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TURBULENT VISCOSITY MODEL ASSESSMENT FOR THE 1D NUMERICAL SIMULATIONS OF VERTICAL ANNULAR FLOWS

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Abstract. *In industrial applications of two-phase flows through pipes, one-dimensional models are commonly used. Among those, the "Slug Capturing" 1D Two-Fluid Model is particularly attractive due to its ability to capture the transition between different regimes (such as from stratified to slug flow). However, in vertical pipes, the standard Two-Fluid Model becomes unconditionally ill-posed, which will lead to the inability to obtain a grid independent solution. Therefore, simulations with capturing schemes are potentially problematic in such situations depending on mesh resolution requirements. To address this issue, several authors have proposed different physical closure relations to render the Two-Fluid Model well posed. In the present work, we investigate the impact of introducing axial diffusion coupled with different proposition of turbulent viscosity models, as well as its combined effect with the momentum flux parameter. To evaluate the effectiveness of the models, comparisons with experimental data of different data sets are carried out.*

Keywords: *Two Fluid Model, turbulence modeling, Vertical Annular Flows, Closure Models*

1 INTRODUCTION

Two-phase flows can be encountered in several industrial processes such as refrigeration, steam power generation, boiling water nuclear reactors, and pipeline transport of oil and gas (Bestion, 1990; Nieckele and Carneiro, 2017). In pipe flows, different flow patterns may occur, depending heavily on the pipe geometry, properties and phase flowrates. For vertical pipes with high gas flow rates, e.g., in natural gas production lines, annular flows may occur.

The one-dimensional Two-Fluid model is widely used for solving multiphase flows industrial problems. It strikes a good balance between computational cost and accuracy, enabling the prediction of complex phenomena associated with various flow configurations. When employed with sufficiently fine meshes, the Two-Fluid model effectively captures interfacial instabilities within the numerical domain and their subsequent evolution into waves and slugs, opening the possibility to simulate the transition between different flow regimes using a single numerical framework (Fullmer et al., 2014; Galleni and Issa, 2015; Nieckele and Carneiro, 2017; Fontalvo et al., 2020; Castello Branco et al., 2022).

Although the 1D Two-Fluid model is widely used in the industry, it still faces several challenges, especially regarding hyperbolicity. Due to the averaging processes to obtain the 1D formulation, information about the flow field and momentum transfer are lost from the system of equations. As a result, the standard formulation is known to be conditionally well-posed for horizontal and inclined geometries and unconditionally ill-posed for vertical geometries. In an ill-posed scenario, the high frequency instabilities captured by the numerical system are amplified at an unbounded rate, and eventually contaminate the physical solution. This process may render the results meaningless and will manifest as an inability to obtain a mesh convergent solution.

During the averaging process information is lost, but they can be reinjected through closure relations. However, these relationships may also dampen short wavelength perturbations. Several formulations have been proposed for closure models such as the wall and interfacial friction factors (Whalley and Hewitt, 1978; Belt et al. 2009), dynamic pressure models related to the phase velocities differences (Bestion, 1990; Fontalvo et al., 2020), and momentum flux parameters due to the non-uniformity of the velocities at the cross section (Song and Ishii, 2000; Fontalvo et al., 2020; Castello Branco et al., 2023).

Issa and Montini (2010) state that artificial viscosity is considered more appropriate for dissipating non-physical disturbances with short wavelengths. Holmås et al. (2008) also introduced artificial diffusion to solve the ill-posedness of the standard model. The addition of diffusive terms stabilizes short wavelengths without significantly affecting longer wavelength mechanisms. However, including diffusion only in the momentum equation was shown to be insufficient to achieve convergent numerical solutions.

Fontalvo et al. (2020) discussed the importance of considering a modeling approach for the momentum flux parameter in vertical annular flows. Furthermore, they discuss the need to consider a non-constant momentum flow parameter for the liquid phase. Recently, Castello Branco et al. (2023) proposed a velocity profile model to estimate the variable flow parameter of the liquid phase. The shape factor models have been shown to improve the stability of the Two-Fluid model. In the standard methodology of the 1D Two-Fluid Model, diffusion is neglected. However, diffusion is a mechanism with stabilizing effects on the system of equations, expanding the range of applicability for which the model is well-posed. Furthermore, including a viscosity model can yield a more physical representation of the flow.

The present work aims to expand on previous studies by introducing axial diffusion and turbulent viscosity models in numerical simulations of annular flow cases. Three turbulent viscosity models are evaluated. Furthermore, the combination of the novel formulations with a variable momentum is also explored in the present work. Results for pressure drop and mean film thickness are used to evaluate the models against experimental results from three different databases.

2 MODELING

In this study, the 1D Two-Fluid Model (Ishii and Hibiki, 2011) was selected as the mathematical approach to determine the velocity, pressure and volumetric fraction fields, thus estimating the behavior of isothermal, two-phase, annular flow in long vertical pipes. The Two-Fluid Model consists of the solution of a set of conservation equations for each phase present in the system, characterized by the volume fraction

$$\alpha_k = \frac{\forall_k}{\forall} \qquad ; \qquad \sum_k \alpha_k = 1 \tag{1}$$

where $\forall_k$ is the volume occupied by phase k (G for gas and L for liquid) and $\forall$ is the total volume.

To obtain the 1D formulation, the conservation equations are integrated along the cross section to determine area averaged quantities. The resulting equations of conservation of mass and momentum are:

$$\frac{\partial(\alpha_k \rho_k)}{\partial t} + \frac{\partial(\alpha_k\, \rho_k\, U_k)}{\partial x} = 0 \tag{2}$$

$$\frac{\partial(\alpha_k \rho_k U_k)}{\partial t} + \frac{\partial\left(C_k \alpha_k\, \rho_k U_k^2\right)}{\partial x} = -\alpha_k\, \rho_k\, g - \alpha_k \frac{\partial P_{Gi}}{\partial x} + \alpha_k \frac{\partial\left(P_{G_i} - P_{ki}\right)}{\partial x} - \frac{\partial \alpha_k (P_k - P_{ki})}{\partial x} - \frac{\tau_{wk} S_k}{A} \pm \frac{\tau_i S_i}{A} + \alpha_k \frac{\partial}{\partial x}\left(\tau_{xx_k} + \widetilde{\tau'_{xx\,k}}\right) \tag{3}$$

where t and x represent the time and axial coordinate, respectively. ρ_k and U_k are the density and average phase velocity. C_k is the momentum flux parameter for phase k , P_k and P_{k_i} are the phase average pressure and their corresponding values at the interface, respectively. g is the gravity acceleration, τ_{wk} and τ_i are the wall-phase and interfacial shear stresses, S_k is the wetted perimeter. For annular flows, the liquid film wetted perimeter is defined $S_L = \pi D$, there is no gas wetted perimeter ($S_G = 0$) and the interface perimeter is equal to $S_i = \pi(D - 2\, h_L)$, where the liquid film thickness is $h_L = D/2\, \left(1 - \sqrt{\alpha_G}\right)$ and D is the pipe diameter, with cross section area, $A = \pi\, D^2/4$.

Viscous stresses τ_{xx_k} and turbulent $\widetilde{\tau'_{xx\,k}}$ are usually neglected. However, as mentioned in the bibliographic review, it is known that due to its diffusive character, the presence of these terms can help to make the render of equations well-posed and will be considered in this work. The viscous stress for a Newtonian fluid and the turbulent Reynolds stress based on the Boussinesq approximation (Pope, 2000) are

$$\tau_{xx_k} + \widetilde{\tau'_{xx\,k}} = \mu_{ef_k} \left\{ \left(\frac{\partial U_k}{\partial x} + \frac{\partial U_k}{\partial x} \right) - \frac{2}{3} \frac{\partial U_k}{\partial x} \right\} - \frac{2}{3}\, \rho_k\, \kappa \qquad ; \quad \mu_{ef_k} = \mu_k + \mu_{t_k} \tag{4}$$

where κ the turbulent kinetic energy. The effective viscosity μ_{ef_k} depends on the molecular μ_k and turbulent viscosity μ_{t_k}. Different turbulence models are proposed in the present work and are presented in the next section.

Due to the averaging process of the 1D Two-Fluid Model, several parameters must be defined/modeled to close the one-dimensional system of equations and compensate for the loss of information. These parameters are referred to as closure relations.

The wall/liquid and interfacial shear stress are defined in terms of a Fanning friction factor as

$$\tau_{wL} = \tfrac{1}{2}\, f_L\, \rho_L\, |U_L\,| U_L \quad ; \quad \tau_i = \tfrac{1}{2}\, f_i\, \rho_G\, |U_G - U_L\,| (U_G - U_L) \tag{5}$$

In the present work, the correlations selected for the friction factors are Kosky and Staub (1971) and Whalley and Hewitt (1978) for the wall/liquid (f_L) and interfacial (f_i) friction factors, respectively.

$$f_L = \begin{cases} \frac{16}{Re_{sL}} & if\ Re_{sL} < 50 \\ \frac{12.7937}{Re_{sL}^{0.9428}} & if\ 50 \leq Re_{sL} \leq 1483 \\ \frac{0.081}{Re_{sL}^{0.25}} & if\ Re_{sL} > 1483 \end{cases} \quad ; \quad f_i = \begin{cases} \frac{16}{Re_{SG}} \left[1 + 12\left(\frac{\rho_L}{\rho_G}\right)^{1/3} \left(1 - \sqrt{\alpha_G}\right)\right] & if\ Re_{SG} < 2100 \\ \frac{0.079}{Re_{SG}^{0.25}} \left[1 + 12\left(\frac{\rho_L}{\rho_G}\right)^{1/3} \left(1 - \sqrt{\alpha_G}\right)\right] & if\ Re_{SG} \geq 2100 \end{cases} \tag{6}$$

Here, the dynamic pressure term, $P_k - P_{k_i}$ is neglected. The interfacial pressure term in the momentum equations accounts for the pressure jump over the interface due to surface tension, and is defined by the Young-Laplace equation

$$P_{Gi} - P_{Li} = \sigma K \ ; \ K = K_1 + K_2 \ ; \ K_1 = \frac{\partial^2 h_L}{\partial x^2} = \left(\frac{D}{4}\frac{1}{\sqrt{\alpha_G}}\right)\frac{\partial^2 \alpha_L}{\partial x^2} + \left(\frac{D}{8}\frac{1}{\alpha_G^{3/2}}\right)\left(\frac{\partial \alpha_L}{\partial x}\right)^2 \ ; \ K_2 = \frac{2}{D - 2h_L} = \frac{2}{D\sqrt{\alpha_G}} \tag{7}$$

where σ is the superficial tension and K is the interface curvature, given by the sum of the longitudinal and cross section curvatures, K_1 and K_2, respectively.

The liquid density is assumed constant and ideal gas is considered to determine the gas density, based on the gas constant R and a reference temperature T_{ref}

$$\rho_G = \frac{P_{G_i}}{R\, T_{ref}} \tag{8}$$

Two approximations were considered for the liquid momentum flux parameter: constant, and as a function of the flow as proposed by Castello Branco (2022).

$$\begin{matrix} C_L = 1.334, & Re_L \leq Re_c \\ C_L = m\, Re_L^n + b, & Re_L > Re_c \end{matrix} \quad ; \quad m = 1.3703 \ ; \ n = -0.12517 \ ; \ b = 0.66361 \ ; \ Re_c = 303 \tag{9}$$

2.1 Turbulent viscosity models

To propose a turbulent viscosity model, first it is necessary to introduce a model for the turbulent kinetic energy κ. One can assume that the velocity fluctuation is proportional to the average velocity, with the intensity of turbulence as the proportionality parameter ($u'_k = I_k u_k$). Thus, for the 1D case, κ can be estimated as

$$\kappa_k \ \frac{\overline{u_k'^2}}{2} = \frac{1}{2} I_k^2 u_k^2 \tag{10}$$

This model corresponds to an instantaneous relaxation of turbulent kinetic energy with respect to changes in mean flow (or mean axial velocity). Typically, the turbulent intensity I_k ranges from 5% to 10%.

The turbulent viscosity can be estimated based on a characteristic velocity V_{c_k} and characteristic length l_{mk}

$$\mu_{t_k} = \rho_k\, V_{c_k}\ l_{mk} \tag{11}$$

Three models were proposed here, with different definitions of these two variables.

Model I is based on the logarithmic universal velocity profile, depending on $K = 0.4$, the von Kármán constant. Model II employs the same formulation for the gas variables, but, for the liquid variables, it is based on the turbulent profile of the liquid film proposed by Castello Branco (2023). Lastly, Model III, employs an empirical β_k parameter to estimate the characteristic length, while the characteristic velocity is based on the turbulent kinetic energy, $V_{c_k} = \sqrt{(2/3)\kappa_k}$. The characteristic variables of each model are shown in Table 1.

Table 1. Turbulent characteristic velocity and characteristic length for both phases.
Turbulent viscosity: $\mu_{t_k} = \rho_k\, V_{c_k}\ l_{mk}$.

Model	V_{c_L}	l_{mL}	V_{c_G}	l_{mG}
Model I	$u_{\tau_L} = \sqrt{\tau_{w_L}/\rho_L}$	$K\, h_L$	$u_{\tau G} = \sqrt{\tau_i/\rho_G}$	$K\ (D - 2\, h_L)/2$
Model II	$u_{\tau_L} = \sqrt{\tau_{w_L}/\rho_L}$	$l_{mL} = \frac{0{,}14}{\tau_i^+} h_L \ ; \ \tau_i^+ = \frac{\tau_i}{\rho_L u_\tau^2}$	$u_{\tau G} = \sqrt{\tau_i/\rho_G}$	$K\ (D - 2\, h_L)/2$
Model III	$\sqrt{(1/3)}\, I_L\, U_L$	$\beta_L\, D$	$\sqrt{(1/3)} I_G\, U_G$	$\beta_G\, D$

2.2 Numerical Model

The conservation equations were discretized with the finite volume method, with a staggered arrangement for the velocities in relation to all other quantities, employing first order Euler implicit time integration and second order TVD (Total Variation Diminishing) van Leer scheme (Versteeg and Malalasekera, 2007).

The conservation equations were solved in a sequential order, at each time step, with a similar procedure to the PRIME algorithm (Simões et al., 2014), employing a tolerance equal to 10^{-6} for the residue of all equations. The TDMA algorithm was used in the solution of each algebraic conservation equation system.

The time step Δt was defined based on the maximum flow velocity u_{max}, so that the Courant number, $Co = u_{max}\, \Delta t/\Delta x$, was always 0.5.

The pressure gradient and liquid film thickness were determined after the flow had attained a statistically steady state regime, i.e., after the time average of flow variables reached approximately constant values (typically 100s). The time average quantities were determined from the data obtained during the subsequent 30 s, with a 0.001 s time interval.

To determine the statistically steady state regime, the flow was initialized as a non-perturbed flow, i.e., with uniform velocities and pressure as prescribed at the boundaries, and uniform volume fraction corresponding to equilibrium. Perturbations evolve at the gas-liquid interface as a natural outcome of the numerical solution of the equations system.

3 RESULTS

To evaluate the impact of different turbulence models in the Two-Fluid model, two configurations were selected, and their set-up are summarized in Table 2 and Table 3. For all cases, a turbulent intensity of 10% was used for both phases ($I_k = 0.1$). The characteristic length of Model III was defined as $l_{m_k} = 0.1D$ for both phases.

Table 2. Summary of the experimental set-ups selected.

Configuration	Geometry		Gas		Liquid	
	Diam. D (mm)	Length L (m)	Density (kg/m^3)	Viscosity (cP)	Density (kg/m^3)	Viscosity (cP)
Case 1 - Zhao *et al*. (2013)	34.5	2.00	1.18	0.0179	998.2	1.00
Case 2 - Fore Dukler (1995)	50.8	3.50	1.18	0.01827	999.0	1.05

Table 3: Experimental database

Configuration	Re_{sL}	U_{sG} (m/s)	$Re_{sG} \times 10^{-4}$	h_L (mm)	dp/dx (Pa/m)
Case 1 - Zhao *et al*. (2013)	569	22.30	4.14	0.2477	1712
Case 2 - Fore Dukler (1995)	300	36.5	12.0	0.272	539

Figure 1and Figure 2 show the impact of the turbulence models in the predictions of the pressure gradient and liquid film thickness for Cases 1 and 2, with $C_L = 1$, respectively. Note that for Case 1 mesh convergence was not attained for Model I with respect to the pressure gradient, while a tendency to stabilization can be seen for the liquid film thickness. Reasonable predictions of the pressure gradient were obtained with Models II and III, while a larger error was obtained for the liquid film thickness. For Case 2, the liquid film thickness predictions of Models I and III are very similar, with a tendency to converge with mesh refinement. However, the results for the pressure gradient are not acceptable: large errors and absence of mesh convergence. Moreover, the numerical simulations with Model II did not converge.

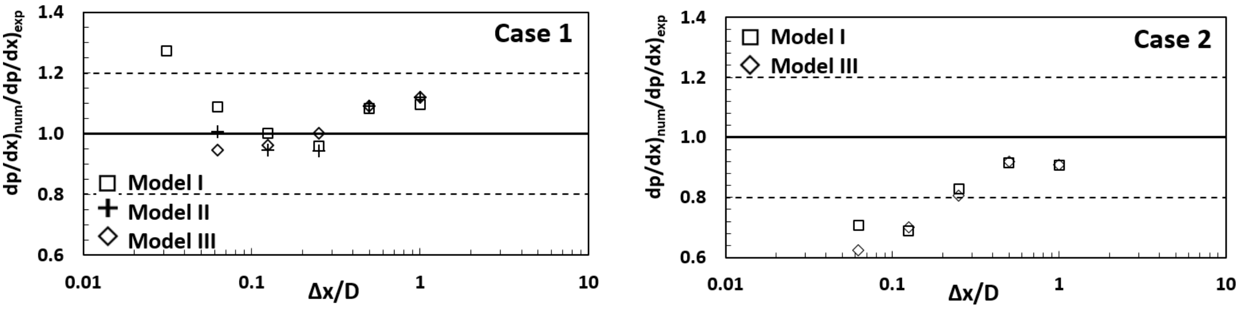

Figure 1 - Influence of turbulence models on pressure gradient with $C_L = 1$. Cases 1 e 2.

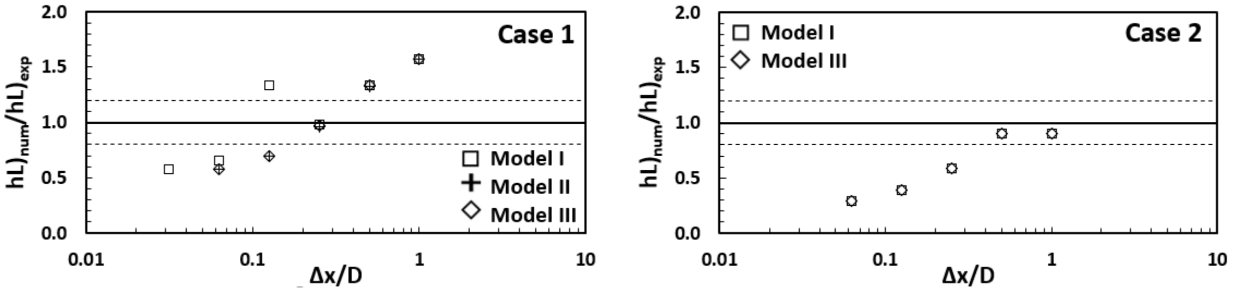

Figure 2 - Influence of turbulence models on liquid film thickness with C_L =1. Cases 1 e 2.

Although the obtained results did not agree well with experimental data, an improvement in the modeling has been attained since the numerical simulations, in the absence of the turbulence models, did not converge.

These results show that the additional diffusion of the turbulence model is positive, but it is not enough to render the models well-posed. Thus, in Figure 3 and Figure 4, the same tests were performed in combination with the variable C_L model. The solutions for the three turbulent viscosity models are compared with the predictions obtained without any turbulence modeling. Note that practically the same results were obtained in both cases for the pressure gradient and liquid film thickness, where mesh convergence is attained. These results show that the contribution of the turbulence models to stabilize the solution is too small in relation to the variable flux parameter.

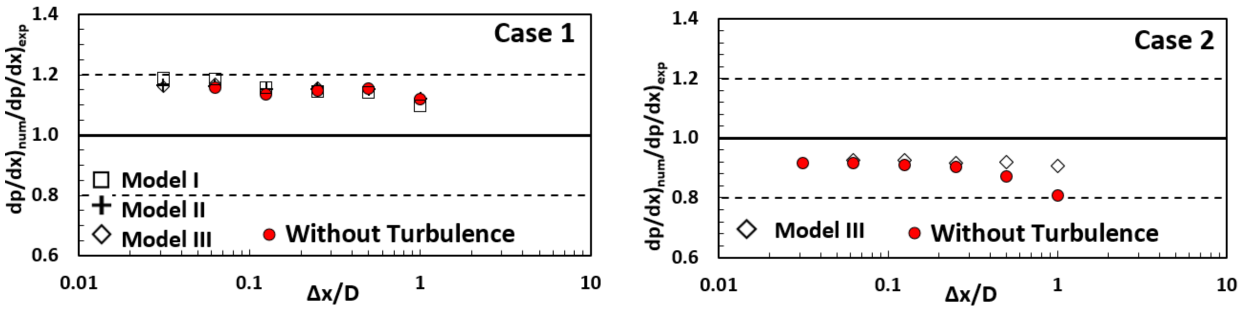

Figure 3 - Influence of turbulence models on pressure gradient with variable C_L. Cases 1 e 2.

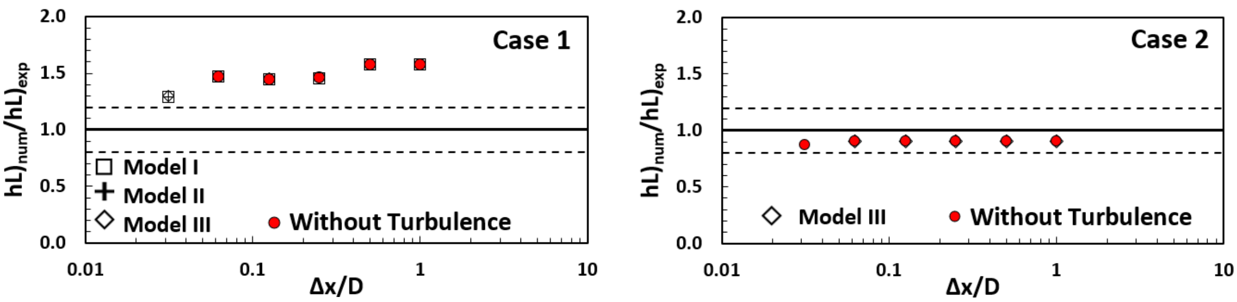

Figure 4 - Influence of turbulence models on liquid film thickness with variable C_L. Cases 1 e 2.

4 FINAL REMARKS

For the three turbulence models, it was noted that for all cases where the momentum flux parameter was equal to 1, the introduction of the turbulent viscosity was not enough to render the system of equations well posed, and for the variable C_L, the turbulence models had no effect on the solution.

For the performed tests, it is clear that although the contribution of the additional diffusion due to turbulence is positive and more realistic, the proposed turbulent viscosities are not enough to stabilize the solution. To improve the turbulence models, it is possible to determine the turbulent kinetic energy by its conservation equation. To aid in the stabilization of the models, one can also consider other closure parameters, such as the momentum flux parameter.

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