_{Ij} + P_i \cdot \frac{\sqrt{2}}{\pi \cdot \rho^2} \cdot \frac{\sqrt{\Gamma_{Ci} + \Gamma_{CE} + 2.3 \cdot \Gamma_{BE}}}{\Gamma_{Ci}}$
  end
  $L_{KI} \leftarrow K_{Ij}$, insert K_{Ij} in the list L_{KI}
end

Figure 4. Algorithm to compute K_I on crack front using 2D point load weight function.

4. NUMERICAL CALCULATION THE STRESS GRADIENT FACTOR

The study involves defining the geometry of a cruciform welded joint made of 350 W steel. This joint comprises four similar weld fingers, which are susceptible to crack initiation due to stress concentrations and the presence of weld defects in both the width (w) and thickness (t) directions. Along with describing the component's geometry, a semi-elliptical crack, as depicted in Figure 5, starting from the weld tip, is modeled to calculate K_{gr}. The modified geometry accounts for the implementation of High-Frequency Mechanical Impact (HFMI) treatment to estimate the enhancement in fatigue life. The inclusion of the term "e" indicates the mechanical impact associated with HFMI. The input data for the analysis includes: Geometry names, specifying the type of element being analyzed for cross-welding (tension-weld-simple-cross-sharp, or tension-single-weld-cross-soft-weld, or tension-weld-single-cross-notch), nominal stress and material characteristics: Steel 350 W. The names of the geometries indicate the specific configurations being studied regarding cross-welding.

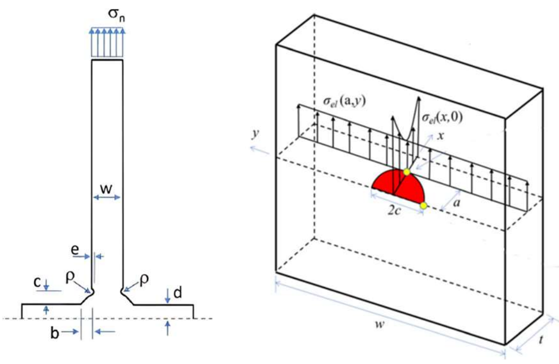

Figure 5. Cross-section with a typical semi-elliptical surface crack.

Table 1 provides the input data for the welded and HFMI-treated specimens, which are essential for the analysis.

Table 1. Input data for the treated and welded specimens.

Dimensions	tension-weld-single-cross-smooth	tension-weld-single-cross-notch (HFMI)
neck width	9.5	9.5
thickness	25.4	25.4
b	6.25	6.25
c	6.4	6.4
r	1	1.93
d	4.75	4.75
e	-	0.177

The mesh concentration at the weld points is generated using the "*genSIF.exe*" program, developed by the author. This program produces a "*.frd*" file that can be opened in the graphical interface of CalculiX, which utilizes the Finite Element Method (FEM) to visualize the stress flow. The mesh concentration is achieved by increasing the number of nodes at the welding points. Another output generated by "*genSIF.exe*" is a file with the extension "*.q2d*," which can be opened in the graphical interface of the Quebra 2D program to visualize stress results at each point in the mesh.

K_{gr} is defined as the ratio between the stress intensity factor in each section affected by a stress concentrator and the stress intensity factor that would exist in that section without the presence of the stress concentrator. K_{gr} differs from the classical stress concentration factor (K_t) by considering the concentration at failure based on the Stress Intensity Factor (SIF). For the 2D model with a crack opening "a" in the thickness direction and "c" in the width direction, the distribution of K_{gra} and K_{grc}, according to the fitted curves, is presented in Figure 6 for both as-received and as-treated conditions. This analysis helps evaluate the effect of treatments on stress concentration and, consequently, on fatigue life. Typically, such curves show how K_{gra} and K_{grc} values change with varying parameters, loads, or material properties.

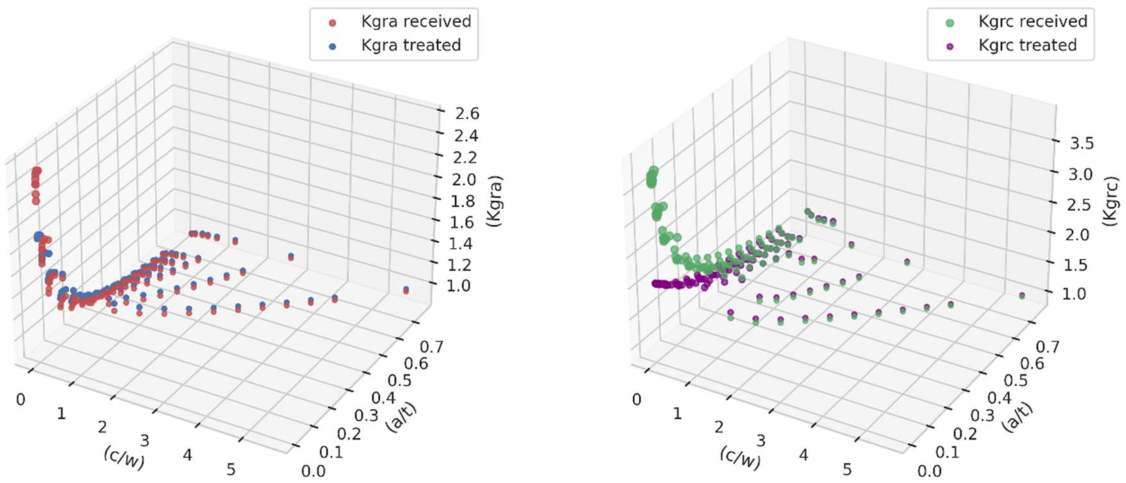

Figure 6. Fitted curves of K_{gra} and K_{grc} as received and treated.

In the 2D analysis, stress distributions generated in the normalization directions $^{a}/_{t}$ and $^{c}/_{w}$ are described by non-linear equations. To obtain results based on these equations, computer programs for numerical calculations are necessary. The equation to determine K_{gra}, with curve-fitting parameters, as a function of crack growth for welding as received, is provided by Eq. (17). This equation is used to establish the relationship between K_{gra} and the crack growth, providing insights into how the stress concentration factor changes as the crack extends. It's a critical factor in understanding the behavior of welds under fatigue conditions and can inform decisions related to maintenance and safety.

$$K_{gra} = A + \frac{B}{C + D\left(\frac{a}{t}\right)\left(\left(\frac{E + \left(\frac{a}{t}\right)}{\left(\frac{c}{w}\right)} + F\right)\left(\left(\frac{a}{t}\right) + \left(\left(G + H\left(\frac{a}{t}\right)\right)\left(\frac{c}{w}\right) + I\right) + J\right)\right)} \tag{17}$$

The fit parameters in the K_{gra} curve is: A=0.8742, B=11395,08 C= 4446.6, D=9786.5, E= -0.000667, F= -0.008, G=-12.37, H=1.72, I= 5.87, J=24.88.

The K_{grc} presents different equations and parameters for each of the analyzed sections. The K_{grc} value as a function of the value of $^{a}/_{t}$ and for the interval $^{c}/_{w} \leq 0.08$ is presented in Eq. (18).

$$K_{grc} = \frac{\left(\frac{a}{t}\right)^2}{-0.003 + \left(\frac{c}{w}\right)} + \frac{\left(\frac{c}{w}\right) + 2.95}{6.52\left(\frac{a}{t}\right) + 0.84} + \frac{0.0023}{-0.004 + \left(\frac{a}{t}\right) - 2.4\left(\frac{a}{t}\right)\left(\frac{c}{w}\right)} \tag{18}$$

The K_{grc} value as a function of the value of $^{a}/_{t}$ and for the interval $0.08 < {}^{c}/_{w} \leq 1.0$ is presented in Eq. (19).

$$K_{grc} = 2.2 - 0.13\left(\frac{a}{t}\right) + \frac{0.11 - a}{0.05 + \left(\frac{a}{t}\right)} + \frac{-0.004 + 0.06\left(\frac{a}{t}\right) - 0.2\left(\frac{a}{t}\right)^2}{-0.06 + \left(\frac{c}{w}\right)} + \frac{0.006\left(\frac{a}{t}\right)^2 + 0.06\left(\frac{a}{t}\right)^3}{\left(-0.06 + \left(\frac{c}{w}\right)\right)\left(\frac{c}{w}\right)} \tag{19}$$

The K_{grc} value as a function of the value of $^{a}/_{t}$ and for the interval with ratio $^{c}/_{w}$ greater than 1 (one) is presented in Eq. (20).

$$K_{grc} = 0.9 + \frac{0.055}{\frac{0.013}{\left(\frac{c}{w}\right) - 0.43} + \left(\frac{a}{t}\right)} + \frac{0.15}{\left(\left(\frac{c}{w}\right)^2 - 7.2\left(\frac{a}{t}\right)^2 + 7.2\left(\frac{c}{w}\right)\left(\frac{a}{t}\right)^3 + 8.5\left(\frac{a}{t}\right)^4\right)\left(\frac{c}{w}\right)^3} \tag{20}$$

For the specimens welded with HFMI treatment, the K_{gr} value showed a reduction. In this case, the K_{gra} equation, correlated with the adjustment parameters, as a function of crack growth, is determined by Eq. (21).

$$K_{gra} = 0.87 - \frac{74 + 131.8\left(\frac{a}{t}\right)^2\left(\frac{c}{w}\right) + 79\left(\frac{a}{t}\right)^3 + 97.24\left(\frac{a}{t}\right)^2}{3500.84\left(\frac{c}{w}\right) + 438} - \frac{739.65}{-8795\left(\frac{a}{t}\right) - 516.63} \tag{21}$$

The K_{grc} presents different equations and parameters for each of the treated sections analyzed. The K_{grc} value as a function of $^{a}/_{t}$ for $^{c}/_{w}$ ratio values less than 1 (one) is presented by the exponential function in Eq. (22).

$$K_{grc} = e^{-2\left(\frac{a}{t}\right)^3 + 3.56\left(\frac{a}{t}\right)^2 + 0.08\left(\frac{c}{w}\right) - 3.45\left(\frac{a}{t}\right)} + 1.07 \tag{22}$$

The values of K_{grc} for the interval with ratio $^{c}/_{w}$ greater than 1 (one) are shown in Eq. (23).

$$K_{grc} = 0.73 + \frac{0.11 + 0.4\left(\frac{a}{t}\right)^2\left(\frac{c}{w}\right) - 0.1\left(\frac{a}{t}\right)^3 - 0.44\left(\frac{a}{t}\right)^2}{3.7\left(\frac{c}{w}\right) - 3.4} + \frac{16.4}{80.5\left(\frac{a}{t}\right) + 19.2} \tag{23}$$

5. PROJECTION OF PROBABILISTIC S-N CURVES

The deterministic analysis model is employed to project S-N (stress versus cycles to failure) curves, which are typically available in structural design codes. These curves help estimate the fatigue life of components under specific stress conditions. In a comparative study, the deterministic analysis can be used to evaluate how different factors affect the fatigue life of materials or structures. As described by Miranda et al. (2015), the synthesis of the Monte Carlo Simulation (MCS) analysis involves studying stresses at various locations along a crack. In this analysis, a range of stress levels is selected, and laboratory tests are conducted to establish an S-N curve for each stress range level. This process is repeated for all stress range levels of interest. For each set of interval levels, a series of trials is performed, with the key difference being that the input data is based on probabilistic inputs. The MCS analysis considers the uncertainties and variations in input parameters, allowing for a more comprehensive assessment of fatigue life under real-world conditions. Table 2 provides a summary of the statistical distributions assumed for different input parameters in the probabilistic analysis. In this analysis, Monte Carlo simulation (MCS) was employed, and critical input parameters were assigned probabilistic distributions. These distributions were determined based on a review of available data and studies conducted by Righiniotis and Chryssanthopoulos in 2003.

Table 2. Input parameters for probabilistic analysis.

Parameter	**Mean**	**Dev**	**Distribution**
LN(C)	27.5	0.4	Lognormal
ΔKth	60	0.07	Lognormal
m	3	-	Uniform
K'	812	0.05	Lognormal
N'	0.108	0.05	Lognormal
a_0	0.4	0.2	Lognormal
c_0	0.4	0.2	Lognormal
Sy	1	0.07	Lognormal
Su	1	0.05	Lognormal

The analysis is conducted using the deterministic fracture mechanics model, and as a result, a series of cycles to failure is obtained for evaluation. For each stress range, a histogram can be plotted to visualize the distribution of failure cycles. Alternatively, the results can be plotted on an S-N (stress versus cycles to failure) graph, and these data can be used to generate graphs that illustrate the probability of failure versus the number of applied cycles for different stress ranges. In Figure 7, the Monte Carlo simulation (MCs) analysis results for 350 W steel are used to design survival curves with a 50% confidence level and a 95% confidence level for both treated and welded samples. These curves demonstrate that the treated samples exhibit longer fatigue lives compared to the received (untreated) samples, indicating the effectiveness of the studied model in predicting fatigue behavior. Furthermore, the modeling results were compared with experimental data presented by Ghahremani (2015) and Ranjan (2019), which provides a validation of the model's accuracy and its ability to predict fatigue life under varying stress conditions.

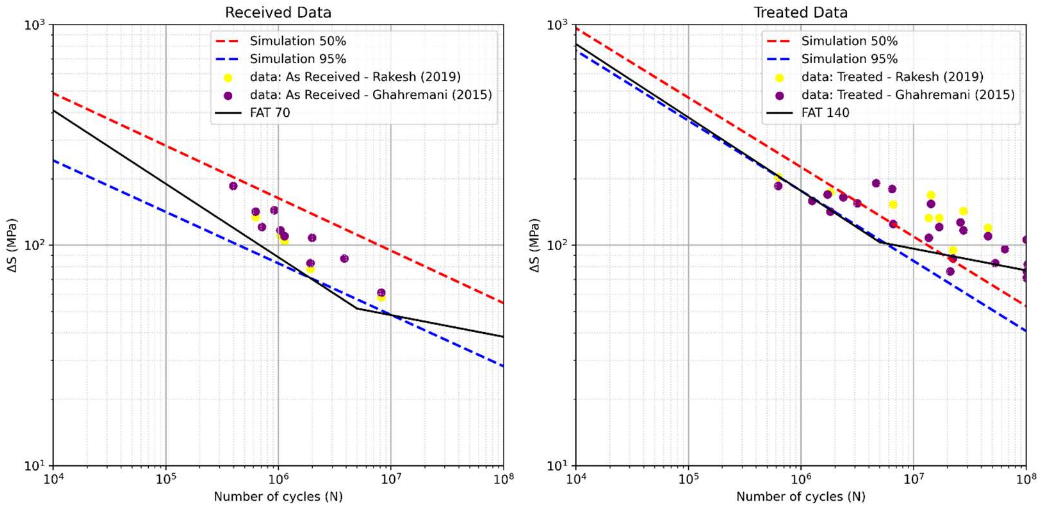

Figure 7. Comparison of test data with Eurocode 3 and design curves as received (left) and as treated (right).

The curve presented in Figure 7 indicates that the model's results are quite conservative when compared to the test data. Specifically, the S-N (stress versus cycles to failure) curves obtained from the analysis align closely with the Eurocode 3, FAT 70, and FAT 140 (m=3) curves for received and treated specimens. For treated specimens, the analysis results closely resemble the FAT 140 curve, indicating good agreement. This observation is in line with findings from Ranjan and Walbridge's studies in 2021. On the other hand, received samples exhibit similarity with the FAT 70 curve for up to two million cycles but with a slope of m=5. This discrepancy suggests that the model may provide a more conservative estimate for the fatigue behavior of received specimens. In summary, the 2D Strain-Based Fracture Mechanics (SBFM) model showcased in this application demonstrates its capability to validate the accuracy of fatigue curve codes provided by industry standards. It can be a valuable tool for assessing the fatigue life of welded elements and treated specimens.

6. CONCLUSIONS

The development of precise computational tools, particularly for fatigue analysis in engineering projects, has become essential to prevent potential issues such as structural failures or machinery breakdowns. Many of the leading computational tools for mechanical fatigue design were developed abroad, making their adoption in Brazilian teaching and research institutions cost-prohibitive. The model described in this article addresses this challenge by offering a means to estimate the fatigue performance of welded components and cruciform welded joints treated with High-Frequency Mechanical Impact (HFMI) under various loading conditions using the Stress Gradient Factor (Kgr). The numerical predictions obtained through this methodology are then compared with alternative approaches and experimental data. The results of this study reveal a strong agreement between the predictions generated by the proposed methodology and the actual experimental data. Consequently, the gensif.exe program, as presented, is a reliable tool for engineers conducting fatigue analysis of welded components. This research contributes to the field of engineering by providing a cost-effective and accurate tool for fatigue analysis, ultimately enhancing the safety and reliability of welded structures and components.

7. REFERENCES

Anderson, T., 2005. *Fracture mechanics: fundamentals and applications*, CRC Press.

Barsom, J. M., Rolfe, S. T., 1999. Fracture and Fatigue Control in Structures. 3ª ed., ASTM.

Dabayeh, A. A., Xu, R.X., Du, B.P., Topper, T.H., 1996. Fatigue of cast aluminium alloys under constant and variable-amplitude loading Int J Fatigue, 18, pp. 95-104

Dowling, N.E., 2007. *Mechanical behavior of materials*. 3rd ed. Pearson Education Inc., Upper Saddle River.

El Haddad, M., Topper, T.H., Smith, K.N., 1979. A strain based intensity factor solution for short fatigue cracks initiating from notches Fract. Mech, ASTM STP 677, pp. 274-289

El Haddad, M. H., Topper, T. H., Topper, T. N., 1981. Fatigue Life Predictions of Smooth and Notched Specimens Based on Fracture Mechanics. Journal of Engineering Materials and Technology 103(2): 91-96.

Ghahremani, K., 2015. *Fatigue Assessment of Repaired Highway Bridge Welds Using Local Approaches*. Doctoral thesis, University Waterloo, Ontario, Canada.

Ghahremani, K., Safa, M., Yeung, J., Walbridge, S., Haas, C., and Dubois S., 2015. *Quality Assurance for High-Frequency Mechanical Impact (HFMI) Treatment of Welds Using Handheld 3D Laser Scanning Technology*. Welding in the World.

Glinka, G. and Reinhardt, W., 2000. *Calculation of stress intensity factors for cracks of complex geometry and subjected to arbitrary nonlinear stress field*s. Fatigue and Fracture Mechanics, 31, ASTM STP 1389:348–370.

Ingraffea, A. R., 2004. "Computational Fracture Mechanics," in Encyclopedia of Computational Mechanics, ed: John Wiley & Sons, Ltd.

Jankowiak, A. and Jakubczak, H., Glinka, G., 2009. *Fatigue crack growth analysis using 2-D weight function*. International Journal of Fatigue, p.1921-1927.

Khalil. M. and Topper, T.H., 2003. *Prediction of crack-opening stress levels for 1045 as-received steel under service loading spectra*. Int J Fatigue 25:149–157.

Miranda, A. C. O., Antenus, M. A., Alarcón, M. V. G., Meggiolaro, M. A., De Castro, J. T. P., 2019. Use of the stress gradient factor to estimate fatigue stress concentration factors K_f. Engineering Fracture Mechanics 206. 250–266.

Miranda, A. C., Gerlich, A., and Walbridge, S., 2015. Aluminum friction stir welds: Review of fatigue parameter data *and probabilistic fracture mechanics*. Engineering Fracture Mechanics .Volume147,October2015,Pages 43-260analysis.https://doi.org/10.1016/j.engfracmech.2015.09.007

Oore, M. and Burns, D. J., 1980. *Estimation of stress intensity factors for embedded irregular cracks subjected to arbitrary normal stress fields*. J Press Vessel Technol ,102/211.

Ranjan, R., 2019. *Probabilistic strain-based fracture mechanics analysis of weldments*. Doctoral thesis, University Waterloo, Ontario, Canada.

Ranjan, R. and Walbridge, S., 2021. *2D fracture mechanics analysis of HFMI treatment effects on the fatigue behaviour of structural steel welds.* Weld World 65, 1805–1819. https://doi.org/10.1007/s40194-021-01120.

Rêgo, E. M. F., 2017. "*Fracture mechanics based on strain applied in fretting fatigue"* (in Portuguese). Doctoral Thesis at the University of Brasilia, Faculty of Technology, Structures and Civil Construction.

Rêgo, E. M. F., Antunes, M. A., and Miranda A. C. O., 2018. *A methodology for fretting fatigue life estimation using strain-based fracture mechanics*. Engineering Fracture Mechanics 194: 24-41.

Righiniotis, T. D. and Chryssanthopoulos, M. K., 2003. *Probabilistic fatigue analysis under constant amplitude loading*. J Constr Steel Res 59:867–886. https://doi.org/10.1016/S0143-974X(03)00002-6

Sandor, B., 1972. *Fundamentals of Cyclic Stress and Strain.* U. of. Wisconsin.