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RELIABILITY INVESTIGATION FOR lOAD SHARING MODELS APPLIED TO REFRIGERATION SYSTEM

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Abstract. *A method to increase the reliability of engineering systems is the use of redundant components. In many of these systems, the system load is shared by these components when operating simultaneously. Therefore, when a component fails, the system continues to function, but the load on the remaining components becomes greater, which leads to an increase in the failure rate of these components. To model the reliability of these systems and, thus, estimate their failure probability, a model based on a stochastic process may be used. Stochastic processes can be defined as a sequence of random variables indexed to time and events. A particular case of a stochastic process is the Markov Chain which the main property is the absence of memory. This property establishes that the behavior of future probabilities is determined only by the current state, widely applied to engineering and reliability problems. Aiming to study models of load-sharing and reliability in redundant mechanical components, a numerical model based on Markov Chains was implemented to describe the reliability of components and systems using two different load-sharing models to calculate failure rates. An industrial application in refrigeration systems with compressors working in redundancy was analyzed. From the results obtained via simulation, it is noted that the studied models are efficient in describing the load-sharing behavior in refrigeration systems with more than one compressor operating in redundancy, estimating load-sharing parameters and calculating system reliability.*
Keywords: *load-sharing, reliability, redundancy, Markov chain*

1. INTRODUCTION

Nowadays, reliability has become indispensable for the market due to competition and consumer demands for better products at a lower cost. Therefore, projects have to consider factors that may harm the final product outcome. In some cases, redundant systems are necessary, as even if component failures occur, the system continues to function. A particular form of redundancy is the K-out-of-N structure, in which at least K out of N components must be functioning for regular system operation. In several cases, the independence of component failures is assumed in reliability calculations (Asadi and Bayramoglu, 2006) and (Boddu and Xing, 2013), meaning that the failure of one component does not affect the failure rate of the surviving components.

However, in reality, when a component fails, the load is transferred. As a consequence, there is an increase in the load on the surviving components. In this sense, various studies have been conducted over time. Daniels et al. (Daniels, 1945) described how fiber stress increases through breaks in others within the bundle, using the equal load-sharing rule. A different load-sharing model was proposed by Bhattacharyya et al. (Bhattacharyya and Soejoeti, 1989): the Tampered Failure Rate (TFR). This statistical model is suitable for accelerated life testing by stress, since it considers that a change in stress has a multiplicative effect on the failure rate function over the remaining component's life. Amari et al. (Amari *et al.*, 2006) transformed general failure distribution on a TFR model into an equivalent model with exponential failure distribution. Then, they provided a close-form analytical calculation for the system reliability. Thus, existing results for load-sharing systems with exponential distributions can be used to analyze the TFR model of a system with general failure distributions. Another failure dependency model was proposed by de Paula et al. (de Paula *et al.*, 2019). Their model considers a linearity parameter and a proportionality parameter. This study explored repairable systems to maximize reliability.

Reliability studies involve stochastic events, meaning it operates in the field of probability. Because of that, mathematical models for reliability evaluation may be applied to random events that evolve with time, i.e. stochastic process. Some of the basic processes used in reliability are renewal process, alternating renewal process, Markov processes, semi-

Markov process, and semi-regenerative process (Birolini, 2017). In this work, it was assumed that a future state of the system depends only on the present state and not on other previous states, so The Markov process was used, since it has this memorylessness characteristic. This method is widely used in modeling the reliability of complex systems and can be used in repairable systems, in availability analysis.

2. METHODOLOGY

Regarding the relevance of a proper reliability assessment to guarantee engineering systems operation, different load-sharing models were proposed, as described in the Introduction section. In addition, it is important to consider the probabilities dependence on time and events. Therefore, this section presents how to evaluate reliability as a stochastic process, presents two different load-sharing models and then makes a comparison between them. This theory will be the basis for the case study presented in Section 3.

2.1 RELIABILITY ANALYSIS THROUGH STOCHASTIC PROCESS

Stochastic processes can be considered as a family of random variables dependent on time, or as a random function of time. As such, their theoretical basis is based on probability theory. Stochastic processes can be continuous, with $T = t : 0 \leq t \leq \infty$, or they can be discrete if observed at regular intervals $T = t : 1, 2, 3....$ The set of possible states, the state space, is considered a subset of the set of real numbers. Thus, the state of the system at a given time t_0 is a random variable $X(t_0)$. Consequently, a stochastic process is a collection of random variables $X(t)$, indexed by a set of time parameters (T). Thus, the stochastic process is given by:

$$X(t), t \in T. \tag{1}$$

A special type of stochastic process is the Markov chain, which is characterized by its main property of memorylessness. This property establishes that the behavior of future probabilities is determined solely by the present state, such that knowledge of the future is independent of the past, as shown in Equation 2.

$$P(X_n + 1) = P\left\{X_{n+1} = x \mid X_0, X_1, ..., X_n\right\} = P\left\{X_{n+1} = x \mid X_n\right\} \qquad \forall_{x \in E}. \tag{2}$$

This work will use the principles of Markov theory according to (Ramakumar, 1993). The Markov model is defined in terms of the set of transition probabilities p_{ij}, where i is the current state and j is the next state. Chain modeling requires determining the transition probabilities between the states. Thus, for a system with n states, the probabilities can be organized in the form of a matrix:

$$P_{ij} = \left[p_{ij}\right], \tag{3}$$

where P_{ij} is the transition matrix, which is a square matrix and can be written as:

$$P_{ij} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{2n} \end{bmatrix}, \tag{4}$$

where:

$$\sum_{j=1}^{n} p_{ij} = 1. \tag{5}$$

It may be necessary to know the transition probability between the two states. For this case, the transition matrix, $P_{ij} = [p_{ij}]$, is defined as the transition probability between states in n phases $p_{ij}^{(n)}$.

$$p_{ij}^{(n)} = P\left\{X(t+n) = j \mid X(t) = i\right\} = P\left\{X(n) = j \mid X(0) = i\right\} \qquad \forall_{t=0,1,2...}\ \forall_{i,j \in E}. \tag{6}$$

Therefore, the transition matrix for n time steps is given by powers of the one-step transition matrix.

$$P_{ij}^{(n)} = [p_{ij}^{(n)}] = (P_{ij})^n. \tag{7}$$

The possible combinations of operational components in a system is the first step when applying Markov theory in reliability. Index 1 is assigned for functioning components and 0 for failed components. The sets of combinations will provide the origin to the state space of the process S. The number of possible state combinations is given by:

$$N_e = 2^n, \tag{8}$$

where n is the number of components. The information combination of the system components number n and the number of states N_e generate the state space matrix, shown in Table 1.

Table 1. Markov process state matrix

State {S} → **Components number (n) ↓**	**1**	**2**	**3**	**4**	⋯	**n+1**	**n+2**	⋯	$\mathbf{2^n}$
1	1	0	1	1	⋯	1	0	⋯	0
2	1	1	0	1	⋯	1	0	⋯	0
3	1	1	1	0	⋯	1	1	⋯	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
n	1	1	1	1	⋯	0	1	⋯	0

Table 1 presents the combination vectors of operational or failed condition of each component. The first state (S_1) represents all components functioning, and the last state (S_{2^n}) represents all components failed. Between states (S_1) and (S_{2^n}), there are combinations of components failures, where failures follow the number of components in the system $(1, 2, 3, \cdots, 2^{n-1})$. One approach to verify the communication between states is to use a fault and repair diagram. An example of a two-components system transition diagram is shown in Figure 1. The numbers inside the circle indicate the system states, and the arrows represent communication between states. Furthermore, the direction of the arrow identifies whether the transition between states is occurring due to failure (λ) or repair (μ), of a given component, identified by the index (for example, λ_1 indicated the failure of component 1).

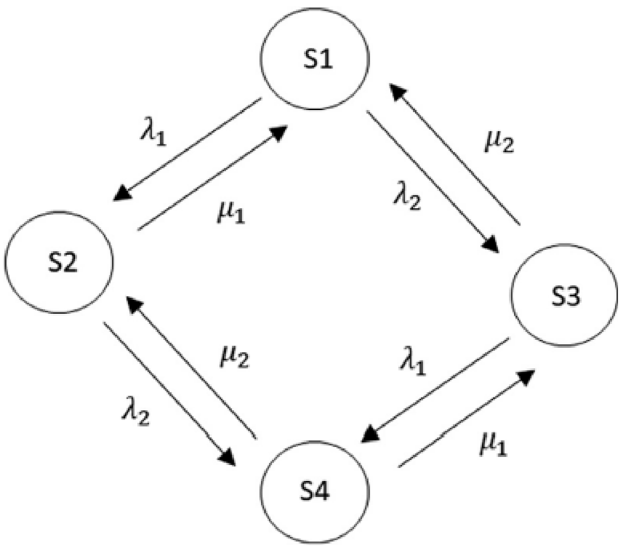

Figure 1. Transition state diagram for two components.

Let $\{X(t), t \geq 0\}$ be a stochastic process, where $X(t)$ is a random variable that describes the system state after a certain time t, and the states belong to the state set S. There are some definitions:

$$\rho_{ij} = state\ change\ rate\ from\ S_i\ to\ S_j \qquad i,\ j\ \in\ S, \tag{9}$$

$$P_{ij} = \rho_{ij}(t), \tag{10}$$

$$P_i(t) = P\left\{X(t) = i \mid X(0) = S_1\right\}, \tag{11}$$

where P_{ij} is the transition probability from state S_i to state S_j in a time interval t and $P_i(t)$ is the probability of the system is in state S_i at time t. The probability of each state is obtained by solving the following differential equation:

$$\begin{bmatrix} p_1'(t) \\ p_2'(t) \\ \vdots \\ p_n'(t) \end{bmatrix} = \begin{bmatrix} -\sum_{j=2}^{n} \rho_{1j} & \rho_{21} & \cdots & \rho_{n1} \\ \rho_{12} & -\sum_{\substack{j=1 \\ j\neq 2}}^{n} \rho_{2j} & \cdots & \rho_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1n} & \rho_{2n} & \cdots & -\sum_{j=1}^{n-1} \rho_{nj} \end{bmatrix} \begin{bmatrix} p_1(t) \\ p_2(t) \\ \vdots \\ p_n(t) \end{bmatrix}. \tag{12}$$

Equation 12 can be written as:

$$P_i'(t) = Q.P(t). \tag{13}$$

Matrix Q, also known as the differential equations system coefficient matrix or the generator matrix, is obtained based on the failure and repair rates of the system under analysis. It is considered that time interval between two states is so small that only one failure or repair can occur in the transition.

The reliability or availability (when repair is considered) values are then determined from $P_i(t)$ values using Equations 14 and 15.

$$R(t) = \sum_{i \in S_o} P_i(t), \tag{14}$$

$$A(t) = \sum_{i \in S_o} P_i(t). \tag{15}$$

2.2 LOAD-SHARING MODELS

In reliability engineering, a common practice is to use redundancy techniques to increase the system's reliability. In systems that use redundant components, it is assumed that the failure of one component affects the failure properties of the surviving components. Thus, when a component fails, the workload is shared among the remaining components, resulting in an increase in the workload on each surviving component.

The model known as TFR (Tampered Failure Rate), proposed by (Bhattacharyya and Soejoeti, 1989), expresses that a component's failure rate depends solely on the current applied load and the component's age. We can say that the failure rate is independent of the component's applied load history. According to Amari et al. (Amari *et al.*, 2006), the acceleration of failure when stress is increased from a lower level to a higher level is reflected in the failure rate function.

Given a component subject to a sequence of loads, with load $\tilde{z}_i(i = 1, 2, ..., n - k)$ applied over a time interval $t_1 + t_{i+1}$, where $t_0 = 0$, the failure rate is:

$$\lambda(t) = \delta(\tilde{z})\lambda_0(t), \tag{16}$$

where $\tilde{z}$ is the load at time t, δ is the tampering factor, and λ_0 is the failure rate of the lowest load $\tilde{z}_0$. The tampering factor follows a power law:

$$\delta(\tilde{z}) = \tilde{z}^{\theta}. \tag{17}$$

Amari et al. (Amari *et al.*, 2006) also states that the equal load distribution, which is widely used, is appropriate when all components are of the same type. Additionally, it is reasonable to assume that the loads are independent and identically distributed (i.i.d), meaning that the random variables have the same probability distribution as the others and are all mutually independent. Considering these assumptions it is possible to properly relate the initial load of each component, $\tilde{z}_0$, and the load of each surviving component, $\tilde{z}_i$:

$$\tilde{z}_0 = \frac{L}{n}; \qquad \tilde{z}_i = \frac{L}{n - i_f} = \tilde{z}_0 \left(\frac{n}{n - i_f} \right), \tag{18}$$

where L is the total load applied to the system, n is the number of components, and i_f is the number of failed components. When the system load does not vary during its lifetime, the parameter $\tilde{z}_0 = 1$. Hence:

$$\lambda_i(t) = \left(\frac{n}{n - i} \right)^{\theta} \lambda_0. \tag{19}$$

The load-sharing model presented by De Paula et al. (de Paula *et al.*, 2019) is defined by the parameters γ and α, where the former describes the linear or non-linear dependence of failure with the number of failed components, and the latter represents the proportionality of this relationship:

$$\lambda_i(t) = \lambda_0[1 + \alpha(i_f)^{\gamma}], \tag{20}$$

where i_f is the number of failed components. Therefore, Equation 20 may be rewritten as:

$$\lambda_i(t) = \lambda_0 + \alpha(i_f)^{\gamma}\lambda_0, \tag{21}$$

2.2.1 load-sharing MODEL COMPARISON

Considering that the load on the system does not change during operation, the considered TFR equation is Equation 19. The failure rate λ_i is proportional to the initial failure rate. This proportionality may be non-linear when the load-sharing factor θ is different from 1. For the De Paula model, equation 21, λ_i is always a function of λ_0 plus a proportional factor of λ_0.

In order to compare both TFR and De Paula models results, two simulations were used. Table 2 shows the De Paula model parameters in each simulation. Table 3 shows the component failure rate λ_i when i system components are failed, considering a system with 10 components.

Table 2. Simulation parameters for De Paula model

Parameters	**Simulation 1**	**Simulation 2**
α	1	0.5
γ	1	1.5
λ_0	0.15	0.15

Table 3. Failure rate values calculated by De Paula model

Number of failed components i_f	**λ_i - Simulation 1**	**{λ_i - Simulation 2**
0	0.15	0.15
1	0.30	0.23
2	0.45	0.36
3	0.60	0.54
4	0.75	0.75
5	0.90	0.99
6	1.05	1.25
7	1.20	1.54
8	1.35	1.85
9	1.50	2.18

To match the failure rate of the TFR model to the De Paula model, the parameter θ was changed for each failure condition i_f. This is because the difference between the models. In TFR model (Equation 19) the failure rate λ_i is proportional to the initial failure rate. This proportionality can be non-linear when the load-sharing factor θ is different from 1. On the other hand, in the De Paula model (Equation 21) λ_i is always a function of λ_0 plus a proportional factor of λ_0.

Figures 2 and 3 show the convergence between the models for both simulations.

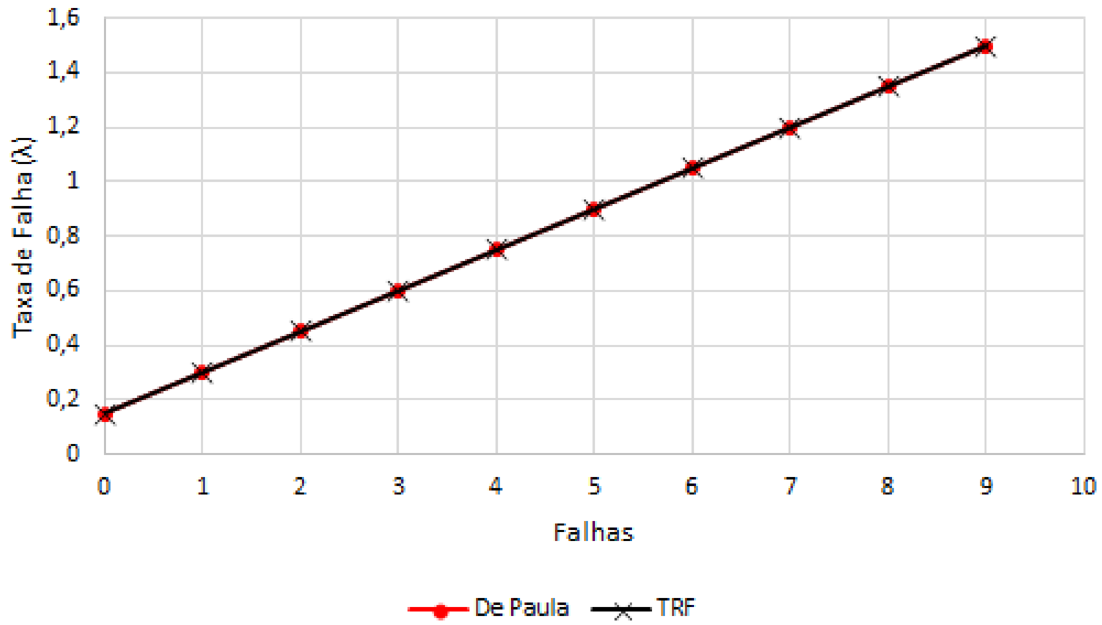

Figure 2. Comparison between models for Simulation 1

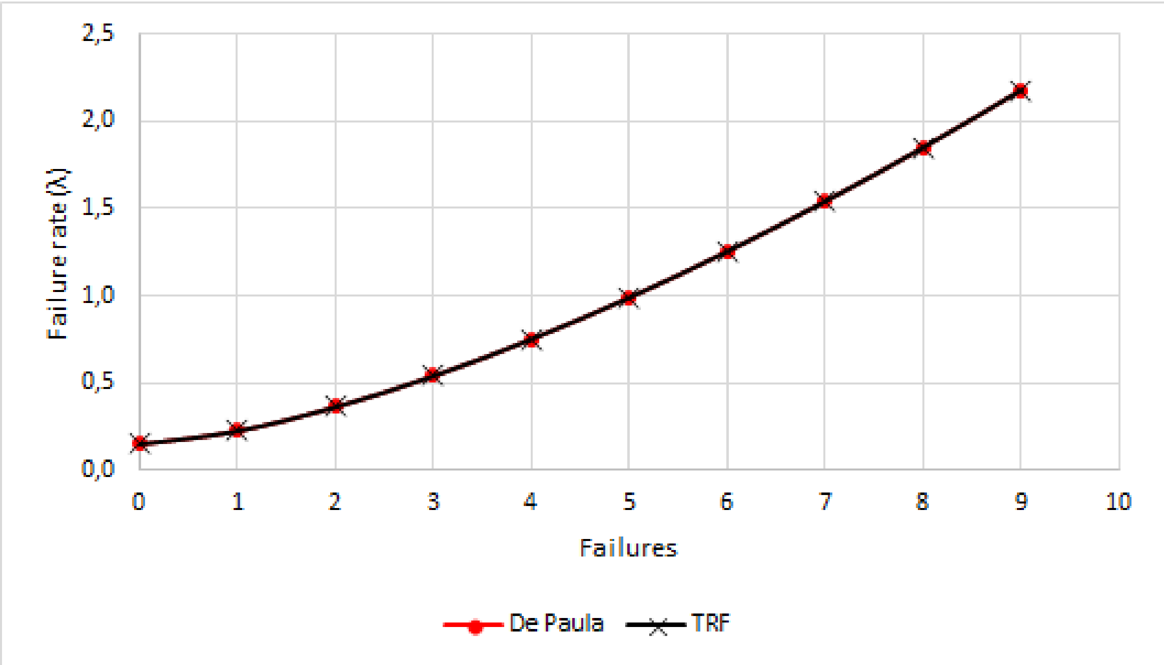

Figure 3. Comparison between models for Simulation 2

3. INDUSTRIAL APPLICATION

A system which usually operates with load-sharing redundant components is the refrigeration system. It basically consists of compressor, condenser, capillary tube or expansion valve and evaporator. A tactic aimed at enhancing the reliability of refrigeration systems involves the integration of redundant compressors. These compressors function collaboratively, effectively distributing the workload, thereby promoting smoother operation and mitigating the potential for failures. An example of a refrigeration system with 3 parallel compressors is shown in Figure 5.

The study explores a system featuring three compressors operating in parallel, where the successful operation necessitates the functioning of at least one compressor. The initial compressor failure rate conforms to a Weibull distribution, deduced from warranty return data, characterized by a shape parameter of $\beta = 1.2$ and a scale parameter of $\alpha = 3.98 \times 10^7$. It's worth noting that the shape and scale parameters correspond to specific aspects of the Weibull distribution.

In the context of the case study, the assessment focused on varying the values of the TFR parameter (1, 1.5 and

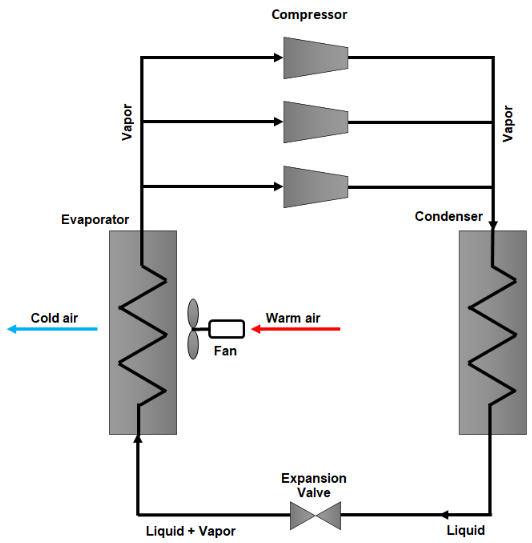

Figure 4. Escheme of a refrigeration system with parallel load-sharing compressors

2), denoted as θ, with a unitary initial state, represented as $\tilde{z}_0$. The choice of this model for the case study stemmed from its single parameter that was subject to investigation. This particular model configuration facilitates a focused and comprehensive analysis of the system's behavior under different TFR parameter values.

The examination of load-sharing factor variation, denoted by θ, exerts a substantial influence on the ultimate reliability outcome of the system, as graphically demonstrated in Figure 5. Importantly, with the escalation of this load-sharing factor θ, the system's reliability experiences a notable reduction. This effect becomes particularly conspicuous when contrasting the system's reliability performance in the presence of load-sharing against its performance without such an influence.

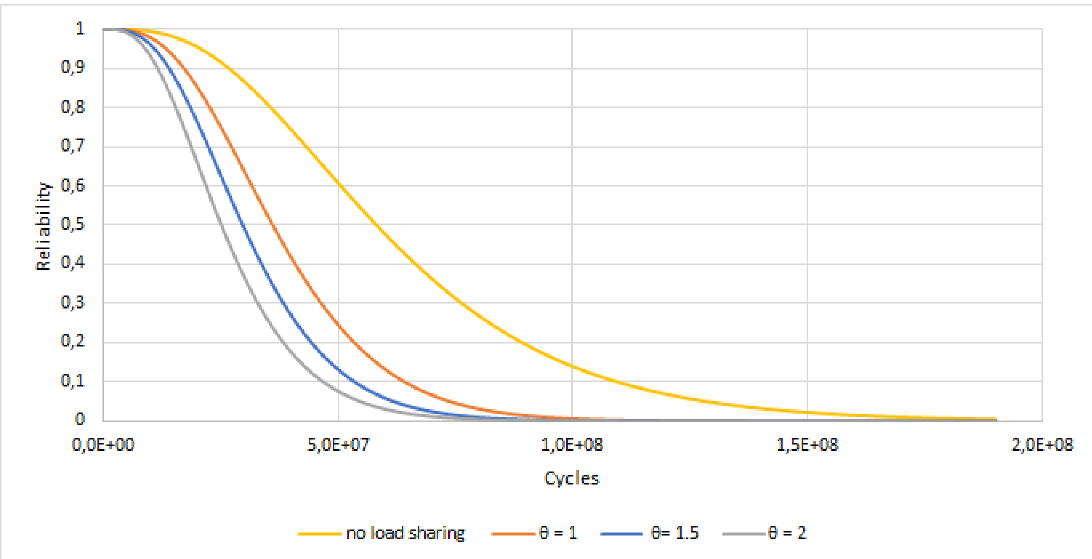

Figure 5. load-sharing analysis on a refrigeration system

Incorporating the load-sharing factor θ into the reliability outcome of a redundant system holds significant importance and realism. As demonstrated, the system responds to the impact caused by the failure of a component. The influence of an increased load-sharing factor prompts a reduction in the system's reliability. For instance, considering 90% realiability, the number of cycles required for systems with θ values of 1, 1.5, and 2 amounts to 1.73E+07, 1.41E+07, and 1.17E+07 cycles, respectively. In contrast, a system devoid of the load-sharing factor demands 2.69E+07 cycles for the same reliability.

In this context, the disparity between systems with and without the load-sharing factor can lead to a variance of up to 130% in the reliability value. This discrepancy underscores the substantial impact that the presence of the load-sharing factor can exert on system reliability, serving as a compelling reminder of the need for meticulous consideration in practical redundancy scenarios.

4. CONCLUSION

This work presented an evaluation of load-sharing models based on a stochastic process. For this, a computer simulation was implemented to calculate the reliability through the Markov chain.

The load-sharing models studied were TFR and De Paula. Both models represent the load-sharing factor for redundant systems and can be used to represent the failure rate behavior of the system when the item fails.

The case study presented in this work represents an application in the refrigeration area, in which the initial compressor failure rate data were based on part data with field return. The model chosen to describe the load-sharing was the TFR model applied to a system with three compressors. The results obtained in the simulation show the effect of the load-sharing factor on the reliability result.

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