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DESIGN OF A VISCOELASTIC PENDULUM ABSORBER FOR ENHANCING FATIGUE LIFE IN DYNAMICALLY EXCITED STRUCTURES

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Abstract. *Low frequency vibrations are a significant contributor to fatigue damage within dynamically excited structures. In many cases, designers try to avoid the superposition of excitation and natural frequencies, that reduces the response of the structure. On the other hand, it may not be possible to make sufficient changes in the design to ensure structural integrity. In such cases, dynamic absorbers can be used to suppress vibration responses. It can be done by tuning them according to the system's modal parameters, reducing the stresses on the structure and enhancing its fatigue life. This article proposes a viscoelastic pendulum absorber design, tuned to the first natural frequency of a cantilever beam, based on the Fixed Points Theory. A FEM of the system is built and submitted to a white-noise force in a range of 0 to 500 Hz, then fatigue analyses are performed in the frequency domain, based on Lalanne's approach. The viscoelastic material is represented using a Fractional Derivative Model, where the Complex Shear Modulus is represented as a function of the frequency and temperature. The analyses showed promising results, with the viscoelastic pendulum absorber reducing almost 20 dB of vibration amplitude in the first natural frequency and improving the fatigue life by 60 times.*

Keywords: *Fatigue of Materials, Random Vibrations, Passive Control of Vibrations, Dynamic Absorbers, Viscoelastic Materials*

1. INTRODUCTION

The design of mechanical structures depends on its application and the structural requirements imposed to it. When the loads have static behaviour, there are many failure criteria that can be used as reference to verify the structural integrity, as the well-known criterion proposed by Mises (1913), often applied to ductile metals to avoid plastic strain in static situations. However, when the loads have cyclic characteristics, the cracks can apper even if the stresses are below the material's yielding, due to the fatigue phenomenon.

Schijve (2008) describes the fatigue in two different periods: the crack initiation and the crack growth. In the first step, the cyclic loads initially cause plastic deformations in a micro scale, limited to a few grains of the microstructure. After some amount of cycles, these dislocation activities create what is called slip bands, that are planes where the plastic deformations are accumulated. As a consequence, intrusions are created on the free surface of the material, which will lead to micro-high stress concentration in that locations. This means that the crack initiation is, in fact, a surface phenomenon. On the other hand, in the crack growth period, the fatigue is no longer depending on the surface conditions. Moreover, in this case, the crack grows preferably ortogonal to the direction of the maximum tensile stress, that intends to open the crack.

Cyclic loads often carry energy on a band of frequencies, that can be narrow or wide. For most part of real applications, these signals are random and functions of time, which can be defined as stochastic processes. One simple model to deal with wideband random loads was proposed by Steinberg (1973), which assumes that the stress response is a zero-mean gaussian variable and it remains 68.3% of time within a range of 2σ, where σ is the standard deviation, 27.1% within a range of 4σ and 4.3% in a range of 6σ. More sophisticated apporaches were proposed by Dirlik (1985) and Lalanne (2002), where the number of cycles related to a particular stress range is described as a probabilistic density function, depending on the statistical parameters of the stress Power Spectral Density (PSD). It is important to mention that all these fatigue theories, related to random signals, assume that the loads are stochastic, stationary (statistical parameters does not vary significantly over time), Gaussian (respect a Gaussian distribuition) and ergodic (ensemble statistical properties can be estimated based on the statistics of one realization over time).

In cases of wideband random excitations, shifting the structures' natural frequencies can be a challenging task. Due to the broad band of frequencies, probably it would be required either a significant increase of the stiffness or a decrease of the mass to guarantee a safety margin between range of excitation and system's natural frequencies. Rather than this, dynamic absorbers can be designed to damp vibrations amplitudes, without the need of changing the equipment design. Hartog (1956) was a pioneer in the design of dynamic absorbers. The author observed that attaching a secundary structure, composed by a mass and a stiffness and tuning it properly, the vibration in a main body could be supressed in a particular natural frequency. However, it would be divided into two different peaks in the Frequency Response Function (FRF), that remain with a high dynamic amplification. To avoid this problem, a source of damping (e.g. viscous damper, viscoelastic material, etc) can be included in the tuned damper, that is responsible for reducing these peaks.

Snowdon (1959) was one of the pioneers when it comes to dynamic absorbers including rubber-like materials. His article was used as reference for several works. Espíndola and Silva (1992) introduced a innovative methodology called "Generalized Equivalent Parameters", which was responsible for simplifing the numerical model of the composed structure (main structure and dynamic absorber), reducing the number of degrees of freedom. Espíndola and Bavastri (1997) applied nonlinear optimization algorithms to find the optimum configurations of viscoelastic neutralizers. Espíndola *et al.* (2008) used the Fractional Calculus Theory to characterize the viscoelastic material in terms of frequency and temperature. This model turned out very attractive to define to represent the dynamic behavior of rubber-like materials and it is used in the present article.

Espíndola *et al.* (2010) presented a mathematical model to deal with viscoelastic pendulum absorbers, including its Genenalized Equivalent Parameters, showing the effectiveness of this type of neutralizer for frequencies below 30 Hz. Moreover, Bavastri *et al.* (2014) developed a optimum viscoelastic absorber for a nonlinear system, with a cubic stiffness. The authors showed that the proposed device not only reduced the vibration amplitudes, but also forced the system to have a linear behavior.

The present article combines the knowledge related to passive control of vibration and fatigue analysis to propose a viscoelastic pendulum absorber focused on the fatigue life enhancement, which is usually applied to others vibrations purposes. Others articles as Li and Sau-Lon James Hu (2002), Pipinato (2019) and Ju and Huang (2020) have been studied the relation between vibration supression and fatigue life improvement. This particular absorber configuration was chosen due to two main characteristics: the frequencies that cause the most part of the damage are usually low, where the structure's displacements and strains are higher. A pendulum has the ideal design flexibility, in terms of inertia, to be tuned in low frequencies (below 15Hz). Moreover, rubber-like viscoelastic materials show good intrinsic damping, combined to small Modulus value, that aids to supress fatigue-damaging frequencies.

This device is designed to reduce the vibration amplitude of the first natural frequency of a cantilever beam. This cantilever beam is then submitted to a white noise load, that excites a broad band of frequencies, and causes fatigue damage in the structure. The structure's damage is compared with and without the tuned damper to verify its efficiency in terms of enhancing the fatigue life of the main structure.

2. THEORY BACKGROUND

In this section, relevant theory background, including main mathematical equations, is described. The information provided in this section is essential for the comprehension of the article.

2.1 Modal Reduction

The dynamic equilibrium equation of a multi-DOF system may be described as (Rao, 2006)

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = \{f(t)\}, \tag{1}$$

where $[M]$, $[C]$, $[K]$ are the mass, damping and stiffness matrices, respectively, $\{u(t)\}$ is the generalized displacement vector and $\{f(t)\}$ is the generalized force vector. Solving the homogeneous portion of the Eq. (1), neglecting damping effects

$$[M]\{\ddot{u}(t)\} + [K]\{u(t)\} = \{0\}, \tag{2}$$

the solution is given as $\{u(t)\} = \{\phi\}e^{i\omega t}$ and represents the mode shapes $\{\phi\}_i$ and the natural frequencies ω_i. The modal reduction assumes that the generalized displacements $\{u(t)\}$ can be approximated as a linear combination of m modal coordinates $\{p(t)\}$. This relation is defined in terms of m mode shapes.

$$\{u(t)\} = [\widehat{\Phi}]\{p(t)\}, \tag{3}$$

where $[\widehat{\Phi}]_{nxm}$ is the truncated modal matrix with n rows, related to the physical degrees of freedom, and m columns, that is the number of mode shapes included in the reduction. It is important to note that $m << n$ (Silva and Bavastri, 2019).

Substituting Eq. (3) in Eq. (1), and multipling both sides of the equation by $[\widehat{\Phi}]^T$

$$[\widehat{\Phi}]^T[M][\widehat{\Phi}]\{\ddot{p}(t)\} + [\widehat{\Phi}]^T[C][\widehat{\Phi}]\{\dot{p}(t)\} + [\widehat{\Phi}]^T[K][\widehat{\Phi}]\{p(t)\} = [\widehat{\Phi}]^T\{f(t)\}. \tag{4}$$

Equation Eq. (4) can also be described in frequency domain, applying the Fourier Transform in both sides of the equation

$$(-\omega^2[\widehat{\Phi}]^T[M][\widehat{\Phi}] + i\omega[\widehat{\Phi}]^T[C][\widehat{\Phi}] + [\widehat{\Phi}]^T[K][\widehat{\Phi}])P(\omega) = [\widehat{\Phi}]^T\{F(\omega)\}. \tag{5}$$

Finally, taking advantage of the orthogonality characteristics of the modal matrix $[\widehat{\Phi}]$, Eq. (5) can be rewrited in function of the generalized displacement in frequency domain $\{Q(\omega)\}$

$$[\widehat{\Phi}](-\omega^2[\widehat{\Phi}]^T[M][\widehat{\Phi}] + i\omega[\widehat{\Phi}]^T[C][\widehat{\Phi}] + [\widehat{\Phi}]^T[K][\widehat{\Phi}])[\widehat{\Phi}]^T\{U(\omega)\} = \{F(\omega)\} \tag{6}$$

where

$$[H(\omega)] = ([\widehat{\Phi}](-\omega^2[\widehat{\Phi}]^T[M][\widehat{\Phi}] + i\omega[\widehat{\Phi}]^T[C][\widehat{\Phi}] + [\widehat{\Phi}]^T[K][\widehat{\Phi}])[\widehat{\Phi}]^T)^{-1} \tag{7}$$

It can be observed that $[H(\omega)]$ gives a relation between displacement response $\{U(\omega)\}$ and input forces $\{F(\omega)\}$. However, sometimes the stress response can be more interesting, mainly when dealing with fatigue. The relation between nodal displacements and element stresses can be described as (Liu and Quek, 2013)

$$\{\sigma(t)\} = [D][B]\{u(t)\}, \tag{8}$$

where $\{\sigma(t)\}$ is the stress vector, $[D]$ the constitutive matrix of a linear isotropic material and $[B]$ the matrix with the derivative of the element shape functions. It is important to note that stresses are tensor quantities, however Voigt notation is used to simplify the mathematical equations. Expressing Eq. (8) in the frequency domain and substituting the relation of Eq. (6), the Frequency Response Matrix, in terms of stress over force, $[H^\sigma(\omega)]$ is

$$[H^\sigma(\omega)] = [D][B][H(\omega)]. \tag{9}$$

2.2 Random Vibrations

When a pattern cannot be identified in the vibration signals, statistical parameters must be used to characterize a stochastic process. Assuming a stationary and ergodic behavior of a stochastic process $x(t)$, the expected value of $x(t)$ can be estimated as (Shin and Hammond, 2008)

$$E[x(t)] = \widehat{\mu}_x = \frac{1}{T}\int_0^T x(t)dt, \tag{10}$$

where $E[]$ is the expected value operator, $\widehat{\mu}_x$ is the time average, that also represents the essemble average due to the egordicity, and T is the total time of the signal window. Similarly, it can be defined others expected values, as

$$E[(x(t) - \widehat{\mu}_x)^n] = M_n = \frac{1}{T}\int_0^T (x(t) - \widehat{\mu}_x)^n dt, \tag{11}$$

where n can assume values from 2 to 4.
Eq. (10) to Eq. (11) show what is called spectral moments, that describe important statistical information. The variance M_2 or $\widehat{\sigma}_x^2$ is related to the signal spread, M_3 indicates the degree of asymmetry of the data when compared with a normal

distribuition, and M_4 shows the degree of flattening of the distribuition. The spectral moments can be used to estimate signal properties for fatigue purposes, as discussed in next sections.

Usually, random signals are represented in frequency domain using its Power Spectral Density (PSD). There are several methods to estimate the PSD of a stochastic process. Welch (2020) proposed a practical methodology to estimate it computationally from a Fast Fourier Transform (FFT) algorithm. First of all, the signal is splited in K windows, that can be overlapped or not. The FFT $X^+(k)$ of $x(n)$ can be written as (Shin and Hammond, 2008)

$$X^+(k) = \sum_{n=0}^{N-1} x(n) e^{(-2\pi i/N)nk}, \tag{12}$$

where N is the number of data points of the discrete signal. The actual Fourier Transform can be calculated from the FFT using the following relation

$$X(k) = \frac{T}{N} X^+(k), \tag{13}$$

where T is the total time of the signal. The peridiogram of the splited signal, considering a window with duration of T_r is then

$$\widehat{S}_{xx}(f_k) = \frac{1}{T_r} |X(k)|^2, \tag{14}$$

where f_k is the k-nth frequency in Hz. The estimated PSD is given by

$$S_{xx}(f_k) = \frac{1}{K} \sum_{i=1}^{K} \widehat{S}_{xx_i}(f_k). \tag{15}$$

The PSD has a important interpretation: it shows the frequencies where the random signal carries power. If a random force is applied in a multi-DOF linear system, it can be described using its PSD $[S_{ff}(f_k)]$, and the system's response $[S_{uu}(f_k)]$ can be calculated as (Shin and Hammond, 2008)

$$[S_{uu}(f_k)] = [H^*(f_k)][S_{ff}(f_k)][H(f_k)]^T, \tag{16}$$

where $[H^*(f_k)]$ is the complex conjugate of $[H(f_k)]$.

2.3 Viscoelastic Material Model

Rubber-like materials have dual properties: they show mixed solid-fluid behavior. The solid behavior is responsible for the deformation resistence, while fluid behavior gives energy dissipation, that can be interpreted as an internal damping (Silva and Bavastri, 2019). To represent it, the Shear Modulus $G(\omega, T)$ of a viscoelastic material can be written as a complex value, in function of frequency and temperature

$$G(\omega, T) = G_r(\omega, T)[1 + i\eta(\omega, T)]. \tag{17}$$

where G_r is the real part of $G(\omega, T)$ and $\eta = {}^{G_i}/_{G_r}$ is the loss factor. The superposition of frequency and temperature dependency can be modeled using the Williams-Landel-Ferry (WLF) relation (Espíndola *et al.*, 2008)

$$\log \alpha_T(T) = \frac{-\theta_1(T - T_0)}{\theta_2 + (T + T_0)}, \tag{18}$$

where θ_1 and θ_2 are material constants, that must be extracted experimentally, and T_0 is the reference temperature.

Moreover, using fractional derivative with four parameters, the Shear Modulus can be estimated as

$$G(\omega,T) = \frac{G_0 + G_\infty b_1 (i\omega\alpha_T)^\beta}{1 + b_1(i\omega\alpha_T)^\beta}, \tag{19}$$

where $G_0 = \lim_{\omega\to 0} G_r(\omega,T)$, $G_\infty = \lim_{\omega\to\infty} G_r(\omega,T)$, b_1 is the Relaxation Modulus and the exponent β is the fractional derivative order.

2.4 Design of Dynamic Absorbers

Dynamic absorbers are external devices that are designed and attached to a structure to add impedance and reduce its vibrations. A classical dynamic absorber is the MCK (mass-damping-stiffness), that containts, as the name suggests, mass, stiffness and a damping source, e.g. viscous damper. In this case, the FRF of a single-DOF system $H(\lambda)$, with a attached MCK absorber can be written as (Wong, 2016)

$$H(\lambda) = \sqrt{\frac{(\kappa^2-\lambda^2)^2 + (2\kappa\lambda\xi)^2}{[(1-\lambda^2)(\kappa^2-\lambda^2) - \mu\kappa^2\lambda^2]^2 + [2\kappa\lambda\xi(1-\lambda^2-\mu\lambda^2)]^2}}, \tag{20}$$

where $\mu = m/M$ is the ratio between the absorber and system's mass, $\kappa = \omega_a/\omega_n$ is the ratio between the natural frequency of both absorber and system,$\lambda = \omega/\omega_n$ is the ratio between the frequency and system's natural frequency and ξ is the critical damping factor. To tune the neutralizer, the classical fixed points approach can be used (Hartog, 1956). In this theory, the optimal values of κ and ξ for a MCK absorber are

$$\kappa_{opt} = \tfrac{1}{1+\mu} \quad \text{and} \quad \xi_{opt} = \sqrt{\tfrac{3\mu}{8(1+\mu)}} \tag{21}$$

where μ is suggested to assume values between 10% and 25% of the main structure mass.

In the case of a viscoelastic absorber, the rubber-like material stiffness can be modeled as a complex value and proportional to the complex Shear Modulus. Therefore, $\overline{k}(\omega,T) = k[1 + i\eta(\omega,T)]$. In this particular case, the FRF of the composed system is (Wong, 2016)

$$H(\lambda) = \sqrt{\frac{(\kappa^2-\lambda^2)^2 + (\kappa^2)^2\eta^2}{[(1-\lambda^2)(\kappa^2-\lambda^2) - \mu\kappa^2\lambda^2]^2 + [\kappa(1-\lambda^2-\mu\lambda^2)]^2\eta^2}}. \tag{22}$$

However, the fixed points approach must be adapted for viscoelastic absorbers. In this case, η is not a design variable, because its value is intrinsic to the rubber-like material. It only changes if the material is changed. Thus, the damping cannot be a optimization variable, and only κ can be used to tune tha dynamic absorber.

2.5 Fatigue Analysis in Frequency Domain

The fatigue evaluation in frequency domain was developed mainly to deal with stochastic processes. In these cases, the signal is better represented in frequency domain in terms of its PSD. However, if it is known only the excitation PSD, all the exact time history has been lost, and then the reconstruction of the events information must be rebuilt. As already mentioned, several authors have proposed different ways to recover cycles information from signals PSD. Lalanne (2002) proposed that, if the stress history is stochastic, stationary, ergodic and gaussian, its stress amplitude should follow a specific probability density function, that can be seen as a weighted average between gaussian and rayleigh distributions.

Fist, it is required the calculation of the spectral moments of the stress response PSD. Considering a single-DOF linear system, the stress response PSD $S^\sigma(\omega)$ can be calculated as

$$S^\sigma(\omega) = [H^\sigma(\omega)]^2 S_f(\omega). \tag{23}$$

Then, from $S^\sigma(\omega)$, the spectral moments can be calculated in frequency domain as

$$M_n = \sum_{i=0}^{n} \omega^i S^\sigma(\omega)\delta\omega, \tag{24}$$

where the spectral moments M_n from 1 to 4 have the meaning already described in a previous section. From the spectral moments, it is possible to estimate some important properties of the original time domain signal (Rice, 1944). For instance, the expected numbers of times that the signal crossed the 0 level $E(0)$ can be estimated as

$$E(0) = \sqrt{\frac{M_2}{M_0}}. \tag{25}$$

Moreover, the expected numbers of times that the signal achieve its peak $E(P)$ can be estimated as

$$E(P) = \sqrt{\frac{M_4}{M_2}}, \tag{26}$$

and then a irregularity factor can be defined as $\gamma = E(0)/E(P)$. Lalanne (2002) proposed that the number of cycles $n(\sigma_a)$ of the signal related to a stress amplitude σ_a can be calculated as

$$n(\sigma_a) = E(P)p(\sigma_a), \tag{27}$$

where $p(\sigma_a)$ is the probability density function proposed by Lalanne, which is

$$p(\sigma_a) = \frac{1}{\sigma_{\text{rms}}}\left[\frac{\sqrt{1-\gamma^2}}{\sqrt{2\pi}}e^{\frac{-\sigma_a^2}{8\sigma_{\text{rms}}(1-\gamma^2)}} + \frac{\sigma_a\gamma}{4\sigma_{\text{rms}}}e^{\frac{-\sigma_a}{8\sigma_{\text{rms}}}}\left(1+\text{erf}\left[\frac{\sigma_a\gamma}{2\sigma_{\text{rms}}\sqrt{2(1-\gamma^2)}}\right]\right)\right]. \tag{28}$$

With $n(\sigma_a)$, the linear accumulated damage can be calculated, based on Miner's rule (Lalanne, 2002).

3. METHODOLOGY

The structure chosen to apply the passive control of vibration is shown in Fig. 1. It is basically a cantilever beam, fixed by bolts in a rigid bracket.

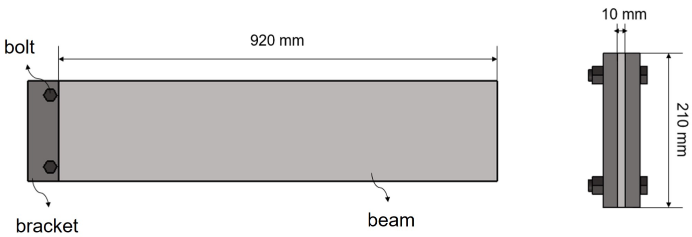

Figure 1: Main structure used in the passive control of vibration.

3.1 Modal Parameters Calculation

To calculate the modal parameters of the main system, e.g. natural frequencies and mode shapes, a Finite Element Model was built. Fig. 2 shows the details of the numerical model. In this case, the random force was applied in the middle of the cantilever beam.

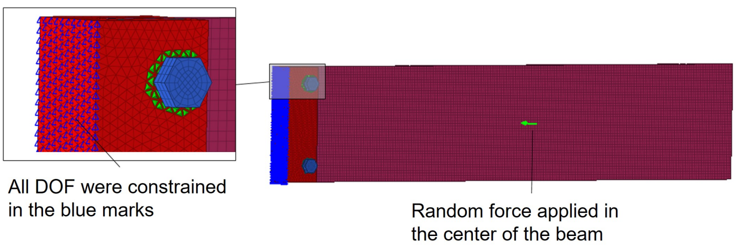

Figure 2: FE model of the main structure.

This same model, excluding the random force, was used to run a modal analysis. Then, the first 5 natural frequencies of the system were extracted. However, only the first natural frequency ω_1 was chosen to be controlled by the dynamic absorber. To isolate the first natural frequency in the passive control design, a single-DOF equivalent model was created. The equivalent mass m_{eq} and stiffness k_{eq} were defined as

$$m_{eq} = 0.203m \quad \text{and} \quad k_{eq} = \omega_1^2 m_{eq}, \tag{29}$$

where m is the beam total mass. With this single-DOF model, κ_{opt} can be calculated to tune the dynamic absorber. Furthermore, Eq. (27) can be used to describe the composed system FRF.

3.2 Viscoelastic Pendulum Absorber

The rubber-like material used in this study is the elastomer BT-806/55. Its constants of the fractional derivative model is shown in Tab. 1

Table 1: Elastomer BT-806/55 viscoelastic properties.

G_0 [Pa]	G_∞ [Pa]	b_1	β	WLF Factor			
				T [K]	T_0 [K]	θ_1	θ_2
2.4033x10^6	1.523x10^8	0.0223	0.417	293	243	7.78	81.7

The general configuration of the proposed viscoelastic pendulum absorber is shown in Fig. 3. The torsional stiffness $k_\theta(\omega, T)$ related to the viscoelastic elements can be defined as

$$k_\theta(\omega, T) = \vartheta G_r(\omega, T)[1 + i\eta(\omega, T)], \tag{30}$$

where ϑ is a shape factor, that is related to the geometrical characteristics. In this case, numerical simulations were used to extract ϑ in function of the ratio between the diameters D/d. The calculated expression is

$$\vartheta = 6\text{x}10^{-5} \left(\frac{D}{d}\right)^{-1.666} \tag{31}$$

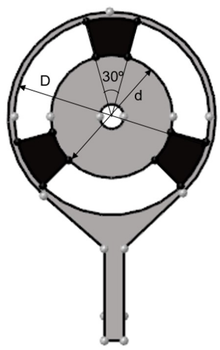

Figure 3: Viscoelastic pendulum absorber general configuration.

In this absorber design, as mentioned before, only κ is used as variable in the fixed points approach, due to the rubber-like material intrinsic damping. Moreover, the viscoelastic pendulum natural frequency ω_a can be calculated as

$$\omega_a = \sqrt{\frac{\vartheta G_r(\omega, T)}{m_a L^2 + I_G}}, \tag{32}$$

where m_a is the mass in the end of the pendulum, L is the pendulum length and I_G is the pendulum's arm inertia.

With the optimum factor κ_{opt}, the variables ϑ, m, L and I_G can be calculated to achieve the required natural frequency to tune the dynamic absorber to the first natural frequency of the structure.

3.3 Fatigue Evaluation

In the fatigue analysis, the material of the cantilever beam was considered as a generic steel, with a Ultimate Strength of 400 MPa. Furthermore, a surface finishing factor was included to represent a hot rolled component, according to the definitions of the software nCode DesignLife®.

A white-noise force was generated, with a random phase between -2π and +2π. Then, the PSD of this force history was calculated to be used in the frequency domain analysis. This procedure was chosen to verify the forces amplitudes and make sure that no plastification occurs in the beam, due to the assumption of linear relation between stress and strain in the S-N theory.

The force time history and its PSD are shown in Fig. 4.

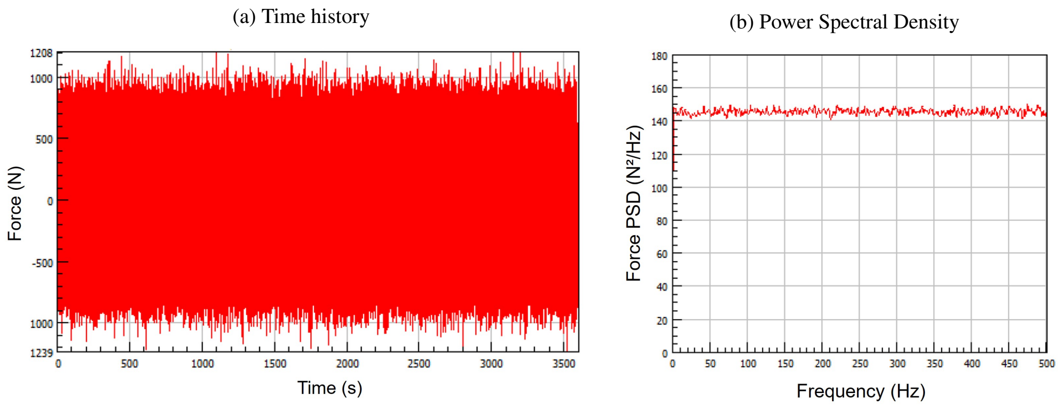

Figure 4: Force applied in the fatigue analysis.

With the excitation PSD, the procedure proposed by Lalanne (2002) was used to estimate the fatigue life using the frequency domain procedure, estimating the accumulated damage based on the statistical properties of the PSD of the system's response. This procedure was done for the cantilever beam with and without the dynamic absorber to verify its contribuition in the fatigue life improvement.

4. RESULTS

The first mode shape of the cantilever beam is shown in Fig. 5. The natural frequency related to this mode shape was approximately 9.6 Hz. Based on this first mode shape and natural frequency, the optimum parameters of the dynamic absorber was calculated and written in Tab. 2. With the optimum parameters for the dynamic absorber, a pendulum

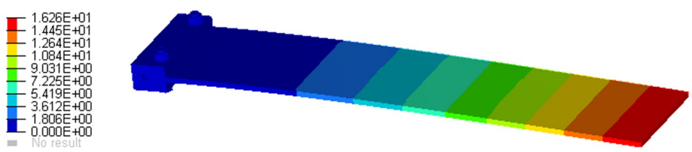

Figure 5: Mode shape related to the first natural frequency of 9.6 Hz.

viscoelastic absorber design was proposed. This design and its modal analysis results are shown in Fig. 6. The modal analysis was conducted to verify if the proposed design is effectively tuned to the first natural frequency of the cantilever beam. The first natural frequency of the proposed pendulum absorber is 8.38Hz, which is sufficient close to the optimum value.

Table 2: Optimum tuned parameters for the dynamic absorber.

Parameter	Symbol	Optimum Value
Tuned ratio	κ_{opt}	0.8663
Absorber natural frequency	ω_a	8.33 Hz
Shape factor	ϑ	2.5974x10^{-5}

(a) Proposed dynamic absorber. (b) Absorber first mode shape.

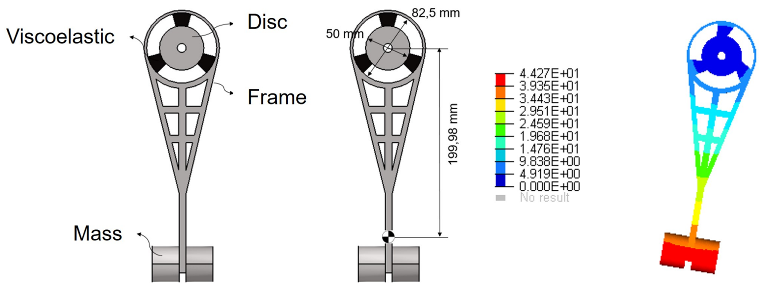

Figure 6: Viscoelastic pendulum absorber design.

With the absorber design, it was attached to the cantilever beam on the unconstrained end. Then, the FRF of the system with and without the viscoelastic pendulum absorber were calculated. This was done simulating a unity excitation over a range of frequencies. Fig. 7 shows the FRF amplitude comparison.

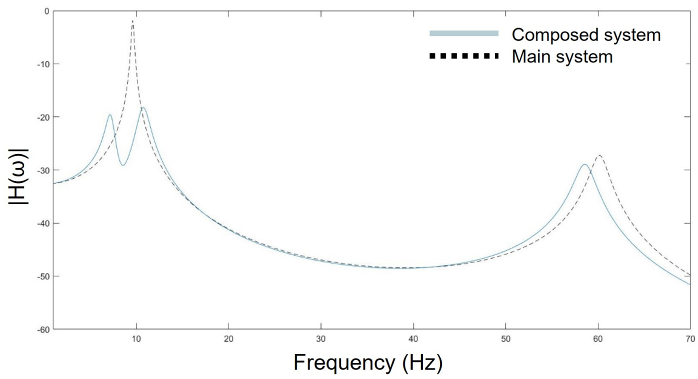

Figure 7: FRF amplitude comparison of the cantilever beam with and without the absorber.

It can be observed that the viscoelastic pendulum absorber reduced almost 20 dBs in the fist natural frequency of the cantilever beam, which represents a significant supressing in the vibration amplitude. Furthermore, the comparison of accumulated damage and life are shown in Fig. 8. The vibration reduction of 20 dBs was converted in an increase of the fatigue life from 28.2 hours to 1696 hours. In other words, the fatigue life was improved by 60 times due to the decrease of vibration provided by the viscoelastic pendulum absorber.

(a) Proposed dynamic absorber. (b) Dynamic absorber first mode shape.

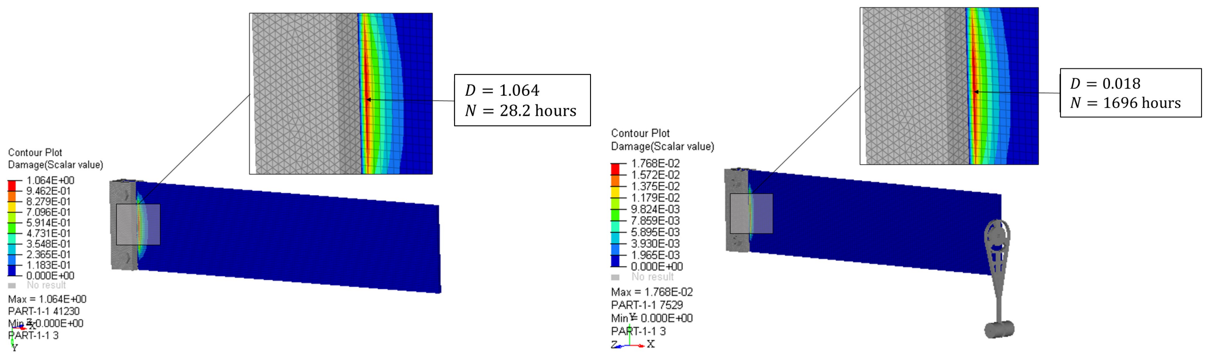

Figure 8: Viscoelastic pendulum absorber design.

5. CONCLUSIONS

The present work showed the development of a viscoelastic pendulum absorber, tuned to the first natural frequency of a cantilever beam, aiming to enhance its fatigue life when submitted to a random load. The viscoelastic material was modeled using Fractional Derivative Models, with a Complex Shear Modulus in function of frequency and temperature. Furthermore, the design of the dynamic absorber was done based on the Fixed Points theory, considering a SDOF model of the main system.

For the fatigue analysis, a frequency domain approach was chosen. In this approach, the PSD of the response is calculated and its statistical properties are used to estimate the fatigue damage caused by a broadband white noise load. After the calculation of the absorber optimum parameters, it was proposed a design, that was modeled using FEM and attached to the unconstrained and of the cantilever beam.

The comparative results showed that the viscoelastic pendulum absorber reduced around 20 dBs the vibration amplitude on the first natural frequency of the main system. Furthermore, in terms of fatigue, it represents a life improvement of almost 60 times. Therefore, the results showed considerable enhancing of the fatigue life of dynamic excited structures when using passive vibration control through viscoelastic pendulum absorbers.

6. REFERENCES

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