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HYBRID INTEGRAL TRANSFORMS FOR ANALYSIS OF COMBINED LAMINAR FORCED CONVECTION AND THERMAL RADIATION

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Abstract. *The present work aims to analyze the heat transfer in laminar flow of participating fluids inside circular and rectangular ducts. Its about a problem of simultaneous heat transfer by convection and radiation, hydrodynamically developed and thermally developing. The fluid is treated as gray, purely absorber and emitter of thermal radiation. The duct wall is considered gray and perfectly diffused, being maintained at constant temperature. In the present study, a differential approximation is used to model radiative transference, based on the method of moments, in accordance with some works found in the specialized literature. A hybrid method, numerical-analytical, known as GITT, is applied to simultaneously solve the energy equation and the radiative transfer equation. Three different strategies are used to solve the problem, all of them through integral transformations, that are compared among themselves and show versatility of the GITT. The results, presented in the form of tables and graphs, allow to analyze the influence that the parameters conduction-radiation, optical thickness and contour surface emissivity exert in the temperature field and in the total Nusselt number. The results obtained in the present work were confronted with existing ones in the open literature, in order to validate the presented model.*

Keywords: *participating fluids, GITT, temperature field, total Nusselt number.*

1. INTRODUCTION

Heat transfer in the flow of participating fluids is extremely relevant in many engineering applications, among them we can give prominence: high temperature gas-cooled reactors (HTGR), combustion chambers, steam generators and power generation equipment, industrial furnaces for processing materials, engines and gas turbines, among others. This problem is recognized as important within thermal engineering and has been widely studied in the specialized literature (Pearce, 1970; Echigo, 1975; Jeng, 1976; Nakra, 1977; Smith, 1985; Campo, 1988a; Schuler, 1988b; Tsay, 1989; Yang, 1991; Yang, 1992; Seo, 1994; França, 1995; França, 1998; Baek, 1999; Bergero, 1999; Sediki, 2002; Sediki, 2003; Zheng, 2003; Diniz et al., 2005; Galarça, 2006; Talukdar, 2007 and Al-amri, 2010).

At high temperature levels, thermal radiation can significantly affect the thermal characteristics of a participant fluid that moves inside a duct. Under these conditions, thermal radiation plays a fundamental role in the internal process of fluid heat transfer, as well as in the heat exchange between the fluid and the surrounding surfaces. The omission of the radiative effect offers an appreciable error in computing the heat transfer rate. For high temperature conditions, it was established that the conventional correlations of the Nusselt number for pure convection are no longer valid. In addition, there are still no correlations that include the share of radiant energy in the heat transfer rate (Galarça, 2006).

Studies to include radiation can follow two different directions: experimental measurements or numerical solutions, based on fundamental equations. Although complex, the basic theory of heat transfer by radiation and convection in ducts is well established today. This makes the numerical solution possible and desirable in relation to the experimental one, because the costs involved in simulating the various conditions of interest are much lower (França, 1995).

It is a well-known fact that heat transfer considering the coupling of thermal radiation and forced convection is a very complex and difficult to solve problem. These difficulties are mainly related to the modeling of transport processes. In addition to the continuity equations, the amount of movement and energy, it is necessary to add one more equation in the mathematical modeling: the radiative transfer equation (RTE). For hydrodynamically developed problems, once the pressure and velocity fields are already established, remains to the simultaneous resolution of the energy equations and of the radiative transfer ones.

The equations of the energy and of the radiative transfer ones are coupled and of non-linear nature, subject to boundary conditions involving the exchange of heat by radiation and convection between the fluid and the medium. In the present work, a mathematical modeling is done so that the energy equation and the radiative transfer equation are simultaneously solved using the GITT methodology, presented by (Cotta,1993 and Cotta, 1998).

2. MATHEMATICAL MODELING

The problem to be studied is about the fully developed flow of a participating fluid inside a channel of parallel flat plates or circular tube, as shown in figure 1. The fluid is considered to have a uniform inlet temperature (Te) and that the wall temperature (Tw) of the tube walls / flat plates is kept fixed. The fluid is treated as a gray medium, purely absorber and emitter of radiation.

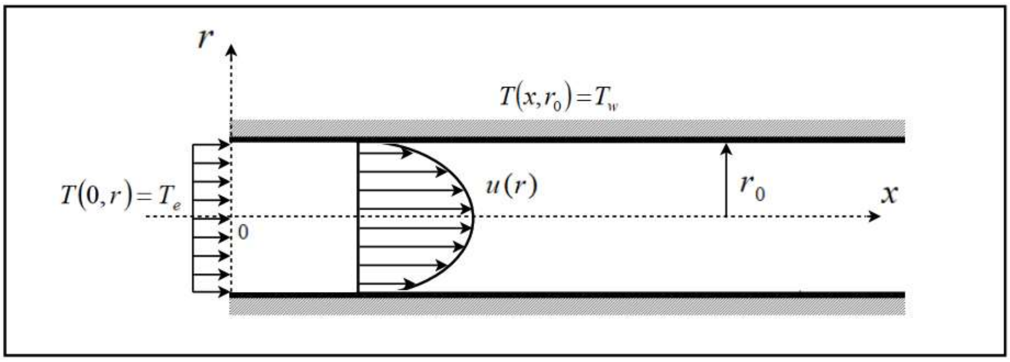

Figure 1. Problem illustration.

For the mathematical modeling of the proposed physical problem, the following considerations were made:

- Laminar flow, in steady state;
- Incompressible fluid;
- The thermal properties of the fluid are considered constant;
- The speed profile is fully developed at the thermal input;
- The effects of viscous dissipation will not be considered;
- Impermeability and non-slip on the walls;
- Disregarding body forces;
- Uniform pressure gradient in the axial direction;
- The axial length of the channel / tube is much greater than its radial length;
- The fluid is gray, which emits, absorbs and does not disperse radiation, so that all its radiative properties are independent of the wave frequency.
- The duct wall is gray and perfectly diffuse, being kept at a constant temperature (Tw).

Taking into account the simplifying hypotheses presented, the governing equations that describe the heat transfer for the convection / radiation coupling of the proposed physical problem, in the dimensionless form, can be written as follows:

Dimensionless Energy equation

$$U(R)\frac{\partial\Theta(X,R)}{\partial X}=\frac{1}{Pe^2}\frac{\partial^2\Theta(X,R)}{\partial X^2}+\frac{4(2-m)^2}{R^m}\frac{\partial}{\partial R}\left(R^m\frac{\partial\Theta(X,R)}{\partial R}\right)+\frac{\tau^2}{N}\left[G^*(X,R)-\Theta^4(X,R)\right] \quad (1)$$

Dimensionless equation of Radiative Transfer

$$\frac{1}{Pe^2}\frac{\partial^2 G^*(X,R)}{\partial X^2}+\frac{4(2-m)^2}{R^m}\frac{\partial}{\partial R}\left(R^m\frac{\partial G^*(X,R)}{\partial R}\right)=3\tau^2\frac{\beta}{\kappa}\left[G^*(X,R)-\Theta^4(X,R)\right] \quad (2)$$

Dimensionless boundary conditions

$$\frac{\partial\Theta(X,R)}{\partial R}=0\ ;\ X>0 \quad and \quad R=0 \quad (3)$$

$$\frac{\partial G^*(X,R)}{\partial R}=0\ ;\ X>0 \quad and \quad R=0 \quad (4)$$

$$\Theta(X,R)=1\ ;\ X>0 \quad and \quad R=1 \quad (5)$$

$$\frac{\partial G^*(X,R)}{\partial R}=-\frac{3\xi_w\tau}{4(2-m)(2-\xi_w)}\left[G^*(X,R)-1\right]\ ;\ X>0 \quad and \quad R=1 \quad (6)$$

Dimensionless inlet conditions

$$\Theta(X,R)=\Theta_e=\frac{T_e}{T_w}\ ;\ X=0 \quad and \quad 0<R<1 \quad (7)$$

$$\frac{\partial G^*(X,R)}{\partial X}=-\frac{3}{2}\tau.Pe\left[G^*(X,R)-\Theta_e{}^4\right]\ ;\ X=0 \quad and \quad 0<R<1 \quad (8)$$

Dimensionless output conditions

$$\Theta(X,R)=1\ ;\ X=\infty \quad and \quad 0<R<1 \quad (9)$$

$$\frac{\partial G^*(X,R)}{\partial X}=0\ ;\ X=\infty \quad and \quad 0<R<1 \quad (10)$$

For the analysis of the problem were defined the following dimensionless parameters, given by equations (11a-j), with the objective of solving not only a particular problem, but a class of problems that are defined by the same proposed model.

$$X=\frac{x}{D_h Pe} \qquad R=\frac{r}{r_0} \qquad U(R)=\frac{u(r)}{u_m} \qquad G^*(X,R)=\frac{G(x,r)}{4\sigma T_w{}^4} \qquad \tau=\kappa.D_h \quad (11a\text{-}e)$$

$$\Theta(X,R)=\frac{T(x,r)}{T_w} \qquad N=\frac{k.\kappa}{4\sigma T_w{}^3} \qquad R_e=\frac{D_h.u_m}{\nu} \qquad \Pr=\frac{\nu}{\alpha} \qquad P_e=R_e.\Pr=\frac{D_h u_m}{\alpha} \quad (11f\text{-}j)$$

where ν represents the kinematic viscosity, α is the thermal diffusivity of the fluid, r_0 is the characteristic length, D_h= 2.(2-m).r_0 is the hydraulic diameter, τ is the optical thickness, N is the conduction-radiation parameter, Pr, Re and Pe are respectively, the numbers Prandtl, Reynolds and Peclet, k is the thermal conductivity, κ the volumetric coefficient of radiation absorption, β is the spectral coefficient of extinction, σ is the Stefan-Boltzmann constant and ξ_w is the emissivity coefficient of the wall.

In the present work, the method of moments, described in (Özisik, 1973 and Modest, 1993) and used by (Schuller, 1988b; Campo, 1989; Yang, 1991; Yang, 1992 and Seo et al., 1994), is used to compute the effect of thermal radiation. The boundary conditions given by equations (3) and (4) refer to the symmetry of the temperature and radiation intensity profiles. In equation (6) a boundary condition of the 3rd type is verified, considering the participation of the radiative effect. At the entrance opening of the duct, it is considered that the radiation is not reflected, being entitled to the assumption of a “pseudo-black wall” for the radiation, as stated in equation (8). At the outlet of the duct, the uniform temperature distribution suggests that the total radiation intensity will not change with the axial location, as given in equation (10).

The velocity field fully developed, u(r), to meet a wide variety of physically possible situations, is adopted in the form:

$$u(r)=u_m\left\{\frac{1+(2+m)n}{1+n}\left[1-\left(\frac{r}{r_0}\right)^{\frac{1+n}{n}}\right]\right\}^p \;;\; \begin{cases} p=0 \rightarrow piston\ type \\ p=1 \rightarrow power\ law \end{cases} ;\; \begin{cases} m=0 \rightarrow flat\ \ plates \\ m=1 \rightarrow circular\ tube \end{cases} \tag{12}$$

Being u_m the average flow velocity. When $p=0$ the flow is of the piston type and if $p=1$ is of the power law type, associated with the hydrodynamically developed from non-Newtonian fluids. For the velocity profile in the power law model, if $n=1$ the fluid will be Newtonian (parabolic model), if $1<n<\infty$ the fluid will be non-Newtonian of the dilating type and if $0\leq n<1$ the fluid will be non- Newtonian of pseudoplastic type. The factor m is associated with geometry, if $m = 0$ the flow will be between flat plates, and if $m =1$ it will be inside a circular duct.

3. APPLICATION OF THE GENERALIZED INTEGRAL TRANSFORMING TECHNIQUE

3.1 Introduction of mathematical filters

The original problem has inhomogeneous boundary conditions in the radial direction, given by equations (5) and (6). In this direction, following the Generalized Integral Transform Technique (GITT) methodology (Cotta, 1993 and Cotta, 1998), auxiliary problems that need to have homogeneous boundary conditions will be formulated, we will introduce mathematical filters to homogenize these boundary conditions in order to apply the GITT properly, as well as improve computational performance in terms of acceleration of convergence. The proposed filter for the temperature field and for the radiation field is simple, and will be of the form:

$$\Theta(X,R)=\Theta^*(X,R)+1 \tag{13}$$

$$G^*(X,R)=G^{**}(X,R)+1 \tag{14}$$

The GITT will be applied to obtain the solution from $\Theta^*(X,R)$ e $G^{**}(X,R)$. With these, equations (13) and (14) will be used to find the general solution to the proposed physical problem.

3.2 Auxiliary eigenvalue problems

The proposed physical problem was solved by adopting three different formulations for the auxiliary problem, and consequently for the transformed-inverse pair, of the temperature field. For the total radiation intensity field, the same formulation was used in all cases analyzed.

3.2.1. Modeling 1

In Modeling 1, the auxiliary problem chosen for determining the temperature field is written as follows:

$$\frac{1}{R^m}\frac{\partial}{\partial R}\left(R^m\,\frac{\partial\Psi_i(\mu_i,R)}{\partial R}\right)+\mu_i^2.\Psi_i(\mu_i,R)=0 \;;\qquad 0<R<1 \tag{15}$$

$$\frac{\partial\Psi_i(\mu_i,R)}{\partial R}=0\;;\qquad R=0 \tag{16}$$

$$\Psi_i(\mu_i,R)=0\;;\qquad R=1 \tag{17}$$

3.2.2. Modeling 2

In modeling 2, the following auxiliary problem was chosen to determine the thermal field:

$$\frac{1}{R^m}\frac{\partial}{\partial R}\left(R^m\frac{\partial\Psi_i(\mu_i,R)}{\partial R}\right)+\mu_i^2U(R)\Psi_i(\mu_i,R)=0\quad;\qquad 0<R<1 \tag{18}$$

$$\frac{\partial\Psi_i(\mu_i,R)}{\partial R}=0\;;\qquad R=0 \tag{19}$$

$$\Psi_i(\mu_i,R)=0\;;\qquad R=1 \tag{20}$$

3.2.3. Modeling 3

In modeling 3, the auxiliary problem chosen for the thermal field is independent of the duct geometry (does not depend on *m*), being written as follows:

$$\frac{\partial^2\Psi_i(\mu_i,R)}{\partial R^2}+\mu_i^2.\Psi_i(\mu_i,R)=0\quad;\qquad 0<R<1 \tag{21}$$

$$\frac{\partial\Psi_i(\mu_i,R)}{\partial R}=0\;;\qquad R=0 \tag{22}$$

$$\Psi_i(\mu_i,R)=0\;;\qquad R=1 \tag{23}$$

In the three models, to determine the total radiation intensity field, the same auxiliary problem was chosen, given by:

$$\frac{1}{R^m}\frac{\partial}{\partial R}\left(R^m\frac{\partial\phi_i(\beta_i,R)}{\partial R}\right)+\beta_i^2\phi_i(\beta_i,R)=0\quad;\qquad 0<R<1 \tag{24}$$

$$\frac{\partial\phi_i(\beta_i,R)}{\partial R}=0\;;\qquad R=0 \tag{25}$$

$$\frac{\partial\phi_i(\beta_i,R)}{\partial R}+\frac{3\xi_w\tau}{4(2-m)(2-\xi_w)}\phi_i(\beta_i,R)=0\;;\qquad R=1 \tag{26}$$

3.3 Transformed / Inverse pairs

3.3.1. Modeling 1

The transform-inverse pair of the temperature field in modeling1 is given by:

$$\overline{\Theta}_i(X)=\frac{1}{N_i^{1/2}}\int_0^1 R^m.\Psi_i(\mu_i,R)\Theta^*(X,R)dR,\qquad \mathit{Transformed} \tag{27}$$

$$\Theta^*(X,R)=\sum_{i=1}^{\infty}\frac{\Psi_i(\mu_i,R)\overline{\Theta}_i(X)}{N_i^{1/2}},\qquad \mathit{Inverse} \tag{28}$$

3.3.2. Modeling 2

In Model 2, the transform-inverse pair of the temperature field is given by:

$$\overline{\Theta}_i(X)=\frac{1}{N_i^{1/2}}\int_0^1 R^mU(R)\Psi_i(\mu_i,R)\Theta^*(X,R)dR,\qquad \mathit{Transformed} \tag{29}$$

$$\Theta^*(X,R)=\sum_{i=1}^{\infty}\frac{\Psi_i(\mu_i,R)\overline{\Theta}_i(X)}{N_i^{1/2}},\qquad \mathit{Inverse} \tag{30}$$

3.3.3. Modeling 3

The transform-inverse pair of the temperature field in modeling 3 is given by:

$$\overline{\Theta}_i(X)=\frac{1}{N_i^{1/2}}\int_0^1 \Psi_i(\mu_i,R)\Theta^*(X,R)dR, \qquad \textit{Transformed} \tag{31}$$

$$\Theta^*(X,R)=\sum_{i=1}^{\infty}\frac{\Psi_i(\mu_i,R)\overline{\Theta}_i(X)}{N_i^{1/2}}, \qquad \textit{Inverse} \tag{32}$$

In the three models, the transform-inverse pair of the total radiation intensity field are equal, given by:

$$\overline{G}_i(X)=\frac{1}{K_i^{1/2}}\int_0^1 R^m.\phi_i(\beta_i,R)G^{**}(X,R)dR, \qquad \textit{Transformed} \tag{33}$$

$$G^{**}(X,R)=\sum_{i=1}^{\infty}\frac{\phi_i(\beta_i,R)\overline{G}_i(X)}{K_i^{1/2}}, \qquad \textit{Inverse} \tag{34}$$

3.4 Integral transformation of the temperature field and radiation field - Obtaining of the transformed problem

By applying integral operators to the governing equations of the problem, with the aid of auxiliary problems and predefined transform-inverse pairs, the original PDE's system can be transformed into a system of ordinary differential equations, commonly called the transformed problem, indicated below for each modeling considered.

3.4.1. Modeling 1

In Modeling 1, the transformed energy equation became:

Transformed energy equation

$$\frac{1}{Pe^2}\frac{d^2\overline{\Theta}_i(X)}{dX^2}-\sum_{j=1}^{\infty}CT1_{ij}\frac{d\overline{\Theta}_j(X)}{dX}-4(2-m)^2\mu_i^2\overline{\Theta}_i(X)+\frac{\tau^2}{N}\sum_{k=1}^{\infty}CT2_{ik}\overline{G}_k(X)+$$
$$-\frac{\tau^2}{N}\sum_{j=1}^{\infty}CT3_{ij}\overline{\Theta}_j^{\,4}(X)-\frac{\tau^2}{N}\sum_{j=1}^{\infty}CT4_{ij}\overline{\Theta}_j^{\,3}(X)-\frac{\tau^2}{N}\sum_{j=1}^{\infty}CT5_{ij}\overline{\Theta}_j^{\,2}(X)-\frac{4\tau^2}{N}\overline{\Theta}_i(X)=0 \tag{35a}$$

Subject to transformed entry and exit conditions:

$$\overline{\Theta}_i(X=0)=(\Theta_e-1)\bar{f}_i \tag{35b}$$

$$\overline{\Theta}_i(X=\infty)=0 \tag{35c}$$

The integral coefficients generated from the integral transformation process are defined as:

$$CT1_{ij}=\int_0^1 R^m U(R)\frac{\Psi_i(\mu_i,R)\Psi_j(\mu_j,R)}{N_i^{1/2}N_j^{1/2}}dR \qquad CT2_{ik}=\int_0^1 R^m\frac{\Psi_i(\mu_i,R)\phi_k(\beta_k,R)}{N_i^{1/2}K_k^{1/2}}dR \tag{35d-e}$$

$$CT3_{ij}=\int_0^1 R^m\frac{\Psi_i(\mu_i,R)\Psi_j^{\,4}(\mu_j,R)}{N_i^{1/2}N_j^2}dR \qquad CT4_{ij}=\int_0^1 4R^m\frac{\Psi_i(\mu_i,R)\Psi_j^{\,3}(\mu_j,R)}{N_i^{1/2}N_j^{3/2}}dR \tag{35f-g}$$

$$CT5_{ij}=\int_0^1 6R^m\frac{\Psi_i(\mu_i,R)\Psi_j^{\,2}(\mu_j,R)}{N_i^{1/2}N_j}dR \qquad \bar{f}_i=\int_0^1 R^m\frac{\Psi_i(\mu_i,R)}{N_i^{1/2}}dR \tag{35h-i}$$

3.4.2. Modeling 2

In Modeling 2, the transformed energy equation became:

Transformed energy equation

$$\frac{1}{Pe^2}\sum_{j=1}^{\infty} CT1_{ij}\frac{d^2\overline{\Theta}_j(X)}{dX^2}-\frac{d\overline{\Theta}_i(X)}{dX}-4(2-m)^2\mu_i^2\overline{\Theta}_i(X)+\frac{\tau^2}{N}\sum_{k=1}^{\infty}CT2_{ik}\overline{G}_k(X)+$$
$$-\frac{\tau^2}{N}\sum_{j=1}^{\infty}CT3_{ij}\overline{\Theta}_j^{\,4}(X)-\frac{\tau^2}{N}\sum_{j=1}^{\infty}CT4_{ij}\overline{\Theta}_j^{\,3}(X)-\frac{\tau^2}{N}\sum_{j=1}^{\infty}CT5_{ij}\overline{\Theta}_j^{\,2}(X)-\frac{\tau^2}{N}\sum_{j=1}^{\infty}CT6_{ij}\overline{\Theta}_j(X)=0 \tag{36a}$$

Subject to transformed entry and exit conditions:

$$\overline{\Theta}_i(X=0)=(\Theta_e-1)\bar{f}_i \tag{36b}$$

$$\overline{\Theta}_i(X=\infty)=0 \tag{36c}$$

The integral coefficients generated from the integral transformation process are defined as:

$$CT1_{ij}=\int_0^1 R^m\frac{\Psi_i(\mu_i,R)\Psi_j(\mu_j,R)}{N_i^{1/2}N_j^{1/2}}dR \qquad CT2_{ik}=\int_0^1 R^m\frac{\Psi_i(\mu_i,R)\phi_k(\beta_k,R)}{N_i^{1/2}K_k^{1/2}}dR \tag{36d-e}$$

$$CT3_{ij}=\int_0^1 R^m\frac{\Psi_i(\mu_i,R)\Psi_j^{\,4}(\mu_j,R)}{N_i^{1/2}N_j^{2}}dR \qquad CT4_{ij}=\int_0^1 4R^m\frac{\Psi_i(\mu_i,R)\Psi_j^{\,3}(\mu_j,R)}{N_i^{1/2}N_j^{3/2}}dR \tag{36f-g}$$

$$CT5_{ij}=\int_0^1 6R^m\frac{\Psi_i(\mu_i,R)\Psi_j^{\,2}(\mu_j,R)}{N_i^{1/2}N_j}dR \qquad CT6_{ij}=\int_0^1 4R^m\frac{\Psi_i(\mu_i,R)\Psi_j(\mu_j,R)}{N_i^{1/2}N_j^{1/2}}dR \tag{36h-i}$$

$$\bar{f}_i=\int_0^1 R^m U(R)\frac{\Psi_i(\mu_i,R)}{N_i^{1/2}}dR \tag{36j}$$

3.4.3. Modeling 3

In Modeling 3, the transformed energy equation became:

Transformed energy equation

$$\frac{1}{Pe^2}\sum_{j=1}^{\infty}CT2_{ij}\frac{d^2\overline{\Theta}_j(X)}{dX^2}-\sum_{j=1}^{\infty}CT1_{ij}\frac{d\overline{\Theta}_j(X)}{dX}-4(2-m)^2\mu_i^2\overline{\Theta}_i(X)+4(2-m)^2\sum_{j=1}^{\infty}CT3_{ij}\overline{\Theta}_j(X)+$$
$$+\frac{\tau^2}{N}\sum_{k=1}^{\infty}CT4_{ik}\overline{G}_k(X)-\frac{\tau^2}{N}\sum_{j=1}^{\infty}CT5_{ij}\overline{\Theta}_j^{\,4}(X)-\frac{\tau^2}{N}\sum_{j=1}^{\infty}CT6_{ij}\overline{\Theta}_j^{\,3}(X)-\frac{\tau^2}{N}\sum_{j=1}^{\infty}CT7_{ij}\overline{\Theta}_j^{\,2}(X)-\frac{\tau^2}{N}\sum_{j=1}^{\infty}CT8_{ij}\overline{\Theta}_j(X)=0 \tag{37a}$$

Subject to transformed entry and exit conditions:

$$\overline{\Theta}_i(X=0)=(\Theta_e-1)\bar{f}_i \tag{37b}$$

$$\overline{\Theta}_i(X=\infty)=0 \tag{37c}$$

The integral coefficients generated from the integral transformation process are defined as:

$$CT1_{ij}=\int_0^1 R^m U(R)\frac{\Psi_i(\mu_i,R)\Psi_j(\mu_j,R)}{N_i^{1/2}N_j^{1/2}}dR \qquad CT2_{ij}=\int_0^1 R^m\frac{\Psi_i(\mu_i,R)\Psi_j(\mu_j,R)}{N_i^{1/2}N_j^{1/2}}dR \tag{37d-e}$$

$$CT3_{ij}=\int_0^1\frac{(1-R^m)}{N_i^{1/2}N_j^{1/2}}\frac{\partial\Psi_j(\mu_j,R)}{\partial R}\frac{\partial\Psi_i(\mu_i,R)}{\partial R}dR \qquad CT4_{ik}=\int_0^1 R^m\frac{\Psi_i(\mu_i,R)\phi_k(\beta_k,R)}{N_i^{1/2}K_k^{1/2}}dR \tag{37f-g}$$

$$CT5_{ij} = \int_0^1 R^m \frac{\Psi_i(\mu_i, R)\Psi_j^4(\mu_j, R)}{N_i^{1/2} N_j^2} dR \qquad CT6_{ij} = \int_0^1 4R^m \frac{\Psi_i(\mu_i, R)\Psi_j^3(\mu_j, R)}{N_i^{1/2} N_j^{3/2}} dR \tag{37h-i}$$

$$CT7_{ij} = \int_0^1 6R^m \frac{\Psi_i(\mu_i, R)\Psi_j^2(\mu_j, R)}{N_i^{1/2} N_j} dR \qquad CT8_{ij} = \int_0^1 4R^m \frac{\Psi_i(\mu_i, R)\Psi_j(\mu_j, R)}{N_i^{1/2} N_j^{1/2}} dR \tag{37j-k}$$

$$\bar{f}_i = \int_0^1 \frac{\Psi_i(\mu_i, R)}{N_i^{1/2}} dR \tag{37l}$$

In the three models, as the same auxiliary problem and the same transformed-inverse pair were used for to obtain the total radiation intensity, the same transformed radiative transfer equation was obtained, given by:

Transformed radiative transfer equation

$$\frac{1}{Pe^2}\frac{d^2\bar{G}_i(X)}{dX^2} - 4(2-m)^2\beta_i^2\bar{G}_i(X) - 3\tau^2\frac{\beta}{\kappa}\bar{G}_i(X) + 3\tau^2\frac{\beta}{\kappa}\sum_{j=1}^{\infty}CR1_{ij}\bar{\Theta}_j^4(X) +$$
$$+3\tau^2\frac{\beta}{\kappa}\sum_{j=1}^{\infty}CR2_{ij}\bar{\Theta}_j^3(X) + 3\tau^2\frac{\beta}{\kappa}\sum_{j=1}^{\infty}CR3_{ij}\bar{\Theta}_j^2(X) + 3\tau^2\frac{\beta}{\kappa}\sum_{j=1}^{\infty}CR4_{ij}\bar{\Theta}_j(X) = 0 \tag{38a}$$

Subject to transformed entry and exit conditions:

$$\left.\frac{d\bar{G}_i(X)}{dX}\right|_{X=0} + \frac{3}{2}\tau.Pe\bar{G}_i(X)\Big|_{X=0} = -\frac{3}{2}\tau.Pe\left[1-\Theta_e^4\right]\bar{g}_i \tag{38b}$$

$$\left.\frac{d\bar{G}_i(X)}{dX}\right|_{X=\infty} = 0 \tag{38c}$$

The integral coefficients generated from the integral transformation processo of the RTE are defined as:

$$CR1_{ij} = \int_0^1 R^m \frac{\phi_i(\beta_i, R)\Psi_j^4(\mu_j, R)}{K_i^{1/2} N_j^2} dR \qquad CR2_{ij} = \int_0^1 4R^m \frac{\phi_i(\beta_i, R)\Psi_j^3(\mu_j, R)}{K_i^{1/2} N_j^{3/2}} dR \tag{38d-e}$$

$$CR3_{ij} = \int_0^1 6R^m \frac{\phi_i(\beta_i, R)\Psi_j^2(\mu_j, R)}{K_i^{1/2} N_j} dR \qquad CR4_{ij} = \int_0^1 4R^m \frac{\phi_i(\beta_i, R)\Psi_j(\mu_j, R)}{K_i^{1/2} N_j^{1/2}} dR \tag{38f-g}$$

$$\bar{g}_i = \frac{1}{K_i^{1/2}}\int_0^1 R^m\phi_i(\beta_i, R)dR \tag{38h}$$

4. RESULTS

For the purpose of benchmarking, the results of the present study were compared with results found in the specialized literature, specifically in (Yang and Ebadian, 1991; Diniz et al., 2005). In figure 2 the analysis is made by comparing the results obtained through the 3 modeling proposed in the present study with the results obtained by (Yang and Ebadian, 1991), who considered a parabolic velocity profile (*p=1* e *n=1*) inside a circular tube (*m=1*), $\varepsilon_w = 1$, N = 0.25, $\beta/\kappa = 1$, *Pe* >>1, $\Theta_e = 0.1$ and the optical thickness assuming the values τ = 1, 3 and 20. The results obtained allow us to verify that the 3 models reproduce approximate solutions to the problem, which have good agreement with each other and present small variations about the reference. In figure 3 it is compared with the results found by (Diniz et al., 2005), who considered a uniform flow (*p=0*) between flat plates (*m=0*), $\varepsilon_w = 1$, $\beta/\kappa = 1$, *Pe* >>1, $\Theta_e = 0$, $\tau = 4$ and the conduction-radiation parameter assuming the values N = 0.1 ; 0.5 and 10. As in the case discussed in Figure 3, the three models converge to one, this unique result of the present work is compared with the result of (Diniz et al., 2005), and the methodology applied in this study is once again validated.

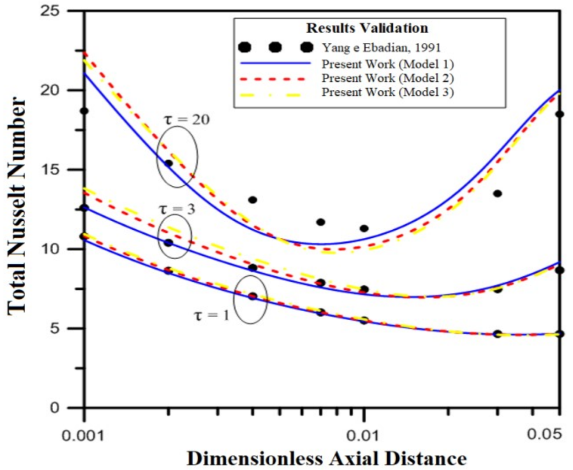

Figure 2. Total Nusselt number for the laminar flow of Newtonian participant fluids inside a circular tube.

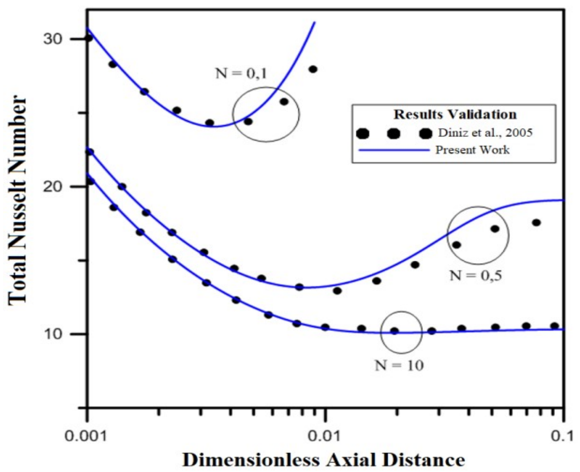

Figure 3. Total Nusselt number for piston-type flow of participating fluids between flat plates.

For a more systematic and detailed analysis of the parametric influence on the thermal and radiative fields, the results are divided into subsections, where in each one of them one of the parameters is varied and the others are considered constant. In all the cases presented below, a parabolic velocity profile is considered and $\beta/\kappa = 1$. All the simulations carried out below take into account the situations of flow between flat plates or a circular duct, and allow the comparison of the three adopted models.

4.1- Effect of emissivity of the wall

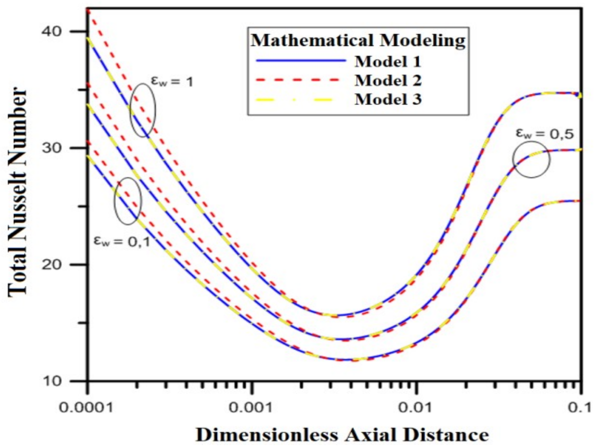

Figure 4. Total Nusselt as a function of axial distance for laminar flow between flat plates.

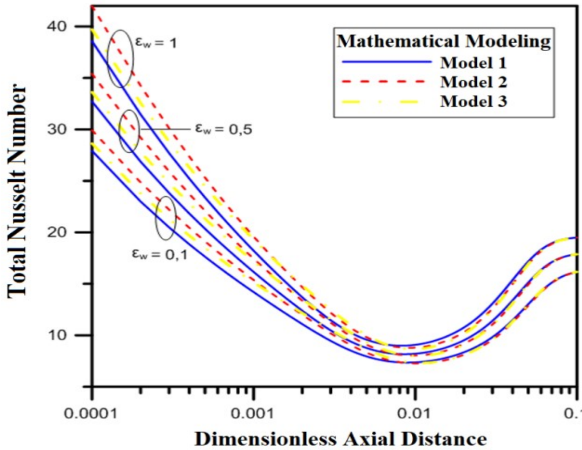

Figure 5. Total Nusselt as a function of axial distance for laminar flow inside a circular tube.

4.2- Optical thickness effect

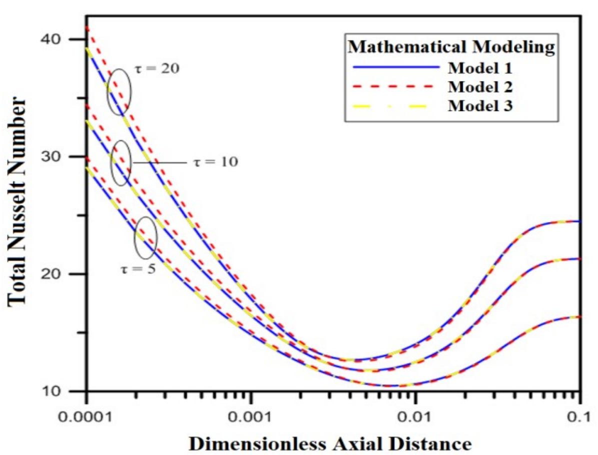

Figure 6. Total Nusselt as a function of axial distance for laminar flow between flat plates.

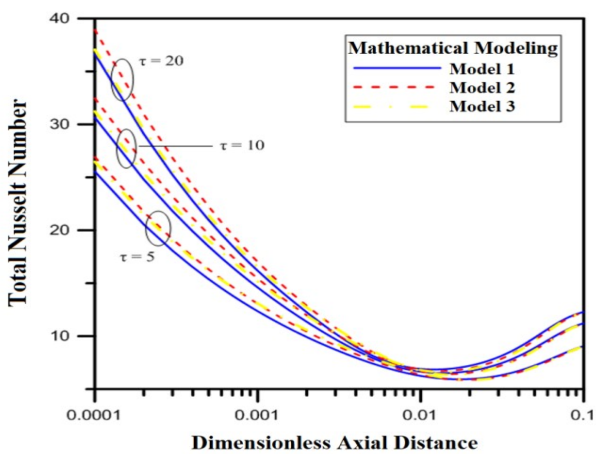

Figure 7. Total Nusselt as a function of axial distance for laminar flow inside a circular tube.

4.3- Effect of the conduction-radiation parameter

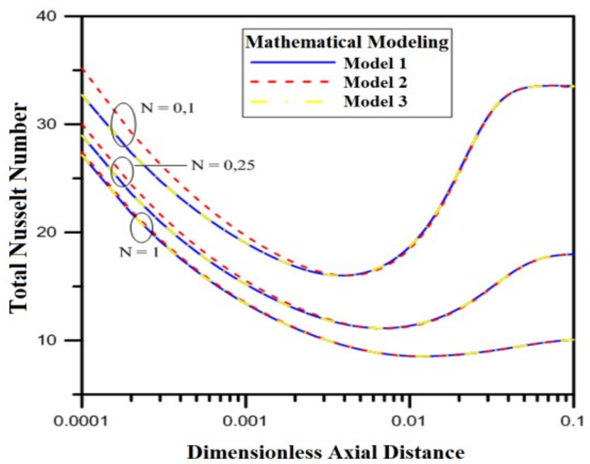

Figure 8. Total Nusselt as a function of axial distance for laminar flow between flat plates.

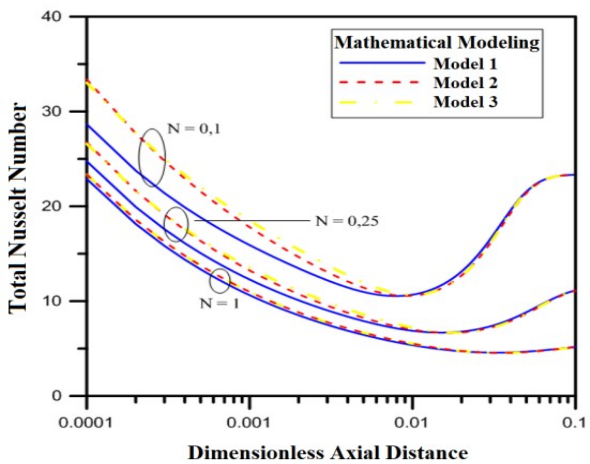

Figure 9. Total Nusselt as a function of axial distance for laminar flow inside a circular tube.

In Figures 4 and 5 analyze the influence of the wall emissivity on the Total Nusselt number, for this analysis it is considered N = 0.25, $\tau = 10$, $\Theta_e = 0$ and the emissivity of the wall assuming the values $\varepsilon_w = 0.1$; 0.5 e 1; it is possible to verify a good approximation in the values obtained and to conclude that the increase in the emissivity of the wall produces an increase in the rates of heat transfer. In Figures 6 and 7 investigate the influence of optical thickness on the development of the total Nusselt number, for this analysis it is considered N = 0.5, $\varepsilon_w = 1$, $\Theta_e = 0$ and the optical thickness assuming the values $\tau = 5$, 10 and 20; the results presented allow us to conclude that the values reached by the Nusselt number decrease together with the optical thickness, so that the curves tend to an asymptotic curve (case of pure convection) in the limit where $\tau \rightarrow 0$. In Figures 8 and 9 investigate the influence of the conduction-radiation parameter on the total Nusselt number, for this analysis we considered $\tau = 3$, $\varepsilon_w = 1$, $\Theta_e = 0$ and the conduction-radiation parameter assuming the values N = 0.1; 0.25 and 1; the results obtained allow us to conclude that the radiative effect is reduced with the increase in the value of the conduction-radiation parameter, so that the curve tends to no longer show an increasing behavior in view of an asymptotic behavior, similarly to the case of pure convection which occurs at the limit where $N \rightarrow \infty$.

Finally, it is concluded, from the results obtained, that the application of GITT in the simultaneous solution of the energy and radiative transfer equations proves to be effective in the analysis of the proposed problem, since the presented formulation was validated with the results found in the specialized literature. The GITT proved to be extremely versatile with regard to the choice of the auxiliary problem, leaving the choice of the basis on which the solution will be expanded to the author's discretion. The study carried out in the present work is of great relevance, since it aims to provide parameters for a better sizing of thermal equipment, as well as providing them with energy optimization in the heat transfer process.

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