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TOPOLOGY OPTIMIZATION OF ELASTIC INTERNAL RESONATORS FOR SANDWICH METASTRUCTURES

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Abstract. *Metastructures have gained attention in recent years due to their noise and vibration isolation properties in tunable frequency ranges, basing their behaviour on the principle of phononic crystals. These phononic crystals exhibit frequency ranges known as bandgaps, in which elastic and acoustic waves do not propagate. Metastructures are achieved from the addition of resonator elements, which act as vibration absorbers and are periodically distributed in host structures. The advantage of applying metastructures is that, unlike phononic crystals, the bandgaps are generated based on the properties of the resonator elements, this paves the way towards compact and lightweight vibroacoustic solutions for the lower frequency ranges, since the waves can be affected by the incorporation of resonator elements of sizes smaller than the wavelength. An approach in the investigation of metastructures is the search for optimal geometry of resonators, which can provide a reduction of vibrations in a greater range of frequencies, or allow the damping of relatively distant resonance peaks. That is why this research work proposes a topological optimization to find the geometry of an elastic resonator, as a building block of a sandwich type metastructure. In the development, the topology optimization based on the Bi-directional Evolutionary Structural Optimization method, together with the objective function and the constraints, which represent the sensitivity of the resonator with respect to its properties and its coupling with the host structure, are implemented in a modeling tool based on finite elements. Numerical results show that the proposed modified method with penalization and the objective functions generate an efficient resonator geometry.*

Keywords: *Topology optimization, Metastructure, Control of vibrations*

1. INTRODUCTION

Sandwich structures, which have a very attractive weight-resistance ratio due to their low-density core, have been used in practice as structural damping mechanisms to control resonance amplitude, wave attenuation and sound propagation. For this low-density core, that is achieved through lattice-like periodic microstructures, the majority of its volume is void space (Mukhopadhyay *et al.* (2023)). The use of that void space, to modulate properties of the structure, has generated interest among researchers. As a solution, Sun and Chen (2011) proposed using the low-density core space of these structures to accommodate periodic resonators, thus adopting the concept of sandwich structures in conjunction with metastructures.

According to Hussein *et al.* (2014), metastructures are defined as periodic structures, similar to phononic crystals, that exhibit unique properties related to vibration and sound attenuation. These periodic structures allow the suppression of the propagation of elastic waves in frequency ranges known as bandgaps, which are achieved through the interaction between the resonator frequency and host structure natural frequencies. The possibility of tuning these bandgaps and enhancing the propagation capability of waves and perturbations in these ranges has been associated with the topology of the resonators. In other words, modifying the mass and stiffness of these resonators allows for the adaptation of the attenuation characteristics of the metastructure as required (Sharma and Sun (2016)).

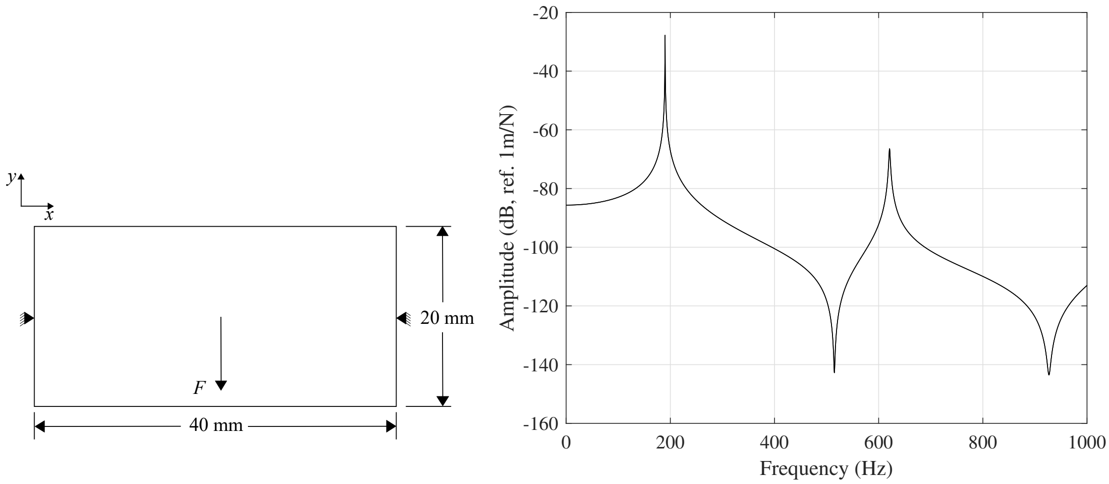
In the literature, several research works focus on strategies to improve the structural vibration control performance under known conditions through variations in topology or resonator properties (Li *et al.* (2020), Igusa and Xu (1994), Alsaffar *et al.* (2021)). For example, Murer *et al.* (2023) proposed a local resonance mechanism by considering infinitely dimensioned resonators embedded in a cellular structure, inspired by a spider web-like geometry with a central mass. Their work provided interesting insights into the behavior and robustness of the stop band. Liu *et al.* (2011), implemented chiral lattices integrated with metamaterial inclusions and investigated their vibration absorption properties. In addition to finding that this configuration exhibits enhanced energy dissipation and low-frequency bandgaps, they numerically demonstrated that the frequency of the bandgaps can be adjusted by varying the parameters of the employed topology.

Following the motivation of these research works, the present study aims to explore the geometry of an elastic resonator as a constitutive block of a sandwich structure. To achieve this, a topology optimization approach is implemented, as it provides greater freedom in creating highly efficient conceptual designs for continuum structures (Huang and Xie (2010)). Based on finite element analysis (FEA) and the discretization of the design domain, this solution process determines the topology of a structure by evaluating whether each point in the domain should contain material (solid element) or not (void element). The Bi-directional Evolutionary Structural Optimization Method (BESO) is employed to select the elements to be considered in the design domain. Sensitivity analysis offers an efficient means of predicting the performance of modified models. In this way, the BESO methodology uses results obtained by FEA to identify the elements with the highest sensitivity number and incorporates them into the final topology.

2. FORMULATION OF OPTIMIZATION METHODOLOGY

2.1 Problem description

The resonator was considered to be made of PolyJet DM 9860, with mass density ($\overline{\rho}$) 1130 kg m^{-3}, Young's modulus ($\overline{E}$) 3.605 MPa and Poisson's ratio (ν) 0.4. Figure 1 shows the boundary conditions considered for the optimization procedure. The Finite Element Method is used to calculate the response of the system. For this, it is considered plane stress and constant thickness over the domain and a regular 200×100 mesh composed of linear quadrilateral elements with edge lengths of 0.2 mm and two degrees of freedom in each node. The external load has a value of -10 N in the y direction, and is applied at the node corresponding to the center of the design domain. For the calculation of the frequency response function (FRF) of the resonator, the displacement in y direction of the node where the load is applied is considered.



(a) Design domain (b) FRF of the full design domain

Figure 1: Initial full domain design and its frequency response for a point applied force.

2.2 Material interpolation scheme

For each discretized element of the design domain in Fig. 1, it is assigned a relative binary density x_e, as a design variable. The BESO method is based on a heuristic relation between this relative element density, and the mechanical properties of the material. In this way, the elementary material stiffness and mass can be defined as follows.

$$E(x_e) = E_{\min} + x_e^p(\overline{E} - E_{\min}) \quad (1)$$

$$\rho(x_e) = x_e^q \overline{\rho} \quad (2)$$

$$x_e = x_{\min} \text{ or } 1 \quad (3)$$

being $E_{\min}$ a small stiffness value assigned to prevent the global stiffness matrix of becoming singular, p and q the penalization factors, that in this case are assumed to be 3 and 1 respectively, and $x_{\min}$ a small value used to denote a void element.

2.3 Finite Element procedure

The linear structural response of the dynamic system is formulated by Newton's law as follows.

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t) \quad (4)$$

with $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ representing the global mass, damping and stiffness matrices, $\mathbf{u}(t)$, $\dot{\mathbf{u}}(t)$ and $\ddot{\mathbf{u}}(t)$ denoting the nodal value of time varying displacement, velocity and acceleration of system and $\mathbf{f}(t)$ the prescribed force. It is assumed a proportional damping modeling $\mathbf{C} = \beta_1\mathbf{M} + \beta_2\mathbf{K}$.

Considering an harmonic load as the external excitation and $\mathbf{u}(t) = \mathbf{U}(\omega)e^{i\omega t}$ as the solution of Eq. (4), it can be rewritten as follows.

$$[-\omega^2\mathbf{M} + j\omega\mathbf{C} + \mathbf{K}]\mathbf{U}(\omega) = \mathbf{D}(\omega)\mathbf{U}(\omega) = \mathbf{F} \tag{5}$$

where $\mathbf{U}(\omega)$ is the complex vector of the nodal displacements, $\mathbf{F}$ the amplitude of the prescribed force and $\mathbf{D}(\omega)$ is the dynamic stiffness matrix. The solution of the static problem, $\omega = 0$, will be labeled as $\mathbf{U}_s$.

$$\mathbf{K}\mathbf{U}_\mathrm{s} = \mathbf{F} \tag{6}$$

The global stiffness and mass of Eq. (5) are assembled based on the elementary stiffness and mass matrices and the elastic modulus and mass density of each element in the design domain.

$$\mathbf{K} = \mathscr{A}_{e=1}^{n} E(x_e)\overline{\mathbf{K}} \tag{7}$$

$$\mathbf{M} = \mathscr{A}_{e=1}^{n} \rho(x_e)\overline{\mathbf{M}} \tag{8}$$

where e is the current element, n the total number of elements and $\overline{\mathbf{K}}$ and $\overline{\mathbf{M}}$ are the elementary stiffness and mass matrices factored out of the initial elastic modulus and mass density respectively.

2.4 Topology optimization formulation

The use of metastructures in systems subjected to static or dynamic forces presents the challenge of tuning resonators to attenuate the structural response in a specific frequency or in a frequency range. The proposal of this work is to tune the resonator in a specific frequency ω_t, this involves increasing its frequency response with respect to ω_t in a given volume (fraction of the initial volume). Following the results obtained by Trindade *et al.* (2021), who states that the presence of one or more resonance frequencies within a bandgap could be interesting to provide a larger bandwidth vibration reduction performance.

In topology optimization problems, where the structure is subject to an harmonic external force, the minimization of the dynamic compliance (Silva *et al.* (2019); Yoon (2010); Olhoff and Du (2014)) is usually considered. Following this approach, the aim of the proposed solution is to retain the elements with higher sensitivity number, that is, those that contribute the most to the behaviour of the resonator when it is excited by a predefined frequency.

Jog (2002) suggested that, if the driving frequency of the load is slightly higher than the fun- damental frequency of the initial configuration (considering all domain elements as solid), then the dynamic compliance becomes negative, and minimization of this function drives the system towards resonance. Olhoff and Du (2009) verified that, when a dynamic compliance minimization in harmonic vibration problemns is developed, consideringwith volume constrain, and single ex- citation frequencies immediately after the first resonance of the initial design, the load region of the final design tends to separate from the rest of the structure, making the respective vibration mode similar to a rigid body mode. To avoid this numerical diffulty, the static compliance is considered as part of the objective function, such that

$$\phi = \mathbf{U}_\mathrm{s}^\mathrm{T}\mathbf{K}\mathbf{U}_\mathrm{s} + \alpha\mathbf{U}^\mathrm{T}\mathbf{D}\mathbf{U} \tag{9}$$

2.5 Sensitivity analysis

To solve optimization problem of the Eq. (9), the sensitivities of the objective function ϕ with respect to the design vector $\mathbf{x}$ is calculated as follows

$$\frac{\mathrm{d}\phi}{\mathrm{d}\mathbf{x}} \cong \mathbf{U}_\mathrm{s}^\mathrm{T}\frac{\partial\mathbf{K}}{\partial\mathbf{x}}\mathbf{U}_\mathrm{s} + \alpha\mathbf{U}^\mathrm{T}\frac{\partial\mathbf{D}}{\partial\mathbf{x}}\mathbf{U} \tag{10}$$

Note that, to calculate the partial derivatives of the global stiffness $\partial\mathbf{K}/\partial\mathbf{x}$ and dynamic stiffness $\partial\mathbf{D}/\partial\mathbf{x}$ in Eq. (10), it is necessary to take into account the material interpolation, related in Eq. (1) and Eq. (2).

$$\frac{\partial\mathbf{K}}{\partial\mathbf{x}} = \mathscr{A}_{e=1}^{n}\, p x_e^{p-1}(\overline{E} - E_{\min})\overline{\mathbf{K}} \tag{11}$$

$$\frac{\partial\mathbf{M}}{\partial\mathbf{x}} = \mathscr{A}_{e=1}^{n}\, q x_e^{q-1}\overline{\mathbf{M}} \tag{12}$$

$$\frac{\partial\mathbf{D}}{\partial\mathbf{x}} = -\omega^2\frac{\partial\mathbf{M}}{\partial\mathbf{x}} + j\omega\left(\beta_1\frac{\partial\mathbf{M}}{\partial\mathbf{x}} + \beta_2\frac{\partial\mathbf{K}}{\partial\mathbf{x}}\right) + \frac{\partial\mathbf{K}}{\partial\mathbf{x}} \tag{13}$$

Based on Eq. (10), the BESO method aims to preserve the elements that contribute the most of the strain energy of the resonator. In this way, it seeks to achieve minimal variation in the response of the resonator with a reduced volume.

2.6 Filtering scheme and sensitivity stabilization procedure

In order to ensure existence of solutions to ϕ, avoid checkerboard patterns and prevent mesh-dependency of the obtained designs, it is used a sensitivity filter, that modifies the element sensitivities as follows

$$\widehat{\frac{\mathrm{d}\phi}{\mathrm{d}x_e}} = \frac{1}{x_e \sum_{e=1}^{n} H_{ei}} \sum_{e=1}^{n} H_{ei} \frac{\mathrm{d}\phi}{\mathrm{d}x_e} \tag{14}$$

where H_{ei} is defined as

$$H_{ei} = \max(0, r_{\min} - \Delta_{ei}) \tag{15}$$

with $r_{\min}$ representing the filter radius, centered in the eth element, and Δ_{ei} the center to center distance between the eth element and the ith element.

Later, as an effort to avoid the hard convergence of the BESO method, the averaging of the sensitivity number with its historical information is used.

$$\widehat{\frac{\mathrm{d}\phi}{\mathrm{d}x_e}} = \frac{\left(\widehat{\frac{\mathrm{d}\phi}{\mathrm{d}x_e}}\right)_{k-1} + \left(\widehat{\frac{\mathrm{d}\phi}{\mathrm{d}x_e}}\right)_{k}}{2} \tag{16}$$

where the subscript k refers to the current iteration.

2.7 Design variable update and stop criterion

The BESO method introduces the Evolutionary Rate ER to define the target volume for the next iteration, and later generate the update of the design variable x_e. In this way, the next expression relates the volume fraction of the current (V_k) and next iteration (V_{k+1}) in function of ER.

$$V_{k+1} = V_k(1 - ER) \tag{17}$$

Then, the sensitivity number are sorted from highest to lowest, and using the definition of V_{k+1} it is defined the amount of elements what will be void ($x_e = x_{min}$) and solid ($x_e = 1$). The cycle of FEA and element removal/addition continues until the objective volume (V^*) is reached and the convergence criterion is satisfied.

$$\frac{\sum_{i=1}^{N} \phi_{k-i+1} - \sum_{i=1}^{N} \phi_{k-N-i+1}}{\sum_{i=1}^{N} \phi_{k-i+1}} \leqslant \tau \tag{18}$$

where τ is the convergence tolerance and N the number of iterations considered in the historial average. In this case, theses values are considered to be 0.1% and 10 respectively.

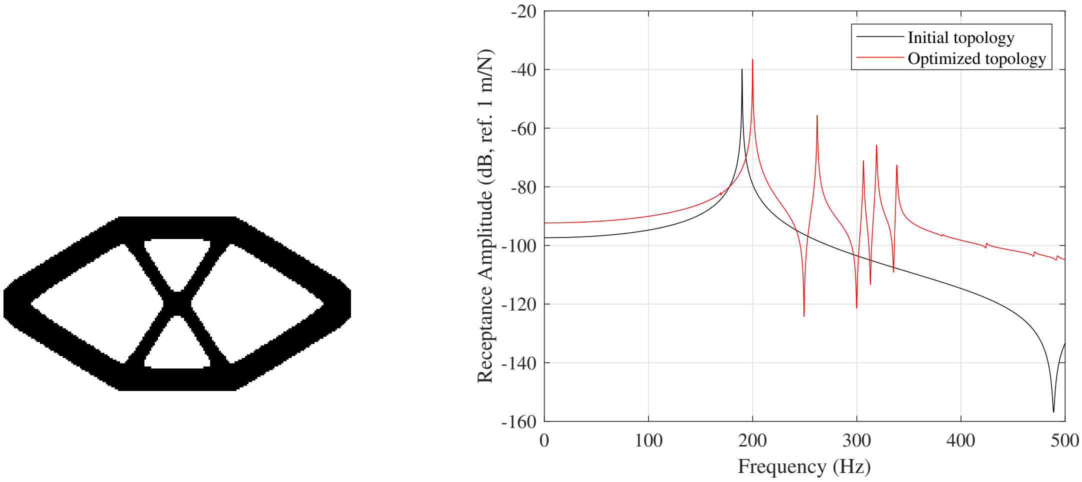
3. NUMERICAL RESULTS

Figure 2a represents the layout adopted after solving Eq. (15), considering $\omega_t = 200$ Hz. Due the boundary conditions applied in the domain, equal sensitivity numbers are observed in the left and the right sides of the domain, reason that explains the symmetry of the final layout. Figure 2b displays the point FRF of the optimal design compared to that of the full domain design. In this case, the desired response is achieved by using $\beta_1 = 1 \times 10^{-3}$ 1/rand, $\beta_2 = 1 \times 10^{-8}$ s/rand a volumetric fraction corresponding to 44% of the domain volume, a weight factor $\alpha = 2$, $ER = 1\%$ and $r_{\min} = 0.6$ mm.

In Fig. 2b one can notice that the final layout of the resonator also exhibits resonances between 250 and 350 Hz. Fig 3 shows the first five vibration modes of the optimal design indicating that this resonator could allow a multimodal performance or satisfactory performance over a wider frequency range.

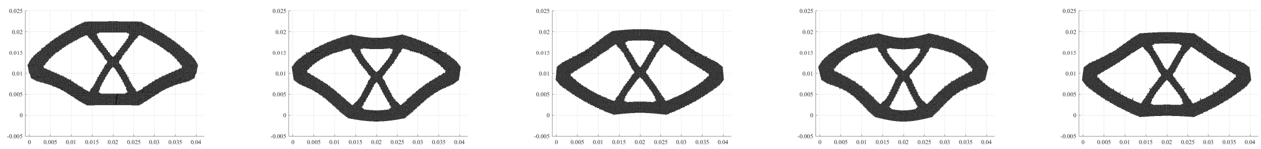
By adjusting the volumetric fraction, α, and ER, it is possible to modify the response of the optimal resonator design and, for instance, tune resonance frequencies both at primary target frequency and other desired frequencies. Figure 4 shows the modifications on the FRF due to variations of the weight factor α.

Figure 5a illustrates the optimal designs obtained using $\alpha = 0$ and $\alpha = 3$. It is noticeable that for a null value of α, the optimization process converges towards a design that is representative of structures subjected to static loads only as can be seen in Fig. 5a. Also, for this design, the resonance tends to be closer to the first resonance of the full design domain (189.87 Hz as seen in Fig. 1b). For the optimal design obtained with $\alpha = 3$ (Fig. 5a), the response near the primary target frequency is very close to the one using $\alpha = 2$, but the response is significantly modified for higher frequencies (Fig. 4).



(a) Optimal design (b) FRF of the optimal design compared to that of the full domain design

Figure 2: Optimal design obtained using topology optimization and its FRF compared to that of the full domain design.



(a) 200 Hz (b) 261.91 Hz (c) 306.32 Hz (d) 319.05 Hz (e) 338.31 Hz

Figure 3: First five vibration modes of the optimal resonator design.

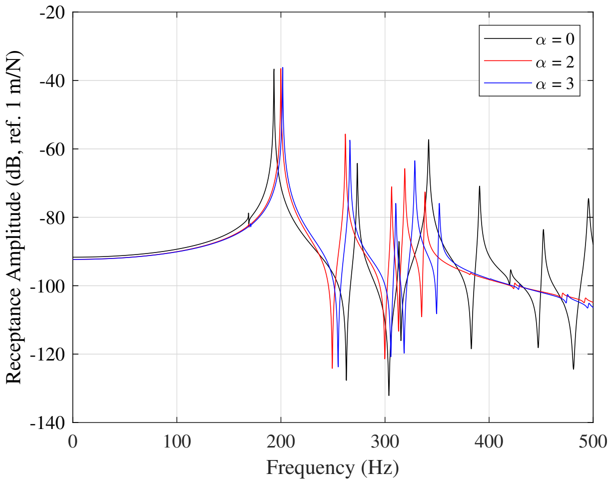
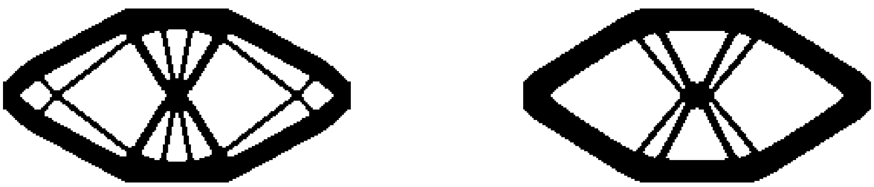


Figure 4: FRF of optimal resonator designs for three different weight factors α.



(a) Layout corresponding to $\alpha = 0$ (b) Layout corresponding to $\alpha = 3$

Figure 5: Optimal resonator designs obtained using different weight factors α.

4. CONCLUSION

This paper discusses the frequency tuning of elastic resonators using topology optimization as a method for volume reduction, and considering a forced vibration problem in steady-state. This work included the definition of a preliminary objective function, implemented with the BESO method, which is able to tune into frequencies close to the first resonance of the initial configuration, where the full domain is considered. This is because the topology optimization method is based on the sensitivity of the model, and its objective is to predict the performance of modified models, from the minimum change of its variables of design. The dependence on the full domain design response imposes limitations on finding a suitable solution. Hence, other strategies to define the objective function will be studied in future research.

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