COBEM 2023 27TH INTERNATIONAL CONGRESS OF MECHANICAL ENGENEERING
ABCM Brazilian Society of Mechanical Sciences and Engineering
27th ABCM International Congress of Mechanical Engineering
December 04-08, 2023, Florianópolis-SC, Brazil

COB-2023-1094
A MATHEMATICAL MODEL FOR POROUS PARALLEL FINS WITH CONTACT RESISTANCE AT THEIR BASE

Vinícius Vendas Sarmento
Maria Laura Martins-Costa
Mechanical Engineering Department, Universidade Federal Fluminense, Rua Passo da Pátria, 156, 24210-320, Niterói, RJ, Brazil
viniciusvendas@id.uff.br maria_laura@id.uff.br

Rogério M. Saldanha da Gama
Mechanical Engineering Department, Universidade do Estado do Rio de Janeiro, Rua São Francisco Xavier, 524, 20550-013, Rio de Janeiro, RJ, Brazil
rsggama@gmail.com

Abstract. *This research aims to find the impact of convection and radiation on the temperature distribution between parallel porous fins in a large array, accounting for contact thermal resistance at their base and heat exchange at the fins' tip. The methodology consists in using a dimensionless form associated with the steady-state heat transfer problem to propose a new yet simple method that transforms the highly non-linear Integro-Differential problem into a linear problem that can be used with a range of methods, including the Finite Differences Method as well as the Finite Volumes Method, for example. The linear problem then becomes a sequence of elements solved through a Gauss-Seidel scheme along the fin's length that must be iteratively computed until the problem converges according to a chosen convergence criteria. However, depending on the selected parameters, an improperly chosen convergence criteria could wrongly stop the computation before the sequence of elements is truly convergent. To avoid this trouble, instead of analyzing the latest two iterations (the most commonly used criterion), sufficiently distant iterations of the sequence are analyzed.*

Keywords: *Nonlinear Heat Transfer, Parallel Porous Fins, Contact Thermal Resistance at Fin's Base, Numerical Simulation.*

1. INTRODUCTION

Fins, especially porous fins, possess various applications in the industrial sector, aerospace, and electronics. One of these applications occurs in the chemical industry, where using fins to solidify materials as studied by Haq et al. (2019), in which the behavior of the heat transfer was analyzed during the solidification process of a mixture using V-shaped fins inside an enclosure, concluding that the increase of the fins' angles increased the necessary time for solidification. Similarly, the use of fins in a solidification process was also studied by Ganji et al. (2019), where it was verified in a duct that by employing V-shaped fins along nanoparticles, it was possible to minimize the time required for solidification and among the parameters that influenced the time, the main ones were the length of the fins as well as the fins' angles.

Park et al. (2016) studied the numerical optimization of a composite V-shaped fin to maximize the efficiency when compared to an X-shaped fin, concluding that the change of the profile from X to V significantly increased efficiency and reduced the total weight. Recent publications such as Sarmento and Martins-Costa (2018), Sarmento et al. (2020), as well as Sarmento et al. (2022), studied the heat transfer for porous fins without resistance at the fin's base through various means such as Chebyshev polynomials, variational methods and an iterative methodology respectively.

This paper focuses on parallel fins in a large array of fins interacting amongst themselves through radiation by applying a novel numerical methodology, whereas Sarmento and Martins-Costa (2018) focus on a single fin by using spectral collocation method as well as finite differences, while Sarmento et al. (2022) focus on a pair of porous fins interacting only between themselves and Sarmento et al. (2020) focus on finding an exact solution through the minimization of convex functionals for the problem of a porous fin under conduction, convection as well as radiation.

Additionally, extensive bibliographic research was conducted and was unable to encounter similar problems in the Literature that allowed better comparisons, as the Literature generally focuses on a single fin, as can be seen in Ma et al. (2016), which employed the Spectral Collocation Method, Finite Volumes Method as well as the Homotopy Perturbation Method, the Spectral Collocation method was also explored by Shivanian et al. (2020), while in Gorla and Bakier (2011) an adaptation of the Runge-Kutta methodology was used. The behavior of the parameters representing the effects of convection, along with the parameter representing the effects of radiation, were similar between all the previously mentioned papers as well as in the currently proposed one.

2. METHODOLOGY

Figure 1 portrays the case studied in this work:

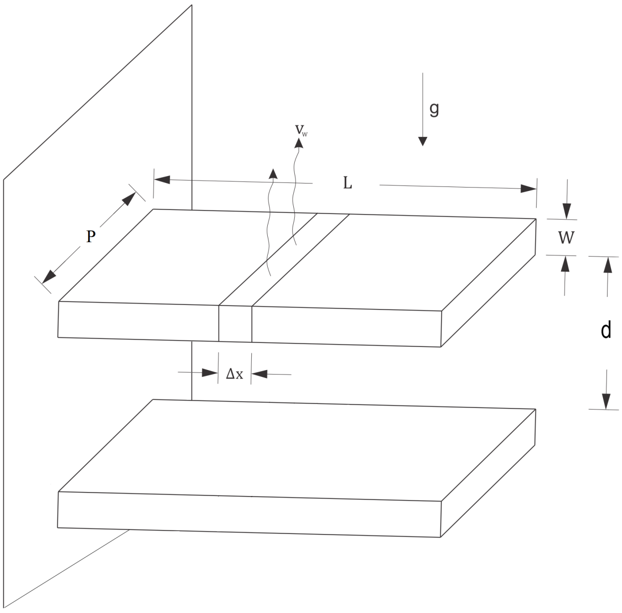


Figure 1. Illustration of the studied problem.

The methodology used for this research is the same as presented in Martins-Costa et al. (2022) and Sarmento et al. (2023), where the portrayed problem follows the given formulation:

$$\frac{d^2T}{dx^2} - f\left(T\right) + \Upsilon\left[T\right] + g\left(x\right) = 0\,, \tag{1}$$

Where T is the temperature, x is the coordinate along the fin's length, $f(T)$ is the heat transfer, $g(x)$ is the heat generation, and $\Upsilon[T]$ is the heat exchange between two parallel fins which involves all the temperatures along the fin's length. In the present work, both convection and radiation are considered, as seen in Sarmento et al. (2023) and Martins-Costa et al. (2022), which gives rise to the following:

$$\frac{d^2T}{dx^2} - \frac{2gK\beta\rho c_P}{\nu k_{eff} w}\left|T - T_F\right|\left(T - T_F\right) - \frac{\sigma\varepsilon}{k_{eff} w}\left[2\left|T^3\right|T - 2\int_0^L \left|T^3\right| T\, F_{12} d\xi\right] = 0\,, \tag{1}$$

Where K is the permeability of the solid matrix, g is the gravity, ρ is the specific mass, c_P is the specific heat at constant pressure, β is the fluid's expansion coefficient, ε is the emissivity, σ is the Stefan-Boltzmann constant, w is the thickness of the fin, ν is the dynamic viscosity and the effective conductivity k_{eff} is given by the following expression:

$$k_{eff} = \left(1-\phi\right)k_S + \phi k_F\,, \tag{3}$$

Where ϕ is the porosity, k_S is the solid's conduction coefficient, and k_F is the fluid's conduction coefficient.

The next step is to make the energy balance equation dimensionless by using the following equations:

$$\theta = \frac{T}{T_B}\,, \tag{4}$$

$$\theta_F = \frac{T_F}{T_B}\,, \tag{5}$$

$$S_h = \frac{D_a R_a^*}{k_R} c_P \left(\frac{L}{w} \right)^2 , \tag{6}$$

$$G = \frac{2\sigma\varepsilon}{k_{eff} w} L^2 \left(T_B \right)^3 , \tag{7}$$

$$R_a = \frac{g\beta}{k_F \nu} \left(T_B \right) w^3 , \tag{8}$$

$$D_a = \frac{K}{w^2} , \tag{9}$$

Where D_a is the Darcy number and R_a is the Rayleigh number, and that S_H represents a convective parameter, while G represents a radiation parameter.

By applying these dimensionless variables, the balance equation (2) will have the following form:

$$\frac{d^2\theta}{dX^2} - S_h \left| \theta - \theta_F \right| \left(\theta - \theta_F \right) - G \left[\left| \theta \right|^3 \theta - 2\int_0^1 \left| \theta(\xi) \right|^3 \theta(\xi) \; F_{12}(\xi) d\xi \right] , \tag{10}$$

The view factor F_{12} is given by:

$$F_{12}^*\left(X, \xi \right) = \frac{\left(\frac{d}{L} \right)^2}{2\left(\left(X - \xi \right)^2 + \left(\frac{d}{L} \right)^2 \right)^{3/2}} , \tag{11}$$

The numerical methodology used to solve the resulting balance equation follows the formulation:

$$\frac{d^2\varphi^{i+1}}{dx^2} - \alpha\varphi^{i+1} + \gamma = 0 \tag{12}$$

With:

$$\gamma = \alpha\varphi^i - f\left(\varphi^i \right) + g(x) \tag{13}$$

Applying the finite differences method:

$$\frac{\varphi_{j+1}^{i+1} - 2\varphi_j^{i+1} + \varphi_{j-1}^{i+1}}{\Delta X^2} - \alpha\varphi_j^{i+1} + \gamma_j^i = 0 \tag{14}$$

Applying the boundary conditions:

$$\frac{-2\varphi_2^{i+1} + 2}{\Delta X^2} - \alpha\varphi_N^{i+1} + \gamma_N^i = 0 \tag{15}$$

$$\frac{-2\varphi_N^{i+1} + 2\varphi_{N-1}^{i+1}}{\Delta X^2} - \alpha\varphi_N^{i+1} + \gamma_N^i = 0 \tag{16}$$

Rewriting the expressions:

$$\varphi_1^{i+1} = \frac{2+\Delta X^2 \gamma_1^i}{2+\alpha \Delta X^2} \tag{17}$$

$$\varphi_j^{i+1} = \frac{\varphi_{j+1}^{i+1}+\varphi_{j-1}^{i+1}+\Delta X^2 \gamma_j^i}{2+\alpha \Delta X^2} \tag{18}$$

$$\varphi_N^{i+1} = \frac{2\varphi_{j-1}^{i+1}+\Delta X^2 \gamma_N^i}{2+\alpha \Delta X^2} \tag{19}$$

For the studied case of parallel rectangular fins:

$$f\left(\varphi^i\right) = S_h \left|\varphi_j^i - \theta_F\right|\left(\varphi_j^i - \theta_F\right) + G\left[\left|\varphi_j^i\right|^3 \varphi_j^i - 2\int_0^1 \left|\varphi^i(\xi)\right|^3 \varphi^i(\xi)\; F_{12}\;(X,\xi)\,d\xi\right] \tag{20}$$

$$g(x) = 0 \tag{21}$$

$$\gamma_1^i = \alpha\varphi_1^i + Bi\left(1-\varphi_1^i\right) \tag{22}$$

$$\gamma_N^i = \alpha\varphi_N^i \tag{23}$$

The condition γ_1^i represents the thermal resistance at the fin's base, expressed through a heat transfer formulation similar to convection (in which *Bi* is the Biot number), whereas γ_N^i represents the insulated tip.

3. RESULTS

One of the significant results of this research is to evaluate how the distance between two parallel porous fins influences the temperatures. In order to do so, the graphic depicted in Fig. 2 was obtained without considering convection and considering $G = 10, \theta_F = 0.4, Bi \to \infty$:

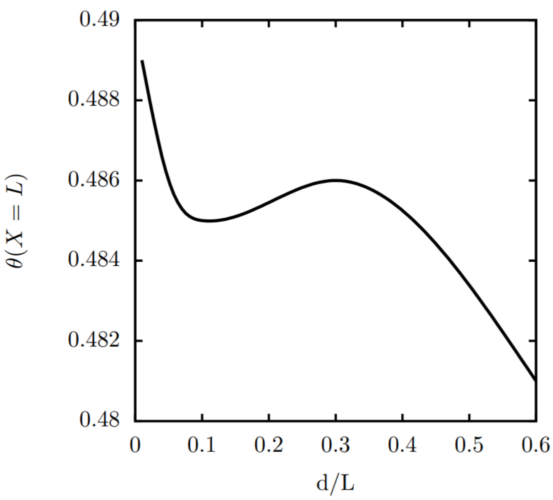
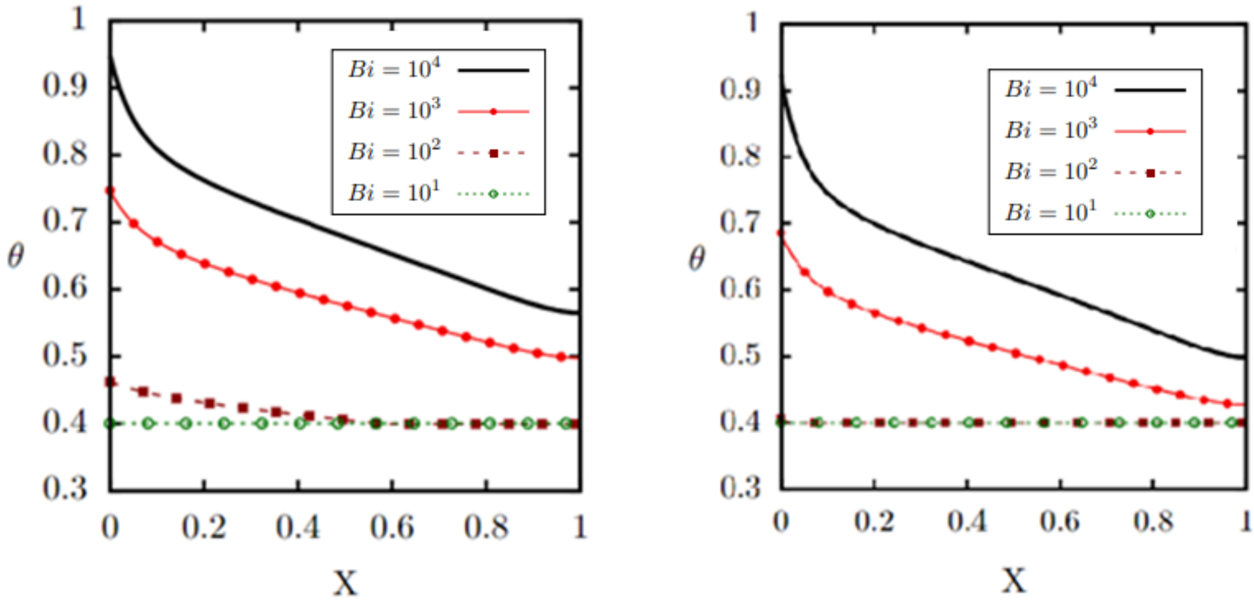


Figure 2. Temperature variation at the insulated tip due to distance-to-length ratio changes.

As shown in Figure 2, at minimal ratios of the distance between parallel fins to length, the porous fins will behave as a single thicker fin and slowly decrease the temperature the further away they are. Nevertheless, it will increase again due to the non-linear nature of heat exchange through radiation, which causes a local maximum around the ratio of 0.3.

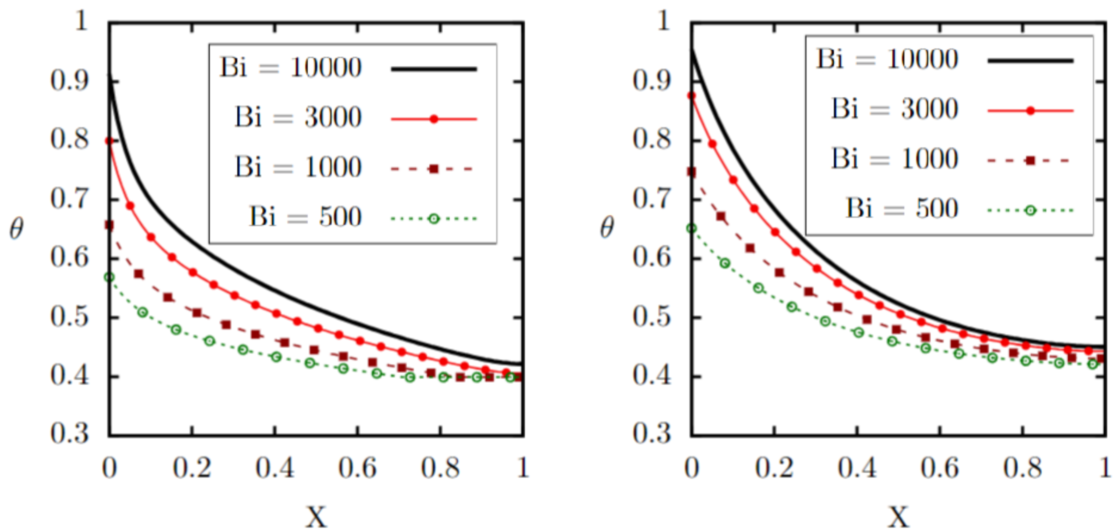
However, after that local maximum, the behavior returns to that of a steady decrease in temperature at the fin's insulated tip.

In Figure 3, it can be noted that overall the shape of the temperature's distribution along the fin is mainly that of a straight line, indicating that for low values of the radiation parameter (G) and zero convective parameter (S_H), the main form of heat transfer is conduction inside the porous fins. Additionally, it can be observed that the decrease of the thermal resistance at the base, represented by the increasing Biot numbers, dramatically impacts the temperatures as expected. Additionally, it can be noted that the change from G=10 to G=100, from (a) to (b), significantly impacted the temperature distribution, especially at low Biot numbers.



(a) (b)

Figure 3. Temperature distribution changes along the length due to thermal resistance variation at the base considering (a): $S_H = 0, G = 10, \theta_F = 0.4, d/L = 0.1$ and (b): $S_H = 0, G = 100, \theta_F = 0.4, d/L = 0.1$.



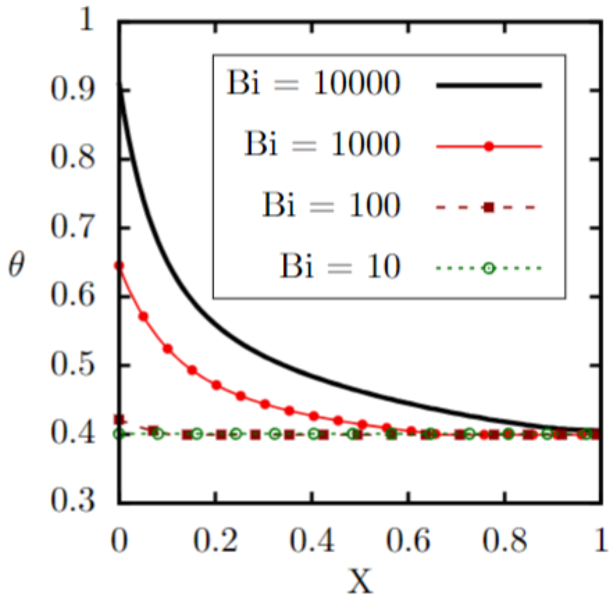
(a) (b)

Figure 4. Temperature distribution changes along the length due to thermal resistance variation at the base considering (a): $S_H = 10, G = 100, \theta_F = 0.4, d/L = 0.1$ and (b): $S_H = 10, G = 10, \theta_F = 0.4, d/L = 0.1$.

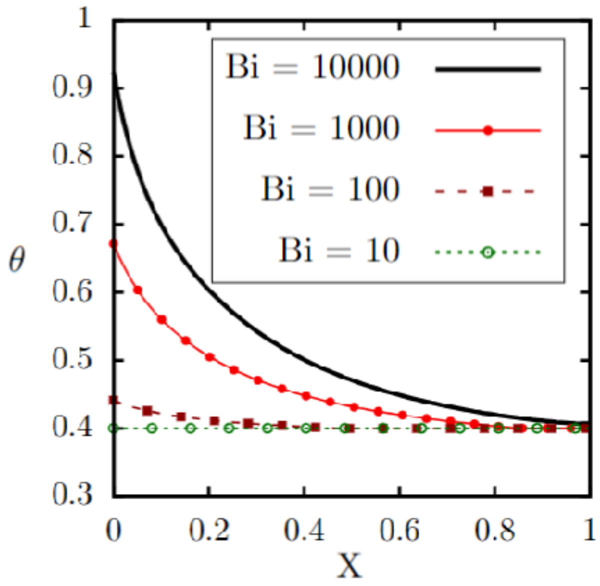
Figure 4 shows the impacts of accounting for convection effects, using two distinct radiation parameters.

From the comparison between Figure 4 and Figure 3, it can be inferred that greater values of the convective parameter (S_H) and the radiation parameter (G) cause temperatures to decrease faster along the fin and change the shape of the temperature distribution along the length. Moreover, once more, the thermal resistance at the base was demonstrated to be a primary factor in determining the temperature distribution. Once more, it can be observed that changing from G=100 to G=10, from (a) to (b), significantly altered the temperature distribution, especially at low Biot numbers.

Figure 5 considers the radiation parameter with half its value in Figure 4. At the same time, the distance-to-length in Figure 5 is three times the distance-to-length in Figure 4. Consequently, comparing Figure 5 and Figure 4 shows that a further increase in the convective parameter, combined with a decrease in the radiation, changes the shape of the distribution more than an increase in the radiation parameter. However, the change from d/L=0.3 to d/L=0.1, from (a) to (b), caused a minimal rise in the temperatures along the fin.



(a)



(b)

Figure 5. Temperature distribution changes along the length due to thermal resistance variation at the base considering (a): $S_H = 20, G = 50, \theta_F = 0.4, d/L = 0.3$ and (b): $S_H = 20, G = 50, \theta_F = 0.4, d/L = 0.1$.

4. CONCLUSIONS

The results from the proposed methodology agree with the more straightforward problems present in the Literature. However, due to a lack of similar research that adequately explores the effects of radiation on a fin's temperature and efficiency, better comparisons could not be made to validate.

Overall, the results showed that convective effects could easily overwhelm radiation effects, which might cause a lack of exploration of such effects. It can be noted from this work and also in Gorla and Bakier (2011), that convection dominates the heat transfer easily. For radiation to be the main heat exchange mode instead of convection, the radiation parameter G needs to be around ten times bigger than the convection parameter S_H.

Nevertheless, many current applications of fins, composite fins, and porous fins happen in environments where radiation becomes the primary vector of heat transfer from the fins to the environment, such as in spatial applications, rarefied environments, and extreme temperatures.

5. ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support the Brazilian Agencies CAPES and CNPq provided.

6. REFERENCES

Ganji, D.D., Alizadeh, M., Hosseinzadeh, Kh. and Shahavi, M.H., 2019. Solidification acceleration in a triplex-tube latent heat thermal energy storage system using V-shaped fin and nano-enhanced phase change material, *Applied Thermal Engineering*, Vol. 163, p. 114-436.

Gorla, R.S. and Bakier, A.Y., 2011. Thermal analysis of natural convection and radiation in porous fins*, International Communications in Heat and Mass Transfer*, Vol. 38, n. 5, p. 638-645.

Haq, R., Sheikholeslami, M., Shafee, A. and Li, Z., 2019. Heat transfer behavior of nanoparticle enhanced PCM solidification through an enclosure with V-shaped fins, *International Journal of Heat and Mass Transfer*, Vol. 130, p. 1322-1342.

Park, D.H., Lee, D.B., Seo, E.R. and Park, Y.J., 2023. Study on the heat transfer and fluid flow characteristics in V-shaped corrugated composite fin, *Applied Thermal Engineering*, Vol. 102, pp. 293-301, 2016.

Saldanha Da Gama,, R. M., Saldanha Da Gama, R. P., Sarmento, V. V., Martins-Costa, M. L., 2023. A variational formulation for fins with nonzero contact thermal resistance at the base. *Axioms,* Vol. 12, p. 54.

Sarmento, V. V., Martins-Costa, M. L., Saldanha Da Gama, R. M., 2023. A model for a set of porous fins on a long rotating cylinder considering thermal radiation and natural convection. *Numerical Heat Transfer Part B-Fundamentals*, Vol. 83, p. 1-19.

Ma, J., Sun, Y., Li, B., Chen, H., 2016. Spectral Collocation for Radiative-Conductive Porous Fin with Temperature Dependent Properties. *Journal of Energy Convection and Management*, Vol. 111, p. 279-288.

Martins-Costa, M. L., Sarmento, V.V., Saldanha Da Gama, R. P., Saldanha Da Gama,, R. M., 2022. Solution construction for the nonlinear heat transfer problem in a cylindrical porous fin. *Journal of Porous Media*, Vol. 25, p. 1-19.

Martins-Costa, M. L., Sarmento, V. V., Moraes De Lira, A., Saldanha Da Gama, R. M., 2020. Temperature distribution in porous fins, subjected to convection and radiation, obtained from the minimization of a Convex Functional. *Mathematical Problems in Engineering*, Vol. 2020, p. 1-10.

Sarmento, V. V.; Martins-Costa, M. L., Saldanha da Gama, R. M., 2022. Numerical approximation for porous parallel fins with radiation. In *Proceedings of the 20th Brazilian Congress of Thermal Sciences and Engineering*. ENCIT, Bento Gonçalves.

Sarmento, V.V., Martins-Costa, M. L., Saldanha da Gama, R.M., 2020. Some exact solutions for a class of nonlinear differential equations arising from the mathematical description of porous fins. In *Proceedings of the 18th Brazilian Congress of Thermal Sciences and Engineering*. ENCIT, online.

Sarmento, V. V., Martins-Costa, M. L., 2018. Spectral and finite difference simulations for temperature distribution in a porous fin. In *Proceedings of the 17th Brazilian Congress of Thermal Sciences and Engineering* ENCIT, Águas de Lindóia.

Shivanian, E., Keshtkar, M., and Navidi, H., 2020. Heat transfer from convecting-radiating fin through optimized Chebyshev polynomials with interior point algorithm. *Nonlinear Engineering*, Vol. 9, no. 1, p. 102-110.

7. RESPONSIBILITY NOTICE

The authors are the only ones responsible for the printed material included in this paper.