27th INTERNATIONAL CONGRESS OF MECHANICAL ENGINEERING
COBEM 2023
ABCM Brazilian Society of Mechanical Sciences and Engineering
27th ABCM International Congress of Mechanical Engineering
December 04-08, 2023, Florianópolis-SC, Brazil

COB-2023-1763
TECHNIQUE FOR THE SIMULTANEOUS ESTIMATION OF THERMAL PROPERTIES OF A COMPOSITE MATERIAL

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Abstract. *This article contributes to the simultaneous estimation of thermal properties in composite materials by presenting experimental techniques and comparing different inverse problem methods. Composite materials, which combine multiple materials, offer lighter and more durable alternatives compared to individual materials. Their unique properties can be tailored to meet specific needs, making them increasingly important worldwide. Understanding the thermal properties of materials is crucial in mechanical engineering for designing, analyzing, and optimizing electro-mechanical systems. However, finding accurate values for specific composite materials is challenging due to their varying compositions and limited literature data. Heat transfer is fundamental in engineering applications like engines, turbines, transformers, and power plants, where the use of composite materials is growing. Unfortunately, there is a lack of information on their thermal properties. This study presents a one-dimensional experimental approach using two inverse problem methods, Gauss Methods and Composite Least-Squares Method, to estimate the thermal conductivity and volumetric heat capacity of a composite piece made of epoxy resin and glass fiber reinforcement. The developed thermal model focuses on a one-dimensional composite plate subjected to a nonlinear heat flux at the top surface while being insulated at the bottom and lateral surfaces.*

Keywords: *thermal conductivity, volumetric heatcapacity, composites, gauss method, least-square method, glass fiber.*

1. INTRODUCTION

The importance of investigating composite materials has grown significantly on a global scale, driven by their distinctive qualities and substantial advantages as highlighted by (Sathishkumar *et al.*, 2014) and (Bian *et al.*, 2021). These materials offer a unique combination of qualities, including superior strength, reduce weight, and increased durability. However, to fully harness these benefits, it is crucial to understand their behavior in relation to the characteristics of the constituent materials, matrix-to-reinforcement ratios,geometry, and constituent phases (Callister and Soares, 2008).

There is a greater demand for the use of composite materials in all industrial sectors such as in military helmets, bodies of automotive and maritime means of transport, plastic pipes, floors and industrial structures, aviation structural components, sports gear, transformers, pressure vessels and ballistic products (Faria, 2007). In order to maximize the potential of these composites, it is essential to understand their thermal behaviors and know their thermal properties. Thermal conductivity (k) represents the material's ability to conduct heat, while volumetric heat capacity (ρcp) refers to the amount of energy required to cause a temperature variation. These properties are fundamental to understand and analyze the thermal behavior of the composite under different conditions and applications.

Thermal properties of composite materials can be determined by several different experimental approaches. Usually, none of them is unrestricted and are only suitable for certain properties or temperatures ranges. The recommended thermal characterization techniques for measuring the thermal properties of solid insulating materials, such as glass fiber-reinforced composites, align with the range of their thermal conductivity (Czichos and Daum, 2007). The hot strip method and the guarded hot plate method, for example, are steady-state procedures that rely on Fourier's law. As a result, these methods are limited to providing measurements of thermal conductivity alone (Jannot *et al.*, 2009). For unsteady techniques, there is the transient hot wire and the laser flash method, and when the objective is to measure the heat capacity of materials, Differential Scanning Calorimetry (DSC) is a commonly employed technique.

Nonetheless, these widely recognized techniques have their shortcomings (Palacios *et al.*, 2019).DSC analysis primarily imforms about specific heat, with better performance at high temperatures.It requires small sample size and can be challenging with inhomogeneous materials, as pointed out by (Yang and Liu, 2018).The laser flash method, while capable of measuring thermal diffusivity and thermal conductivity separately, faces difficulties at room temperature and with thermal insulating or inhomogeneous materials. Moreover, these approaches demand specialized, somewhat complex, and potentially expensive equipment with expensive equipment, with extensive preparation efforts required.

In this context, this research introduces a novel approach, aiming to simultaneously estimate thermal conductivity and volumetric heat capacity for a specific composite using cost-effective experimental techniques conducted at room temperature. The composite studied has epoxy resin in its matrix and fiberglass as its reinforcement; such reinforcement is under a longitudinal and transverse distribution with a volume percentage of 62.38%. Through the analysis of a sample that has a rectangular dimension of 10cm x 10cm x 1cm, this work seeks to precisely determine the thermal properties of this material in a specific temperature range using experimental techniques based on the Gauss and compound least squares methods.

This research significantly contributes to the development and optimization of systems utilizing this composite in their applications. Furthermore, it advances the field of composite materials by pioneering the estimation of previously uncharted thermal properties, offering the potential to significantly impact various industrial sectors that rely on fiberglass-reinforced epoxy resin composites, including aviation, automotive and construction.

2. THEORY DEVELOPMENT

This section presents the theoretical foundations used for simultaneous estimation of thermal conductivity and volumetric heat capacity in a composite sample. In addition, it demonstrates the thermal model used, the numerical model necessary to obtain the thermal properties and every analysis made so that it was possible to obtain the desired results in the experiment.

2.1 One-dimensional thermal model

The thermal model used to obtain thermal properties was the one-dimensional model represented by Figure 1, in which the effects of radiation, convection, contact resistance, internal heat generation and phase change were ignored, considering constant thermal properties in relation to temperature.

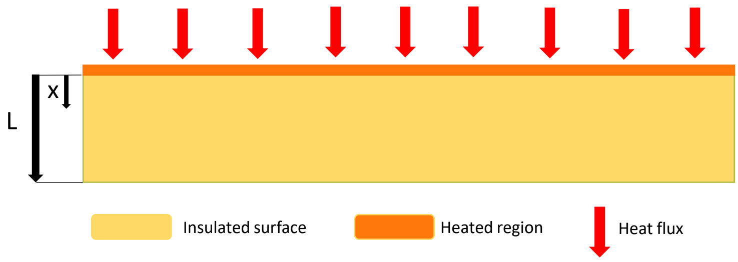


Figure 1: Representation of the one-dimensional thermal model.

Therefore, by using the heat diffusion Eq. (1), it is possible to obtain the heat conduction presented by the one-dimensional, transient and non-linear thermal model:

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{\rho c p}{k}\frac{\partial T(x,t)}{\partial t} \qquad 0 \leq x \leq L \quad t > 0. \tag{1}$$

Where: T is the temperature, ρ is the material density, considered constant for this application, t is the time and x is the direction of heat transfer. Therefore, applying the initial condition and boundary conditions, respectively, by Eq. (2), (3) and (4).

$$T(x,0) = T_0 \qquad 0 \leq x \leq L \quad t = 0. \tag{2}$$

$$-k\frac{\partial T(x,t)}{\partial t} = q(T) \qquad x = 0. \tag{3}$$

$$\frac{\partial^2 T(x,t)}{\partial x^2} = 0 \qquad x = L. \tag{4}$$

Where: q is the heat flow imposed on the sample, L is the sample thickness and T_0 is the initial temperature of the sample.

2.2 One-dimensional (1D) model numerical solution

(Incropera *et al.*, 2009) present the concepts and calculations used to obtain thermal properties involving problems of one-dimensional heat transfer. Thus, in order to determine the temperature distribution throughout the sample, the finite difference method with implicit formulation in the Eq. (1) is applied. This method is characterized as a discretization method, which is based on the replacement of continuous media by points in the space, where each point is called a region of the system and the temperature is obtained by the average of the neighboring regions.

Therefore, the first derivative of temperature in relation to time is given by:

$$\frac{\partial T(x,t)}{\partial t} = \lim_{\Delta t \to 0} \frac{T(x,t+\Delta t) - T(x,t)}{\Delta t} \tag{5}$$

By convention, n is the representative index of temperature variation in time and Δt is the increment of this variation; thus, the Eq. (5) can be rewritten as:

$$\frac{\partial T(x,t)}{\partial t} = \frac{T_i^n - T_i^{\text{n-1}}}{\Delta t} \tag{6}$$

By applying the concepts of Taylor series expansion to an increment of space Δx in the Eq. (6) , the temperature variation in space is obtained:

$$T(x+\Delta x) = T(x) + \Delta x \frac{\partial T}{\partial x} \tag{7}$$

$$T(x-\Delta x) = T(x) - \Delta x \frac{\partial T}{\partial x} \tag{8}$$

When isolating the term from the derivative of the Eq. (7) and (8), it is obtained that:

$$\frac{\partial T(x+\Delta x,t)}{\partial x} = \frac{T^n{}_{\text{i+1}} - T_i^n}{\Delta x} \tag{9}$$

$$\frac{\partial T(x-\Delta x,t)}{\partial x} = \frac{T_i^n - T^n{}_{\text{i-1}}}{\Delta x} \tag{10}$$

Moreover, the Eq. (6) is expanded with an increment of $2\Delta x$; then:

$$T(x+2\Delta x) = T(x) + 2\Delta x \frac{\partial T}{\partial x} + 2\Delta x^2 \frac{\partial^2 T}{\partial x^2} \tag{11}$$

$$T(x-2\Delta x) = T(x) - 2\Delta x \frac{\partial T}{\partial x} + 2\Delta x^2 \frac{\partial^2 T}{\partial x^2} \tag{12}$$

By replacing the Eq (11) and (12) in the Eq. (9) and (10) , it is obtained that:

$$\frac{\partial^2 T}{\partial x^2} = \frac{T(x+\Delta x) + T(x-\Delta x) - 2T(x)}{\Delta x} \tag{13}$$

Thus, the discretization of the Eq. (13) is:

$$\frac{\partial^2 T}{\partial x^2} \approx \frac{T^n{}_{\text{i-1}} - T_i^n + T^n{}_{\text{i+1}}}{\Delta x^2} \tag{14}$$

Therefore, the thermal properties k and ρcp are obtained by the operational derivatives and by isolating the central node:

$$T_i^n \approx \frac{T^n{}_{\text{i-1}} + f^n{}_{\text{i-1}} \times T^n{}_{\text{i-1}} + f^n{}_{\text{i+1}} \times T^n{}_{\text{i+1}}}{1 + 2f_i^n} \tag{15}$$

Where:

$$f_i^n = \frac{k_i^n \cdot \Delta t}{\rho c p_i^n \cdot \Delta x^2} \tag{16}$$

Finally, the entire methodology explained in this section is programmed in a Matlab routine so that the resolution is done automatically. This resolution calculates the thermal properties with the temperatures in a previous iteration where the systems of equations are solved by an iterative process. In this case, the Gauss-Seidel algorithm has the characteristic of allowing the use of the previous temperature at a current point, hence there is a need for a convergence criterion to minimize the experimental errors when obtaining the thermal properties.

2.3 Estimation of thermal properties by Gauss method

As per the mentioned concepts (Beck and Arnold, 1977) and based on the iterative technique of the Gauss minimization method, it is possible to estimate thermal properties simultaneously and sequentially across the temperature domain using a routine in Matlab, where this methodology to obtain the thermal properties will be described in this section. Thus, a vector is determined in reference to the desired properties:

$$b = [k; \rho cp] \tag{17}$$

By using the expansion of the Taylor series with the approximation of the Eq. (18), it is obtained that:

$$T|b + \Delta b = T|b + \frac{\partial T}{\partial \beta}|_b \Delta b \tag{18}$$

The gradient of the Eq. (18) is a sensitivity matrix of the T vector in relation to the estimated parameters β represented by:

$$X_\beta = \frac{\partial T}{\partial \beta} = \begin{bmatrix} \frac{\partial T_1}{\partial \beta_1} & \cdots & \frac{\partial T_1}{\partial \beta_p} \\ \vdots & \ddots & \vdots \\ \frac{\partial T_n}{\partial \beta_1} & \cdots & \frac{\partial T_n}{\partial \beta_p} \end{bmatrix} \tag{19}$$

Therefore, the expression for increment b becomes:

$$\Delta b = (X_\beta^T X_\beta)^{-1} X_\beta^T (Y - T|_b) \tag{20}$$

Therefore, for the resolution of the iterative technique there is a step-by-step process for the equations cited above: i. Adopt a convergence criterion; ii. Enter with an initial value for b; iii. Calculate the temperature vector T_b; iv. Calculate the values of the sensitivity matrix X_β; v. Calculate the value of the increment Δb with the Eq (20); vi. If the convergence criterion was reached, the solution was found; otherwise update the vector as follows:

$$b^{\mathrm{n+1}} = b^n + \Delta b^n \tag{21}$$

Go back to step iii and repeat until the convergence criterion is reached in step vi. Therefore, each iteration calculates the properties k and ρcp at its given time. Therefore, to obtain the results it is necessary to use the concept of future times, that is, it is assumed that the thermal properties remained constant. Finally, the convergence criterion adopted in this work is obtained by:

$$conv = abs[(\frac{\Delta b_1}{\rho cp}); (\frac{\Delta b_2}{k})] \tag{22}$$

However, this convergence may fail as cited in (Aguiar, 2012). Therefore, it is necessary that the initial guess of the desired properties is close to the actual value of the studied property or values found in existing literatures.

2.4 Analysis of sensivity coefficients

According to the concepts of reverse engineering when there are inverse problems involving heat conduction, the sensitivity coefficients represent how sensitive the temperature is in relation to the variation of the desired thermal properties. In other words, high sensitivity refers to small variations in the analyzed properties, which result in large variations in temperature. Therefore, this means that, in case of high sensitivity, the problem studied will be well-conditioned, that is, such properties can be estimated through temperature measurements. However, when there is correlation between the sensitivity coefficients studied, such properties will not be possible to be estimated because the coefficients tended to infinity. Consequently, this represents that the problem is ill-conditioned. Thus, the sensitivity coefficients desired by this work are obtained by:

$$X_k = k\frac{T(k + \delta k, \rho cp) - T(k, \rho cp}{\delta k} \tag{23}$$

$$X_{\rho\ \mathrm{cp}} = \rho cp \frac{T(k, \rho cp + \delta \rho cp) - T(k, \rho cp)}{\delta \rho cp} \tag{24}$$

Therefore, it is extremely important to analyze the sensitivity coefficients to have an adequate estimation of the thermal properties. In addition, it is of paramount importance to obtain the place of the thermocouple that will bring the best result to the estimation.

2.5 Estimation of thermal conductivity by Fourier's Law

In a one-dimensional thermal system, temperature gradients exist along only a single direction, with heat transfer and thermal flow occurring only in this direction. Although they are essentially simple, one-dimensional thermal models can be used to reliably represent numerous engineering problems (Incropera *et al.*, 2009).

Fourier's Law governs one-dimensional conduction in a permanent regime; it is a phenomenological law developed from experimentally observed phenomena rather than being deduced from principles. This heat conduction equation is therefore the result of a large sum of experimental evidence. According to Fourier's Law, the equation of heat rate transferred by permanent conduction is described by:

$$q = -kA\frac{\partial T}{\partial x} \tag{25}$$

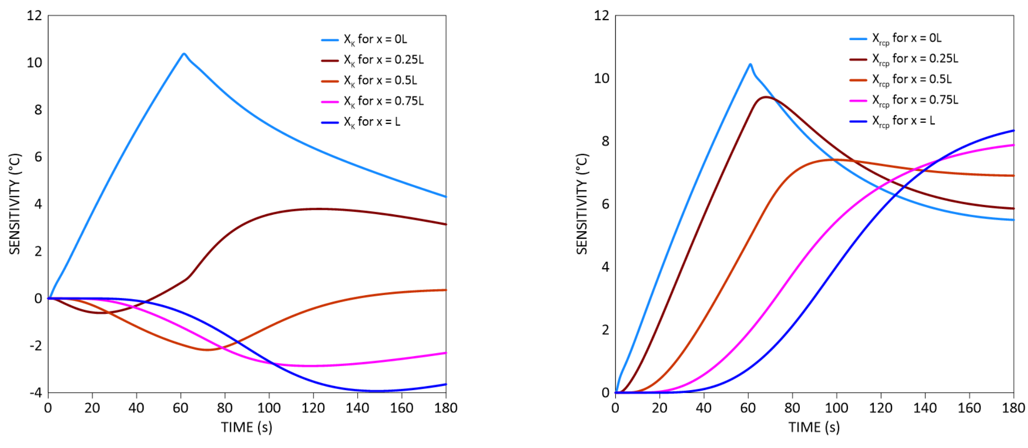
Where: q is heat transferred by conduction; x is the direction of thermal flow; A is the area perpendicular to the heat direction; T is the temperature; $\frac{\partial T}{\partial X}$ it is the thermal gradient; and k is the thermal conductivity of the material.

Heat always flows in the direction of higher temperature to lower temperature, which explains the negative signal in Eq. (25). The thermal conductivity of a material indicates the rate of thermal energy transferred by diffusion, which depends on the chemical composition, molecular structure and physical state of the material. In general, thermal conductivity is much less pressure dependent than temperature, so pressure dependence can be ignored. However, the variation of conductivity according to temperature can be ignored when the temperature range in consideration is not too large. Therefore, in Eq. (25), the thermal conductivity represents a constant of proportionality. In the case of the guarded hot plate method, a resistive heater is used to heat one side of the sample until the permanent regime is reached, that is, when the heat passing through the sample becomes constant, the temperature distribution can also be considered practically constant.

3. EXPERIMENTAL PROCEDURES

3.1 Experimental aspects

The analysis of sensitivity coefficients allows us to determine the experimental aspects which have the greatest relevance for the experiments to be carried out in the most efficient way. Such aspects are: thermocouple position in the sample, intensity of the heat flow and the warm-up time of the sample. The purpose of the sensitivity analysis is to obtain an experimental arrangement that maximizes the sensitivity coefficients, that is, the best positioning of thermocouples is in the region that obtains the highest sensitivities. Therefore, in Figure 2 the behaviors of the modified sensitivities for the composite sample are shown in different positions.



(a) (b)

Figure 2: (a) conductivity sensibility for different sample spaces. (b) volumetric heat capacity sensibility for different sample spaces.

In composite materials such as the one studied in this work, it is noticeable that on the surface of the sample $x = 0$, the coefficients are higher, because the thermocouple to be measured will be near the heated region. However, it is not feasible to measure the temperature in these places, since the coefficients are correlated, which means it would be impossible to estimate the thermal properties simultaneously in this region. However, the intermediate regions of the sample, shown that had lower results than the end of the sample, since to measure the internal temperature it would be necessary to introduce the thermocouples into the sample, which consequently would bring a discontinuity effect in the sample as stated in (Martínez *et al.*, 2017).

Therefore, in this application, the lower surface of the sample $x = L$, had the most appropriate behavior for the maximization of the sensitivity coefficients. As a result, throughout this work, in the experimental procedures mentioned below, the thermocouples were glued to the lower surface of the samples.

Also, in Figure2, the coefficients of volumetric heat capacity, $X_{\rho cp}$, are maximized with the increase of heat intensity imposed in the sample; on the other hand, X_k varies according to each position of the thermocouples, meaning it is sensitive both by the flow variation and its intensity. Therefore, the higher the flow imposed in the experiment, the greater the sensitivities of the parameters desired.

For the studied composite, the maximization of sensitivity coefficients was achieved by the use of thermocouples on the lower surface of the sample and by the maximization of the flow, and the limitation of the flow was due to the specifications of the resistive heater and transducer manufacturers. However, the durations of the experiments are limited by the maximum desired temperature difference.

3.2 Hot plate method experimental procedure

In order to measure the thermal conductivity of the composite, an experimental bench is mounted, represented by Figure3.

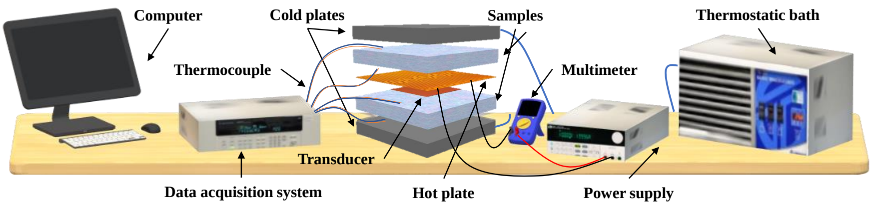


Figure 3: Experimental representation of the hot plate method.

The hot plate used is a great differential, it is made of Kapton and its thickness is similar to that of a sheet of paper, with dimensions of 100x100mm and a resistance of approximately 13 Ω, which is connected to a direct current source. To avoid air interstices between the samples and the heater, two wooden supports are used pressed against each other by two C-clamps, with due care not to compress the sample and consequently cause a change in its thickness. In addition, four Type T thermocouples are used. A thermocouple is used only to monitor room temperature, while the others are placed on the samples and on the hot plate, as shown in Figure 3. All are connected to a data acquisition system, controlled by a computer. After the assembly is completed, tests are performed to determine the power required to be supplied to the heater to achieve the desired thermal gradient in the permanent regime.

The experimental assembly of the hot plate method carried out at the Heat Transfer Laboratory (LabTC) of the Federal University of Itajubá is symmetrical to ensure that the thermal flow occurs in one direction, but in two senses. The samples are square section with about 100mm and a thickness of 4mm. One of the surfaces of the samples was placed in contact with a cold plate to help obtain a unidirectional heat transfer. However, the other surface was heated by a resistive heater that is between the two samples, responsible for providing the heat flow.

Therefore, there is a symmetrical experiment in relation to the heater, in an attempt to provide heat equally for both samples. For greater reliability, a heat flow transducer was used to measure the steady flow received by the samples. In addition, to mitigate the lateral heat leakage, a polystyrene insulation was used in the assembly. The leakage is natural and occurs due to heat exchange with the environment through convection and radiation. However, such a phenomenon should be minimized to the maximum, since a pure model of heat conduction is being considered.

The temperatures of the two surfaces were collected using Type T thermocouples, and the difference of these temperatures is used to calculate the thermal conductivity of the composite. In this experiment, 60-point data were collected at a frequency of 0.1Hz after 4 hours of the beginning of the experiment, and the permanent regime was reached at about 1h, which reinforces the importance of using the cold plate. That way, it was ensured that the information used accurately transmitted the stationary thermal behavior of the experimental assembly. In this work, two cold plates and a polystyrene insulation were also used to maintain the heat flow generated by the resistive heater as one-dimensional as possible.

3.3 Experimental procedure using Gauss

In order to simultaneously obtain the thermal conductivity and volumetric heat capacity of the studied composite, it was necessary to use the experimental assembly represented in Figure 4 to obtain the experimental temperatures used in the thermal models. This assembly was carried out in the Heat Transfer Laboratory (LabTC) of the Federal University of Itajubá.

The main components of the experimental assembly are shown in Figure4: data acquisition system, which collects data from the thermocouple and the transducer; computer, which controls the parameters obtained by the data acquisition

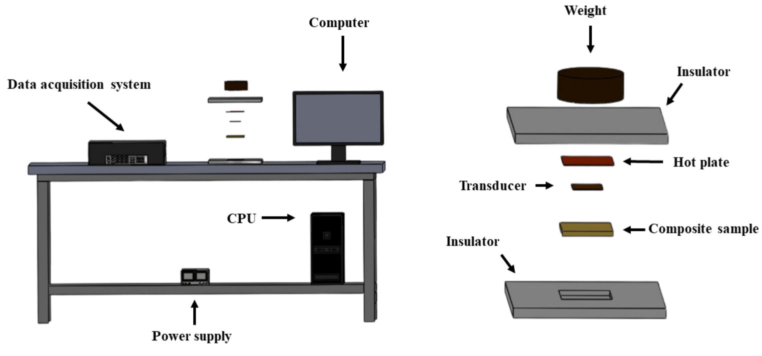


Figure 4: Representation of the experimental model used in the Gauss numerical model.

system; Type T thermocouple, which conducts temperature data to the data acquisition system; transducer, which conducts flow data to the data acquisition system; Kapton resistive heater, which heats the sample with the power provided by the power supply; power supply, which provides constant power to the resistive heater. However, since the material is a composite, with the characteristics of an insulating material, there is much loss, making it necessary to use a transducer to measure the actual flow being used in the sample. Thus, as seen in Figure 5, the behavior of the flow over time is not linear.

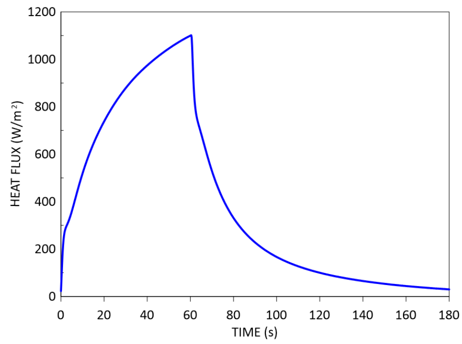


Figure 5: Flow used in composite sample.

Besides, Figure 5 shows that the heating time was 60 s and the total time of the experiment was 180 s, which resulted in an adequate temperature gradient to be possible to estimate the thermal properties of the material. Also, because the material has an insulating behavior, the heat is trapped on the surface of the sample after the source is turned off and, throughout the experiment, such heat generates the heating flow of the sample. The composite sample has the same dimension as the heater, 100x 100x 10 mm, this ensures that the entire surface of the sample is heated, that is, the heat transfer is one-dimensional in this application. Also, in order to minimize the existing losses in the experiment, polystyrene was used to isolate the sample, and with the combination of two insulating materials, consequently, there was a portion of heat loss to the environment.

4. RESULTS

4.1 Results using the hot plate method

The detailed results for measuring the thermal conductivity of the composite sample using the hot plate method are shown below.

Table 1: Thermal conductivity measurements obtained by the hot plate method.

Experiment	1	2	3	4	5
Thermal Conductivity $Wm^{-1}K^{-1}$	0.158	0.156	0.158	0.157	0.157

Therefore, based on the experimental results shown in Table 1, the composite sample presents thermal conductivity at room temperature with average value and standard deviation, respectively, 0.157 $Wm^{-1}K^{-1}$ and 0.000837 $Wm^{-1}K^{-1}$.

Therefore, because there is some degree of uncertainty in the measurements due to the sample not being exactly uniform, with lateral heat loss occurring in the experiment, fiberglass wool was used around the assembly. However, due to the small discrepancy in the average value, it can be stated that the technique was conducted satisfactorily and that the results are reliable, this fact is proven by the small magnitude of the standard deviation value.

4.2 Results using Gauss

In (Holman, 2012) it is stated that, in order to do a statistical analysis of the experimental data obtained, taking into consideration the errors within the experiment, it is necessary to obtain at least 20 measurements to have a reliable analysis. That said, using the statistical concepts cited in (Guimarães and de Azevedo Júnior, 2020), a descriptive statistics test was performed using the Minitab software to determine the total number of measurements of the experiment. With a margin of error of 10%, a confidence level of 95% and a standard deviation of 0.22, it was obtained that the number of measurements to be performed in the experiment is 22.

The sensitivity coefficients of the estimated properties can be analyzed in Figure 6, which shows that the magnitude of $X_{\rho cp}$ increases proportionally as the intensity of the heat flow proposed rises in the sample, Moreover, X_k had the same behavior as the sensitivity of the volumetric heat capacity, but its magnitude had a lower value. Therefore, the experiment was able to estimate thermal properties simultaneously, because there is no correlation between these sensitivities.

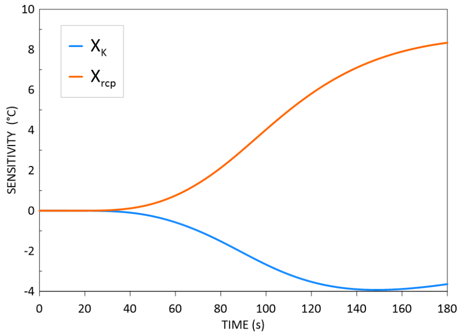
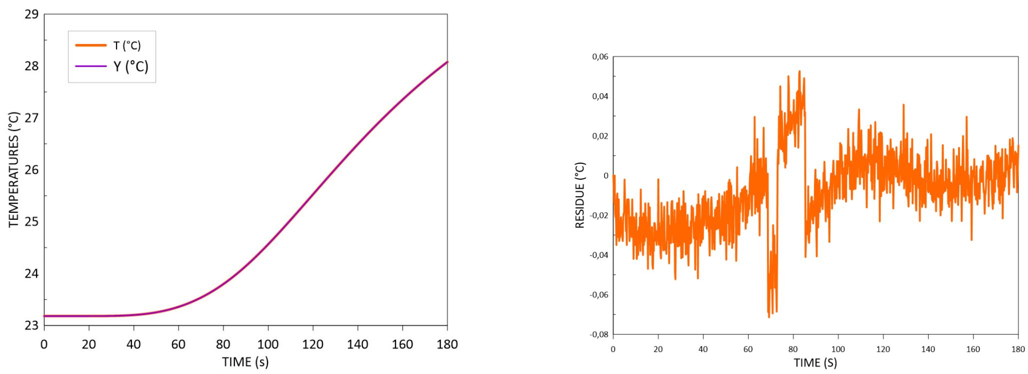


Figure 6: Sensitivity coefficients obtained on the lower surface of a composite.

On the other hand, in Figure 7(a)is shown the experimental and numerical temperatures measured on the lower surface of the sample, $x = L$. The numerical temperature was calculated using the Gauss-Seidel methodology, using the values obtained in one of the experiments. This way, when comparing these temperatures, it can be concluded that they produced great results, given that there were few noises and an optimal agreement between them. These facts are proven by the residue of temperatures, which presented a maximum divergence of 0.07°C, as shown in Figure 7(b).



(a) (b)

Figure 7: (a) Comparison betweennumerical temperature and experimental temperature. (b)Residue between temperatures on the lower surface of the sample.

Table 2 shows the results obtained for thermal conductivity and volumetric heat capacity of the studied composite. In Table 3, the average and standard deviation of the measurements presented are represented.

Therefore, when analyzing the results in Table 3, it is concluded that for the two properties estimated simultaneously, the values obtained showed low standard deviation, assuring the reliability and repeatability of the experimental technique used.

Table 2: **Measurements of thermal properties by Gauss method.**

Measurements	Thermal Conductivity [$Wm^{-1}K^{-1}$]	volumetric heat capacity [$Jm^{3}kg^{-1}K^{-1}$]
01	0.1530	1.0852
02	0.1391	0.9871
03	0.1505	1.0192
04	0.1242	0.8738
05	0.1345	0.9216
06	0.1227	0.8622
07	0.1312	0.9044
08	0.1410	0.9443
09	0.1274	0.8841
10	0.1400	0.9342
11	0.1288	0.8910
12	0.1283	0.8947
13	0.1483	0.9542
14	0.1333	0.9187
15	0.1400	0.9342
16	0.1391	0.9611
17	0.1324	0.9144
18	0.1342	0.9050
19	0.1352	0.9294
20	0.1325	0.9190
21	0.1359	0.9224
22	0.1363	0.9273

Table 3: Statistical analysis of the results obtained using Gauss.

	Average	**Standard deviation**
Thermal Conductivity $Wm^{-1}K^{-1}$	0.1356	0.0076
volumetric heat capacity 10^{6} [$Jm^{3}kg^{-1}K^{-1}$]	0.9312	0.0496

5. CONCLUSION

This study introduces a novel technique for the simultaneous estimation of thermal conductivity and volumetric heat capacity in insulating materials, focusing on a composite material utilizing epoxy resin as its matrix and fiberglass as its reinforcement. What sets this method apart is its remarkable simplicity, efficiency, and cost-effectiveness, particularly when contrasted with the traditional hot plate method that relies on Fourier's Law.

One key differentiator in our approach is the utilization of a relatively straightforward and low-cost experiment.However, it's essential to note that insulating materials like the one we examined pose specific challenges.These challenges stem from the material's low thermal conductivity, which hinders the efficient diffusion of heat through the sample. Also, the use of imperfect insulation results in inevitable heat losses that can impact the accuracy of the results.

In contrast, the hot plate method offers a distinct advantage.By incorporating cold plates, this method promotes the efficient diffusion of heat, and the design of the hot plate's guard ring maintains uniform temperatures during the experiment, minimizing heat losses and enhancing accuracy.

An essential aspect of our work was the careful selection of experimental parameters, given the limitations imposed by the resistive heater's temperature range. Sensitivity coefficients played a critical role in ensuring the accuracy of our results. Additionally, data analysis post-experiment was necessary due to the insulating nature of the material.

Our results revealed that both methods employed showed a reasonable level of agreement, with a difference of approximately 13%. The low standard deviation values in both cases underscore the reliability of the estimates. The variance in results can be attributed to the experimental insulation utilized in the Gauss method, as isolating one insulator with another led to greater heat leakage compared to the hot plate method.

In conclusion, our approach effectively enables the simultaneous estimation of thermal conductivity and volumetric heat capacity in composite materials. As a potential avenue for improvement, we suggest exploring alternative insulation methods to further enhance the accuracy of our experimental approach. These improvements could involve the use of a vacuum oven to minimize heat losses and the exploration of different thermal models to enhance sensitivity coefficients. Additionally, a study of the influence of temperature variations on thermal properties would contribute to a deeper understanding of the material's behavior.

6. ACKNOWLEDGMENTS

We thank the Minas Gerais Research Funding Foundation (FAPEMIG), the National Council for Scientific and Technological Development (CNPq) and the Coordination for the Improvement of Higher Education Personnel (CAPES) for the financial support during the research. In addition, we thank the Federal University of Itajubá for the support during the research and for the space assigned to LabTC.

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