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NUMERICAL INVESTIGATION OF CROSS INFLUENCE BETWEEN PILES IN THE DYNAMIC BEHAVIOR OF PILE FOUNDATIONS

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Abstract. *This paper presents a formulation for analysis of the dynamic behavior of two piles under vertical harmonic loads. The soil was considered as a viscoelastic, transversely isotropic half-space and was modeled by the indirect boundary element method (IBEM). The piles were modeled as one-dimensional bar finite elements with two nodes and one translational degree of freedom per node. The results of this formulation were compared to those obtained using the extended half-space method for different geometric parameters and materials. The results showed that there is a divergence in the solutions obtained by the two methods and that this divergence increases as the proximity between the piles increases. This divergence occurs due to the absence of crossed stress terms in the formulation of the expanded half-space method and shows that despite being numerically less expensive, it is not adequate to represent the behavior of two or more piles.*

Keywords: *Pile groups, dynamic soil-structure interaction, coupled methods*

1. INTRODUCTION

Pile groups are commonly used as the foundations for structures subject to static and dynamic loads, such as bridges, offshore platforms and skyscrapers. The behavior of a group of piles is a result of the individual characteristics of the piles and their arrangement. Each pile in a group is influenced by the presence of other piles, which affects its individual stiffness and dynamic response. Several researchers have investigated the additional settlement experienced by a pile when it is in close proximity to another pile. Poulos (1968) and Cooke *et al.* (1980) studied this relationship, using the term "interaction factor" to describe the relationship between the induced settlement in a nearby pile and its own settlement when loaded. Novak (1991) provides extensive references on some of the developments in pile dynamics. His paper reviews the progress in the analysis of single piles and groups of piles, field and laboratory experiments, and soil-pile-structure interaction.

A method that has been widely used to analyze the dynamic behavior of groups of piles is the impedance matrix method developed by Kaynia and Kausel (1991). In this formulation, the piles are modeled as beam elements and the interaction with the soil is represented by distributed loads applied along the shaft and at the tip of the pile. This formulation uses the extendend half-space method in which the continuous medium is decomposed into two parts: an "extended soil" and a "fictitious pile" with Young's modulus E' and density ρ' such that the material properties of the fictitious pile are those of the real pile subtracted from those of the soil. The great advantage of this method is the need to determine only the displacements caused by the stresses applied by the pile, dispensing with the calculation of stresses, which are more costly to determine numerically. Rajapakse and Shah (1987) have shown that the extended half-space strategy may not be adequate to model piles undergoing dynamic excitation, due to their difficulty in representing the inertial effects of the embedded body of the pile. In this paper, we show that this is especially true for the case of more than one pile.

2. PROBLEM STATEMENT

This work presents a formulation for dynamic analysis of the behavior of two piles, installed in a homogeneous half-space subjected to harmonic vertical loads.

The piles are cylindrical and vertical; characterized by the radius $a^{(i)}$ of the cross section, length $l^{(i)}$, density $\rho_p^{(i)}$ and elasticity modulus $E_p^{(i)}$, in which i = 1, 2 denotes pile i. The vertical external forces $F^{(i)}$ are applied at the top of the piles and s is the horizontal distance between the axes of the piles. The half-space is characterized by its elastic constants c_{11}, c_{12}, c_{13}, c_{33}, c_{44}, and density ρ_s.

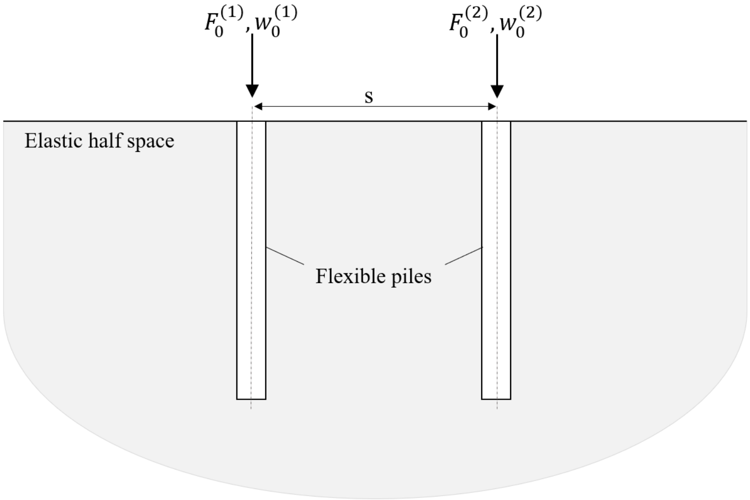

Figure 1. Two piles in a elastic half-space.

3. NUMERICAL MODEL

3.1 Soil modeling by IBEM

The elastic half-space in which the piles are installed is modeled with the indirect formulation of the Boundary Element Method (IBEM). The soil-piles interface is divided into elements, with nodes at the midpoints of each element. Fictitious loads $\mathbf{q}_s^{(i)}$ are applied along the elements of each pile, which together cause stresses $\mathbf{t}_s$ and displacements $\mathbf{u}_s$ at the nodes, given by:

$$\begin{Bmatrix} \mathbf{t}_s^{(1)} \\ \mathbf{t}_s^{(2)} \end{Bmatrix} = \begin{bmatrix} \mathbf{T}_s^{(11)} & \mathbf{T}_s^{(12)} \\ \mathbf{T}_s^{(21)} & \mathbf{T}_s^{(22)} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_s^{(1)} \\ \mathbf{q}_s^{(2)} \end{Bmatrix} \tag{1}$$

$$\begin{Bmatrix} \mathbf{u}_s^{(1)} \\ \mathbf{u}_s^{(2)} \end{Bmatrix} = \begin{bmatrix} \mathbf{U}_s^{(11)} & \mathbf{U}_s^{(12)} \\ \mathbf{U}_s^{(21)} & \mathbf{U}_s^{(22)} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_s^{(1)} \\ \mathbf{q}_s^{(2)} \end{Bmatrix} \tag{2}$$

where $\mathbf{T}_s$ and $\mathbf{U}_s$ are influence functions of soil stresses and displacements, respectively. A full description of the soil influence used in this model is given by Rajapakse and Wang (1993).

3.2 Modeling of piles by MEF

The piles are modeled by linear bar finite elements, with two nodes and one degree of freedom per node. The stiffness and mass matrices of the element are given by:

$$\mathbf{K}_e = \frac{A_p E_p}{l_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{3}$$

$$\mathbf{M}_e = \frac{A_p \rho_p l_e}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{4}$$

where A_p is the cross-sectional area of the pile, E_p is the modulus of elasticity of the pile, ρ_p is the density of the pile and l_e is the length of the element.

The dynamic stiffness matrix of the pile is given by:

$$\bar{\mathbf{K}}^{(i)} = \mathbf{K}^{(i)} - \omega^2 \mathbf{M}^{(i)} \tag{5}$$

where $\mathbf{K}^{(i)}$ and $\mathbf{M}^{(i)}$ are the global matrices of stiffness and mass of the pile *i* assembled from the elementary matrices $\mathbf{K_e}$ and $\mathbf{M_e}$ and ω is the excitation frequency.

The equation of motion for pile *i* is given by:

$$\bar{\mathbf{K}}^{(i)} w^{(i)} = \mathbf{f}^{(i)} - \mathbf{f}_s^{(i)} \tag{6}$$

where $w^{(i)}$ is the vector of nodal displacements of the pile *i*, $\mathbf{f}^{(i)}$ is the vector of external forces applied at the nodes and $\mathbf{f}_s^{(i)}$ is the vector of nodal forces equivalent to the stresses $\mathbf{t}_s$ applied by the soil along the pile *i*. It is assumed that the soil reaction $\mathbf{t}_s$ is constant along each element.

3.3 Soil-piles coupling IBEM-FEM

Since the nodes of the pile model do not coincide with the nodes of the soil model, it is necessary to use a transformation that relates the displacements and stresses at the nodes of both. The displacements $\mathbf{u}^{(i)}$ and the equivalent nodal loads $\mathbf{f}_s^{(i)}$ can be written as:

$$\mathbf{u}^{(i)} = \mathbf{D}^{(i)}\mathbf{w}^{(i)} \tag{7}$$

$$\mathbf{f}_s^{(i)} = \mathbf{A}^{(i)}\mathbf{t}^{(i)} \tag{8}$$

where $\mathbf{D}$ and $\mathbf{A}$ are transformation matrices.

By imposing conditions of equilibrium and kinematic compatibility at the soil-pile interface, we obtain:

$$\begin{bmatrix} \bar{\mathbf{K}}^{(1)} & \mathbf{0} & \mathbf{A}^{(1)}\mathbf{T}_s^{(11)} & \mathbf{A}^{(1)}\mathbf{T}_s^{(12)} \\ \mathbf{0} & \bar{\mathbf{K}}^{(2)} & \mathbf{A}^{(2)}\mathbf{T}_s^{(21)} & \mathbf{A}^{(2)}\mathbf{T}_s^{(22)} \\ -\mathbf{D}^{(1)} & \mathbf{0} & \mathbf{U}_s^{(11)} & \mathbf{U}_s^{(12)} \\ \mathbf{0} & -\mathbf{D}^{(2)} & \mathbf{U}_s^{(21)} & \mathbf{U}_s^{(22)} \end{bmatrix} \begin{Bmatrix} \mathbf{w}^{(1)} \\ \mathbf{w}^{(2)} \\ \mathbf{q}_s^{(1)} \\ \mathbf{q}_s^{(2)} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}^{(1)} \\ \mathbf{f}^{(2)} \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \tag{9}$$

The solution of the system of equations provides the displacements at the nodes of the pile elements and the applied fictitious loads. With the fictitious loads one can determine the displacements and stresses at any point in the half-space.

3.4 Extended half-space method

A method widely used in the elastic analysis of piles, both in static and dynamic analysis, is the extended half-space method. In this method, the real pile is replaced by a fictitious pile with the same dimensions as the real one; but with its modulus of elasticity and density reduced by:

$$E_p' = E_p - E_s \tag{10}$$

$$\rho_p' = \rho_p - \rho_s \tag{11}$$

where E_s is the modulus of elasticity of the soil in the vertical direction and and ρ_s is the density of the soil. This replacement has the purpose of subtracting from the system the stiffness and mass of the soil excavated for the installation of the pile. Then, the stresses are applied directly in the half space, without the need to consider the space occupied by the pile.

The displacement of the nodes along the pile-soil interface are given by

$$\begin{Bmatrix} \mathbf{u}_s^{(1)} \\ \mathbf{u}_s^{(2)} \end{Bmatrix} = \begin{bmatrix} \mathbf{U}_s^{(11)} & \mathbf{U}_s^{(12)} \\ \mathbf{U}_s^{(21)} & \mathbf{U}_s^{(22)} \end{bmatrix} \begin{Bmatrix} \mathbf{t}_s'^{(1)} \\ \mathbf{t}_s'^{(2)} \end{Bmatrix} \tag{12}$$

Hence, the global system of equations is

$$\begin{bmatrix} \bar{\mathbf{K}}'^{(1)} & \mathbf{0} & \mathbf{A}^{(1)} & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{K}}'^{(2)} & \mathbf{0} & \mathbf{A}^{(2)} \\ -\mathbf{D}^{(1)} & \mathbf{0} & \mathbf{U}_s^{(11)} & \mathbf{U}_s^{(12)} \\ \mathbf{0} & -\mathbf{D}^{(2)} & \mathbf{U}_s^{(21)} & \mathbf{U}_s^{(22)} \end{bmatrix} \begin{Bmatrix} \mathbf{w}^{(1)} \\ \mathbf{w}^{(2)} \\ \mathbf{t}_s'^{(1)} \\ \mathbf{t}_s'^{(2)} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}^{(1)} \\ \mathbf{f}^{(2)} \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \tag{13}$$

Note that it is no longer necessary to determine the stress influence matrices $\mathbf{T}_s$. It is also important to note that the first two equations of the system are uncoupled. This means that the cross influence of stress is not considered in the extended half-space approximation.

In order to also consider stresses, the equations of motion can be modified by adding the stress cross term, resulting in:

$$\begin{bmatrix} \bar{\mathbf{K}}'^{(1)} & \mathbf{0} & \mathbf{A}^{(1)} & \mathbf{A}^{(1)}\mathbf{T}_s^{(12)} \\ \mathbf{0} & \bar{\mathbf{K}}'^{(2)} & \mathbf{A}^{(2)}\mathbf{T}_s^{(21)} & \mathbf{A}^{(2)} \\ -\mathbf{D}^{(1)} & \mathbf{0} & \mathbf{U}_s^{(11)} & \mathbf{U}_s^{(12)} \\ \mathbf{0} & -\mathbf{D}^{(2)} & \mathbf{U}_s^{(21)} & \mathbf{U}_s^{(22)} \end{bmatrix} \begin{Bmatrix} \mathbf{w}^{(1)} \\ \mathbf{w}^{(2)} \\ \mathbf{t}_s'^{(1)} \\ \mathbf{t}_s'^{(2)} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}^{(1)} \\ \mathbf{f}^{(2)} \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \tag{14}$$

In this complete formulation, it is necessary to compute the stress influence functions.

4. NUMERICAL RESULTS

4.1 Isolated pile

In order to verify the implementations made, the response of an isolated pile to a vertical load F_0 applied on its top is initially examined. The loading is harmonic with frequency ω. The normalized frequency a_0 of excitation is defined by:

$$a_0 = a\omega\sqrt{\frac{\rho_s}{c_{44}}} \tag{15}$$

In this section, we consider a pile with $L/d = 10$, $E_p/E_s = 1000$, installed in a homogeneous, isotropic half-space, with Poison's ratio $\nu = 0.4$. The results are compared with those obtained with Kaynia and Kausel's method (Figure 2), which uses the extended half-space method. Note that in this case in which only one pile is considered, the methods show good agreement.

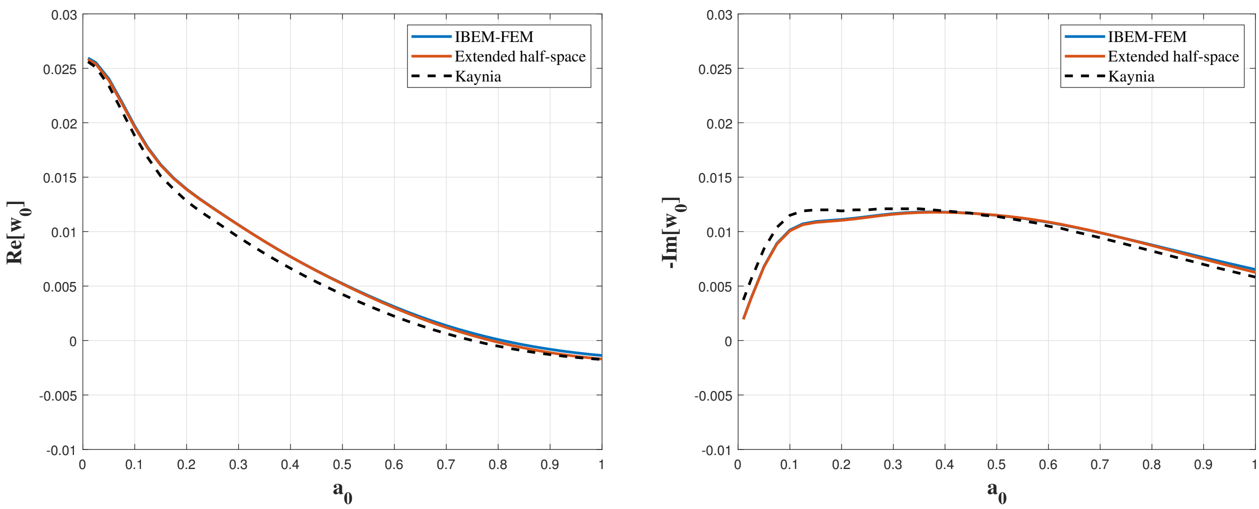

Figure 2. Response of an isolated pile, $s/d = 3$, $L/d = 10$, $E_p/E_s = 1000$.

4.2 Interaction factor α

In the case of two piles that are close together, the mutual influence between them can be expressed by the interaction factor α defined by Poulos (1968) as:

$$\alpha = \frac{w_0 - w_0'}{w_0'} \tag{16}$$

where w_0 is the displacement of the top of each of them when both are subjected to the same vertical load $F_0^{(1)} = F_0^{(2)}$ and w_0' is the vertical displacement of an isolated pile under the same loading.

Figures 3 to 6 show results with the variation of the Interaction factor α with the frequency a_0, obtained with the formulations developed in this work (IBEM+FEM), with and without the cross-stress components $\mathbf{T}_s$ compared to the traditional formulation of the extended half-space method and to a modification of the extended half-space method when the cross components of stress are added.

Figure 3 shows the Interaction factor α between two piles with $s/d = 3$, $L/d = 10$, $E_p/E_s = 1000$. There is agreement between the IBEM-FEM and extended half-space methods plus stress cross components and between the extended half-space and IBEM-FEM methods without stress cross components, mainly for low frequencies of excitation. As the frequency increases, so does the difference between the results. This is a reflection of the limitations of the extended half-space method.

In Figure 4, the stiffness of the piles was modified through the modulus of elasticity ($E_p/E_s = 10$). In Figure 5, the distance between the piles was modified ($s/d = 6$). In Figure 6, the length of the piles was modified ($L/d = 20$). The agreement between all curves is observed up to approximately $a_0 = 0.2$ for all cases.

5. CONCLUSIONS

In this paper, a formulation was presented that allows analyzing the dynamic behavior of piles considering the interaction with the soil. The soil was modeled with the indirect boundary element method (IBEM) and the piles with the

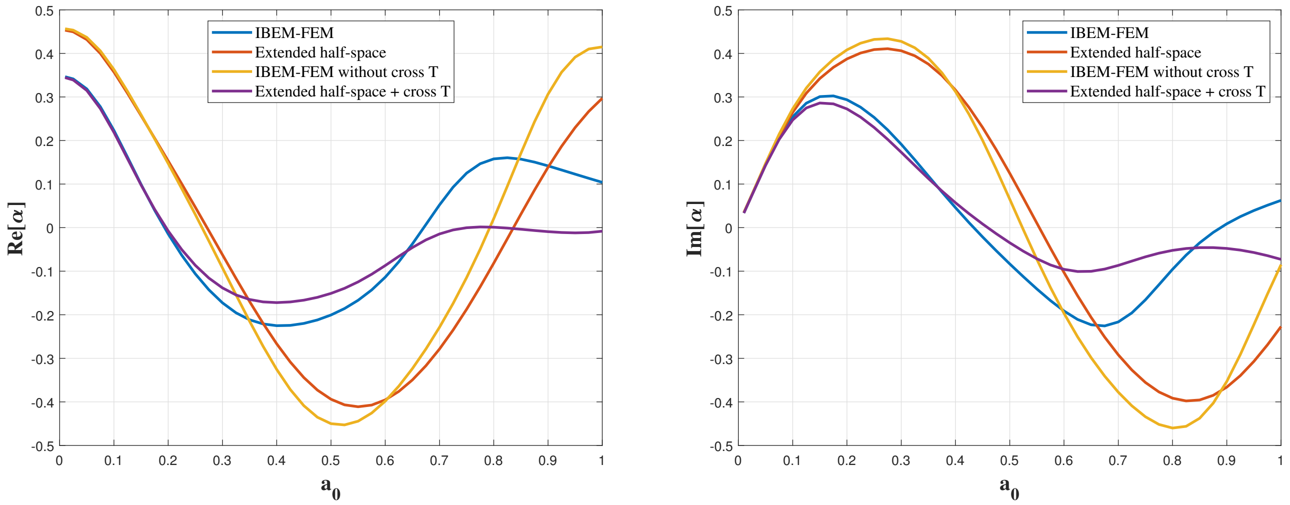

Figure 3. Interaction factor, $s/d = 3$, $L/d = 10$, $E_p/E_s = 1000$.

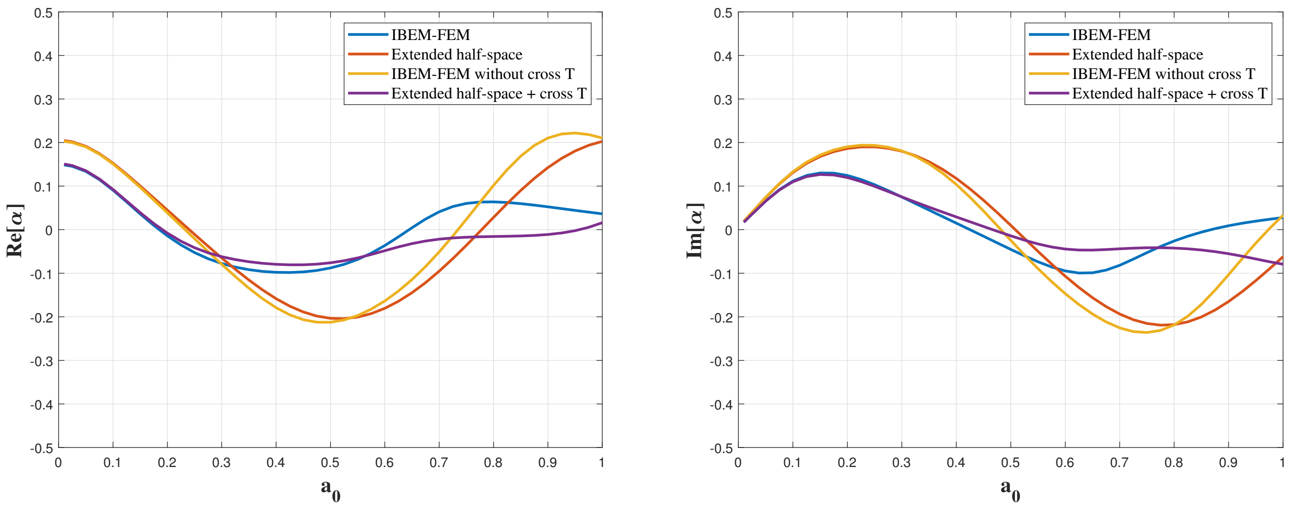

Figure 4. Interaction factor, $s/d = 3$, $L/d = 10$, $E_p/E_s = 10$.

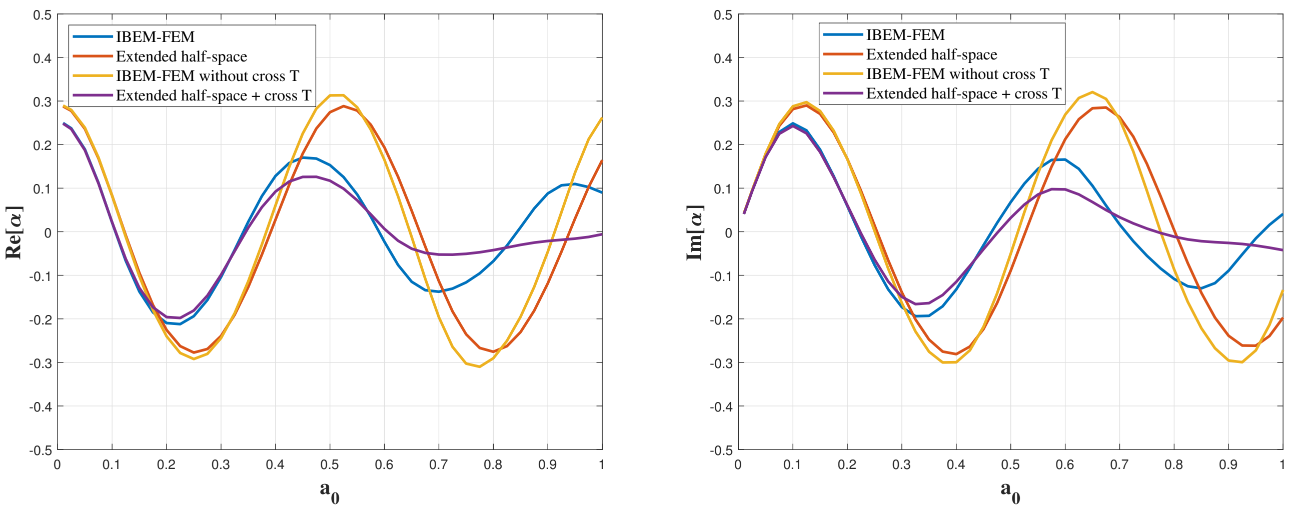

Figure 5. Interaction factor, $s/d = 6$, $L/d = 10$, $E_p/E_s = 1000$.

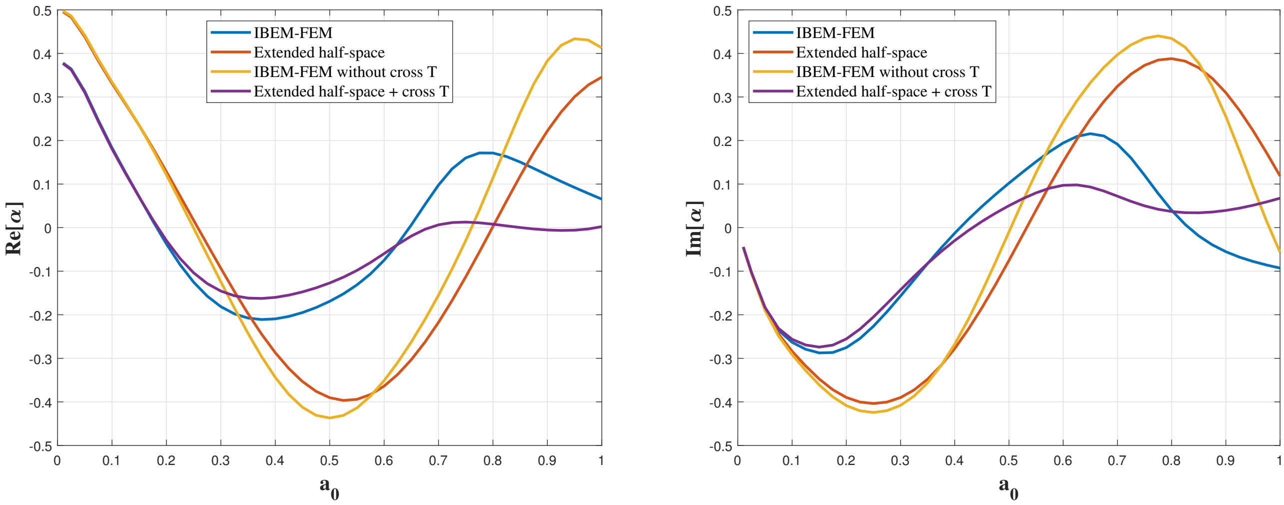

Figure 6. Interaction factor, $s/d = 3$, $L/d = 20$, $E_p/E_s = 1000$.

finite element method (FEM). A comparison was made with a method that is widely used to model pile group interaction problems, which is the extended half-space method. The main difference between the methods is that the latter does not consider the cross stress components of the pile equilibrium equations. The paper shows a comparison between the formulation developed in this work (IBEM+FEM), with and without the cross-stress components, with the traditional formulation of the extended half-space method as well as with a modification of it in which the cross-stress components were incorporated. The results show that although the extended half-space method is efficient to represent the behavior of an isolated pile, it may in some cases not be suitable to be used in the case of two or more piles, as it neglects the stresses caused in the soil by adjacent piles.

6. ACKNOWLEDGEMENTS

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