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# STATIC ANALYSIS AND FAILURE MODES OF HUMANOID ROBOTS IN CONTACT WITH THE ENVIRONMENT

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**Abstract.** *Humanoid robots (HR) can be developed to interact with people and work in an unstructured environment, where external contact forces may arise intentionally or accidentally, changing the static and dynamic balance. When in contact with the environment, several HR tasks are executed at low velocities and the kinetostatic approach is useful. The fundamentals and concepts of static analysis of mechanisms in general and robots in particular is a topic well established in the literature. But a deeper analysis of humanoid robots statics shows that there are still open points to be addressed in tasks where the robot has to push an object. Such kind of tasks may take place in situations where HRs interacts with people, doing activities like pushing a wheelchair or helping an elder people to move an object. Knowing all the external forces influences and how it reflects in the internal structure of the HR can be the difference between success or failure to accomplish such kind of tasks that can potentially threaten humans life's. In this work, the static analysis of HR that considers gravity, slipping, tumbling, links structural resistance and actuators saturation in a whole integrated solving approach is presented and the different failures modes that can occur are discussed. Since the humanoid robot in contact with the environment can be understood as a closed kinematic chain with redundant actuation, this paper presents an approach that does not use a pseudo-inverse matrix to solve the problem. Important simulation results about how friction and external contact forces influences the force capabilities of humanoid robots are presented. Results obtained from this work can be useful for the design of new HR or for task planning in actual robot where the contact with the environment occurs.*

**Keywords:** *humanoid robots, static analysis, failure modes, environment, contact.*

## 1. INTRODUCTION

Recent advancements in robotics are noteworthy, particularly in the field of humanoid robots (HR), primarily due to its versatility. In the past years, there have been numerous investigations on this subject, leading to a large amount of publications and experimental tests. Even though several fields of knowledge are studying humanoid robots, from the mechanical engineering perspective, the majority of research efforts focus on biped motion and handling problems (Torricelli et al., 2016) (Saeedvand et al., 2019). However, because human resources were mainly created to interact with people and work in an unstructured environment, external contact forces could happen either on purpose or by accident and modify the static and dynamic balance of the robot. Many HR tasks are carried out at low speeds when in contact with the environment, thereby making the kinetostatic approach useful.

When the HR performs tasks that require the application of known forces on the environment, it is necessary to check whether it is possible to apply these forces without incurring in one of the failure modes. If the imminence of a failure is identified, actions can be taken in order to increase the force capacity of the HR, allowing the execution of the task. As the humanoid robot presents kinematic redundancy, the possible actions to increase the strength capacity include changing the posture of the HR, modifying the contact points of the feet and hands and changing the contact forces.

In Saeedvand et al. (2021), an algorithm is developed that learns a HR how to drag heavy objects, identifying the type of floor surface to avoid slippage. After the training phase, the authors successfully did experimental tests in an adult-size HR.

Yang et al. (2018) proposes a real time Quadratic Programming (QP) solver to optimize the joint torques when a HR pushes a heavy object, without slipping. The contact force between the robot and the environment are obtained using a virtual mass method. The HR capabilities for executing these tasks are verified by performing simulations.

HR ability to push an object executing a whole body interaction is present in Murooka et al. (2015). The proposed algorithm properly selects the best contact point in order to produce a large pushing force. To verify the effectiveness of the method, the authors show experimental results with an adult sized HR.

When in contact with the environment through its four limbs, the humanoid robot can be understood as a closed kinematic chain. Unlike most closed-chain mechanisms, the humanoid robot has actuators at all joints and therefore presents actuation redundancy (Mattioli and Vendittelli, 2017). The traditional approach to solving robot statics uses the Jacobian matrix, which maps external forces into torques at the joints (Yokokohji et al., 2002). However, in the presence of actuation redundancy, the solution is not unique and, in general, a pseudo-inverse matrix is used to enable the mapping (Mattioli and Vendittelli, 2017). In addition, in the studies found in the literature that deal with the application of a pushing force, the authors consider that the two arms of the robot are in the same configuration and the point of contact occurs at the same height in the object (Konno et al, 2005)(Yang et al., 2018). This simplifies the evaluation of the HR statics, since the total wrenches obtained on the arms can simply be divided by two to obtain the efforts on each arm.

And no approach was found in the literature that evaluates the force capabilities of a HR taking into account all possible failure modes. Accurately identify this failure modes in a HR can be the difference between success and failure when carrying out tasks like pushing a wheelchair or assisting an elderly person to stand up

The objective of this paper is to provide a method to evaluate the static analysis of HR, that considers gravity, slipping, tumbling, links structural resistance and actuators saturation in a whole integrated solving approach. This method based on Davies' approach allows the configuration of each of the arms to be distinct and does not make use of a pseudo inverse matrix to find a possible solution. Additionally, the results obtained shown that changes in contact forces can increase the robot's force capacity without incurring in one of the failure modes. Considerations regarding the contact wrenches, necessary to allow the statics of the robot's with redundant actuation to be solved, are presented. The influence of external forces and Zero Moment Point (ZMP) variation in the different failures modes are also discussed.

The approach taken to solve the HR statics in contact with the environment will be described in Chapter 2. The failure modes that can occur when an HR performs an object-pushing task and their implications on the calculation of the static equilibrium of the system are covered in Chapter 3. Chapter 4 will present the results obtained through simulation and analyze the possible failure modes. The conclusions of the work carried out are presented in Chapter 5.

## 2. HR STATIC ANALYSIS

Robot statics can be solved by obtaining the Jacobian matrix, which maps the external forces into joint wrenches. In the presence of kinematic or actuation redundancy, where there can be infinitely different solutions, the Jacobian matrix does not have an inverse (Mattioli and Vendittelli, 2017). In these situations, one of the approaches presented in the literature is to use a pseudo-inverse matrix, which seeks to optimize some user-defined quantity. However, the Jacobian only provides the active forces in the joints. In order to obtain the passive forces in each joint and, in turn, in the links, it is necessary to perform the static equilibrium for each of the links. The Davies' method provides all the forces in each joint explicitly, without the need for further calculations (Davies, 1983). In addition, with the considerations discussed in section 2.2 regarding the forces acting in the contacts of the feet and hands with the environment, the approach presented uses an external loop which allows the calculation of the HR statics, without using a pseudo inverse matrix.

### 2.1 HR geometry and features

The humanoid robot employed in this research has the same geometric and physics characteristics as presented in Pierezan et al. (2012). It contains twelve rotational joints and nine links, as shown in Figure 1, and their masses and lengths have predefined values, as described in Table 1. The dimensions do not correspond to any real robot, but are used for comparative purposes with other works done in the area. Generic units could have been used instead, since what really matters for the analysis carried out in this work are the relative values found and not the absolute values.

Table 1 – Humanoid links features.

Links' Name	Symbols	Mass (kg)	Length (m)
Forearms	{1,3}	1.0	1.0
Biceps	{2,4}	1.0	1.0
Trunk	5	2.0	2.0
Thighs	{6,8}	2.0	1.2
Calfs	{7,9}	1.5	1.2

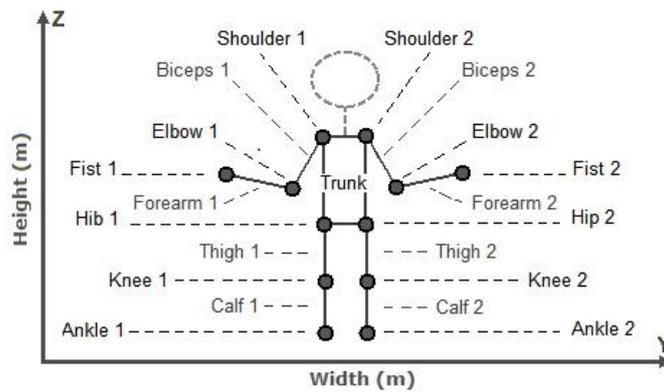


Figure 1 – The humanoid robot model studied [1].

In this work, a two dimensional analysis in the sagittal plane of the HR will be evaluated. And only static tasks, where the HR applies forces to push objects but without moving them, will be considered.

When performing a task of pushing a wall or an object, the HR has 4 contacts points with the environment, one in each of the hands and one in each of the feet, as shown in Figure 2. In this condition and considering that there will be no movement of the object and no change of the contact points, the system can be understood as a closed kinematic chain.

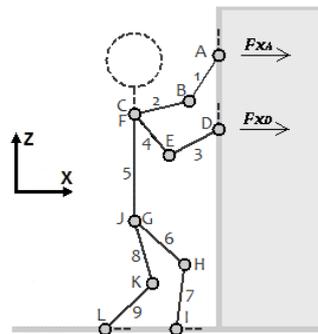


Figure 2 – The humanoid robot making contact with the environment.

If the contact points are not fixed, although restricted to certain limits that allow the closure of the kinematic chain, the HR will present kinematic redundancy. And, even if the contact points are fixed, the system will still present kinematic redundancy because it is possible to change the Cartesian position of the torso and its inclination in the  $x$ - $z$  plane, as shown in Figure 3, without changing the contact points. Therefore, in order to solve the position kinematics of the HR, it is necessary to define the contact points, the position and the inclination of the robot torso.

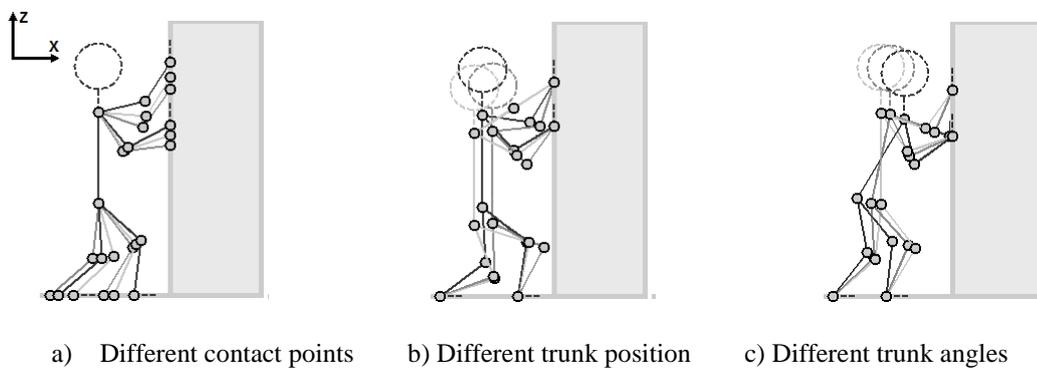


Figure 3 – Kinematic position redundancy.

## 2.2 Hand and feet contact analysis

When the HR makes contact with the environment, the geometry of the feet and hands at the contact points deserves special attention.

As the proposed method only considers contacts on flat surfaces, the hands of the robot can be considered as having also a flat geometry. Even if the robot has hands with a higher dexterity, with for example the ability to hold and manipulate objects, this feature would not be useful for the type of task evaluated in this work.

In addition, the reaction forces from the wall will always act in a direction that intersects the axis of the fist joint. If the forces were to act, for example, at the fingertips, a torque would arise which would have to be balanced by a moment applied to the fist, resulting in a torque on the actuator with no benefit in terms of increased applied force. That represents unnecessary additional energy consumption. And the distance in  $x$  direction between the contact point of the hand and the fist joint can be assumed to be zero. As a result, the forces in the  $z$  direction at the contact point intercept the fist joint, generating no moment, which is a good approximation of what happens in reality. Therefore, a punctual contact occurs, where the torque on the fists in this kind of task must be zero. Thus, at contact, only forces in the  $x$  and  $z$  direction will be considered.

The forces in the  $z$  direction, if any, should be balanced by the friction forces between the object and the robot hands. Considering these issues, there is no need to consider the hands as links in the HR model, since the closure of the chain happens at the point of contact of the fists joints with the object.

A similar analysis can be performed for the feet, with the following consequences: i) flat feet will be considered in this work; ii) the feet are not considered in the model as additional links; iii) at static equilibrium, the torques at the ankle joints are zero; iv) the forces in the  $x$  direction are balanced by the friction that exists in the contact between the feet and the ground.

These considerations are important because they provide tools to eliminate the HR redundant actuation, allowing the evaluation of its static analysis.

## 2.3 Task specific features

Regarding the task to be performed, three different situations can be considered.

In the first one, the objective is to determine whether the robot can exert a desired force in the  $x$  direction on the object, while in a specific posture and with fixed contact points. Failure modes are tracked and show whether or not the task can be completed successfully.

In the second situation, also with a defined posture of the robot and with fixed contact points, it is desired to find the maximum force that the robot can apply in the  $x$  direction without failing. The failure modes are tracked, and the maximum force is attained right before the first failure occurs.

In the third situation, we seek to optimize the posture and the location of the contact points between the robot and the object, in order to find the maximum force that can be applied in the  $x$  direction. Despite the freedom of positioning, limits must be respected so that the closure of the kinematic chain can occur.

In this paper only situation 1 will be considered, as the results obtained provide sufficient information to support the concepts presented. Interested in details about situations 2 and 3 can consult the work of Pierezan et al. (2017).

But, regardless of the situation under analysis, it is necessary to solve the position kinematics and calculate the static equilibrium of the system.

## 2.4 Position kinematics

Since the HR presents kinematic redundancy, the approach used in this work to obtain its position kinematics is defining the trunk position. In situations 1, described in Subsection 2.3, the position of the trunk and the contacts of the hands and feet are defined. Therefore, from the position of the trunk, as introduced in Pierezan et al. (2012), it is possible to directly define the position kinematics, obtaining the position of the other joints and links. As the orientation at contact are free, there are actually two solutions for each limb, usually known in the literature as elbow up and elbow down. In this paper we will consider only the configurations that most closely represent the limits of movement of the elbow and knee joints of a human being. That is, in the upper limbs the elbows will always be down and in the lower limbs the knees will always be forward. In this way the position kinematics of the HR is uniquely defined.

Once the position kinematics has been defined, the static balance of the system can be calculated.

## 2.5 Static analysis

The Davie's method is used to establish the static balance because it is simple to manipulate in terms of inputs and outputs (referred to as primary and secondary variables, respectively), allows for the addition of new external forces and moments and has a visual graph representation (Davies, 1983).

Figure 4a (Pierezan et al., 2012) shows the links and joints in the sagittal plane and the Cartesian coordinate system used along this work. The humanoid graph (Figure 4b) is generated based on these joints and links. Given that only the sagittal plane is under consideration (i.e., a planar robot), the forces are decomposed in their  $x$  and  $z$  components with a moment along the  $y$  axis.

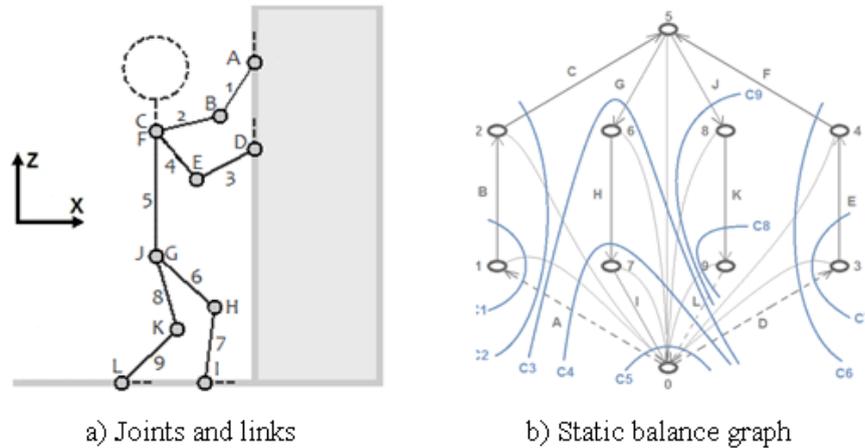


Figure 4 – HR joints, links and graph.

The humanoid has nine links (1–9), which result in nine gravitational forces. Feet and hands are not considered links, as explained in Subsection 2.2. As a result, there are four joints (A, D, I, and L) in contact with the environment with three wrenches (forces and moments) each. And there are eight internal revolute joints (which do not make contact with the environment) with three wrenches each. The humanoid trunk is represented with the number 5 and the ground (environment) with number 0 respectively in the graph.

According to Pierezan et al. (2017), this mechanism has a total of 45 constraints, which corresponds to all the wrenches (12 from the contact joints, 24 from the internal joints, and 9 from the links weights). The total number of independent equations extracted from the graph is 27, and the number of independent variables required to determine the static balance (primary variables) is 18.

To evaluate the proposed method, it is necessary to identify the internal HR wrenches and the external ones. External wrenches are the contact forces in the  $x$  and  $z$  direction that may arise at each of the hands, the gravitational forces due to the weight of the body links, the forces in the  $x$  and  $z$  direction that appear at each of the feet and the four moments at the points of contact. The forces acting in the  $z$  direction on the hands and in the  $x$  direction on the feet are the external frictional forces. Thus, there are in total 21 external forces. The internal forces are the torques of the actuators and the forces in the  $x$  and  $z$  direction acting on each of the joints.

As the 9 link weight forces are always known and the moments in the four contact joints are assumed null, as explained in Subsection 2.2, 13 of the 18 primary variables are automatically defined. Other five variables must be known in order to determine the humanoid static balance. The five additional primary variables needed to solve the statics can be either internal or internal variables, or even a composition of them.

At this point, the approaches to evaluate the statics analysis differ somewhat according to the situation. In situation 1, which is the one of interest in this paper, the 5 additional primary variables must be external. Two of them are precisely the known contact forces on the hands in the  $x$  direction. And the friction forces on the hands must also be known, providing two more primary variables. At a first glance it might seem appropriate to consider these frictional forces on the hands to be zero, but it will be shown in Subsection 3.3 and 3.4 that these forces can be important to improve the maximum force that a robot can apply.

The last primary variable must come from the contact of the feet with the floor. In a first analysis, both the friction force and the normal force on one of the feet can be chosen. There is therefore a dependency between these four variables. Once one of them is known, together with the primary variables mentioned above, the other three can be calculated.

As the weights of each of the links are known, the resultant gravitational force  $G$  can be represented as being located at the center of gravity ( $GC$ ) of the HR. Note that the friction forces are assumed to act intercepting the axes of the ankle joints, as defined in Subsection 2.2. The normal forces  $F_{zL}$  and  $F_{zI}$  acting on the ankle joints of each foot can be determined using the Zero Moment Point (ZMP) approach. ZMP is defined as the projection of the robot's center of gravity on the ground, calculated as in Eq. (1):

$$ZMP = \frac{-M \cdot |g| \cdot Cx - \sum_{\rho} (F_x P_z) + \sum_{\rho} (F_z P_x)}{-M \cdot |g| + \sum_{\rho} F_z} \quad (1)$$

where  $M$  is the robot total mass,  $g$  is the gravitational acceleration,  $Cx$  is the robot's center of gravity coordinate in the  $x$ -axis,  $\rho$  is the set of the upper joints in contact with the environment,  $F_x$  is the vector of the forces in the  $x$  axis,  $P_z$  is the set of its respective coordinates in the  $z$  axis,  $F_z$  is the vector of the forces in the  $z$  axis and  $P_x$  is the set of its respective coordinates in the  $x$  axis. Once the position of the ZMP is obtained,  $F_{z_L}$  and  $F_{z_I}$  can be calculated as being inversely proportional to their distance from the ZMP and their sum being equal to the sum of all the other external forces acting in  $z$  direction.

But, despite the calculation of the fifth primary variable, which could be  $F_{z_L}$  and  $F_{z_I}$ , the static equilibrium is still not solved. This is because the friction forces are related to the normal forces through the Coulomb equation of static friction, as in Eq. (2):

$$F_f = \mu F_n \quad (1)$$

where  $\mu$  is the coefficient of static friction,  $F_f$  is the friction force and  $F_n$  is the resultant normal force to the ground. The concept of a friction cone, which establishes the maximum reaction force vectors that can arise when a robot comes into contact with the environment without causing slippage, is well established in the literature. The opening angle of the cone is a function of the maximum coefficient of friction of the objects in contact. Analyzing the friction cone, it is possible to state that the coefficient of static friction can take any value between 0 and its maximum value, which depends on the materials of the surfaces in contact. Since we are only analyzing the robot in the sagittal plane and the task executed by the HR requires that there are forces in the  $x$  direction observed in the contact between hands and environment, it is possible to say that the total friction force on the feet must have the same magnitude and opposite direction of these contact forces so that the robot does not slip. Therefore, the total friction force can be obtained, but it is necessary to define the friction force on each of the feet, in order to allow the calculation of the static equilibrium of the system. If the system is at the limit of slippage, the  $\mu$  will be equal to the maximum at each of the feet and the friction force will also be maximum at each of the feet for a given normal force. However, if it is not the case, the friction force should be distributed between the two feet. It is done following the classical approach presented in Physics textbooks, where static friction is considered proportional to the existing normal force.

Thus, since  $F_{z_L}$  and  $F_{z_I}$  are known,  $\mu$  is evaluated so that the friction forces on each of the feet are proportional to these normal forces. From  $F_{z_L}$ ,  $F_{z_I}$  and  $\mu$  we obtain  $F_{x_L}$  and  $F_{x_I}$ . Any one of these four variables can be chosen as the last primary variable required to solve the static analysis. Therefore, this balance of external forces and moments is an initial procedure that must be done for the calculation of the static equilibrium of the HR before using Davies' method. The continuation of the method is presented in details in Weihmann (2011). As a result of the method, the wrenches in each of the joints that make up the system are obtained. Through the joint torques, it is possible to verify the failure modes of joint saturation and mechanical strength of the links. Note that the ZMP and the static friction coefficient  $\mu$  are obtained from the external balance, indicating whether the failure modes of tumbling and slipping can occur or not. Note that this approach was only possible because of the result provided in the analysis of the feet's contact with the ground, where it was concluded that the torque must be zero. Otherwise, there would be actuation redundancy and this method could not be used.

### 3. FAILURE MODES

When performing contact with the environment by applying forces and moments, the HR is in danger of failing to perform a particular task if any of the limits of its capabilities are reached.

In the case under study, where the robot in static equilibrium applies a force with its hands in the direction of pushing an object, four failure modes can arise. The first failure mode is related to the maximum torque that the actuators can provide. If the task requires a moment greater than that specified for any of the active joints, the task cannot be completed. The second failure mode is related to the mechanical strength of the links. A structural failure will occur if the wrenches on each of the links are greater than those supported due to their geometry and material. The third failure mode is related to slippage. If the friction on the feet is not sufficient to support the external loads applied in the  $x$  direction, the HR will slip, not being able to perform the task. And the 4th failure mode is related to the robot tumbling. If, due to the external forces, the ZMP is at a point located further away from the object than the back foot of the HR, the robot will tumble.

How to identify these failure modes using Davies' method is explored in the following subsections. Possibilities to increase the robot's ability to resist slipping and tumbling failures by changing the contact forces on the hands are also discussed.

### 3.1 Actuators saturation

In revolution joints such as those of the HR under study, saturation can be understood as the maximum torque that an actuator can apply or withstand when in use. This information can be obtained, in general, from manufacturers' catalogs. As the Davies method explicitly provides the torques at each of the actuated joints, the verification of this failure mode is done directly by comparing the values obtained in the static analysis with the maximum values provided by the actuators manufacturer.

### 3.2 Mechanical Resistance

The static calculation using the Davies method explicitly provides the wrenches in each of the joints that make up the HR and the external forces and moments. As the gravitational forces are also known, the wrenches in each of the links of the mechanism can be obtained directly from the statics evaluation. So, it can be verified by analytical methods or using finite element analysis if the links resist the imposed stresses. In this work, the mechanical strength of the links is not checked. However, the methodology presented provides all the necessary information to support this analysis.

### 3.3 Tumbling

Tumbling is one of the failure modes commonly treated in the literature. The analysis is done by calculating the ZMP and checking if it is within the HR stability polygon. Considering only the HR sagittal plane, the stability polygon is reduced to a straight line, which can be obtained in the external loop of the static analysis, presented in section 2.5. In the task of pushing an object, if the ZMP is located at a point between the rear foot of the HR and the object, there will be no tumbling. Figure 5 shows the center of gravity (GC) of the whole system and possible ZMP locations, indicating where tumbling occurs or not.

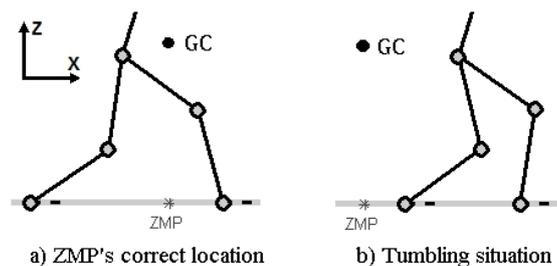


Figure 5 – ZMP location.

The forces that contribute to the position of the ZMP are the contact forces on the hands and the gravitational forces, as shown in Eq. (2).

If the maximum force capacity is being constrained by tumbling, it is important to check if there are ways to overcome these limitations. One straight modification would be to increase the mass of the robot, increasing the normal force on the robot's center of gravity (GC), and, as a result, shifting the ZMP forward. But increasing the mass of the links generally implies in increasing the torque on the actuators and higher energy consumption to perform tasks that require dynamics moves. Another alternative, which does not imply a physical change in the HR, is to apply contact force between the hands and the environment in the upward  $z$  direction. These contact forces, although limited by the coefficient of friction between the hands and the environment, contribute to moving the ZMP forward, bringing it to a point closer to the object, thus allowing an increase of the force in the  $x$  direction. This type of action depends only on the existence of friction in the contact between the hands and the surface and on the availability of torque in the actuators. Therefore, it is possible to modify the location of the ZMP and the imminent tumbling failure by applying hand contact forces in the  $z$  direction, allowing an increase of the forces applied by the robot in the  $x$  direction.

### 3.4 Slipping

Slip failure occurs when the reaction force vectors that arise when a robot comes into contact with the environment are outside the friction cone. As the HR is being considered only in the sagittal plane, the concept of the cone of friction can be simplified, and it can be stated that slippage occurs when the friction force at the contact of the feet with the ground is not sufficient to resist the contact forces of the hands with the object in the  $x$  direction. This friction forces are obtained directly from the external loop of the static analysis presented in Subsection 2.5. As already discussed, the maximum friction force depends on the normal force acting of the feet and the maximum friction coefficient, which in turn depends on the nature of the surfaces in contact. Note that this maximum friction force is independent of how the normal force is distributed on each foot. Just as for the tumbling failure, one way to increase the force applied by the

hands in the  $x$  direction would be to increase the mass of the links, increasing the normal force and, as a consequence, the maximum friction force. Yet, the same difficulties would arise as those already discussed. Another way to increase the friction forces would be to change the material of the surfaces in contact, but this is not often possible under real conditions. But it is important to note that, as well as for tumbling, the application of contact forces on the hands in the upward  $z$  direction can also improve the slip failure limits. This is because the reaction of the object generates forces on the HR in the downward  $z$  direction, increasing the normal force and, as a consequence, the maximum friction forces. Again, it is possible to increase the normal force and the imminent sliding failure by applying hand contact forces in the  $z$  direction, allowing the increase of the forces applied by the robot in the  $x$  direction.

#### 4. STATIC ANALYSIS RESULTS

As stated in Subsection 2.3, only situation 1 will be evaluated in this paper, where, for the static balance evaluation, the positions of the feet, hands and trunk are known. The contact forces on the hands must also be known, but different study cases will be created to analyze how this contact forces impact the failures modes.

For all simulations, the origin of the coordinate system will be at the front foot, located 1.5m from the wall, and the back foot will be positioned 2.5m from the wall. The contact points of the hands on the walls will occur at heights 3.5m and 4m. The center of gravity of the trunk will be at coordinates  $x=0\text{m}$  and  $z=3\text{m}$ , with an inclination of  $60^\circ$ . Figure 6 shows the HR in this position that will be used in the different study cases.

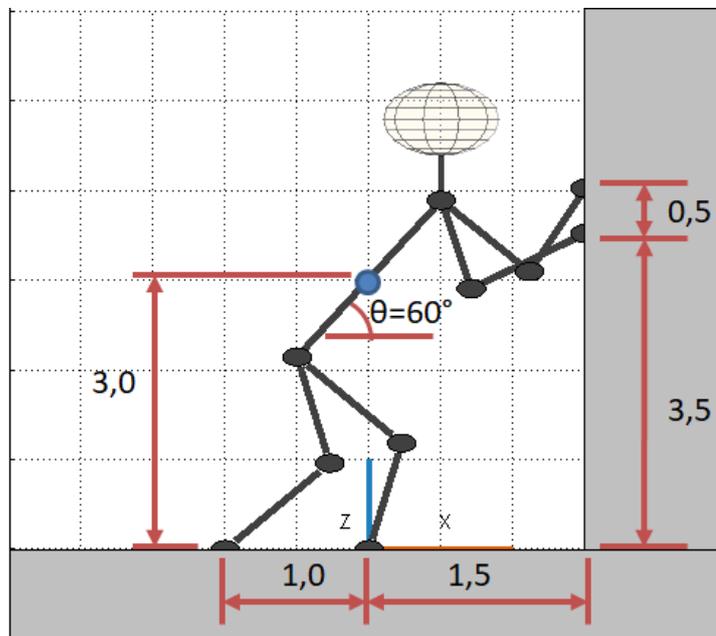


Figure 6 – Study cases HR position and contact points.

Forces in the  $x$  direction will be indicated by  $F_x$ , forces in the  $z$  direction by  $F_z$  and joint torques by  $\tau$ , with the sub-index indicating which joint it refers to, following the notation presented in Figure 4. With this configuration, the ZMP must be located between 0 m and -1.0 m so that the HR does not tumble.

Table 2 shows the results of the static analyses performed. The first column identifies the different study cases, numbered from 1 to 6. The columns  $F_{x_A}$ ,  $F_{x_D}$ ,  $F_{z_A}$  and  $F_{z_D}$  indicate the contact forces on the hands, whose variations allows for the generation of the different case studies. These 4 variables are primary and need to be known a priori for the static calculation.

Table 2 – Static analysis results.

Case	$F_{x_A}$ (N)	$F_{x_D}$ (N)	$F_{z_A}$ (N)	$F_{z_D}$ (N)	$\tau_{max}$ (Nm)	ZMP (m)	$\mu_A$	$\mu_D$	$\mu_L$	$\mu_I$
1	0	0	0	0	$\tau_G = 57.3$	0.15	0.0	0.0	0.0	0.0
2	10	20	0	0	$\tau_K = 40.0$	-0.71	0.0	0.0	0.23	0.23
3	10	20	5	10	$\tau_G = 54.5$	-0.48	0.5	0.5	0.21	0.21
4	20	20	0	0	$\tau_K = 50.3$	-1.02	0.0	0.0	0.31	0.31
5	20	20	10	10	$\tau_K = 41.6$	-0.68	0.5	0.5	0.27	0.27
6	25	25	10	10	$\tau_K = 50.1$	-0.94	0.4	0.4	0.34	0.34

The  $\tau_{max}$  column shows the joint where the highest torque occurs and the value found in modulus. The ZMP column shows the location of the ZMP relative to the origin. The columns  $\mu_A$ ,  $\mu_D$ ,  $\mu_L$ , and  $\mu_I$  show the necessary friction coefficients found in each contact when performing the simulation.

Case study 1 presents the trivial solution where no contact force is applied by the hands in the environment. There are no frictional forces on the feet and the joint torques exist to counterbalance the gravitational forces. Note that the ZMP is situated closer to the wall than the front foot of the HR, which would cause forward tumbling. In a real situation, the hands would bear these forces avoiding this situation. But, since in this case the contact forces were set to zero, the HR would effectively tumble forward.

In case study 2, contact forces on the hands have been defined in the  $x$  direction, but the forces in the  $z$  direction remain zero. The ZMP is within the stability limit and no tumbling occurs. The most stressed joint is the knee joint, having to resist 40 Nm. And, in order to not slip, the coefficient of static friction between the feet and the floor must be at least 0.23.

Compared to case 2, contact forces on the hands in the  $z$  direction are added in case study 3. With the addition of these forces it is possible to reduce the necessary static friction coefficient at the feet by applying the same force in the  $x$  direction as in case 2. The position of the ZMP has also been shifted forward, leaving the HR further away from a possible tumbling failure. In compensation, it is necessary to have a static friction coefficient of 0.5 on the hand contact and the maximum torque supported by joint  $G$  is higher.

In case 4, the total contact force in the  $x$  direction is increased compared to the previous cases studied. In case 5, compared to case study 4, forces in  $z$  direction are added. Comparing case 4 and 5, it is interesting to note that in case 5, as the contact force in the  $z$  direction is increased, the maximum torque at joint  $K$  decreases, the ZMP moves away from its tumbling limit position and the required coefficient of friction at the feet decreases. The only setback is that the coefficient of friction at the hands must be higher, but one can clearly see the benefits that the application of the contact force in the  $z$  direction by the hands can produce.

Case 6 is presented for comparison with the results of case 3. Note that, even with a significant increase in the force applied by the hands in the  $x$  direction, there was a reduction in the maximum torque of the most loaded actuator, still keeping the ZMP within the stability limit.

Concerning the analysis of the actuators saturation and mechanical resistance failure modes, Table 3 presents the forces and moments in each of the HR joints for case study 3, obtained from the static analysis.

Table 3 – Force and moments in each joint for scenario 3.

Joint	$\tau$ (Nm)	Fx (N)	Fz(N)
A	0	10.00	5.00
B	5.41	10.00	14.80
C	-14.58	10.00	24.60
D	0	20.00	10.00
E	0.45	20.00	19.80
F	-24.27	20.00	29.60
G	-54.52	15.67	40.10
H	-3.32	15.67	59.70
I	0	15.67	74.40
J	3.63	14.33	33.70
K	30.35	14.33	53.30
L	0	14.33	68.00

To identify if there was any failure due to the saturation of the actuators, it is enough to compare the second column values with the joint torques limits, obtained in general from the manufacturers. Concerning the mechanical failure of the links, it is necessary to compose the forces acting on the links from the forces on their joints. If we want to obtain, for example, the wrenches acting on link 2 (biceps), it is possible to accomplish this by using the calculated wrenches in joints B and C, together with the gravitational force on the respective link. As the points of application of these wrenches are also known, it can be checked whether the link can resist these loads or not.

## 5. CONCLUSIONS

In this paper a methodology for the evaluation of HR statics that considers all possible failure modes in an integrated approach was presented. In addition, differing from the approaches presented in the literature, the two robot arms can have different configuration and the point of contact can occur at distinct heights regarding the object. The analysis of the contact specificities made it possible to eliminate the actuation redundancy and obtain a unique explicit solution. To the best of the authors' knowledge, no similar approach had been presented in the literature before.

It was also shown how the contact force on the hands in the  $z$  direction (friction forces) can be used to increase the ability of the robot to apply force in the  $x$  direction, minimizing the possibility of failure by slipping and tumbling and, in certain situations, even reducing the maximum torque that the internal joints have to withstand.

The method was applied to a two-dimensional representation of the HR, but can be extended to a three-dimensional analysis. Tasks where the robot needs to push an object were used as an example, but the methodology can be extended to other situations. Results obtained from this work can be useful for the design of new HR or for task planning in actual robot where the contact with the environment occurs.

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## 7. RESPONSIBILITY NOTICE

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