27th ABCM International Congress of Mechanical Engineering
December 04-08, 2023, Florianópolis-SC, Brazil

COB-2023-0844
WAVE ATTENUATION USING 2-D METAMATERIAL THIN PLATES WITH SHUNTED PIEZO-PATCHES

Edson Jansen Pedrosa de Miranda Jr.
Federal Institute of Maranhão, IFMA-EIB-DE, Rua Afonso Pena, 174, CEP 65010-030, São Luís, MA, Brazil
Federal Institute of Maranhão, IFMA-PPGEM, Avenida Getúlio Vargas, 4, CEP 65030-005, São Luís, MA, Brazil
Vale Institute of Technology, Rua Professor Paulo Magalhães Gomes - Bauxita, CEP 35400-000, Ouro Preto, MG, Brazil
edson.jansen@ifma.edu.br

José Maria Campos Dos Santos
University of Campinas, UNICAMP-FEM-DMC, Rua Mendeleyev, 200, CEP 13083-970, Campinas, SP, Brazil
zema@fem.unicamp.br

Amilton Sinatora
Vale Institute of Technology, Rua Professor Paulo Magalhães Gomes - Bauxita, CEP 35400-000, Ouro Preto, MG, Brazil
University of São Paulo, Department of Mechanical Engineering, Avenida Prof. Mello Moraes, 2231, CEP 05508-970, São Paulo, SP, Brazil
amilton.sinatora@itv.org

Abstract. *Recently, the piezoelectric shunt damping combined with the concept of mechanical periodic structures originated the piezoelectric metamaterials (PMs). In terms of elastic wave attenuation, the advantage of using PMs is the formation of both Bragg-type and locally resonant band gaps. These forbidden bands are regions of frequency where there are only evanescent waves. The wave propagation in a 2-D PM Kirchhoff-Love plate (i.e., thin plate theory) with periodic arrays of shunted piezo-patches is investigated in this study. This PM thin plate is capable of filtering the propagation of flexural waves over a specified range of frequency. The dispersion diagrams are obtained by the improved plane wave expansion (IPWE) and extended plane wave expansion (EPWE) approaches. First, the cases of open and short circuits are studied. The dispersion diagrams computed by IPWE and EPWE methods show good agreement for these cases. Furthermore, only the Bragg-type band gaps are observed for open and short circuits. Next, two types of closed electrical circuits are considered, i.e., resistive and resonant circuits. The locally resonant band gap is opened up for the resonant case. The shunt circuits influence significantly the propagating and the evanescent modes. The results can be used for elastic wave attenuation using 2-D PM plates.*

Keywords: *periodicity, band gaps, shunt circuits, thin plate theory.*

1. INTRODUCTION

Periodic structures known as phononic crystals and mechanical metamaterials are artificial structures designed to open up Bragg-type and/or locally resonant band gaps. In these ranges of frequency, there are no propagating waves, only evanescent waves (Miranda Jr. *et al.*, 2019). Such periodic structures are being applied in many branches of science, and have many applications, *e.g.*, passive/active vibration control, acoustic barriers/filters, metamaterials-based enhanced energy harvesting, waveguides, among others.

Recently, the piezoelectric shunt damping combined with the concept of periodic structures originated the piezoelectric metamaterials (PMs). The 1-D (Sugino *et al.*, 2017; Miranda Jr. *et al.*, 2021) and 2-D (Aghakhani *et al.*, 2020; Xiao *et al.*, 2020) PMs have been extensively studied numerically and by experimental techniques.

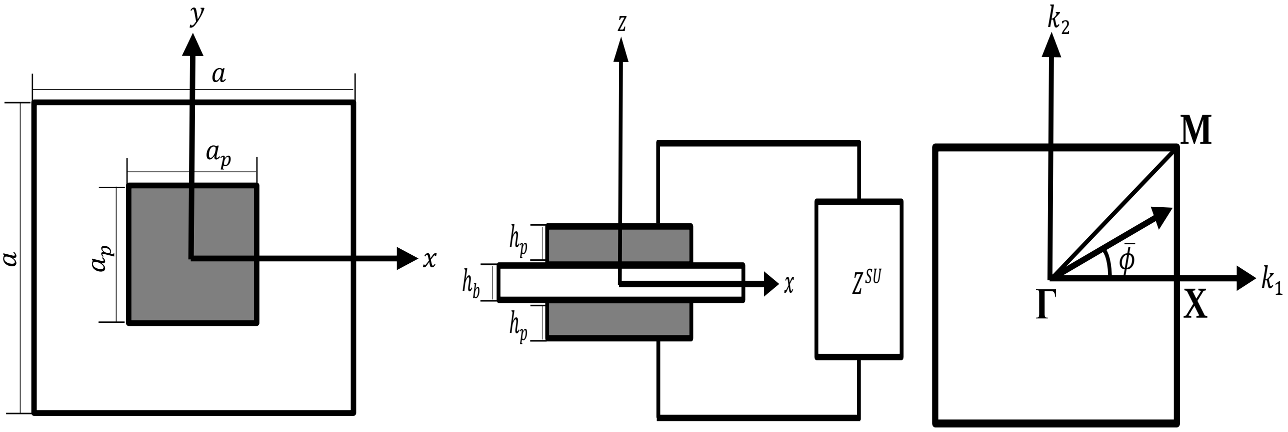
Chen (2018) obtained the dispersion diagrams of 2-D acoustic metamaterials with shunting circuits by using the finite element. He observed an attenuation zone around the band gap location where the wave propagation decayed strongly.

In this study, the dispersion diagrams are obtained by the improved plane wave expansion (IPWE) Cao *et al.* (2004) and extended plane wave expansion (EPWE) Hsue *et al.* (2018) methods. First, the cases of open and short circuits are studied. Next, the resistive and resonant circuits (*i.e.*, closed electrical circuits) are investigated.

2. PIEZOELECTRIC METAMATERIAL PLATE MODELLING

Figure 1 sketches the top (a) and front (b) views of the 2-D PM unit cell, where a is the lattice parameter, and a_p and h_p are the length and thickness of the piezoeclectric patches. The piezoelectric patches with shunting circuits connected in parallel are illustrated in (b) for the cases of resistive $(Z^{SU} = R)$ and resonant $(Z^{SU} = R + i\omega L)$ circuits, where

$i = \sqrt{-1}$, ω is the angular frequency, Z^{SU} is the electrical impedance, R is the resistance, and L is the inductance of the electrical circuit.



(a) (b) (c)
Figure 1. Top (a) and front (b) views of the 2-D PM unit cell, where a is the lattice parameter, a_p and h_p are the length and thickness of the piezoeclectric patches, and Z^{SU} is the electrical impedance. (c) First irreducible Brillouin zone (FIBZ) of the 2-D PM for a square lattice, where $\mathbf{M}$ $(0, 0)$, $\mathbf{\Gamma}$ $(\pi/a, 0)$, and $\mathbf{X}$ $(\pi/a, \pi/a)$ are the FIBZ high-symmetry points.

In Fig. 1 (c), it is shown the first irreducible Brillouin zone (FIBZ) (Brillouin, 1946) of the 2-D PM for a square lattice, where the FIBZ high-symmetry points are $\mathbf{M}$ $(0, 0)$, $\mathbf{\Gamma}$ $(\pi/a, 0)$, and $\mathbf{X}$ $(\pi/a, \pi/a)$.

The IPWE, $\omega(\mathbf{k})$ approach (where $\mathbf{k}$ is the Bloch wave vector, also known as wave number), is used to compute the propagating modes, whereas the EPWE, $\mathbf{k}(\omega)$, can be used to obtain both propagating and evanescent modes of the dispersion diagram. It should be highlighted that the IPWE method has higher convergence than the traditional plane wave expansion (PWE) approach (Cao *et al.*, 2004). Furthermore, the dispersion diagram computed by IPWE shows a considerably lower computational cost (Dal Poggetto and Serpa, 2020), since it is a semi-analytical approach and it is not necessary to consider a large number of degrees-of-freedom as with other methods. The IPWE and EPWE formulations will be derived in a future publication. However, some fundamental issues associated with these approaches can be found in Laude *et al.* (2009) and Miranda Jr. *et al.* (2022).

The Kirchhoff-Love (Kirchhoff, 1850; Love, 1888) thin plate theory is used to model the 2-D PMMs with periodic arrays of shunted piezo-patches with a square cross section area (Fig. 1 (a)). The evanescent modes obtained by the EPWE are related to the wave attenuation in the unit cell, since it is defined as $\Im\{\mathbf{k}\}a$ (Miranda Jr. *et al.*, 2020). In this paper, the media are isotropic and only flexural (*i.e.*, out-of-plane) wave modes are considered.

3. SIMULATED EXAMPLES

The physical parameters (Xiao *et al.*, 2020; Hollkamp, 1994) of the plate (b) and the piezoelectric patches (p) are listed in Table 1. It should be pointed out that the plate and piezoelectric patch loss factors are not considered.

Hereafter, the model assurance criterion (MAC) (Mencik, 2010) is used to estimate the correlation among wave mode shapes for the EPWE. Furthermore, for IPWE and EPWE calculations and comparison, 49 plane waves are regarded, in order to reduce the computational time (*i.e.*, the Fourier series convergence is not verified) and the dispersion diagram is analysed only on the ΓX direction, *i.e.*, $\bar{\phi} = 0$.

Figure 2 shows the complex dispersion diagram of the 2-D PM plate for the case of open circuit ($Z^{SU} \rightarrow \infty$). In Fig. 2 ($a$), one can note that the IPWE (blue circles) can identify only the propagating modes. The evanescent modes with complex wave numbers are obtained by the EPWE (points). A good agreement between the IPWE and EPWE is observed (Fig. 2 (a)). Note that some modes in Fig. 2 (a) cannot be found by the IPWE, since they are evanescent. For EPWE calculation, a $\Delta f = 1$ Hz is regarded. The band gap in Fig. 2 (a) (blue dashed rectangle) is created only by Bragg scattering along $\mathbf{\Gamma X}$ direction (partial band gap), since there is no electrical resonance. This Bragg-type band gap is opened up between 2239-2513 Hz and can be directly observed by the propagating modes from IPWE. The unit cell wave attenuation inside this band gap can be seen in Fig. 2 (b). Moreover, there are also other regions of wave attenuation in higher frequencies that are shown in Fig. 2 (b).

Figure 3 shows the complex dispersion diagram for the case of short ($Z^{SU} = 0$) circuit. Two partial band gaps (Fig. 3 (a)) are created between 1948-2150 Hz and 7823-8127 Hz. A good agreement between the IPWE and EPWE (Fig. 3 (a)) is observed, similar to the open circuit case (Fig. 2 (a)).

Figure 4 illustrates the complex dispersion diagram for the case of resistive ($R = 50$ Ω) circuit.

Table 1. Geometry and material properties (Xiao *et al.*, 2020; Hollkamp, 1994) of the plate (b) and piezoelectric patches (PZT-5H) (p).

Geometry/Property	**Value**
Lattice parameter (a)	0.06 m
Piezoelectric patch length (a_p)	0.03 m
Plate thickness (h_b)	0.0016 m
Piezoelectric patch thickness (h_p)	0.0002 m
Mass density (ρ_b, ρ_p)	1.6×10^3 kg/m^3, 7.5×10^3 kg/m^3
Young's modulus (E_b, $E_p(\omega)$)	181×10^9 N/m^2, Eq. **??**
Poisson's ratio (ν_b, $\nu_p(\omega)$)	0.28, Eq. **??**
Compliance coefficient at constant electric field (s_{11}^E)	16.5×10^{-12} 1/Pa
Compliance coefficient at constant electric field (s_{12}^E)	-4.78 $\times 10^{-12}$ 1/Pa
Piezoelectric strain constant (d_{31})	-2.74 $\times 10^{-10}$ C/N
Dielectric constant (ε_{33}^σ)	$3400\varepsilon_0$
Electromechanical coupling coefficient (k_{31})	0.35
Electrical capacitance of the piezo at constant strain (C_p^ε)	118.87×10^{-9} F

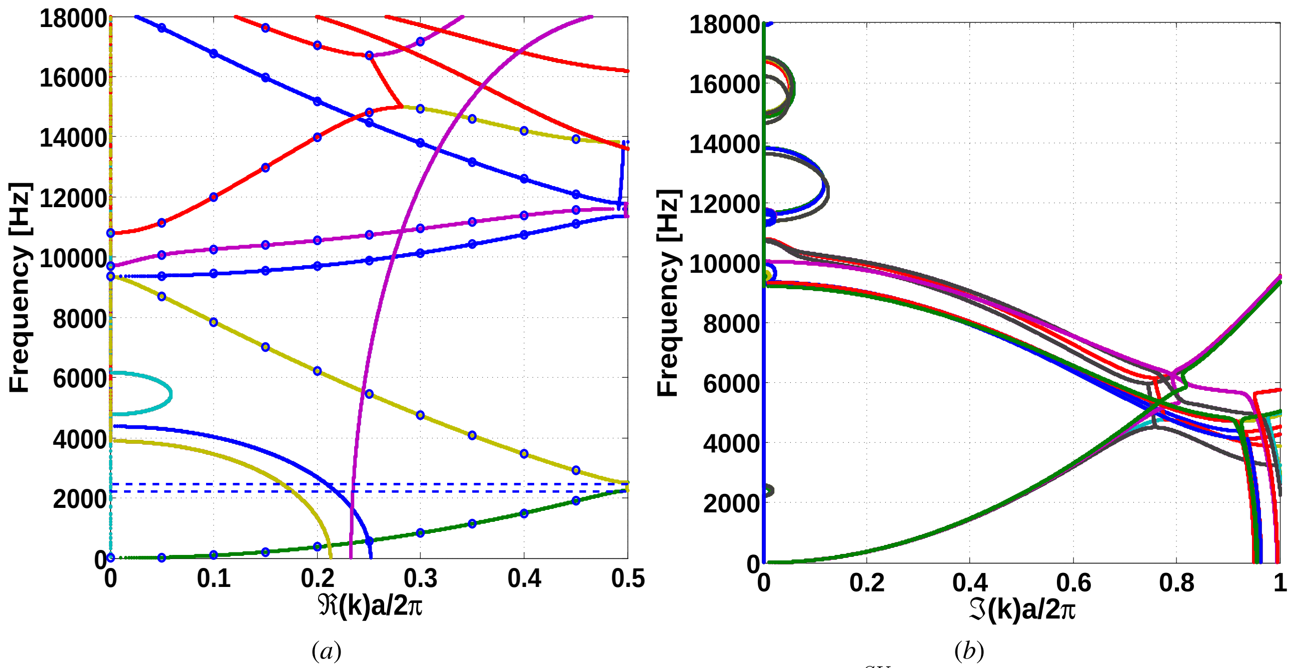
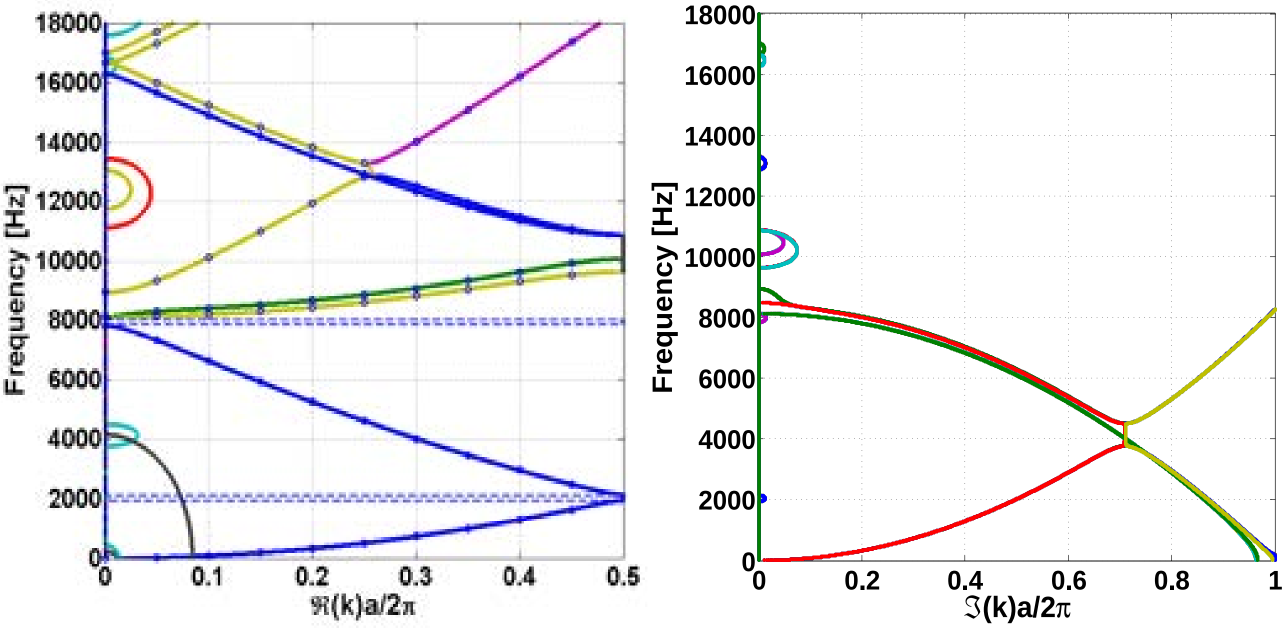


Figure 2. Complex dispersion diagram of the 2-D PM plate with open circuit ($Z^{SU} \to \infty$) computed by (a) IPWE (blue circles) and $(a-b)$ EPWE (points) methods.

This dispersion diagram behaviour is similar to the short circuit case (Fig. 3). However, the resistor slightly increases the total piezoelectric loss factor (Fig. 4).

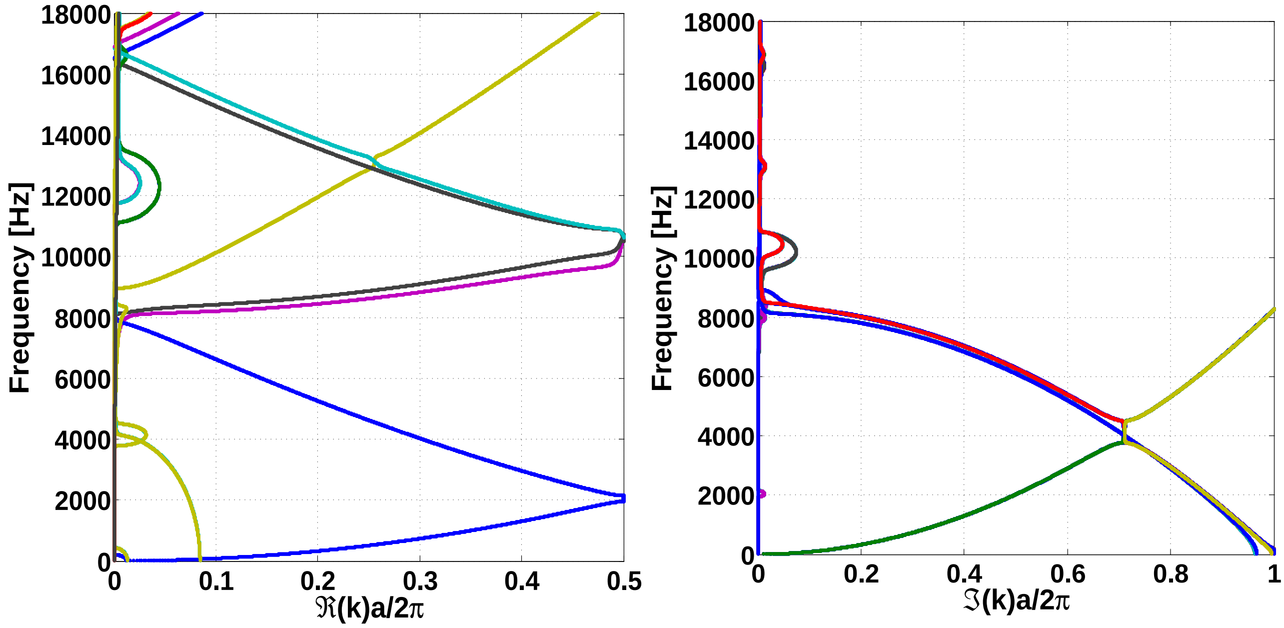
It should be underlined that the IPWE cannot be directly used to compute the dispersion diagrams for the cases of closed circuits (resistive and resonant circuits), since there are some properties depending on the frequency. However, an iterative algorithm can be designed to obtain the band gap structure due to the dependence of elastic constants (Zhao and Wei, 2009).

Figure 5 presents the complex dispersion diagram for the case of resonant circuit ($f_T = 768.786$ Hz), where f_T is the resonance of the electrical circuit. The locally resonant band gap can be observed in Fig. 5 around the resonant frequency. The resonance is easily identified considering for instance only the first four modes (see Fig. 6).



(*a*) (*b*)

Figure 3. Complex dispersion diagram of the 2-D PM plate with short circuit ($Z^{SU} = 0$) computed by (a) IPWE (blue circles) and $(a - b)$ EPWE (points) methods.



(*a*) (*b*)

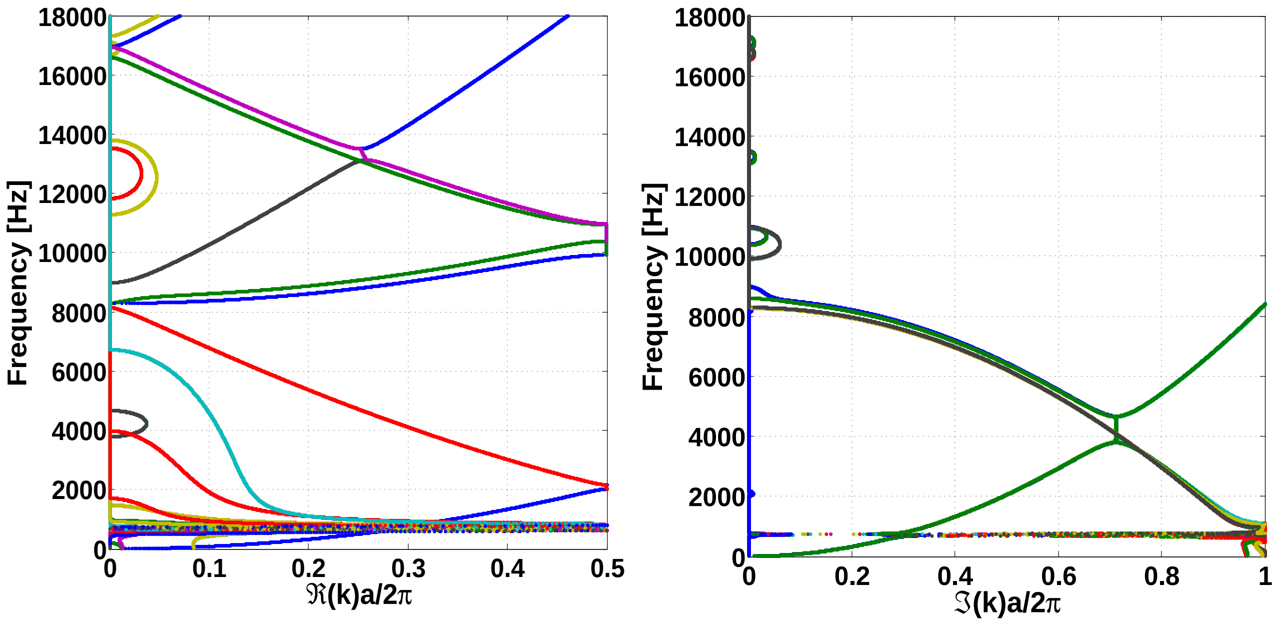
Figure 4. Complex dispersion diagram of the 2-D PM plate with resistive circuit ($R = 50\ \Omega$) computed by EPWE.

4. CONCLUSIONS

The complex dispersion diagrams of a 2-D mechanical metamaterial thin plate with periodic arrays of shunted piezo-patches are investigated. These dispersion diagrams computed by IPWE and EPWE approaches show good agreement. The Bragg-type band gaps are first observed for the open and short circuits. Next, the resistive and resonant circuits are studied and the locally resonant band gap is opened up for the resonant case. The results can be used for elastic wave attenuation using 2-D piezoelectric periodic structures.

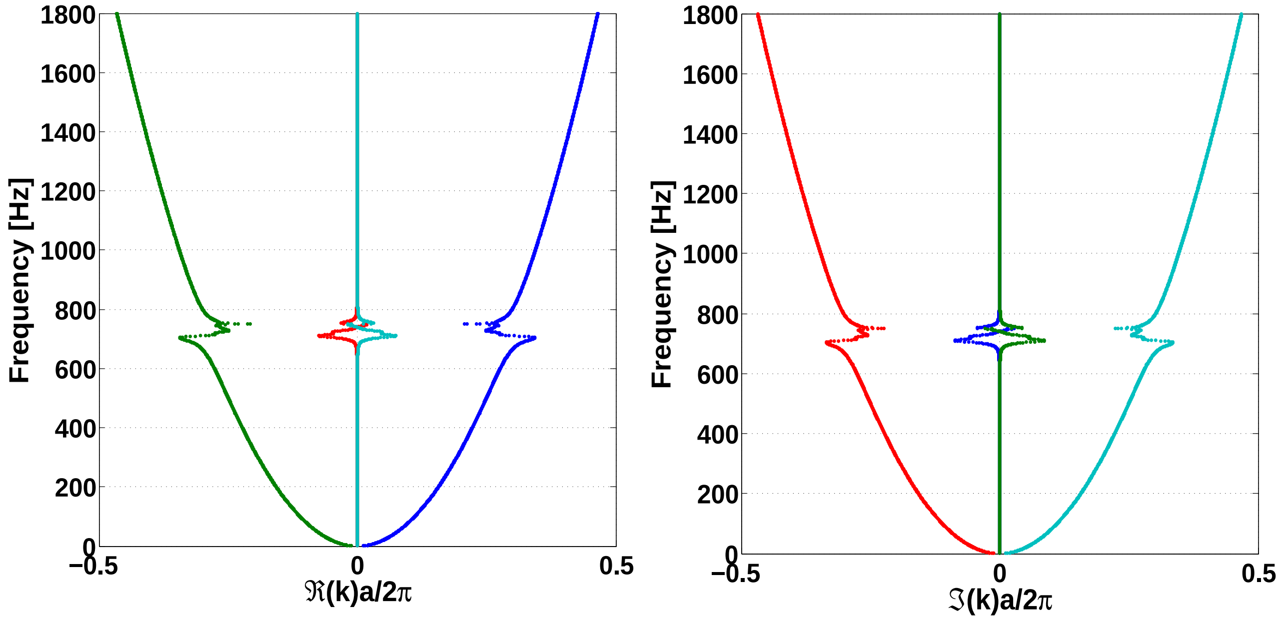
5. ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support of the Federal Institute of Maranhão (IFMA), Brazilian funding agencies CAPES (Finance Code 001), CNPq (Grant Reference Numbers 313620/2018, 403234/2021-2, 405638/2022-1, and



(*a*) (*b*)

Figure 5. Complex dispersion diagram of the 2-D PM plate with resonant circuit ($f_T = 768.786$ Hz) computed by EPWE.



(*a*) (*b*)

Figure 6. Complex dispersion diagram zoom (only the first four modes) around the locally resonant band gap of the 2-D PM plate with a resonant circuit ($f_T = 768.786$ Hz) computed by the EPWE.

300166/2022-2), FAPEMA (Grant Reference Numbers 07168/22 and 09683/22), and FAPESP (Grant Reference Number 2018/15894-0).

6. REFERENCES

Aghakhani, A., Gozum, M.M. and Basdogan, I., 2020. “Modal analysis of finite-size piezoelectric metamaterial plates”. *Journal of Physics D: Applied Physics*, Vol. 53, No. 505340.

Brillouin, L., 1946. *Wave Propagation in Periodic Structures*. Dover Publications, New York.

Cao, Y., Hou, Z. and Liu, Y., 2004. “Convergence problem of plane-wave expansion method for phononic crystals”. *Physics Letters A*, Vol. 327, No. 247-253.

Chen, S., 2018. “Wave propagation in acoustic metamaterials with resonantly shunted cross-shape piezos”. *Journal of*

Intelligent Material Systems, Vol. 29, No. 13, pp. 2744–2753.

Dal Poggetto, V.F. and Serpa, A.L., 2020. "Elastic wave band gaps in a three-dimensional periodic metamaterial using the plane wave expansion method". *International Journal of Mechanical Sciences*, Vol. 184, p. 105841.

Hollkamp, J.J., 1994. "Multimodal passive vibration suppression with piezoelectric materials and resonant shunts". *Journal of Intelligent Material Systems and Structures*, Vol. 5, No. 49-57.

Hsue, Y.C., Freeman, A.J. and Gu, B.Y., 2018. "Extended plane-wave expansion method in three-dimensional anisotropic photonic crystals". *Physical Review B*, Vol. 72, No. 195118.

Kirchhoff, G., 1850. "Über das gleichgewicht und die bewegung einer elastischen scheibe". *Journal für die reine and angewandtle Mathematik*, Vol. 40, pp. 51–88.

Laude, V., Achaoui, Y., Benchabane, S. and Khelif, A., 2009. "Evanescent Bloch waves and the complex band structure of phononic crystals". *Physical Review B*, Vol. 80, p. 092301.

Love, A.E.H., 1888. "The small free vibrations and deformation of a thin elastic shell". *Philosophical Transactions of the Royal Society*, Vol. 179, pp. 491–546.

Mencik, J.M., 2010. "On the low- and mid-frequency forced response of elastic structures using wave finite elements with one-dimensional propagation". *Computers and Structures*, Vol. 88, pp. 674–689.

Miranda Jr., E.J.P., Nobrega, E.D., Ferreira, A.H.R. and Dos Santos, J.M.C., 2019. "Flexural wave band gaps in a multi-resonator elastic metamaterial plate using Kirchhoff-Love theory". *Mechanical Systems and Signal Processing*, Vol. 116, pp. 480–504.

Miranda Jr., E.J.P., Nobrega, E.D., Oliveira, L.P.R. and Dos Santos, J.M.C., 2021. "Elastic wave propagation in metamaterial rods with periodic shunted piezo-patches". In *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*. Washington, D.C., USA, pp. 4303–4311.

Miranda Jr., E.J.P., Nobrega, E.D., Rodrigues, S.F., Aranas Jr., C. and Dos Santos, J.M.C., 2020. "Wave attenuation in elastic metamaterial thick plates: Analytical, numerical and experimental investigations". *International Journal of Solids and Structures*, Vol. 204-205, pp. 138–152.

Miranda Jr., E.J.P., Rodrigues, S.F., Aranas Jr., C. and Dos Santos, J.M.C., 2022. "Plane wave expansion and extended plane wave expansion formulations for Mindlin-Reissner elastic metamaterial thick plates". *Journal of Mathematical Analysis and Applications*, Vol. 505, No. 2, p. 125503.

Sugino, C., Leadenham, S., Ruzzene, M. and Erturk, A., 2017. "An investigation of electroelastic bandgap formation in locally resonant piezoelectric metastructures". *Smart Materials and Structures*, Vol. 26, No. 055029.

Xiao, X., He, Z.C., Li, E., Zhou, B. and Li, X.K., 2020. "A lightweight adaptive hybrid laminate metamaterial with higher design freedom for wave attenuation". *Composite Structures*, Vol. 243, No. 112230.

Zhao, Y.P. and Wei, P.J., 2009. "The band gap of 1D viscoelastic phononic crystal". *Computational Materials Science*, Vol. 46, No. 3, pp. 603–606.

7. RESPONSIBILITY NOTICE

The authors are solely responsible for the printed material included in this paper.