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MULTI-MATERIAL TOPOLOGY OPTIMIZATION WITH SIMP FORMULATION AND HEAVISIDE THRESHOLD FUNCTION FILTER

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Abstract. *Topological optimization of parts and components is a reality today. However, little is explored about the possibility of using more than one type of material in its construction. This is beneficial in generating new possibilities for structural topological optimization, making rational and efficient use of the specific strength properties of each material where necessary, in addition to reducing design and production costs. This work aims to implement for a multi-material topology optimization algorithm with SIMP (Solid Isotropic Material with Penalization) formulation, making use of volumetric constraints for each of the material phases. Furthermore, it is sought to implement a material filter with a Heaviside threshold function for the final densities, in order to make the resulting topology more defined, with as few intermediate densities as possible. Results for benchmark cases from the literature are evaluated in comparison with the results of the implemented code, in addition to performing parametric studies for the variables of the algorithm, and the results shows the use of the Heaviside threshold function reduces drastically the greys areas of the final topologies although the fine tuning of the intrinsic parameters of the method may be required to get best improvements.*

Keywords: *topology optimization, multimaterial, finite elements, SIMP, Heaviside threshold filter.*

1. INTRODUCTION

Topological optimization of parts and components is a reality today. However, little is explored about the possibility of using more than one type of material in its construction. This would be beneficial in generating new possibilities for structural topological optimization, making rational and efficient use of the specific strength properties of each material where necessary, as well as reducing the cost of design and production. This can be verified, according to Huang and Xie (2010), by the increase in the structural use of composite materials, which benefit from optimization algorithms to improve their properties according to the need.

Among the most used methodologies nowadays, we can highlight the SIMP (Solid Isotropic Material with Penalization) methodology which assumes densities and elastic properties of materials as continuous and differentiable and is based on gradients of the objective function (in general Compliance) to perform topological optimization; the BESO (Bi-directional Evolutionary Structural Optimization) methodology that uses the MEF (Finite Element Method) to evaluate portions of the material and simultaneously remove and place material in the domain so that there will be no elements with intermediate densities, removing the need for final filters; and the Level-Set, a method that is based on the movement of borders in the domain, having the advantage of not requiring the parameterization of objects, allowing the analysis of complex geometries. This proves to be of great value for topology-changing shapes, and is also very useful for transient models.

The present work proposes the implementation in Matlab (2012) of a formulation for multimaterial topological optimization with volumetric constraints within a SIMP methodology, based on the implementations made by Tavakoli and Mohseni (2014). As secondary objective, it is intended to implement a final filter for densities, generating the minimum of "gray regions" (Yamazaki et al. (2015) and that meets the volumetric constraints (Cui et al. 2018) in the SIMP framework. Results are evaluated for benchmark cases in the literature in comparison with the results of the implemented code, in addition to parametric tests of the algorithm variables.

2. LITERATURE REVIEW

The topic of topological optimization has been frequently addressed in numerous researches and academic works. Topological optimization can also be applied to dynamic analysis in time resulting from multiple loads, as proposed by Ribeiro (2020). The author implements, through the Matlab software (2012), an algorithm based on the BESO method with volume constraint that allows optimizing two-dimensional structures subject to transient loads, in order to be able to evaluate their safety. According to the author, the method explored proved to be effective despite its simplicity, sometimes obtaining better results than those of the literature used. Furthermore, it is proven that optimization performed for a larger request does not equate to optimization for multiple requests.

Bücker (2019) addressed the topic of multimaterial optimization. To perform the interpolation of different materials within the same domain, the author proposed the use of normal distribution functions. Examples of the L-beam and the free set beam were used to validate the results, both using 4 phases (including the void). In most cases it was successful, but the parameters of the Gauss distribution (mean and standard deviation) must be carefully chosen, otherwise there may be non-convergence in the distribution of some of the materials and a somewhat dispersed distribution.

Tavakoli (2014) proposes a new method for multimaterial topological optimization that consists of dividing the problem into smaller binary subproblems, allowing the sequential calculation for the multiple phases of the structure with existing solvers. The subsequent joining of these incomplete results is performed with an external iteration, based on the Gauss-Seidel block coordinated descent method. The constructed algorithm proved easy to implement, being possible to extend almost any two-phase optimization tool to a multiphase one using the presented structure. In addition, the complexity of the code is not affected by the number of desired phases, that is, it can be easily adjusted, proving the efficiency of the method presented by solving numerous examples.

Huang and Xie (2010) define the foundations for multimaterial topological optimization with a BESO approach. In their book, entitled "Evolutionary Topology Optimization of Continuum Structures Methods and Applications", they make use of rigidity optimization (minimization of Compliance). The modulus of elasticity for the multimaterial cases are allocated in a decreasing fashion in a vector. They also define that, for multiphase structures, one can determine the properties of the material situated between two distinct phases by means of a penalized interpolation.

A multimaterial topological optimization methodology based on the PTO (Proportional Topology Optimization) method was proposed by Cui et al. (2018). The authors claim that the complexity of the problem can be diminished by using a non-gradient-based method. The logistic function is used for the interpolation of densities between phases. In addition, a density filter combined with a Heaviside threshold function allowed the junction between two solid materials at the end of the optimization to be more evident compared to other methods. The accuracy of the proposed method was verified by obtaining final compliances lower than the examples found in the literature, which are resolved by other methods.

3. FUNDAMENTAL CONCEPTS

3.1 SIMP Method

Making use of finite elements, optimization methods are based on the premise of material removal in poorly requested regions of the structure. This withdrawal is carried out according to an imposed restriction, which can be volumetric, related to the limit stress of the material used, among others. Its general formulation is given by Equations (1) to (4).

$$\text{Minimize}_{\mathbf{x}}:\ c(\mathbf{x}) = \mathbf{U}^{\mathrm{T}}\mathbf{K}\mathbf{U} = \sum_{\boldsymbol{e}=\mathbf{1}}^{N} x_e\ \boldsymbol{u}_e^{\mathrm{T}} k_0 \boldsymbol{u}_e \tag{1}$$

$$\text{Subject to:}\ V(\mathbf{x})/V_o = f \tag{2}$$

$$:\ \mathbf{KU} = \mathbf{F} \tag{3}$$

$$:\ \mathbf{0} < \mathbf{x}_{min} \leq \mathbf{x}_e \leq \mathbf{1}, \tag{4}$$

where $c(\mathbf{x})$ is the flexibility of the structure (Compliance),$\mathbf{U}$ the vector of global displacements, $\mathbf{F}$ the vector of global forces, $\mathbf{K}$ the global stiffness matrix, $\boldsymbol{u}_e$ and $\boldsymbol{k}_0$ are the displacement vector and the stiffness matrix of a single finite element, respectively, $\mathbf{x}$ is the vector of the design variables (in this case, the density in each finite element), $\mathbf{x}_{min}$ the vector of minimum allowed densities (assumed here 10^{-6} to avoid the singularity problems), N the number of elements of the discretized domain, $V(\mathbf{x})$ the volume of material in the structure as a function of density, V the volume of the design domain, and f the user-defined volumetric fraction (Sigmund, 2001).

If x_e is modeled by the power law (x_e^p) in order to reduce the stiffness of the elements that are to be removed, we have the SIMP method, where p is the penalty factor of the method. This method differs from the others by not making use of discrete densities, but continuous in the interval [0,1], thus avoiding the problems associated with this formulation, in addition to allowing the use of solution algorithms that use gradients. This is accomplished by parameterizing the constitutive tensor of the material by means of Equation (5):

$$E_e = E_{min} + (x_e)^p (E_0 - E_{min}), \tag{5}$$

Since E_0 is the full elastic modulus of the constitutive matrix of the material, E_{min} is a small value ($E_{min} = 10^{-9}E_0$) to avoid numerical problems andx_e the density of the element. In addition, a penalty factor (usually p=3) applied to density is used, in order to guide the result to the boundaries of the interval [$\mathbf{x}_{min}$, 1], seeking to eliminate the intermediate densities in the final structure, since they do not have physical meaning (Bendsøe and Sigmund, 2003).

According to Bendsøe and Sigmund (1995), by the Optimality Criteria method (OC), based on the 2nd KKT condition, one can formulate a heuristic update system for the variables in order to ensure that the densities meet, in fact, one extreme of the optimality conditions. Thus, Equation (6) indicates how the densities are updated to ensure that in all iterations the volume constraint imposed for each phase of the multiple materials is satisfied

$$x_e^{new} = \begin{Bmatrix} \max(x_{min}, x_e - m) \ if & x_e B_e^{\eta} \leq \max(x_{min}, x_e - m) \\ x_e B_e^{\eta} \ \ if & \max(x_{min}, x_e - m) \leq x_e B_e^{\eta} \leq \min(1, x_e + m) \\ \min(1, x_e + m) \ if & \min(1, x_e + m) \leq x_e B_e^{\eta} \end{Bmatrix}, \tag{6}$$

where m is a positive limit to the motion of the asymptote method, η a damping coefficient (with a value equal to 0.5 whose purpose is a numerical damping of the OC method to avoid sudden variations in the value of B_e) and B_e is given by equation (7), where λ is a Lagrange multiplier (Sigmund, 2001).

$$B_e = \frac{\partial c}{\partial x_e} / \lambda \frac{dV}{dx_e}. \tag{7}$$

The optimization is further implemented numerically by the OC method simplifying the problem into sequential subproblems where an approximation of 1st order of objective function and constraints is used. The sensitivity of the objective function to the design variables are evaluated as:

$$\partial c \backslash \partial x_e = -p(x_e)^{p-1} \boldsymbol{u}_e^{\mathrm{T}} k_0 \boldsymbol{u}_e \ , \tag{8}$$

which is calculated for each element of the finite element mesh and k_0 the stiffness matrix for unit elastic modulus. To assemble the matrices of finite elements one assumes here the constitutive matrix $\mathbf{D}_e$ for plane state of stresses

$$\mathbf{D}_e = E_e \frac{1}{1-\nu} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-\nu/2 \end{bmatrix} = E_e \mathbf{D}_0 \tag{9}$$

Thus, the finite element stiffness matrix is:

$$\boldsymbol{k}_e = t \iint_{\mathrm{A_e}} \mathbf{B}_e^T \mathbf{D}_e \, \mathbf{B}_e \, dA \qquad e = 1, \dots, N, \tag{10}$$

where N is the number of mesh elements, t is the thickness of the element, $\mathbf{B}$ is the matrix that relates deformations and displacements $\boldsymbol{\varepsilon} = \mathbf{B}\,\mathbf{u}$, with $\varepsilon = \{\varepsilon_x, \varepsilon_y, \gamma_{xy}\}^T$ the vector of deformations, and $\mathbf{u} = \{u_x \ u_y\}^T$, the vector of displacements. The stiffness matrix for linear element of 4 nodes is:

$$\boldsymbol{k}_e = \frac{E_e t}{(1-\nu^2)24} \begin{bmatrix} 12 & 3 & -6 & -3 & -6 & -3 & 0 & 3 \\ & 12 & 3 & 0 & -3 & -6 & -3 & -6 \\ & & 12 & -3 & 0 & -3 & -6 & 3 \\ & & & 12 & 3 & -6 & 3 & -6 \\ & & & & 12 & 3 & -6 & -3 \\ & & & & & 12 & 3 & 0 \\ & & & & & & 12 & -3 \\ sym. & & & & & & & 12 \end{bmatrix} + \nu \begin{bmatrix} -4 & 3 & -2 & 9 & 2 & -3 & 4 & -9 \\ & -4 & -9 & 4 & -3 & 2 & 9 & -2 \\ & & -4 & -3 & 4 & 9 & 2 & 3 \\ & & & -4 & -9 & -2 & 3 & 2 \\ & & & & -4 & 3 & 2 & 9 \\ & & & & & -4 & -9 & 4 \\ & & & & & & -4 & -3 \\ sym. & & & & & & & -4 \end{bmatrix} \tag{11}$$

Assembling to the respective degrees of freedom of all $\boldsymbol{k}_e$ of the elements of the finite element mesh results in the global stiffness matrix $\mathbf{K}$. Later the boundary conditions are applied, resulting in the conditioned matrix ($\mathbf{K}_c$) to then be used to solve the linear system for displacements $\mathbf{K_c U} = \mathbf{F}$.

3.2 Sensitivity Filter

In order to avoid the numerous recurring problems in optimization algorithms (such as checkerboard and mesh dependence), it is necessary to employ a filter for these sensitivities. Sigmund (2001) employs a filter given by Equation (12), which allows the solution that is independent of the mesh. However, as pointed out by the author, it has not yet been possible to prove that the filter employed ensures the unique existence of solutions, despite guaranteeing the desired mesh independence.

$$\widehat{\frac{\partial c}{\partial x_e}} = \frac{1}{x_e \sum_{f=1}^{N} \widehat{H}_f} \sum_{f=1}^{N} \widehat{H}_f x_f \frac{\partial c}{\partial x_f} \, , \tag{12}$$

where the weight factor operator ($\widehat{H}_f$) is given by:

$$\widehat{H}_f = r_{min} - dist(e,f), \ \ \{f \in N | dist(e,f) \leq r_{min}\}, \quad e = 1, \dots, N \tag{13}$$

where r_{min} is the size of the influence of the filter on number of elements and the $dist(e,f)$ operator is defined as the distance between the center of the element in question e and the center of the element f. The convolution factor is zero outside the area covered by the filter and decays linearly with increasing distance.

4. METHODOLOGY

4.1 Multimaterial Topology Optimization

According to Huang and Xie (2010), it is assumed that compliance is optimized (minimized) using a certain number of n phases with user-defined volumetric fractions $f_1,\dots,f_n$ so that $\sum_{i=1}^{n} f_i = 1$. The elastic modules of the materials are ranked in descending order $(E_1 > E_2 \dots > E_n)$ and that the prescribed volumes of each material $V_0{}^j$ are in the same order. One of the phases may be the void, which will be given a small value $E_n = E_{min}$ (in this work $10^{-9}\ max(E_j)$). Thus, the optimization problem can be posed as follows:

$$min_{\mathbf{X}} \quad : \quad c(\mathbf{x}) = \mathbf{U}^{\mathrm{T}}\mathbf{K}\mathbf{U} = \sum_{e=1}^{N} x_e^p \boldsymbol{u}_e^{\mathrm{T}} k_0 \boldsymbol{u}_e \tag{14}$$

$$\text{Subject to:} \quad V_0^j - \sum_{i=1}^{N} V_i\, x_{ij} - \sum_{i=1}^{j-1} V_0^i = 0\,,\ \ j = 1,\dots,n-1 \tag{15}$$

$$: \quad \mathbf{KU} = \mathbf{F} \tag{16}$$

$$: \quad \mathbf{0} \le \mathbf{x}_{ij} \le \mathbf{1}, \tag{17}$$

with x_{ij} being the densities for an element i of a phase j. In each element i, the sum of the fractions of the various phases j must be equal to 1 ($\sum_{j=1}^{n} x_{ij} = 1$) and the sum of the volumes of the phases in all elements must result in the predefined fractions ($\sum_{i=1}^{N} V_i x_{ij} = V_0^j$).

Thus, as in the SIMP method, the elastic properties are interpolated, two by two, according to Equation (18).

$$E_e(x_{ij}) = (x_{ij})^p E_j + (1 - x_{ij}^p) E_{j+1}, \tag{18}$$

To calculate the sensitivity, we proceed in a similar way to the SIMP method, previously explained, taking into account the multiple phases:

$$\frac{\partial c}{\partial x_{ij}} = -p\left(x_{ij}\right)^{p-1}(\boldsymbol{u}_i^{\mathrm{T}} k_0^j \boldsymbol{u}_i - \boldsymbol{u}_i^{\mathrm{T}} k_i^{j+1} \boldsymbol{u}_i) \tag{19}$$

To avoid checkboard problems, a mesh-independent filtering based on a minimum radius was on sensitivity vector throughout the iterative optimization process. Moreover, in the algorithm there is an internal loop in which the densities of the materials are distributed according to the volumetric fractions of each phase (two by two), this distribution being confirmed by multiple passes of the algorithm in this loop and controlled by the user with the maximum number of iterations, given by the parameter $iter\_max\_in$.

4.2 Material Filter with Heaviside threshold function

A contribution of this work is in the application of a filter so that the final densities always result in elements closer to full (1) for a certain phase j (and the others tend to zero), a fact not completely achieved with the use of only the penalty factor p of the original SIMP method. According to Cui et al. (2018) this can be achieved with the use of a filter with a Heaviside Threshold Function which preserves the desired volumetric fractions. The design variables need to go through traditional filtering (done at the level of elements and their neighborhoods), and only then go through Heaviside threshold function filtering (done only once, at the end of the convergence of the method). Density values below or above a given threshold gradually tend to 0 and 1 if the function is of type:

$$\bar{x}_{ij} = \frac{\arctan(\beta\phi) + \arctan\left(\beta(x_{ij} - \phi)\right)}{\arctan(\beta\phi) + \arctan\left(\beta(1 - \phi)\right)}, \tag{20}$$

where $\phi \in [0\ 1]$ are the thresholds of the variables, β is a positive number that determines the approximation rate (chosen by the user), $\bar{x}_{ij}$ are the filtered densities. In their work, Cui et al. (2018) states that the $arctan$ function behaves better than the $tanh$ function, also used in some filters proposed in the literature. To keep the volume consistent, after applying this filter, the final volume of that phase must remain the same before the application of the filter, so that the following relationship, for a given phase j, is valid:

$$\sum_{i=1}^{N} x_{ij}\, v_i = \sum_{i=1}^{N} \bar{x}_{ij}\, (\phi) v_i\,, \tag{21}$$

Thus, the ϕ parameter is obtained by the bisection method in order to preserve the volumetric fraction after the application of the filter, that is, obtained by solving Equation (21) for ϕ. Cui et al. (2018) demonstrates that this filter has generated good at results with almost discrete final topologies, indicating a single type of material per region of space for the final results obtained with SIMP. This problem would not need to be circumvented with this filter if the approximation of the problem was made by the BESO method. However, even so in the node regions of the final topology, it is possible that it results in materials that have intermediate densities and proportions between the phases of the chosen materials, and this is still the subject of research in the literature.

5. RESULTS

In this section we will solve some examples of topology optimization found in the literature regarding materials with several phases. For all the problems proposed in this section, the following factors are used: penalty $p = 3$, filter radius $rf = 8$, minimum elastic modulus of $E_{mín} = 10^{-9}$ N/m², unit finite element thickness, maximum of 200 external iterations, filter tolerance of 5% and tolerance for convergence of 0.1%. All loads (F) are unitary. The ratio between mesh elements (n_x and n_y) must be identical to the ratio between the lengths L_x and L_y. In every problem with geometric symmetry, only half of the topology will be simulated to reduce the computational cost. A length of 1 centimeter was adopted for each element, regardless of the direction: n_x =n_y=0.01 [m]. In the topologies covered, the void is represented by the white color.

5.1 Example of Beam: MBB, Tavakoli et al. (2014) with 4 phases

The MBB beam presented in Tavakoli et al. (2014) with 4 phases is optimized. It has a ratio of 2:1 (L_x/L_y). It is used $n_x = 96$ elements horizontally and $n_y = 48$vertically (96x48). The elastic modules of the phases of the materials are: $E_1 = 9$N/m² (red), $E_2 = 3$N/m² (blue), $E_3 = 1$N/m² (green) and $E_4 = E_{mín}$(white). The volumetric fractions are: $f_1 = 0.16$, $f_2 = f_3 = 0.08$ and $f_4 = 0.68$.

Figure 1 shows the resulting topology after convergence to the case solved by TAVAKOLI et al. (2014) and the present work. It is noticed that the result of the literature presents a topology that has a mixture of phases, especially in the nodes, in addition to the occurrence of intermediate densities at the edges of the structure. This is evidenced by the fact that the image appears unsharp, giving a blurred aspect to the generated structure. In contrast, the topology obtained by the present work presents well-defined contours and high sharpness, due to the passage of the Heaviside threshold filter previously mentioned

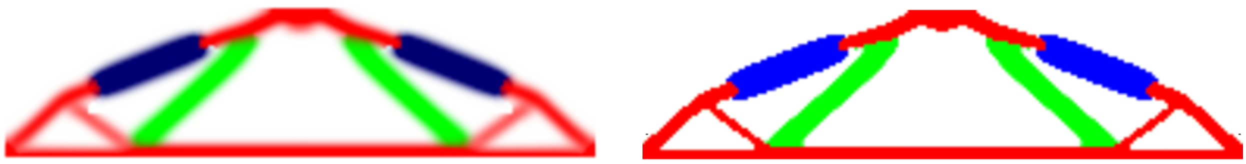

Figure 1. (a) Result of Tavakoli et al. (2014) and (b) of this work (Phases 1 to 4: red, blue, green and white).

Figure 2 shows the history - throughout the iterations - of the objective function (Compliance) for the example solved by Tavakoli et al. (2014) and by this work, along with the volumetric fraction obtained in each phase. It is possible to confirm that the implemented filter effectively corrects the intermediate densities, in addition to keeping unchanged the volumetric fractions throughout the iterations.

The Compliance of the present study resulted in 26 J and the respective value of Tavakoli et al. (2014) was 29.45 J. This again demonstrates the advantage of the proposed filter, which reduces Compliance due to the transformation of the various elements with intermediate density phases into new elements with a single phase.

5.2 Example of Bridge-type Structure Tavakoli et al. (2014) with 4 phases

In this example, a 4-phase Bridge-type structure is topologically optimized. The elastic modules of the phases are given by: $E_1 = 9$N/m² (red), $E_2 = 3$N/m² (blue), $E_3 = 1$N/m² (green) e $E_4 = E_{mín}$(white). The desired volumetric fractions are: $f_1 = 0{,}2$, $f_2 = f_3 = 0{,}1$ e $f_4 = 0{,}6$. A ratio of 1:1 ($L_x = L_y$) is used, and 96 elements are used in each direction for the construction of the mesh ($n_x = n_y = 96$).

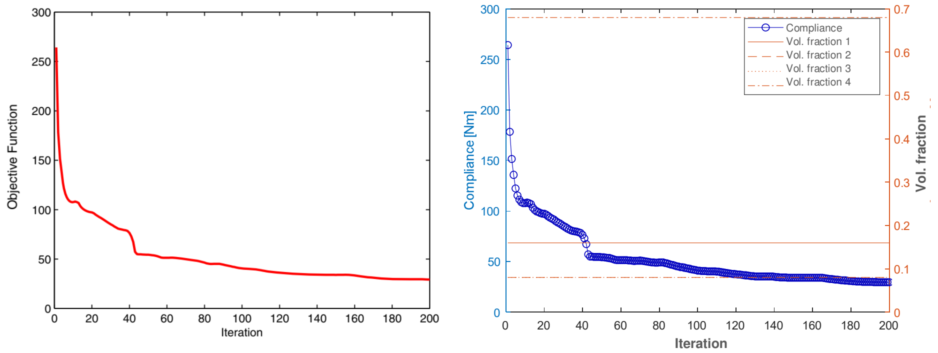

Figure 2. (a) Result of Tavakoli et al. (2014) and (b) obtained with this work for Compliance.

Figure 3 shows the topologies obtained for this case by Tavakoli et al. (2014) and by the present study. Again, it is possible to observe the clearer aspect of the result obtained by the present study in relation to the result of the literature.

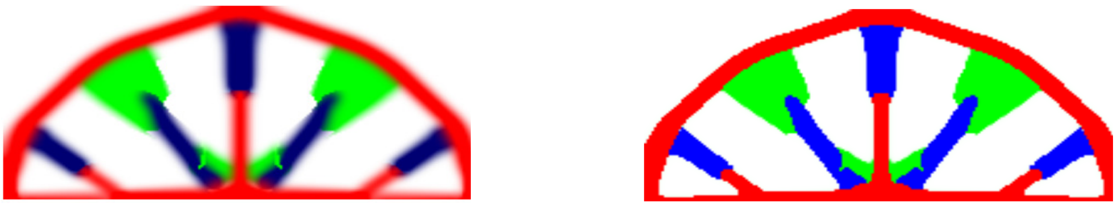

Figure 3. (a) Result of Tavakoli et al. (2014) and (b) of this work (Phases 1 to 4 given by red, blue, green and white).

Figure 4 shows the history of the objective function obtained by Tavakoli et al. (2014) and by this work, along with the volumetric fraction obtained in each phase. In this way, it is confirmed again that the modifications implemented keep the volumetric fractions unchanged, in addition to correcting the intermediate densities in the final topology.

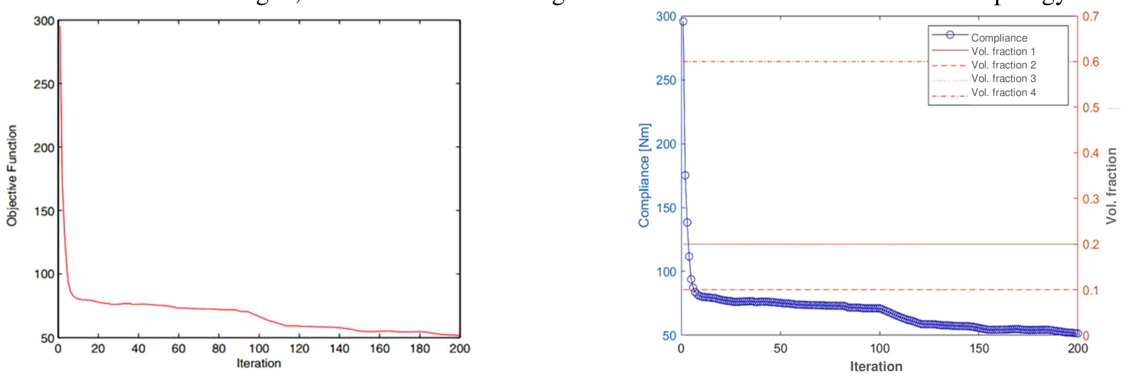

Figure 4. (a) Result of Tavakoli et al. (2014) and (b) obtained with this work for Compliance.

Again the beneficial effect of the passage of the proposed filter is observed. The resulting Compliance for this problem was 45.45 J. The respective value obtained by Tavakoli et al. (2014) was 51.53 J.

5.3 Example of Cantilever Beam by Cui et al. (2018) with 4 phases

The structure has a dimensional ratio of 2:1, making use of a mesh of 96 ×48 elements. The load $\boldsymbol{F}$ is applied to the free end at half height ($L_y/2$). The phases are defined as $E_1 = 2$ N/m² (red), $E_2 = 1$ N/m² (green) and $E_3 = E_{min}$ (white). The volume fraction are: $f_1 = 0.25$, $f_2 = 0.15$ and for the void $f_3 = 0.60$.

Figure 5 shows the result obtained by (a) CUI et al. (2018) and (b) by the present study. The final Compliance calculated by Cui et al. (2018) was 23.77 J, in contrast to 23.38 J, for the present study. It can be verified that, for the present case, a result closer to that of the literature was obtained (if compared to the previous one), both for Compliance and for the final topology. Figure 6 shows the history of Compliance for this example solved by CUI et al. (2018) and by this work, along with the volumetric fraction obtained in each phase, corroborating with the previous results for the modifications implemented.

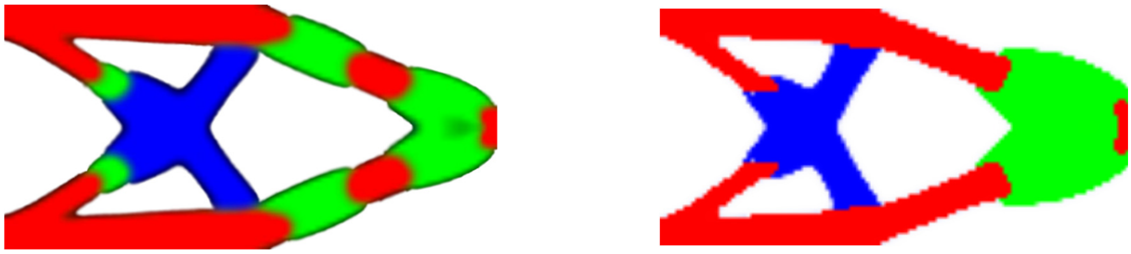

Figure 5. Result of Cui et al. (2018) and (b) obtained with this work (phases 1 to 4 given by red, green, blue and white).

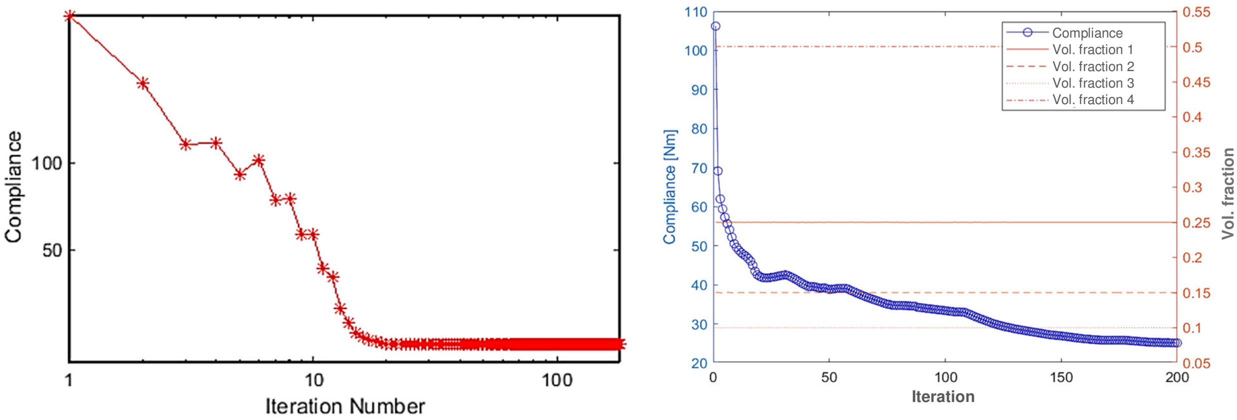

Figure 6. (a) Result of Cui et al. (2018) and (b) obtained with this work for Compliance

In order to allow a complete evaluation of the solution, the different possible parametric combinations between the filter radius (rf) and the number of internal iterations are studied. The simulation is performed by evaluating each rf value for a single value of $iter\_max\_in$, which is incremented in unit steps, repeating the process again. The intervals of each variable are: $4 \leq rf \leq 16$ and $1 \leq iter\_max\_in \leq 12$. Among the different possible combinations (156), Figure 7 (a) shows the best topology obtained (rf=6 and $iter\_max\_in$=1), similar to the result of the literature, with final Compliance of 46.6 J. Figure 10 (b) presents a compilation of all the results obtained. From the analysis of the data, it can be verified that the best results are obtained for the lowest values of $iter\_max\_in$ and rf. Thus, it is possible to minimize the demand for the intervals $4 \leq rf \leq 16$ and $iter\_max\_in \leq 2$, reducing to 26 the total of iterations. If there is interest in the structural viability of the topology, the procedure described should be performed until a satisfactory result occurs.

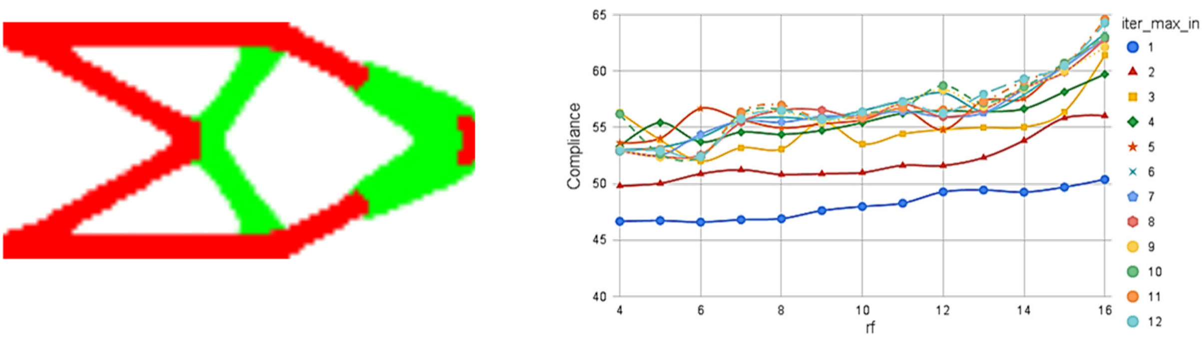

Figure 7. Best result obtained for the combinations of rf, and $iter\_max\_in$, with Compliance of 46.6 J.

5.4 Mesh Dependency Study

This test consists of the repetitive execution of a problem, refining the mesh with each new test, in order to verify if it has an influence on the results. For this case, a 4-phase MBB beam is used. The elastic modules of the phases of the materials are: $E_1 = 4$N/m², $E_2 = 2$N/m², $E_3 = 1$N/m², and $E_4 = E_{min}$. The volumetric adopted fractions are: $f_1 = 0.2$, $f_2 = f_3 = 0.15$, and $f_4 = 0.5$. The meshes applied are: 48 × 24, 96 × 48, and 192 × 96. It is worth mentioning that, according to the established method, one must change rf proportionally to the mesh, in order to isolate it as the only influential factor (the more refined the mesh, the higher the rf value). For the present case, the following values are used: rf=4, 8, 16 respectively. Figure 8 shows a comparison between the three covered meshes.

Figure 8. Result for mesh test for (a) 48×24, (b) 96×48 and (c) 192×96 elements.

For meshes with a larger number of elements, the resulting topology has a higher resolution, which results in a structure with more defined edges and less serrated effect. Meshes with an insufficiently low number of elements can lead to a change in the final topology. Therefore, as long as a consistent mesh-to-object ratio and a sufficient number of elements are maintained, the mesh will not significantly influence the outcome of the final topology.

5.5 Study of the Maximum Number of Internal Iterations

Given the approach used by Tavakoli et al. (2014) depends on the definition of a parameter, it is of interest to verify whether the number of internal iterations ($iter\_max\_in$) affects the final result. To this end, the problem is solved changing this value. It is analyzed the cantilever beam proposed by Cui et al. (2018) and for the bridge-type structure proposed by Tavakoli et al. (2014). For both cases, the original parameters are maintained, varying only the value of $iter\_max\_in$. Both problems are addressed for the case of 3 and 4 phases. Figures 9 and 10 shows the results obtained for the bridge structure with 4 phases (Tavakoli et al. 2014) and for the cantilever beam with 4 phases (Cui et al. 2018), in this order.

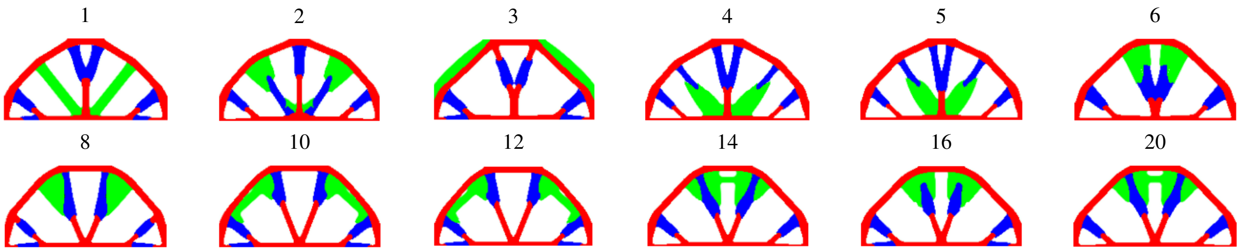

Figure 9. Bridge structure with 4 phases (Tavakoli et al. 2014) for different $iter\_max\_in$ values.

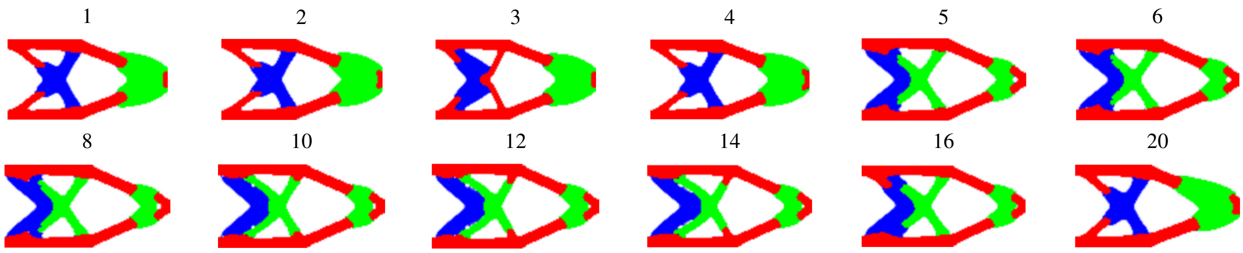

Figure 10. Cantilever beam with 4 phases (Cui et al. 2018) for different $iter\_max\_in$ values.

Based on the results obtained for the six examples addressed, it can be verified that the increase in the number of phases makes the topology more sensitive to variations in $iter\_max\_in$. This is evidenced by analyzing the Figures 9 and 10, which present significant changes in their topology during the study. Another factor accentuated by the increase in the number of internal iterations is the instability (more irregular distribution of materials) of the results. As can be seen, the instability of topologies increases with $iter\_max\_in$, sometimes generating impractical construction structures (e.g., Figure 10, $iter\_max\_in \geq 20$).

Despite the differences highlighted, a trend remained constant in all the problems addressed. Regardless of the number of phases, the higher the number of i $iter\_max\_in$, the greater the resulting compliance. Thus, it can be ensured that the optimal value for this variable will be $iter\_max\_in$=1. However, if there is interest in the feasibility of constructing the topology, it is enough to evaluate a small search interval for the variable, starting from 1 with unit increments. Thus, it can be guaranteed that the best result will be the first stable structure found. This is evident when analyzing the history of compliance for each case.

6. CONCLUSIONS

The present work aimed to investigate and modify the implementations proposed by Tavakoli et al. (2014) for a multi-material topology optimization algorithm with SIMP formulation, in addition to the implementation of a material filter with Heaviside threshold function (Cui et al. 2018), to minimize the existence of intermediate densities. Due to the rational and efficient use of all materials used in the optimization, one has the ability to maximize their individual attributes in the final topology, whether they be stiffness, limit stresses, or even cost.

Given the implementation of the proposed Heaviside threshold function filter and comparing the results obtained for benchmark cases from the literature with those generated by the implemented code, one can verify that the use of the filter is beneficial, both for the topology - reducing the incidence of intermediate densities - and for the final Compliance value, which was reduced by 11.8% for certain cases, as long as the efficient use of parameters is made.

Among the variables studied, it can be confirmed that the mesh does not interfere in the final result, as long as it has a sufficient number of elements to represent the topology and that a consistent mesh-to-object ratio is maintained. The filter radius (rf) and maximum internal iterations ($iter\_max\_in$) have a greater influence on the results. Both variables influence the final topology more the larger the number of phases in the problem. It was also found that the value of the objective function (Compliance) gets worse as the variables increase. As such, its optimal values are the smallest possible among the ranges addressed (rf=4 and $iter\_max\_in$=1). However, sometimes the optimal result is an abstract (unstable) topology. If there is interest in the feasibility of executing the structure, a more cautious approach to the choice of variables is required. The solution found consists in starting the simulation with values at the lower end of the range and gradually increasing them until a feasible topology is obtained that meets the user's criteria

7. ACKNOWLEDGEMENTS

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