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# **A COMPARATIVE STUDY OF PID CONTROLLER TUNING METHODS USING BIO-INSPIRED ALGORITHMS AND IMC APPROACH FOR NONLINEAR TANK SYSTEM**

**Matheus Bawden Silverio de Castro**

**Gabriel da Silva Lima**

**Vinicius Rafael de Freitas**

**Eduardo Liberato**

**José Oniram de Aquino Limaverde Filho**

**Eugenio Fortaleza**

Department of Mechanical Engineering, University of Brasília, Brasília, Federal District, Brazil

matheusbawden@gmail.com

gabriel7bsb@gmail.com

vinicius\_r\_freitas@hotmail.com

eduardo.perez.liberato@gmail.com

joseoniram@ieee.org

efortaleza@unb.br

**Rafael Valladares de Almeida**

Repsol Sinopec Brasil

rafael.valladares@repsolsinopec.com

**Abstract.** Proportional Integral Derivative (PID) control tuning based on Internal Model Control (IMC) is widely used in industrial control loops due to its simplicity and robustness. It is widespread in the petroleum, chemical, pharmaceutical, and food industries regarding tank-level control. For nonlinear plants, the main drawbacks of the IMC tuning technique are: i) it is difficult to determine the optimal value of its only tuning parameter (closed-loop time constant), and ii) the need for a linear model around an operation point, which can lead to significant errors. To overcome these limitations, bio-inspired algorithms such as Particle Swarm Optimization (PSO) and Grey Wolf Optimization (GWO) can be used as tuning methods. These algorithms can determine the PID controller gains only using the knowledge of the nonlinear model of the plant and a cost function, mitigating the errors associated with linearity while satisfying multiple control performance requirements, such as minimization of overshoot and rise time. In this study, to test the effectiveness of PSO and GWO as PID controller tuning methods in comparison to IMC, numerical analyses were conducted to design a water level control for a nonlinear plant consisting of a pump, a cylindrical tank, and a conical tank, both connected in series. Different scenarios with one or more control performance requirements were evaluated for the three algorithms. The findings indicated that designing a PID controller based on PSO and GWO has more options for the performance requirements of the nonlinear plant when compared to the well-known IMC approach, even in the presence of unknown disturbances or model uncertainties. Although it's crucial to determine a fitness function that properly considers the control requirements, with the advancement of the knowledge and the increasing demand for multiple control requirements in PID controller design, bio-inspired algorithms like PSO and GWO could be key players in the future of industrial control systems for nonlinear plants.

**Keywords:** Internal Model Control, Particle Swarm Optimization, Grey Wolf Optimization, PID Tuning Method, Nonlinear Tank System

## **1. INTRODUCTION**

Proportional-integral-derivative (PID) controller is by far the most widely used ones in industrial processes due to ease of implementation and robust performance (Johnson and Moradi, 2005). For example, the PID control was applied in tank systems for chemical processes, food, and pharmaceutical industries in Koo *et al.* (2001); Manisha *et al.* (2018). However, the controller gain tuning problem still presents significant theoretical challenges in order to handle control design requirements.

The literature contains a number of tuning methods for PID controller; possibly the best known are the Ziegler-Nichols method proposed in Ziegler and Nichols (1942). Another approach that stands out for its simplicity is the Internal Model Control (IMC), which was introduced in Rivera *et al.* (1986). This is because all the three parameters of PID controller can

be obtained from a single parameter. Practical experience suggests that this approach minimizes controller interactions and improves overall process disturbance rejection (Fruehauf *et al.*, 1994). As consequence, there are plenty of applications in the industry using the Internal Model Control (IMC) method as tuning method such as steam distillation (Johari *et al.*, 2020) and liquid level control (Vijayan *et al.*, 2017). Nevertheless, many industrial processes depend on nonlinear behaviors coming from the plant's dynamics and its control, while IMC usually requires a linear model of the plant, which leads to the use of linearization strategies that can compromise the performance of the control loop.

As an alternative tuning method, some bio-inspired algorithms have emerged over time. Unlike the IMC tuning technique, which relies on process models and mathematical analysis to determine optimal tuning parameters, bio-inspired algorithms take inspiration from nature to find the best solutions. Bio-inspired algorithms offer a heuristic and adaptive approach to PID tuning, using principles such as self-organization, adaptation, and exploration. In addition, they are capable of handling complex and nonlinear systems, making them effective even when the process model is unknown or hard to obtain, as demonstrated in a study by Mirjalili *et al.* (2014). Among them, two algorithms that have shown good results recently are Gray Wolf Optimization (GWO) and Particle Swarm Optimization (PSO) (Bouallègue *et al.*, 2012; Ting *et al.*, 2015; Faris *et al.*, 2018; Hatta *et al.*, 2019; Negi *et al.*, 2021).

In this study, we propose to demonstrate the applicability of GWO and PSO as PID tuning techniques to design a liquid level control for a nonlinear coupled tanks system, where the first tank is cylindrical and the second tank is conical. In addition, a comparison between the Internal Model Control tuning method and these bio-inspired algorithms is discussed.

## 2. COUPLED TANKS SYSTEM

Coupled tanks represent an important plant model to be studied since it acts as the basic liquid level control problem, that commonly appears in industrial processes and it is a well-known benchmark problem for testing and analyzing the tracking performance of designed controllers, as presented before in (Vijayan *et al.*, 2017). Considering a system for tank level control, the mathematical model of the nonlinear coupled tanks system considered in this paper has three main components in cascade (see Fig. 1): a centrifugal pump, a cylindrical tank, and a conical tank.

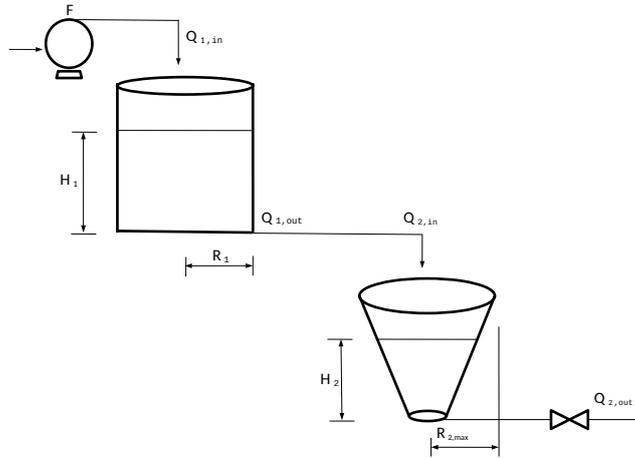


Figure 1. Schematic diagram of a coupled tanks system.

$A_{1,in}$  represents the cylindrical tank's cross-sectional area with a radius of  $R_1$ , with its maximum height  $H_{1,max}$  and this tank is fed by the centrifugal pump with a flow rate  $Q_{1,in}$  given, according to the following affinity law (Takacs, 2017):

$$Q_{1,in} = Q_{nom} \frac{F}{F_{nom}} \quad (1)$$

with  $F$  as the pump motor power supply frequency in Hz, and the pair  $(Q_{nom}, F_{nom})$  represent the nominal flow rate and frequency of the pump, respectively, since these values are normally provided by pump manufacturers.

From Bernoulli's equation for a steady, non-viscous, and not compressible flow, the flow rate exiting the tank ( $Q_{1,out}$ ) can be expressed by:

$$Q_{1,out} = \rho A_{1,out} \sqrt{2gH_1} \quad (2)$$

where  $\rho$ ,  $g$  and  $H_1$  mean the fluid density, the gravitational constant, and the water level in tank 1, respectively. In addition, the cross-sectional area of the pipe from tank 1 to tank 2 is  $A_{1,out}$ .

The mass balance principle of the liquid level in tank 1, which is defined as the tank's inflow and outflow difference, can be written as the following first-order differential equation:

$$\rho \dot{H}_1 A_{1,in} = Q_{1,in} - Q_{1,out} \quad (3)$$

Substituting (1) and (2) in (3), the differential equation of the cylindrical tank can be described as:

$$\dot{H}_1 = C_1 F - C_2 \sqrt{H_1} \quad (4)$$

considering the constant values  $C_1$  and  $C_2$  the following Equation (5):

$$C_1 = \frac{Q_{nom}}{F_{nom} \rho A_{1,in}} \quad \text{and} \quad C_2 = \frac{A_{1,out} \sqrt{2g}}{A_{1,in}} \quad (5)$$

For the conical tank, its cross-sectional area  $A_{2,in}$  is a function that varies according to its water level height  $H_2$ , that is represented mathematically as:

$$A_{2,in} = \pi \left( \frac{H_2 R_{2,max}}{H_{2,max}} \right)^2 \quad (6)$$

where  $R_{2,max}$  and  $H_{2,max}$  represent the maximum radius and height of tank 2, respectively.

In a similar way to the centrifugal tank, the mass balance principle of the liquid level in a conical tank can be written as the following first-order differential equation:

$$\rho \dot{H}_2 A_{2,in} = Q_{2,in} - Q_{2,out} \quad (7)$$

with the input flow rate  $Q_{2,in} = Q_{1,out}$  and the flow rate exiting the tank  $Q_{2,out}$  given by:

$$Q_{2,out} = \rho A_{2,out} \sqrt{2g H_2} \quad (8)$$

where  $A_{2,out}$  is the outlet area of tank 2.

Substituting (2), (6) and (8) in (7), we arrive at the differential equation of the conical tank:

$$\dot{H}_2 = C_3 \frac{\sqrt{H_1}}{H_2^2} - C_4 \frac{1}{H_2^{1.5}} \quad (9)$$

where the constant parameters  $C_3$  and  $C_4$  are calculated as follows:

$$C_3 = \frac{A_{1,out} \sqrt{2g}}{\pi} \left( \frac{H_{2,max}}{R_{2,max}} \right)^2 \quad \text{and} \quad C_4 = \frac{A_{2,out} \sqrt{2g}}{\pi} \left( \frac{H_{2,max}}{R_{2,max}} \right)^2 \quad (10)$$

### 3. PID CONTROLLER DESIGN AND TUNING TECHNIQUES

The PID controller comprises three essential components: the proportional (P) term, the integral (I) term, and the derivative (D) term. The proportional term generates an output proportional to the current error, facilitating a prompt response to changes in the error. The integral term integrates past errors to eliminate steady-state errors and gradually adjust the control signal. The derivative term considers the rate of change of the error and provides stability to the system response.

Let us consider the widely used ISA PID structure (Astrom, 1995):

$$K(s) = K_p \left( 1 + \frac{1}{T_i s} + \frac{s T_d}{1 + s \frac{T_d}{N}} \right) \quad (11)$$

where  $K_p$ ,  $T_i$ , and  $T_d$  represent the gains to be tuned for the proportional, integral, and derivative terms, respectively. In addition, the derivative term has a lower pass filter to assure the implementation of the controller, whose parameter  $N$  is typically fixed at 100, making the controller a proper transfer function.

By suitably tuning the PID controller gains, a balance between stability, responsiveness, and disturbance rejection can be achieved. This enables effective control of the system output, ensuring accurate tracking of the desired setpoint (Visioli, 2006). In the following subsections, we will discuss three tuning methods to calculate the PID controller gains: IMC, GWO and PSO. From the gains obtained by each method, the PID controller is numerically simulated to control the nonlinear coupled tanks system. The results will be compared and analyzed in Section 4.

### 3.1 Internal Model Control

The IMC-PID tuning method is highly appealing to industrial users due to its sole tuning parameter: the closed-loop time constant. This parameter can be adjusted to yield the best compromise between performance and robustness. For example, a lower value reduces robustness but eliminates oscillations and overshoots in the step load response of the closed loop. Meanwhile, a larger value of the tuning parameter results in more robust control which, in turn, makes the designed controllers insensitive to model error (Lee *et al.*, 1993).

IMC structure's objective is, using a process model and a performance specification, to obtain a suitable controller (Teixeira *et al.*, 2010). According to Shah *et al.* (2010), the premise of IMC is that, in reality, there is only an approximation of the real process, never the process itself. Even when the correct model is available, it is not possible to have accurate measurements of its process parameters. Hence, the linearization is obligatory to model an IMC controller. To do so, a common method consists in applying a Jacobian matrix and choosing the operation points to substitute in the Equation 12:

$$\begin{cases} \dot{x} = f(x(t), u(t)) \\ y = g(x(t), u(t)) \end{cases} \quad (12)$$

The dynamics can be non-linear, linear and time variant. To overcome this process it is realized the linearization and considering  $A$ ,  $B$ ,  $C$ , and  $D$  the space state matrices equations and that the transfer function being represented as the matrix multiplication:

$$\begin{cases} \nabla \dot{x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ \dots & \dots & \dots & \dots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \dots & \frac{\partial f_1}{\partial u_n} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \dots & \frac{\partial f_2}{\partial u_n} \\ \dots & \dots & \dots & \dots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \dots & \frac{\partial f_n}{\partial u_n} \end{bmatrix} = Ax + Bu \\ y = Cx + Du \end{cases} \quad (13)$$

Thus, one can use the transfer function as:

$$G(s) = \frac{Y(s)}{R(s)} = C(sI - A)^{-1}B + D \quad (14)$$

Table 1 shows the process model that will be considered for this paper and how the controller parameters are obtained through the IMC.

Table 1. PID tuning through IMC, adapted from (Rivera *et al.*, 1986).

| Process Model                  | $K_p$                                      | $T_I$             | $T_D$  |
|--------------------------------|--|-------------------|--|
| $K$                            | $\frac{\tau_1 + \tau_2}{K \times \lambda}$ | $\tau_1 + \tau_2$ | $\frac{\tau_1 \times \tau_2}{\tau_1 + \tau_2}$ |
| $(\tau_1 s + 1)(\tau_2 s + 1)$ | $\frac{\tau_1 + \tau_2}{K \times \lambda}$ | $\tau_1 + \tau_2$ | $\frac{\tau_1 \times \tau_2}{\tau_1 + \tau_2}$ |

### 3.2 Bio-inspired Algorithms

#### 3.2.1 Grey Wolf Optimizer

As initially presented by Mirjalili *et al.* (2014) in their work, the Grey Wolf Optimizer (GWO) is an optimization algorithm that draws inspiration from the social conduct of gray wolves. The GWO has demonstrated successful applications across a diverse array of optimization problems (Faris *et al.*, 2018; Negi *et al.*, 2021).

The fundamental concept underlying the Grey Wolf Optimization (GWO) algorithm is to replicate the hunting behavior exhibited by grey wolves to effectively discover the optimal solution for an optimization problem. Within a gray wolf pack, distinct roles are observed among the wolves, including the alpha, beta, delta, and omega. Serving as the pack's leader, the alpha wolf assumes the responsibility of making the majority of decisions. Supporting the alpha, the beta wolf acts as the second-in-command and assists in decision-making. As lower-ranking members, the delta and omega wolves typically trail the alpha and beta, respectively.

The circling pattern, when mathematically depicted, exhibited by prey during a hunt can be symbolized using the following equations:

$$\vec{X}(t+1) = \vec{X}_p - \vec{A} \cdot \left| \vec{C} \cdot \vec{X}_p(t) - \vec{X}(t) \right| \quad (15)$$

where  $t$  represents the current iteration,  $\vec{A}$  and  $\vec{C}$  are coefficient vectors,  $\vec{X}_p$  and  $\vec{X}$  indicate the position vector of a prey and a gray wolf, respectively. The vectors  $\vec{A}$  and  $\vec{C}$  are given by:

$$\vec{A} = 2\vec{a} \cdot \vec{r}_1 - \vec{a} \quad (16)$$

$$\vec{C} = 2\vec{r}_2 \quad (17)$$

with the components of  $\vec{a}$  decreasing linearly from 2 to 0 during the iterations. In addition, the vectors  $\vec{r}_1$  and  $\vec{r}_2$  are randomly generated in the range of 0 to 1.

During the encircling of their prey, it is postulated that the alpha (the most promising solution), beta, and delta wolves possess superior awareness regarding the potential prey location. Consequently, the first three optimal solutions obtained are preserved, while the remaining search agents adjust their positions based on the optimal agent's location. The subsequent equations are presented to depict this behavior.

$$\vec{X}_1 = \vec{X}_\alpha - \vec{A}_1 \cdot \left| \vec{C}_1 \cdot \vec{X}_\alpha - \vec{X} \right| \quad (18)$$

$$\vec{X}_2 = \vec{X}_\beta - \vec{A}_2 \cdot \left| \vec{C}_2 \cdot \vec{X}_\beta - \vec{X} \right| \quad (19)$$

$$\vec{X}_3 = \vec{X}_\delta - \vec{A}_3 \cdot \left| \vec{C}_3 \cdot \vec{X}_\delta - \vec{X} \right| \quad (20)$$

The fitness of each wolf is calculated based on its position (which is the vector of variables that are being optimized). The algorithm updates the wolves' position accordingly with the best values of the fitness function. The termination of the algorithm occurs upon meeting a predefined stopping criterion, such as reaching the maximum number of iterations or when the fitness of the best wolf falls below a specified threshold. Ultimately, the algorithm yields the optimal value found, as indicated by:

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \quad (21)$$

### 3.2.2 Particle Swarm Optimization

Particle swarm optimization (PSO) is a stochastic algorithm that draws inspiration from the intelligent collective actions observed in certain animals and evolutionary principles, such as the synchronized movements of bird flocks and fish schools (Eberhart and James Kennedy, 2010; Wang *et al.*, 2018). The utilization of PSO as an optimization technique for continuous non-linear functions was proposed by these researchers. Additionally, the PSO method offers the benefit of computational efficiency and cost-effectiveness, as it avoids the need for intricate computations (Eberhart and James Kennedy, 1999).

For PSO, each possible solution is named a "particle", which represents a point in a  $N$ -dimensional space with the same amount of parameters to be optimized. One may call the  $S$  particles a swarm, which is represented in the following equations.

$$X = \{x_1, x_2, \dots, x_S\} \quad (22)$$

Considering the  $N$ -dimensional position for the  $i^{th}$  particle in  $S$  described by the vector  $x_i$ :

$$x_i = [x_{i1}, x_{i2}, \dots, x_{iD}] \quad (23)$$

When gathering the solution, the new position for the  $i^{th}$  particle is calculated at each iteration (considering  $t$  and  $t+1$  successive iterations in the algorithm, while  $v_{ij}$  represents the velocity vector in  $i^{th}$  particle) following the equation of motion, as presented in Marini and Walczak (2015):

$$x_i(t+1) = x_i(t) + v_i(t+1) \quad (24)$$

The velocity vectors govern the way particles should move across the search space and it can be defined for the  $i^{th}$  particle as:

$$v_i(t+1) = w(t)v_i(t) + c_1U_1(p_{best,i} - x_i(t)) + c_2U_2(g_{best} - x_i(t)) \quad (25)$$

One can notice that the pair  $(p_{best,i}, g_{best})$  represents the coordinates of the best solution obtained so far by the  $i^{th}$  particle and the overall best outcome obtained by the swarm, respectively. The cognitive and social coefficients ( $c_1$  and  $c_2$ ) modulate the magnitude of the steps taken by one particle in its personal and global best direction. The random factors  $U_1$  and  $U_2$  are two diagonal matrices of uniformly distributed random numbers in the range  $[0, 1]$ . Thus, both  $c_1$  and  $c_2$  influences play a role in updating the  $i^{th}$  particle's velocity. It is worth mentioning that the inertia factor  $w$  balances the global and local searches.

### 3.3 Fitness Function

The fitness function serves as a mathematical tool employed to assess the quality of a solution or candidate solution within the search space of a problem. By taking a candidate solution as input, the function produces an output value representing the solution's quality or fitness. The algorithm's objective is to locate the solution with the minimum fitness value, indicating the optimal solution found to the problem.

The fitness function brought in this paper is based on the Integral of Time Squared Error (ITSE) index for preventing the overshooting of  $H_1$  and  $H_2$  (Carrasco and Salgado, 2009):

$$J = \int_{t_0}^{t_f} |\alpha_1 (H_1(\tau) - H_1^*(\tau)) + \alpha_2 (H_2(\tau) - H_2^*(\tau))|^2 \tau d\tau + \alpha_3 * \sum_{t_0}^{t_f} (u(\tau - 1) + u(\tau)) \quad (26)$$

where the weights  $\alpha_1$ ,  $\alpha_2$  can be used to prioritize the settling time between  $H_2$  and  $H_1$  and  $\alpha_3$  to impose the cost function to considerate the change in the actuator signal, making the abrupt change in the system actuator a concern in the optimization problem. All the proposed changes in the cost function were made in order to achieve better results for the algorithms, and it's worth mentioning that these changes were based on the researchers' previous experience with this type of system. In this study, GWO and PSO algorithms use this function to find the PID controller gains  $K_p$ ,  $T_i$ , and  $T_d$ .

It is important to mention that the fitness function can be modified by changing constants, adding term and imposing new conditions as the solution requires. So it is possible to design a solution that can accommodate all the requirements or find the best level of compromise between all necessities imposed by the designer. By example it can be done that the weight  $\alpha_1$  is much greater than  $\alpha_2$  to prioritize the height of the first tank, or make all weights equal to 0 except for  $\alpha_2$  to only optimize with  $H_2$ . And a new term can be added to optimize the flow of tank 1 to tank 2 if needed for example.

## 4. NUMERICAL SIMULATIONS

In this section, a numerical study was conducted to compare the results of the PID tuning methods for the proposed system. The methods and systems were implemented and simulated in MATLAB. Table 2 shows the parameters of the coupled tanks system.

Table 2. Coupled tanks system parameters.

| Parameter   | Value                    |
|-------------|--------------------------|
| $Q_{nom}$   | 16.67 Kg/s               |
| $F_{nom}$   | 60 Hz                    |
| $\rho$      | 998.23 Kg/m <sup>3</sup> |
| $g$         | 9.81 m/s <sup>2</sup>    |
| $H_{1,max}$ | 1.0 m                    |
| $H_{2,max}$ | 1.0 m                    |
| $R_{2,max}$ | 0.176 m                  |
| $A_{1,in}$  | 0.0146 m <sup>2</sup>    |
| $A_{1,out}$ | 0.0020 m <sup>2</sup>    |
| $A_{2,out}$ | 0.0015 m <sup>2</sup>    |

With the these parameters, the equations of Section 2 and Section 3, we chose the values of 0.5 meter for the linearization in the second tank and 0.3 meter as consequence for the first tank, using them as set points. As a result, Equation 27 shows the system transfer function on the desired set point that can be worked through with the IMC method.

$$G(s) = \frac{0.006205}{(s + 0.5423)(s + 0.2145)} \quad (27)$$

As stated in Section 3, both bio-inspired algorithms employed in this study are stochastic. Hence, to ensure a fair comparison between GWO and PSO algorithms, sixteen runs were executed for each algorithm, and the fitness function

outcomes are presented in Table 3. Additionally, the average, median, minimum, and standard deviation of the fitness function for both algorithms were calculated and are displayed in Table 3.

Table 3. Results of the fitness function for GWO and PSO in 16 runs.

|                    | <b>GWO</b>           | <b>PSO</b>           |
|--------------------|----------------------|----------------------|
| 1                  | $1.2325 \times 10^5$ | $1.5011 \times 10^5$ |
| 2                  | $1.2385 \times 10^5$ | $1.5029 \times 10^5$ |
| 3                  | $1.2295 \times 10^5$ | $1.5138 \times 10^5$ |
| 4                  | $1.2441 \times 10^5$ | $1.5018 \times 10^5$ |
| 5                  | $1.2631 \times 10^5$ | $1.5015 \times 10^5$ |
| 6                  | $1.2402 \times 10^5$ | $1.5017 \times 10^5$ |
| 7                  | $1.2395 \times 10^5$ | $1.5014 \times 10^5$ |
| 8                  | $1.2655 \times 10^5$ | $1.5075 \times 10^5$ |
| 9                  | $1.2247 \times 10^5$ | $1.5022 \times 10^5$ |
| 10                 | $1.2748 \times 10^5$ | $1.5019 \times 10^5$ |
| 11                 | $1.2292 \times 10^5$ | $1.5035 \times 10^5$ |
| 12                 | $1.2444 \times 10^5$ | $1.5024 \times 10^5$ |
| 13                 | $1.2687 \times 10^5$ | $1.5169 \times 10^5$ |
| 14                 | $1.2435 \times 10^5$ | $1.5053 \times 10^5$ |
| 15                 | $1.2659 \times 10^5$ | $1.5111 \times 10^5$ |
| 16                 | $1.2465 \times 10^5$ | $1.5068 \times 10^5$ |
| Average            | $1.2469 \times 10^5$ | $1.5051 \times 10^5$ |
| Median             | $1.2438 \times 10^5$ | $1.5026 \times 10^5$ |
| Minimum            | $1.2247 \times 10^5$ | $1.5011 \times 10^5$ |
| Standard Deviation | $1.5783 \times 10^3$ | $4.8941 \times 10^2$ |

Overall, GWO achieved better minimums for the fitness function, as well as smaller averages and medians, losing only when it came to standard deviation, having a higher than the PSO tuning methods. In this way, it is evident that the GWO algorithm exhibited superior performance compared to PSO in almost all statistical parameters, with a 17.15% lower Average, 17.22% lower Median, and 18.41% lower Minimum. However, the Standard Deviation of GWO was 222.49% higher than that of PSO, suggesting that although GWO provides better results for the problem, its outcomes vary more from one run to another.

Thus, the results from run 9 of GWO and run 1 of PSO were chosen for comparison with the IMC, yielding fitness function results of  $1.2247 \times 10^5$ ,  $1.5011 \times 10^5$ , and  $1.7878 \times 10^5$ , respectively. It is noteworthy that the fitness function value obtained by GWO was the best among the cases studied, being 18.41% better than that achieved by PSO and 31.50% better than that obtained by IMC.

Table 4 presents the calculated PID gains for each tuning method. The obtained results for  $K_p$ ,  $T_i$ , and  $T_d$  vary significantly from one method to another. However, directly analyzing these gains can be quite complex. Therefore, in order to better understand their impact on the controller, it is necessary to observe the system's response to a specific reference.

Table 4. Controller gains tuned by the IMC, GWO and PSO.

| <b>Controller gain</b> | <b>IMC</b> | <b>GWO</b> | <b>PSO</b> |
|------------------------|------------|------------|------------|
| $K_p$                  | 244.1276   | 22.7335    | 213.7679   |
| $T_i$                  | 6.5060     | 2.8696     | 5.0000     |
| $T_d$                  | 1.3214     | 0.0472     | 0.0400     |

For this purpose, the decision was made to vary the reference over time and assess the adaptability of the determined gains. Initially, the system output reference ( $H_2$ ) was set to 0.5 meters, aligning with the set-point of IMC linearization. Subsequently, the reference was adjusted to 0.8 meters and then to 0.2 meters. Figure 2 depicts the system response of the PID controller tuned using the IMC, GWO, and PSO methods in the non-linear system.

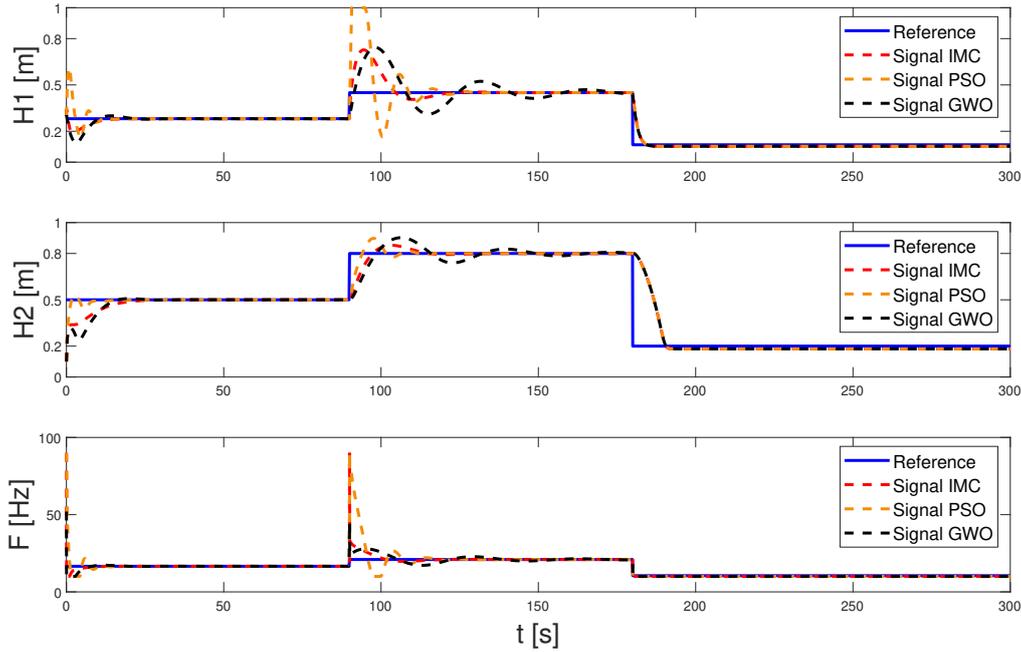


Figure 2. System response for PID with different control strategies.

It is possible to notice that, among all the methods, PSO achieved the best settling time for the first reference (0.5m) in  $H_2$ . In the second reference (0.8m), all methods exhibited overshooting, but unlike the others, GWO failed to achieve the Reference Value. For the last reference (0.2m), the response was almost the same for all methods (the controlled outputs reached the desired value after a 10 s delay in  $H_2$ ).

One unwanted effect occurred in PSO:  $H_1$  level saturated. Nevertheless, IMC and GWO did not come close to saturation in  $H_1$ . Another undesired condition was observed in IMC, where the actuator frequency experienced an abrupt change, which can lead to problems in real systems. This factor was added to the fitness function as a control requirement for the bio-inspired algorithms, and it is possible to state that they performed better in this aspect than the IMC. GWO achieved a slightly more subtle signal than IMC, and PSO managed to mitigate the abrupt change, especially in the second reference.

Therefore, it is expected that, considering the fitness function malleability, the amount of possibilities that one could explore for the aforementioned system, other values inside the fitness function could be tested to gather better control theory.

## 5. CONCLUSION

In the comparative analysis of the three techniques (GWO, PSO, and IMC), it is evident that GWO performed better in terms of achieving lower minimums for the fitness function and exhibiting smaller averages and medians. However, it did show a higher standard deviation when compared to the PSO.

When considering the settling time for the first reference in  $H_2$ , PSO outperformed both GWO and IMC, demonstrating the best response among the three algorithms. In the case of the second reference, all methods exhibited overshooting, but unlike the others, GWO did not converge to the reference. The responses of all methods were quite similar for the last reference, showing a small steady state error.

It should be noted that one undesirable effect was observed in PSO, where the level in  $H_1$  reached saturation. On the contrary, neither IMC nor GWO came close to saturation in  $H_1$ . Additionally, an undesired condition was observed in IMC, where the actuator frequency experienced an abrupt change, potentially causing problems in real systems. GWO achieved a slightly smoother signal compared to IMC, while PSO managed to mitigate the abrupt change, particularly in the second reference. Despite this, the IMC achieved results that were better than the GWO and slightly worse than the PSO in terms of settling time.

During the study, it was possible to notice that the fitness function has a profound impact on the performance of bio-inspired algorithms. A small change in the function could lead to better or worse results. Thus, it's crucial to know how to implement control requirements in the fitness function in order to make bio-inspired algorithms a reliable technique for PID tuning.

Finally, while GWO generally performed well in terms of fitness function measures, PSO exhibited superior settling time and the ability to handle abrupt changes in the actuator. Also, it's possible to mention that, given its practicality and fairly good results, the IMC remains a competitive method for PID tuning, especially in the industry. Further analyses and

considerations of these results will assist in selecting the most appropriate algorithm for the specific requirements of the system under study.

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